Proof

$Q(SSE)$ is defined as:

$$Q = \sum_{i=1}^{n} (y_i - b_0 - b_1 x_i)^2$$

1. Partial derivative with respect to b_0:

$$\frac{\partial Q}{\partial b_0} = \frac{\partial}{\partial b_0} \sum_{i=1}^{n} (y_i - b_0 - b_1 x_i)^2 = -2 \sum_{i=1}^{n} (y_i - b_0 - b_1 x_i) = 0$$

2. Partial derivative with respect to b_1:

$$\frac{\partial Q}{\partial b_1} = \frac{\partial}{\partial b_1} \sum_{i=1}^{n} (y_i - b_0 - b_1 x_i)^2 = -2 \sum_{i=1}^{n} (y_i - b_0 - b_1 x_i) x_i = 0$$

—
From the derivative with respect to b_0:

$$\sum_{i=1}^{n} y_i - n b_0 - b_1 \sum_{i=1}^{n} x_i = 0$$

$$b_0 = \bar{y} - b_1 \bar{x}$$

—
From the derivative with respect to b_1:

$$\sum_{i=1}^{n} x_i y_i - b_0 \sum_{i=1}^{n} x_i - b_1 \sum_{i=1}^{n} x_i^2 = 0$$

Substitute $b_0 = \bar{y} - b_1 \bar{x}$ into the equation:

$$\sum_{i=1}^{n} x_i y_i - n \bar{x} \bar{y} - b_1 \left(\sum_{i=1}^{n} x_i^2 - n \bar{x}^2 \right) = 0$$

Expressing it in terms of S_{xy} and S_{xx}:

$$S_{xy} = \sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y}), \quad S_{xx} = \sum_{i=1}^{n} (x_i - \bar{x})^2$$

Calculations for S_{xy} and S_{xx}

$$S_{xy} = \sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y}) = \sum_{i=1}^{n} x_i y_i - n \bar{x} \bar{y}$$

$$S_{xx} = \sum_{i=1}^{n} (x_i - \bar{x})^2 = \sum_{i=1}^{n} x_i^2 - n \bar{x}^2$$

Thus:

$$b_1 = \frac{S_{xy}}{S_{xx}}$$

—

Final Results

The regression coefficients are given by:

$$b_1 = \frac{S_{xy}}{S_{xx}}, \quad b_0 = \bar{y} - b_1 \bar{x}$$