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E-Record 4

Experiment–4A: Solution of homogeneous system of first order and second order differential equations by matrix method(2 Questions)

Question 1

Aim:

1. Solve the system of differential equations $y_1' = 4y_1 + y_2$, $y_2' = 3y_1 + 2y_2$, with the initial conditions $y_1(0) = 2$, $y_2(0) = 0$.

Mathematical Background:

System of First Order Linear Differential Equations

A system of n linear first order differential equations in n unknowns (an $n \times n$ system of linear equations) has the general form:

$$\begin{cases} x_1' = a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n + g_1(t) \\ x_2' = a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n + g_2(t) \\ \vdots = \vdots + \vdots + \vdots + \vdots + \vdots \\ x_n' = a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n + g_n(t) \end{cases} \quad (1)$$

where the coefficients a_{ij} 's are arbitrary constants, and g_i 's are arbitrary functions of t . If every term g_i is constant zero, then the system is said to be homogeneous.

The system (1) is most often given in a shorthand format as a matrix-vector equation, in the form:

$$X' = AX + G$$

where $X' = [x_i']_{n \times 1}$, $A = [a_{ij}]_{n \times n}$, $X = [x_i]_{n \times 1}$, and $G = [g_i(t)]_{n \times 1}$.

If the coefficient matrix A has two distinct real eigenvalues λ_1 and λ_2 and their respective eigenvectors are X_1 and X_2 , then the 2×2 system

$$X' = AX$$

has a general solution

$$X = C_1X_1e^{\lambda_1 t} + C_2X_2e^{\lambda_2 t}$$

Matlab Code:

```
clc
clear
syms t C1 C2
A=input('Enter A: ');
[P,D]=eig(A);
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```

```

L1=D(1);L2=D(4);
y1=C1*exp(L1*t);y2=C2*exp(L2*t);
Y=[y1;y2];
X=P*Y;
Cond=input('Enter the initial conditions [t0, x10,x20]: ');
t0=Cond(1);x10=Cond(2);x20=Cond(3);
eq1=subs(X(1)-x10,t0);eq2=subs(X(2)-x20,t0);
[C1, C2] = solve(eq1,eq2);
X=subs(X)

```

Code Screenshots:

The screenshot shows the MATLAB Editor interface. The Editor window displays the following code:

```

1 - c1c
2 - clear
3 - syms t C1 C2
4 - A=input('Enter A: ');
5 - [P,D]=eig(A);
6 - L1=D(1);L2=D(4);
7 - y1=C1*exp(L1*t);y2=C2*exp(L2*t);
8 - Y=[y1;y2];
9 - X=P*Y;
10 - Cond=input('Enter the initial conditions [t0, x10,x20]: ');
11 - t0=Cond(1);x10=Cond(2);x20=Cond(3);
12 - eq1=subs(X(1)-x10,t0);eq2=subs(X(2)-x20,t0);
13 - [C1, C2] = solve(eq1,eq2);
14 - X=subs(X)
15

```

The Command Window shows the output of the code:

```

X =

(3*exp(5*t))/2 + exp(t)/2
(3*exp(5*t))/2 - (3*exp(t))/2
>>

```

Output Screenshots:

The screenshot shows the MATLAB Command Window. The user has entered the following input:

```

Enter A:
[4 1;3 2]
Enter the initial conditions [t0, x10,x20]:
[0 2 0]

```

The Command Window also shows the output of the code:

```

X =

(3*exp(5*t))/2 + exp(t)/2
(3*exp(5*t))/2 - (3*exp(t))/2
>>

```

Question 2

Aim:

2. Solve the system of differential equation $y_1'' = 2y_1 + y_2$, $y_2'' = y_1 + 2y_2$, with the initial conditions $y_1(0) = 0, y_1'(0) = 1, y_2(0) = 1, y_2'(0) = 0$.

Mathematical Background:

System of Second Order Linear Differential Equations

Consider the system of second order linear differential equations of the form

$$\begin{cases} x_1'' = a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \\ x_2'' = a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \\ \vdots = \vdots + \vdots + \vdots + \vdots \\ x_n'' = a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n \end{cases} \quad (2)$$

where the coefficients a_{ij} 's are arbitrary constants.

Then, the solution of (2), $X'' = AX$, is

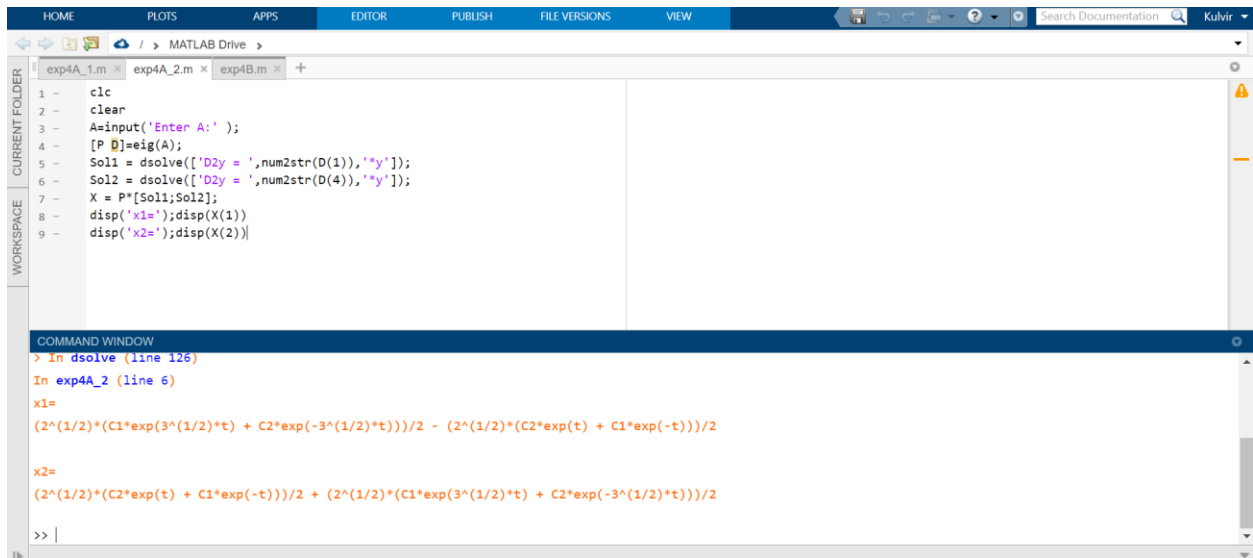
$$X = PY$$

where Y is the solution of $Y'' = DY$, P is the modal matrix of A and D is it's diagonal matrix.

Matlab Code:

```
clc
clear
A=input('Enter A: ');
[P D]=eig(A);
Sol1 = dsolve(['D2y = ',num2str(D(1)),'*y']);
Sol2 = dsolve(['D2y = ',num2str(D(4)),'*y']);
X = P*[Sol1;Sol2];
disp('x1=');disp(X(1))
disp('x2=');disp(X(2))
```

Code Screenshots:



The screenshot shows the MATLAB Editor with a script named 'exp4A_1.m'. The code defines a matrix A, finds its eigenvalues, and solves two differential equations. The Command Window shows the execution of 'dsolve' for both equations, resulting in symbolic solutions for x1 and x2.

```
1 - clc
2 - clear
3 - A=input('Enter A: ');
4 - [P D]=eig(A);
5 - Sol1 = dsolve(['D2y = ',num2str(D(1)), '*y']);
6 - Sol2 = dsolve(['D2y = ',num2str(D(4)), '*y']);
7 - X = P*[Sol1;Sol2];
8 - disp('x1=');disp(X(1))
9 - disp('x2=');disp(X(2))
```

COMMAND WINDOW

```
> In dsolve (line 126)
In exp4A_2 (line 6)
x1=
(2^(1/2)*(C1*exp(3^(1/2)*t) + C2*exp(-3^(1/2)*t)))/2 - (2^(1/2)*(C2*exp(t) + C1*exp(-t)))/2

x2=
(2^(1/2)*(C2*exp(t) + C1*exp(-t)))/2 + (2^(1/2)*(C1*exp(3^(1/2)*t) + C2*exp(-3^(1/2)*t)))/2

>> |
```

Output Screenshots:



The screenshot shows the MATLAB Command Window with the output of the code. It displays the input matrix A, a warning about character vectors, and the symbolic solutions for x1 and x2.

```
Enter A:
[2 1;1 2]
Warning: Support of character vectors and strings will be removed in a future release. Use sym objects to define differential equations instead.
> In dsolve (line 126)
In exp4A_2 (line 5)
Warning: Support of character vectors and strings will be removed in a future release. Use sym objects to define differential equations instead.
> In dsolve (line 126)
In exp4A_2 (line 6)
x1=
(2^(1/2)*(C1*exp(3^(1/2)*t) + C2*exp(-3^(1/2)*t)))/2 - (2^(1/2)*(C2*exp(t) + C1*exp(-t)))/2

x2=
(2^(1/2)*(C2*exp(t) + C1*exp(-t)))/2 + (2^(1/2)*(C1*exp(3^(1/2)*t) + C2*exp(-3^(1/2)*t)))/2

>> |
```

Experiment:4B–Series solutions of ordinary differential equations

Aim:

1. Find the first five terms in the power series solution of the differential equation $y'' + 2y' + y = 0$.

Mathematical Background:

Series Solution when $x = 0$ is an Ordinary Point of the Equation

$$P_0 \frac{d^2 y}{dx^2} + P_1 \frac{dy}{dx} + P_2 y = 0 \quad (1)$$

where P 's are polynomial in x and $P_0 \neq 0$ at $x = 0$.

1. Assume its solution to be of the form

$$y = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n + \cdots \quad (2)$$

2. Calculate $\frac{dy}{dx}$, $\frac{d^2 y}{dx^2}$, from (2) and substitute the values of y , $\frac{dy}{dx}$, $\frac{d^2 y}{dx^2}$ in (1).
3. Equate to zero the coefficients of the various powers of x and determine a_2, a_3, a_4, \dots in terms of a_0, a_1 .
4. Substituting the values of a_2, a_3, a_4, \dots in (2), we get the desired series solution having a_0, a_1 as its arbitrary constants.

Matlab Code:

```
clc
clear all
close all
syms x a0 a1 a2 a3 a4 a5 A B
a=[a0 a1 a2 a3 a4 a5];
y=sum(a.*x.^[0:5])
dy=diff(y)
d2y=diff(dy)
% de= collect(d2y+y,x)
de= collect(d2y+2*dy+y,x)
coef= coeffs(de,x)
A2=solve(coef(1),a2)
A3=subs(solve(coef(2),a3),a2,A2)
A4=subs(solve(coef(3),a4),{a2,a3},{A2,A3})
A5=subs(solve(coef(4),a5),{a2,a3,a4},{A2,A3,A4})

y=subs(y,{a2,a3,a4,a5},{A2,A3,A4,A5})
soln=coeffs(y,[a1,a0])
gs=A*soln(1)+B*soln(2)
```

Code Screenshots:

The screenshot shows the MATLAB Editor interface with the file 'exp4B.m' open. The code defines a polynomial $y = a_5x^5 + a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$, computes its derivative dy and second derivative $d2y$, and then solves the differential equation $d2y + 2*dy + y = B$ for y . The solution is expressed as $y = \text{soln} + A$, where A is a particular solution and soln is the homogeneous solution.

```

1 - c1c
2 - clear all
3 - close all
4 - syms x a0 a1 a2 a3 a4 a5 A B
5 - a=[a0 a1 a2 a3 a4 a5];
6 - y=sum(a.*x.^[0:5])
7 - dy=diff(y)
8 - d2y=diff(dy)
9 - % de= collect(d2y+y,x)
10 - de= collect(d2y+2*dy+y,x)
11 - coef= coeffs(de,x)
12 - A2=solve(coef(1),a2)
13 - A3=subs(solve(coef(2),a3),a2,A2)
14 - A4=subs(solve(coef(3),a4),{a2,a3},{A2,A3})
15 - A5=subs(solve(coef(4),a5),{a2,a3,a4},{A2,A3,A4})
16 -
17 - y=subs(y,{a2,a3,a4,a5},{A2,A3,A4,A5})
18 - soln=coeffs(y,[a1,a0])
19 - gs=A*soln(1)+B*soln(2)

```

COMMAND WINDOW

```

B*(x^5/24 - x^4/6 + x^3/2 - x^2 + x) + A*(x^5/30 - x^4/8 + x^3/3 - x^2/2 + 1)

```

Output Screenshots:

The screenshot shows the MATLAB Command Window displaying the output of the code. It shows the polynomial y , its derivative dy , and the second derivative $d2y$. The output also shows the coefficients of the polynomial and the solution y for the differential equation.

```

y =
a5*x^5 + a4*x^4 + a3*x^3 + a2*x^2 + a1*x + a0

dy =
5*a5*x^4 + 4*a4*x^3 + 3*a3*x^2 + 2*a2*x + a1

d2y =
20*a5*x^3 + 12*a4*x^2 + 6*a3*x + 2*a2

de =
a5*x^5 + (a4 + 10*a5)*x^4 + (a3 + 8*a4 + 20*a5)*x^3 + (a2 + 6*a3 + 12*a4)*x^2 + (a1 + 4*a2 + 6*a3)*x + a0 + 2*a1 + 2*a2

```

