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E-Record 3

Experiment:3A Solution of linear differential equation by method of variation of parameters

Aim:

Submit the e-record for the following.

1. Consider the problem of suspension cable $\frac{d^2y}{dx^2} = \frac{w(x)}{T_H}$ with the conditions $y(0) = 0$, $y'(0) = 0$, where $w(x) = x^2$,$T_H = 10$. Plot shape of the cable in the range $[-4, 10]$.

Mathematical Background:

Method of variation of parameters:

We consider a second order linear differential equation of the form

$$F(D)y \equiv \frac{d^2y}{dx^2} + p(x)\frac{dy}{dx} + q(x)y = f(x) \qquad (1)$$

Let the solutions of the homogeneous problem of $F(D)y = 0$ be $y_1(x)$ and $y_2(x)$.

Then the Complementary function (solution of the homogeneous problem) of (1) is

$$y_c(x) = C_1y_1 + C_2y_2 \qquad (2)$$

Then by the method of variation of parameters the particular integral of (1) is of the form

$$y_p(x) = uy_1 + vy_2 \qquad (3)$$

where the parameters C_1, C_2 of (2) are replaced with functions $u(x)$, $v(x)$ given by

$$u(x) = -\int \frac{y_2 f(x)}{W(x)}dx \text{ and } v(x) = \int \frac{y_1 f(x)}{W(x)}dx,$$

where the wronskian $W(x) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1y_2' - y_2y_1' \neq 0$.

In this experiment we consider the coefficients p, q to be constants.

Matlab Code:

```
clc
clear all
close all
syms x c1 c2
a=input('Enter coeff of d2y');
b=input('Enter coeff of dy');
c=input('Enter coeff of y');
f=input('Enter nonhomogeneous part: ')
r=roots([a b c]);
d=b^2-4*a*c;   %discriminant
if d>0
    disp('Real and distinct')
    disp(r)
    y1=exp(r(1)*x);
    y2=exp(r(2)*x);
else if d==0
        disp('Roots are real and equal')
        disp(r)
        y1=exp(r(1)*x);
        y2=x*y1;
else
    disp('Roots are complex')
    disp(r)
    y1=exp(real(r(1))*x)*cos(abs(imag(r(1)))*x);
    y2=exp(real(r(1))*x)*sin(abs(imag(r(1)))*x);
end
end
yh=c1*y1+c2*y2  %
if f==0
ygs=yh
else
    %method of variation of parameters
    w=det([y1,y2;diff(y1) diff(y2)]); %wronksian
    u=-int(y2*f/w);
    v=int(y1*f/w);
    yp=u*y1+v*y2;
    ygs=yh+yp;
end
IC=input('Enter initial conditions as [a y(a) dy(a)]: ');
eq1=subs(ygs,x,IC(1))-IC(2);
eq2=subs(diff(ygs),x,IC(1))-IC(3);
[c1 c2]=solve(eq1==0,eq2==0) %(nonhomogeneous soln (f) may not be there)
% simultaneous eqN solve
y=subs(ygs)
ezplot(y,[-5,5])
```

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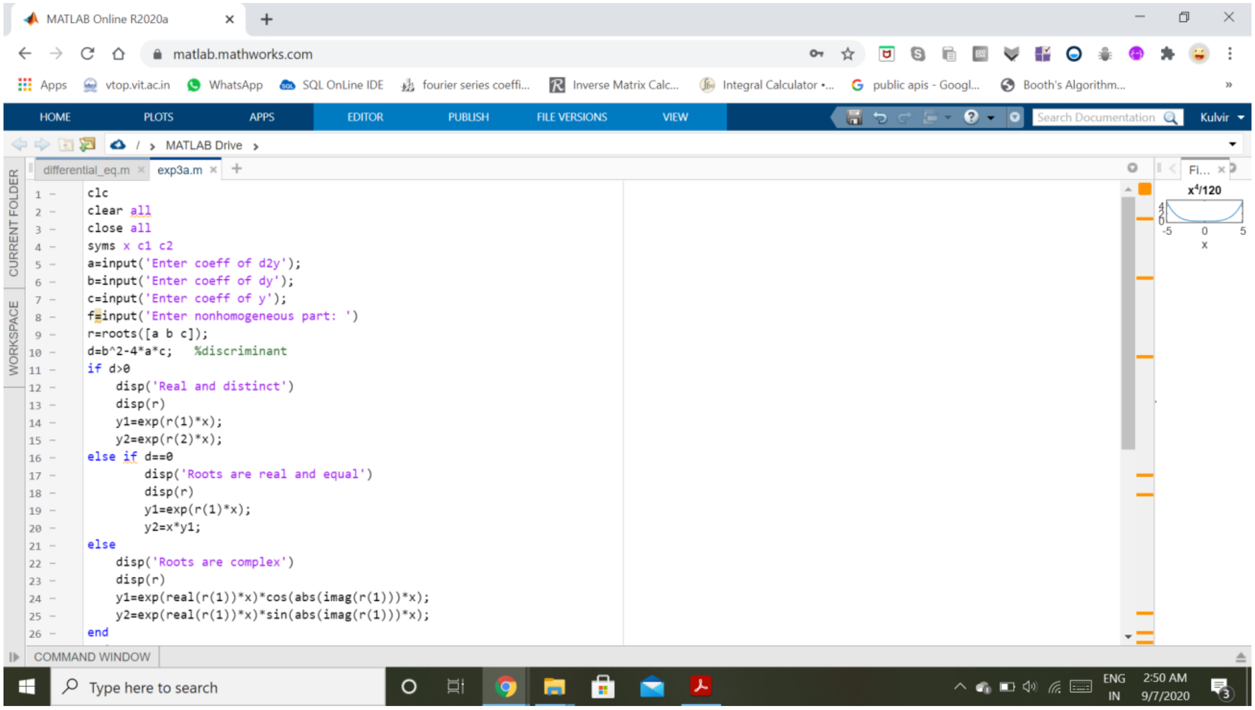
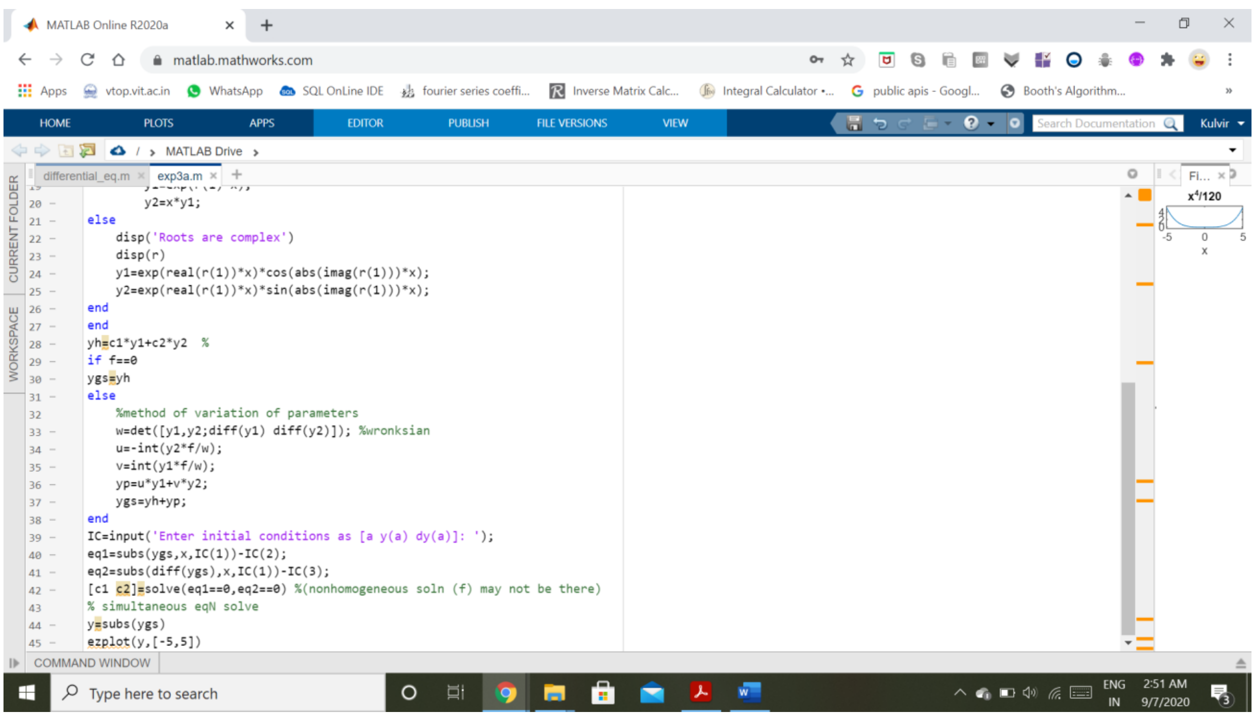
Code Screenshots:



```
clc
clear all
close all
syms x c1 c2
a=input('Enter coeff of d2y');
b=input('Enter coeff of dy');
c=input('Enter coeff of y');
f=input('Enter nonhomogeneous part: ')
r=roots([a b c]);
d=b^2-4*a*c;   %discriminant
if d>0
    disp('Real and distinct')
    disp(r)
    y1=exp(r(1)*x);
    y2=exp(r(2)*x);
else if d==0
        disp('Roots are real and equal')
        disp(r)
        y1=exp(r(1)*x);
        y2=x*y1;
else
    disp('Roots are complex')
    disp(r)
    y1=exp(real(r(1))*x)*cos(abs(imag(r(1)))*x);
    y2=exp(real(r(1))*x)*sin(abs(imag(r(1)))*x);
end
```



```
        y2=x*y1;
else
    disp('Roots are complex')
    disp(r)
    y1=exp(real(r(1))*x)*cos(abs(imag(r(1)))*x);
    y2=exp(real(r(1))*x)*sin(abs(imag(r(1)))*x);
end
end
yh=c1*y1+c2*y2  %
if f==0
ygs=yh
else
    %method of variation of parameters
    w=det([y1,y2;diff(y1) diff(y2)]); %wronksian
    u=-int(y2*f/w);
    v=int(y1*f/w);
    yp=u*y1+v*y2;
    ygs=yh+yp;
end
IC=input('Enter initial conditions as [a y(a) dy(a)]: ');
eq1=subs(ygs,x,IC(1))-IC(2);
eq2=subs(diff(ygs),x,IC(1))-IC(3);
[c1 c2]=solve(eq1==0,eq2==0) %(nonhomogeneous soln (f) may not be there)
% simultaneous eqN solve
y=subs(ygs)
ezplot(y,[-5,5])
```

Output Screenshots:

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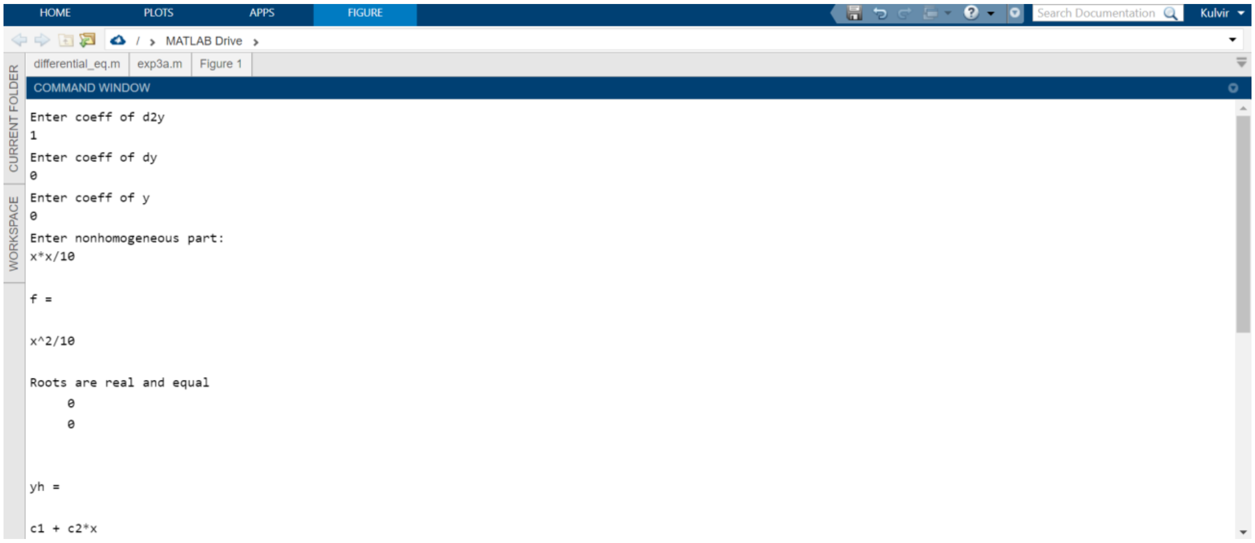

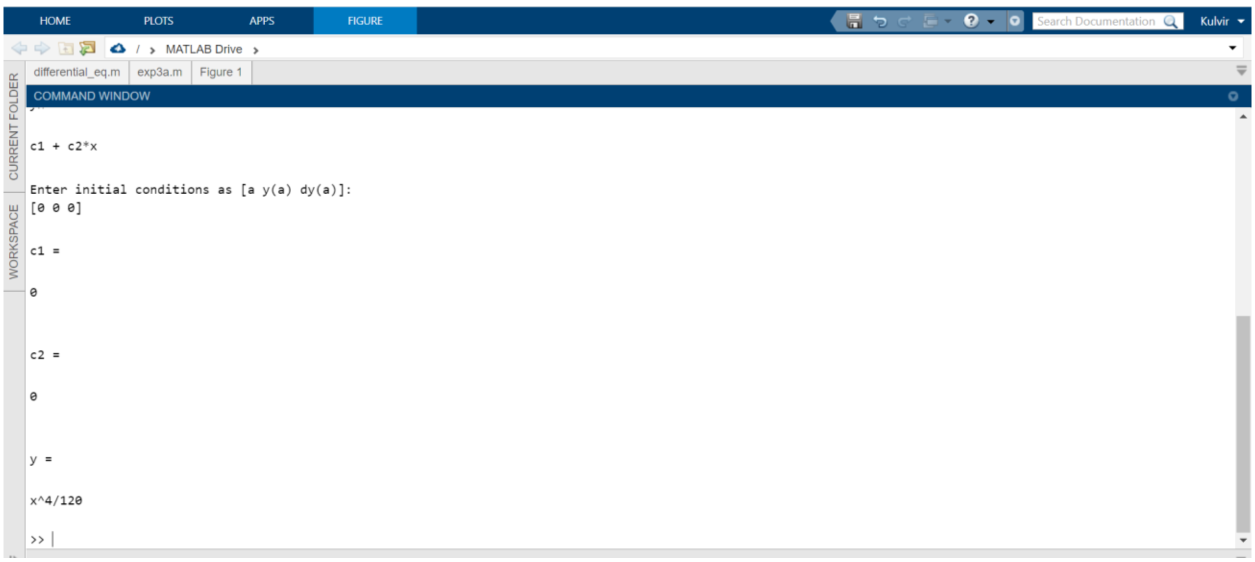

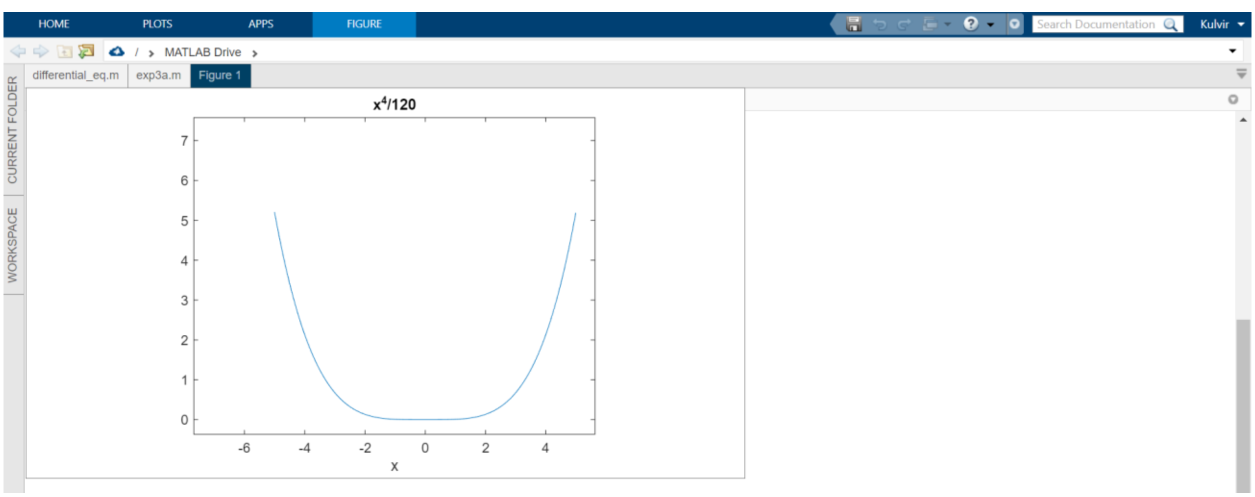

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Experiment:3B Solution of linear differential equation by Laplace Transforms

Aim:

1. Determine the response of the damped mass- spring system under a square wave, modeled by $y'' + 3y' + 2y = r(t)$, where $r(t) = \begin{cases} 0, & 0 < t < 2 \\ 1, & 2 < t < 4 \\ 0, & t \geq 4 \end{cases}$. Subject to the initial conditions $y(0) = y'(0) = 0$. Plot the solution.

Mathematical Background:

To solve and visualize solutions of a second order Linear differential equation using Laplace transform.

Working Procedure:

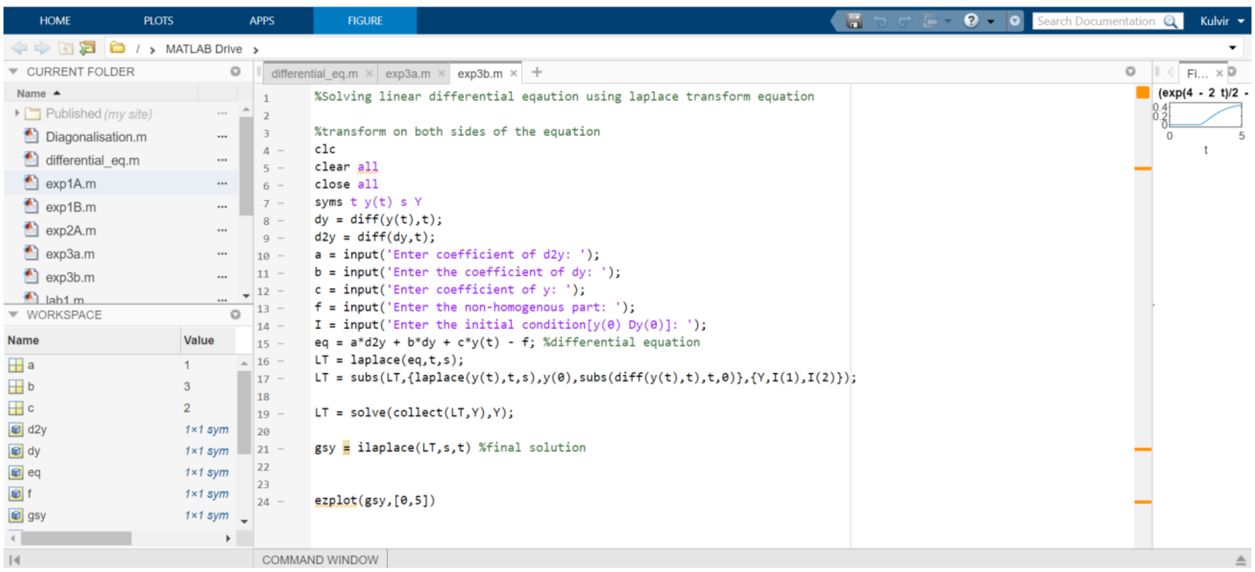
- Input the differential equation coefficients a, b, c and the RHS function $f(x)$ of the differential equation $ay'' + by' + cy = f(x)$.
- Input the initial conditions $y(0)$ and $y'(0)$.
- Apply Laplace Transform and find $Y(s)$.
- Apply inverse Transform and find $y(t)$.

Matlab Code:

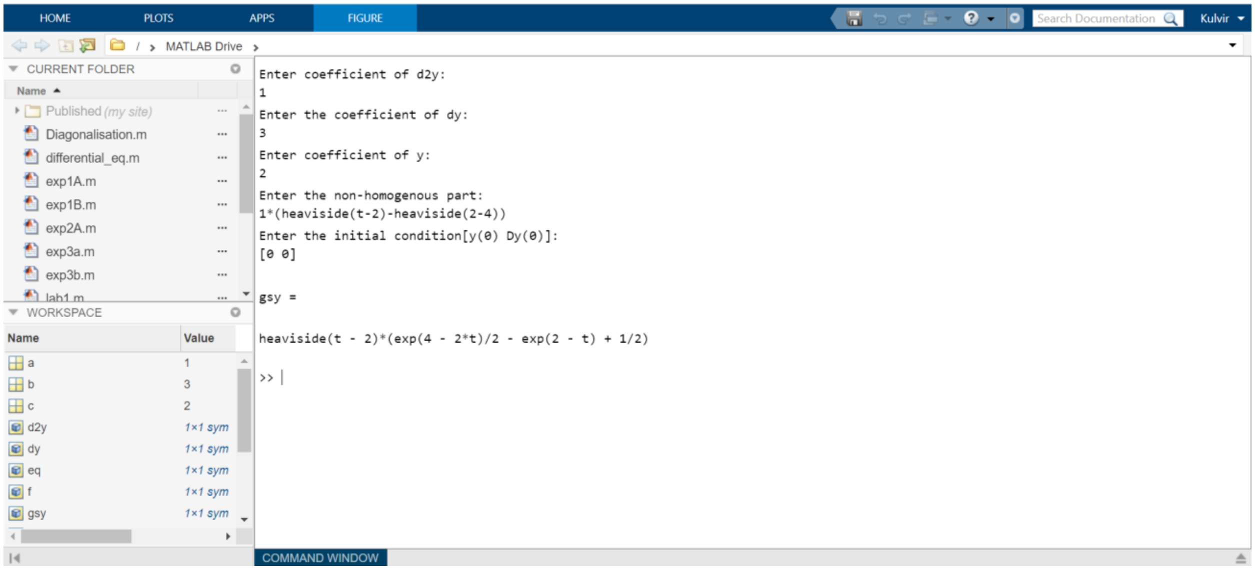
```
%Solving linear differential eqaution using laplace transform equation


%transform on both sides of the equation
clc
clear all
close all
syms t y(t) s Y
dy = diff(y(t),t);
d2y = diff(dy,t);
a = input('Enter coefficient of d2y: ');
b = input('Enter the coefficient of dy: ');
c = input('Enter coefficient of y: ');
f = input('Enter the non-homogenous part: ');
I = input('Enter the initial condition[y(0) Dy(0)]: ');
eq = a*d2y + b*dy + c*y(t) - f; %differential equation
LT = laplace(eq,t,s);
LT = subs(LT,{laplace(y(t),t,s),y(0),subs(diff(y(t),t),t,0)},{Y,I(1),I(2)});
```

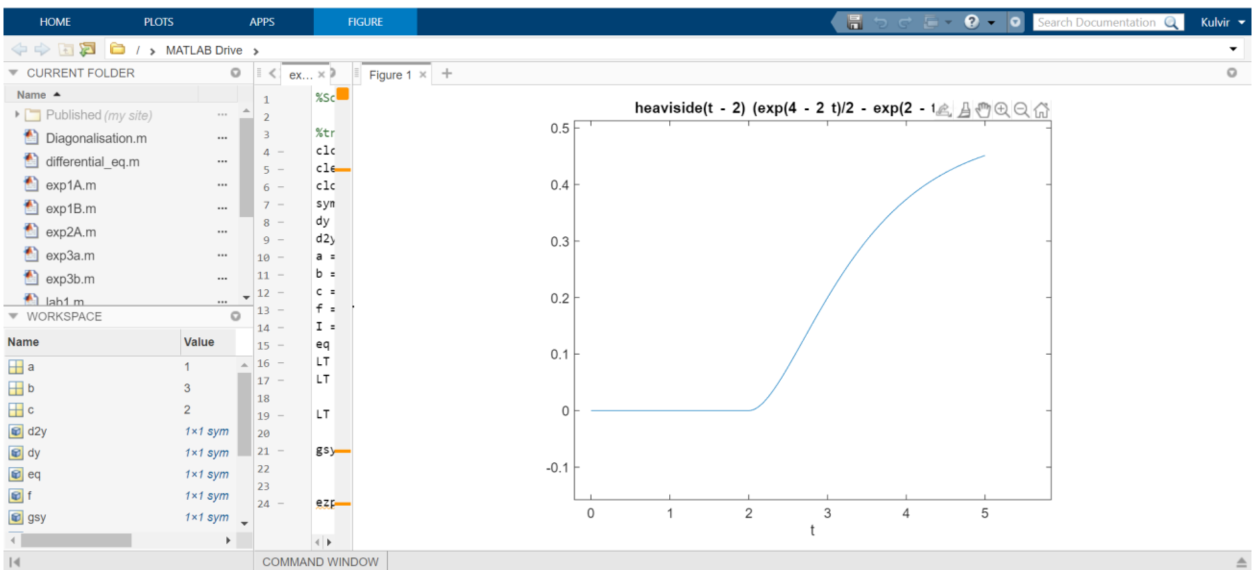
```
LT = solve(collect(LT,Y),Y);

gsy = ilaplace(LT,s,t) %final solution

ezplot(gsy,[0,5])
```

Code Screenshots:



Output Screenshot:





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