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19BCE2074

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FAT-Fall Semester 2020-21 Instructor: Dr. Aruna.K
Applications of Differential and Difference Equations(MAT2002)

SET-II

TITLE

Power Series

1. Find the first four terms of the power series solution of the differential equation $y'' - 9y = 0$ with $y(0) = 0, y'(0) = 1$. Plot the obtained solution in the range $0 < t < 3$

AIM

To find the first four terms of the power series solution of the given differential equation with the initial conditions and the graph of the same solution.

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MATHEMATICAL BACKGROUND

Series Solution

Series solution when $x = 0$ is an Ordinary Point of the Equation.

$$P_0 \frac{d^2y}{dx^2} + P_1 \frac{dy}{dx} + P_2 y = 0 \quad (1)$$

where P's are polynomials in x and $P_0 \neq 0$ at $x = 0$.

→ Assume its solⁿ. to be $\left[y = a_0 + a_1 x + a_2 x^2 + \cdots a_n x^n \right]$ (2)

→ Calculate $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ from (2) and substitute the values of y, $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ in (1).

→ Equate the co-efficients of various powers of x to zero and determine a_2, a_3, a_4 in terms of a_0, a_1

→ Substituting the values of a_2, a_3, a_4 — in (2) we get the desired series solution having a_0, a_1 as the arbitrary constant.

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MATLAB CODE (SCREENSHOT)

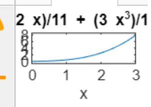
FIGURE

exp4B.m

```
clc
clear all
close all
syms x a0 a1 a2 a3 a4 A B
a=[a0 a1 a2 a3 a4];
y=sum(a.*x.^[0:4]) ;
dy=diff(y);
d2y=diff(dy);
% de= collect(d2y+y,x)
de= collect(d2y-9*y,x);
coef= coeffs(de,x);
A2=solve(coef(1),a2) ;
A3=subs(solve(coef(2),a3),a2,A2);
A4=subs(solve(coef(3),a4),{a2,a3},{A2,A3});

y=subs(y,{a2,a3,a4},{A2,A3,A4});
soln=coeffs(y,[a1,a0]);
disp('power series upto 4 terms : -')
gs=A*soln(1)+B*soln(2)

equation1=subs(gs,x,0);
equation2=subs(diff(gs),x,1);
[A B]=solve(equation1==0,equation2==1);
var=subs(gs);
ezplot(var,[0,3])
```



COMMAND WINDOW

OUTPUT

FIGURE

```
power series upto 4 terms : -

gs =

A*((27*x^4)/8 + (9*x^2)/2 + 1) + B*((3*x^3)/2 + x)

>>
```

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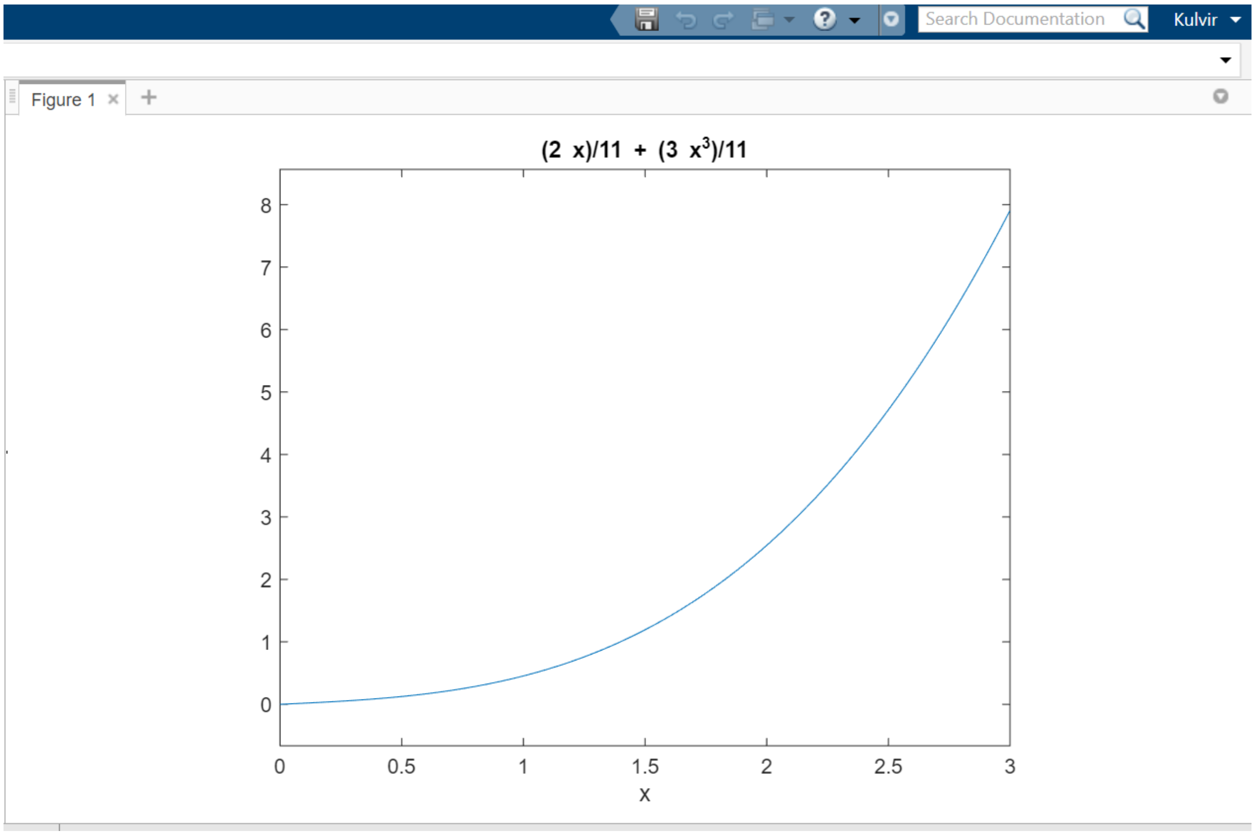

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TITLE

Eigen Values and Diagonalization

2. Verify any three properties of eigen values of $A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$. Also, diagonalize matrix A.

AIM

To verify 3 properties of eigen values of the given matrix and to display the diagonalized matrix of the same.

MATHEMATICAL BACKGROUND

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Eigen Values and Eigen Vectors

We study the problem

$$AX = \lambda X$$

where A is $n \times n$ matrix, X is unknown $n \times 1$ column vector, and λ is scalar. given an $n \times n$ matrix A, find the value of λ such that $|A - \lambda I|X = 0$ admits non-trivial solution, and find those non-trivial solution.

Properties of Eigen Values.

(i) $\det(A)$ = product of eigen values of A

(ii) trace (A) = sum of eigen values of A

(iii) A and A^T [transpose of A] have the same eigen values

(iv) If λ is the eigen value of A then $\frac{1}{\lambda}$ is the eigen value of A^{-1}

(v) If λ is the eigen value of A then λ^n is the eigen value of A^n

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Similarity Transformation

A is said to be similar to B if there exist a non-singular matrix P such that

$$B = P^{-1} A P$$

The transformation of A to B is known as similarity transformation.

Let $x_1, x_2, \dots, x_n$ be the n linearly independent eigenvectors of A corresponding to n eigen values $\lambda_1, \lambda_2 \dots \lambda_n$, $P_{n \times n} = [x_1 \; x_2 \; \dots \; x_n]$ is known as modal matrix.

$$\begin{aligned} AP &= A [x_1 \; x_2 \; \dots \; x_n] \\ &= [Ax_1 \; Ax_2 \; \dots \; Ax_n] \\ &= [\lambda_1 x_1 \; \lambda_2 x_2 \; \dots \; \lambda_n x_n] \\ &= [x_1 \; x_2 \; \dots \; x_n] \begin{bmatrix} \lambda_1 & 0 & \dots & 0 \\ 0 & \lambda_2 & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \dots & & \lambda_n \end{bmatrix} \\ &= P \cdot D \end{aligned}$$

Multiplying P^{-1} on both side

$$P^{-1} A P = (P^{-1} P) D = D$$

where D is the diagonal matrix with eigen values of A as the principal diagonal elements. D is known as spectral matrix.

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MATLAB CODE

EDITOR PUBLISH FILE VERSIONS VIEW

exp2A.m

```
clc
clear all
close all
syms L real
A=input("Enter the square matrix:");
ch = det(A-L*eye(length(A)));
r=solve(ch);
p=poly(A);
r1=roots([p]);
disp('eigen values are :-');
disp(r1);
[p,d]=eig(A);
%%det of A = product of eigen values
disp('determinant of A is equal to the product of Eigen Values');
det_A = det(A)%determinant of A
prod_A= prod(eig(A))%product of eigen values
%%trace of A = sum of the eigen values
disp('trace of A is equal to the sum of Eigen Values');
trace_A = trace(A)
sum_A = sum(eig(A))
%%A and A^T have the same eigen values
disp('A and transpos of A have the same Eigen values');
eig_A=eig(A) %eigen values of A
eig_AT=eig(transpose(A)) %eigen values of A transpose

```

EDITOR PUBLISH FILE VERSIONS VIEW

exp2A.m

```
prod_A= prod(eig(A))%product of eigen values
%%trace of A = sum of the eigen values
disp('trace of A is equal to the sum of Eigen Values');
trace_A = trace(A)
sum_A = sum(eig(A))
%%A and A^T have the same eigen values
disp('A and transpos of A have the same Eigen values');
eig_A=eig(A) %eigen values of A
eig_AT=eig(transpose(A)) %eigen values of A transpose

if A==transpose(A)
    disp('symmetric matrix')
    disp('orthogonal transformation')
    disp('Diagonal Matrix is')
    D=round(transpose(p)*A*p)
else
    disp('non symmetric matirx')
    disp('similarity transformation')
        disp('Diagonal Matrix is')
    D=round(inv(p)*A*p)
end
```

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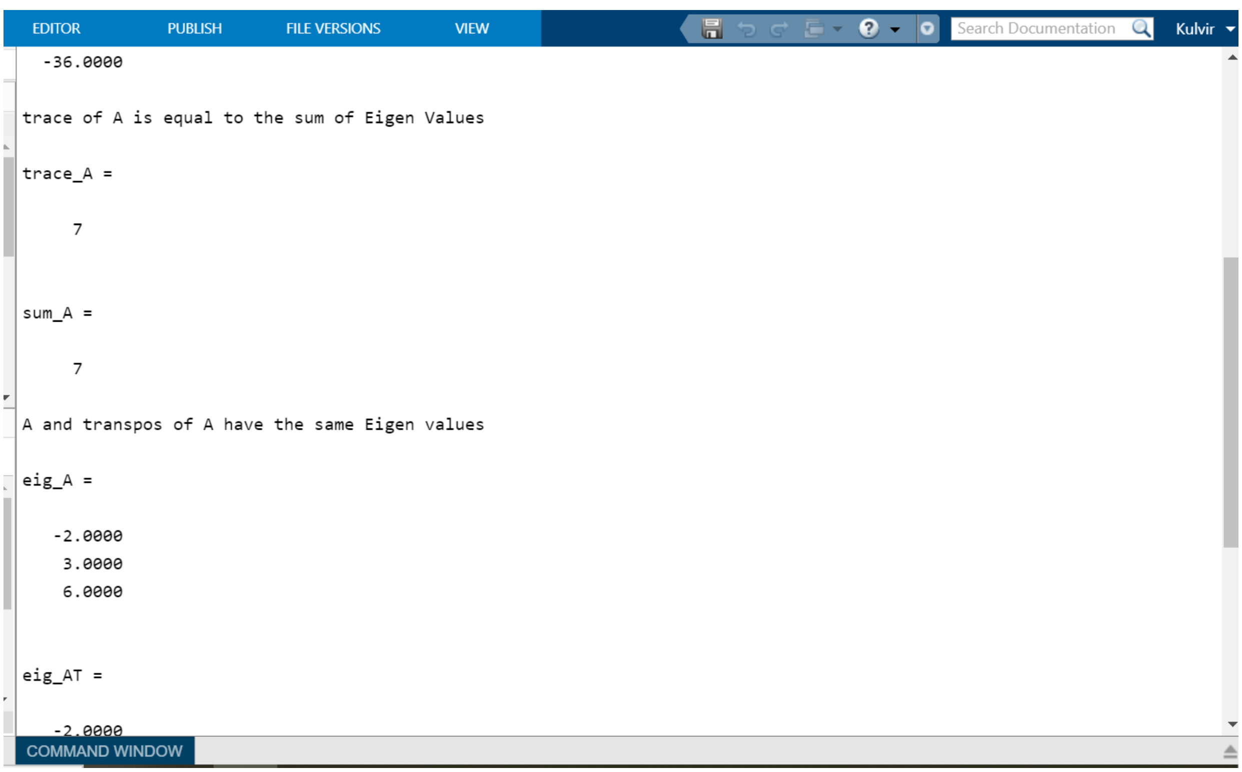
OUTPUT



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```
eig_A =

   -2.0000
    3.0000
    6.0000


eig_AT =

   -2.0000
    3.0000
    6.0000

symmetric matrix
orthogonal transformation
Diagonal Matrix is

D =

    -2     0     0
     0     3     0
     0     0     6

>> 
```