1) $f(t) = t(6-t)^{2/3}$

a) Critical Points

$$f'(t) = (6-t)^{2/3} + \frac{2}{3}t(6-t)^{2/3}(-1)$$

$$= (6-t)^{2/3} - \frac{2}{3}\frac{t}{(6-t)^{1/3}}$$

$$= \frac{3(6-t) - 2t}{3(6-t)^{1/3}}$$

$$= \frac{18-5t}{3(6-t)^{1/3}}$$

$f'(t) = 0$ for C.P

$$\frac{18-5t}{3(6-t)^{1/3}} = 0$$

$$t = \frac{18}{5}.$$

b) Inflection point

$f''(t) = 0$

$$\Rightarrow f''(t) = \frac{1}{3}(-5)(6-t)^{-1/3} + \frac{(18-5t)(6-t)^{-4/3}}{9}$$

$$= \frac{-5}{3(6-t)^{1/3}} + \frac{18-5t}{9(6-t)^{4/3}}$$

$$= \frac{1}{3(6-t)^{1/3}}\left[-5 + \frac{18-5t}{3(6-t)^{4/3}}\right]$$

$$= \frac{-15(6-t) + 18 - 5t}{9 \times (6-t)^{4/3}}$$

$$f''(t) = \frac{-72 + 10t}{9 \times (6-t)^{4/3}}$$

$f''(t) = 0.$

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$$\frac{-72+10t}{9(6-t)^{4/3}} = 0$$

$$t = \frac{72}{10} = \frac{36}{5}$$

Inflection Point $\Rightarrow t = \frac{36}{5}$

c) increasing & decreasing

$$f'(t) = \frac{18-5t}{3(6-t)^{1/3}}$$

→ increasing $f'(t) > 0$

case 1		case 2	
$18-5t > 0$	$6-t > 0$	$18-5t < 0$	$6-t < 0$
$t < \frac{18}{5}$	$t < 6$	$t > \frac{18}{5}$	$t > 6$
$t \in \left(-\infty, \frac{18}{5}\right)$		$t \in (6, \infty)$	

$\therefore$ increasing in $t \in \left(-\infty, \frac{18}{5}\right) \cup (6, \infty)$

→ decreasing $f'(t) < 0$

case 1		case 2	
$18-5t < 0$	$6-t > 0$	$18-5t > 0$,	$6-t < 0$
$t > \frac{18}{5}$	$t < 6$	$t < \frac{18}{5}$,	$t > 6$
$t \in \left(\frac{18}{5}, 6\right)$		$t \in \phi$	

$f(x)$ is decreasing in $t \in \left(\frac{18}{5}, 6\right)$

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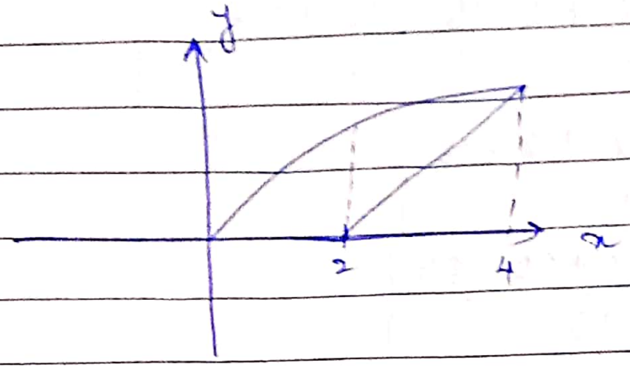
2) $y = \sqrt{x}$

$y = x - 2$



$\sqrt{x} = x - 2$

$\Rightarrow x = (x-2)^2$

$\Rightarrow x = x^2 + 4 - 4x$

$\Rightarrow x^2 - 4x - x + 4 = 0$

$\Rightarrow x(x-4) - 1(x-4) = 0$

$\Rightarrow (x-1)(x-4) = 0$

$\Rightarrow x = 1$ or 4

$x = 4$ will be considered as its only point that satisfies given condition (above x-axis)

$$\text{Area} = \int_0^2 \sqrt{x}\,.dx + \int_2^4 \sqrt{x} - (x-2)\,dx$$

$$= \left.\frac{x^{3/2}}{3/2}\right|_0^2 + \left.\frac{x^{3/2}}{3/2}\right|_2^4 - \left.\left(\frac{x^2}{2} - 2x\right)\right|_2^4$$

$$= \left(\frac{2^{3/2}}{3/2} - 0\right) + \frac{4^{3/2}}{3/2} - \frac{2^{3/2}}{3/2} - \left(\frac{4^2}{2} - 2.4 - \left(\frac{2^2}{2}\right.\right.$$

$$= \cancel{\frac{2^{3/2}}{3/2}} + \frac{8\times 2}{3} \cancel{- \frac{2^{3/2}}{3/2}} - \left(\cancel{8} - \cancel{8} - 2 + 4\right)$$

$$= \frac{16}{3} - 2 = \frac{10}{3} \text{ units}^2$$

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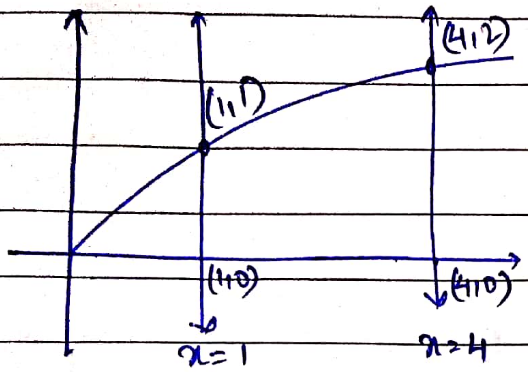
3) $y = \sqrt{x}$, $x = 4$ about $x = 1$



$g(y) = x = y^2$

$f(y) = x = 1$

$$\text{volume} = \int_0^2 \pi \{g(y)\}^2 dy - \int_0^2 \pi (f(y))^2 . dy$$

$$\text{Volume} = \int_0^2 \pi (y^4 - 1) \, dy$$

$$= \pi \int_0^2 (y^4 - 1) \, dy$$

$$= \pi \left[\frac{y^5}{5} - y\right]\Big|_0^2$$

$$= \pi \left(\frac{32}{5} - 2\right)$$

$$= \pi \frac{22}{5}$$

$$\text{Vol} = \frac{22}{5}\pi \text{ units}^3$$

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4 (i) $L\left(\int_0^t \frac{e^{-t}\sin t}{t}\right)$

$= L(\sin t)$

$= \frac{a}{s^2+a^2}$

$= L\left(\frac{\sin t}{t}\right)$

$= \int_s^{\infty} \frac{a}{s^2+a^2} ds$

$= a \tan^{-1}\left(\frac{s}{a}\right)\Big|_s^{\infty}$

$= a\left(\frac{\pi}{2} - \tan^{-1}\left(\frac{s}{a}\right)\right)$

$= a \cot^{-1}\left(\frac{s}{a}\right)$

$= L\left(e^{-t}\frac{\sin t}{t}\right)$

$= a \cot^{-1}\left(\frac{s+1}{a}\right)$

$= L\left(\int_0^t e^{-t}\frac{\sin t}{t}\right)$

$= \boxed{\frac{a \cot^{-1}\left(\frac{s+1}{a}\right)}{s}}$

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(iii) $L\left(\int_0^t e^{-t}\cosh t\right)$

$= L(\cosh t)$

$= \dfrac{s}{s^2-a^2}$

$= L(e^{-t}\cosh t)$

$= \dfrac{s+1}{(s+1)^2-a^2}$

$= L\left(\int_0^t e^{-t}\cosh t\right)$

$= \boxed{\dfrac{(s+1)}{s(s+1)^2-a^2}}$

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4) (iii)

$$f = \begin{cases} \sin t & 0 \le t \le \pi \\ \sin 2t & \pi \le t < 2\pi \\ \sin 3t & t \ge 2\pi \end{cases}$$

$$f(t) = \sin t\,(u(t-0) - u(t-\pi)) + \sin 2t\,(u(t-\pi) - u(t-2\pi)) + \sin 3t\,(u(t-2\pi))$$

$$= \sin t\, u(t) - \sin t\, u(t-\pi) + \sin 2t\, u(t-\pi) - \sin 2t\, u(t-2\pi) + \sin 3t\, u(t-2\pi)$$

$$= \sin t\, u(t) - \sin(t-\pi)\, u(t-\pi) + \sin 2(t-\pi)\, u(t-\pi) - \sin 2(t-2\pi)\, u(t-2\pi) + \sin 3(t-2\pi)\, u(t-2\pi)$$

$$= e^{-0s} L(\sin t) - e^{-\pi s} L(\sin t) + e^{-\pi s} L(\sin 2t) - e^{-2\pi s} L(\sin 2t) + e^{-2\pi s} L(\sin 3t)$$

$$= \frac{1}{s^2+1} + e^{-\pi s}\left(\frac{-1}{s^2+1} + \frac{2}{s^2+4}\right) - e^{-2\pi s}\left(\frac{2}{s^2+4} - \frac{3}{s^2+9}\right)$$

$$= \frac{(1-e^{-\pi s})}{s^2+1} + \frac{2(e^{-\pi s} - e^{-2\pi s})}{s^2+4} + \frac{3e^{-2\pi s}}{s^2+9}$$

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5) (i) $L^{-1}\left[\dfrac{s^{-2}}{(s^2+1)(s^2+4)}\right]$

$= L^{-1}\left[\dfrac{1}{s^2(s^2+1)(s^2+4)}\right]$

$= L^{-1}\left[\dfrac{1+s^2-s^2}{s^2(s^2+1)(s^2+4)}\right]$

$= L^{-1}\left[\dfrac{\cancel{s^2+1}}{s^2\cancel{(s^2+1)}(s^2+4)}\right] - L^{-1}\left[\dfrac{\cancel{s^2}}{\cancel{s^2}(s^2+1)(s^2+4)}\right]$

$= \dfrac{L^{-1}}{2}\left[\dfrac{1}{s^2}\cdot\dfrac{(1\times 2)}{(s^2+4)}\right] - \dfrac{L^{-1}}{2}\left[\dfrac{1\times 2}{(s^2+1)(s^2+4)}\right]$

$= \dfrac{1}{2}(t) * (\sin 2t) - \dfrac{1}{2}(\sin t) * (\sin 2t)$

$= \dfrac{1}{2}\int_0^t (t+u)\cdot \sin 2t \, \cancel{dt}\, du - \dfrac{1}{2}\int_0^t \sin(t+u)\sin 2t \cdot \cancel{dt}\, du$

$= \dfrac{1}{2}\int_0^t (u-t)\sin 2u\, du - \dfrac{1}{2}\int_0^t \sin(t+u)\cdot \sin 2u \cdot du$

$= \dfrac{1}{2}\left[I_1 - I_2\right]$

Now: $I_1 = \int_0^t (u-t)\sin 2u du$

$= \int (u-t)\int \sin 2u - \int \dfrac{d(u-t)}{du}\int \sin 2u\, du \cdot du$

$= \dfrac{(u-t)\cos 2u}{2} - \int \dfrac{\cos 2u}{2} du$

$= \dfrac{(u-t)\cos 2u}{2} - \dfrac{(-\sin 2u)}{4}$

$= \left[\dfrac{(u-t)\cos 2u}{2} + \dfrac{\sin 2u}{4}\right]_0^t$

$= \dfrac{\sin 2t}{4} - \cancel{4}\left(-\dfrac{t}{2}\right) = \left(\dfrac{\sin 2t}{4} + \dfrac{t}{2}\right)$

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$$I_2 = \int_0^t \sin(t+u) \cdot \sin 2u \, du$$

$$= \frac{1}{2} \int_0^t 2\sin(t+u) \sin 2u \, du$$

$$= \frac{1}{-2} \int_0^t -2\sin(u-t) \sin 2u \, du$$

$$= -\frac{1}{2} \int_0^t \cos(u-t+2u) - \cos(u-t-2u) \, du$$

$$= -\frac{1}{2} \left[\int_0^t \cos(3u-t) \, du - \int_0^t \cos(-u-t) \, du \right]$$

$$= -\frac{1}{2} \left[\frac{\sin(3u-t)}{3} \Big|_0^t - \frac{\sin(-u-t)}{-1} \Big|_0^t \right]$$

$$= -\frac{1}{2} \left[\frac{\sin(3t-t)}{3} - \frac{\sin(-t)}{3} - \sin(u+t) \Big|_0^t \right]$$

$$= -\frac{1}{2} \left[\frac{\sin 2t + \sin t}{3} - (\sin 2t - \sin t) \right]$$

$$= -\frac{1}{2} \left[\frac{\sin 2t + \sin t - 3\sin 2t + 3\sin t}{3} \right]$$

$$= \frac{-1}{6} (4\sin t - 2\sin 2t)$$

$$= \frac{-1}{3} (2\sin t - \sin 2t)$$

$$\therefore L^{-1}\left(\frac{s^{-2}}{(s^2+1)(s^2+4)} \right) = \frac{1}{2} \left[\frac{\sin 2t}{4} + \frac{t}{2} + \frac{+1}{3}(2\sin t - \sin 2t) \right]$$

$$= \frac{1}{2} \left[\frac{\sin 2t}{4} + \frac{t}{2} + \frac{2\sin t}{3} - \frac{\sin 2t}{3} \right]$$

$$= \frac{1}{2} \left[\frac{t}{2} + \frac{2\sin t}{3} - \frac{\sin 2t}{12} \right]$$

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5) (ii)

$$L^{-1} \frac{s}{(s+1)(s-3)(s+5)}$$

$$\bar{f}(s) = \frac{s}{(s+1)(s-3)} = \frac{A}{(s+1)} + \frac{B}{(s-3)}$$

$$\Rightarrow \frac{s}{(s+1)(s-3)} = \frac{s(A+B) - 3A + B}{(s+1)(s-3)}$$

$$A = \frac{1}{4}, \quad B = \frac{3}{4}$$

$$\bar{f}(s) = \frac{1}{4(s+1)} + \frac{3}{4(s-3)}$$

$$f(t) = \frac{e^{-t}}{4} + \frac{3}{4}e^{3t}$$

$$\bar{g}(s) = \frac{1}{s+5}$$

$$g(t) = e^{-5t}$$

$$L^{-1}(\bar{f}(s)\,\bar{g}(s)) = f(t) * g(t)$$

$$= \int_0^t g(t-u)\cdot f(u)\,du$$

$$= \int_0^t e^{-5(t-u)} \left[\frac{1}{4}e^{-u} + 3e^{+3u}\right] du$$

$$= \frac{1}{4}\int_0^t e^{4u-5t} + 3e^{8u-5t}\,du$$

$$= \frac{1}{32}\left| 2e^{-t} - 2e^{-5t} + 3e^{+3t} - 3e^{-5t} \right|$$

$$= \frac{1}{32}(2e^{-t} - 5e^{-5t} + 3e^{3t}$$

$$= \frac{e^{-t}}{16} - \frac{5}{32}e^{-5t} + \frac{3}{32}e^{3t} \leftarrow \text{Ans}$$

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