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E-Record 2

Experiment:2A Properties of Eigen values and Eigen vectors, Cayley Hamilton theorem

Aim:

1. Let $A=\begin{pmatrix} 3 & 1 & -1 \\ 1 & 3 & -1 \\ -1 & -1 & 5 \end{pmatrix}$

   (a) Find characteristic equation of A(without using **poly** command).

   (b) Find eigen values by finding the roots of characteristic equation.

   (c) Find eigen vector X of A by solving the equation $AX = \lambda X$.

   (d) Verify the properties of Eigen values.

   (e) Verify Cayley-Hamilton theorem and find inverse

Mathematical Background:

Eigenvalues and Eigenvectors

We study the problem

$$AX = \lambda X$$

where A is given $n \times n$ square matrix, X is an unknown $n \times 1$column vector, and λ is an scalar.
given an $n \times n$ matrix A, find the value of λ such that $[A - \lambda I]X = 0$ admits non-trival solution, and find those non-trival solution.
This is called the **Eigenvalue Problem**

Solving **characterstic equation** $|A-\lambda I| = 0$, we get n values of λ. These values are known as **eigenvalues**. The vectors corresponding to each of these n values of λ are known as **eigenvectors**.

Properties of Eigenvalues

1) Any square matrix A and its transpose A^T have the same eigen values.
2) The eigenvalues of triangular matrix are just the diagonal elements of the matrix.
3) The eigenvalues of an idempotent matrix are either 0 or 1.
4) The sum of the eigen values of a matrix is the sum of the elements of the principal diagonal.
5) The product of the eigenvalues of a matrix A is equal to its determinant.
6) If λ is an eigenvalues of a matrix A, then $\frac{1}{\lambda}$ is the eigenvalue of A^{-1}.
7) If λ is an eigenvalue of an orthogonal matrix, then $\frac{1}{\lambda}$ is also its eigenvalue.
8) If $\lambda_1, \lambda_2, \cdots, \lambda_n$ are the eigenvalues of a matrix, then A^m has the eigenvalues $\lambda_1^m, \lambda_2^m, \cdots, \lambda_n^m$ (m being a positive integer).

Cayley-Hamilton Theorem

Every square matrix satisfies its own characteristic equation.

Matlab Code:

```
clc
clear all
close all
syms L real
A=input("Enter the square matrix:");
ch = det(A-L*eye(length(A)));
disp(ch);
r=solve(ch);
p=poly(A);
r1=roots([p]);
disp(r1);
for i=1:length(A)
    x=null(A-r(i)*eye(length(A)))
end
[p,d]=eig(A)
```

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```
%%det of A = product of eigen values
det_A = det(A)%determinant of A
prod_A= prod(eig(A))%product of eigen values


%%trace of A = sum of the eigen values
trace_A = trace(A)
sum_A = sum(eig(A))


%%A and A^T have the same eigen values
eig_A=eig(A) %eigen values of A
eig_AT=eig(transpose(A)) %eigen values of A transpose


%%A-lambda and A^-1 (1/lambda)
inv_A=eig(inv(A))


%A-lambda and A^n -lambda^n
power_A=eig(A^2)


n=length(p);
cht= p(1)*A^(n-1);
for i=2:n
    cht=cht+p(i)*A^(n-i);
end
cht=round(cht);
disp(cht)


%inverse using C H theorem
inv_A=p(1)*A^(n-2);
for i=2:n-1
    inv_A=inv_A+p(i)*A^(n-i-1);
end
inv_A=inv_A/(-p(n))
```

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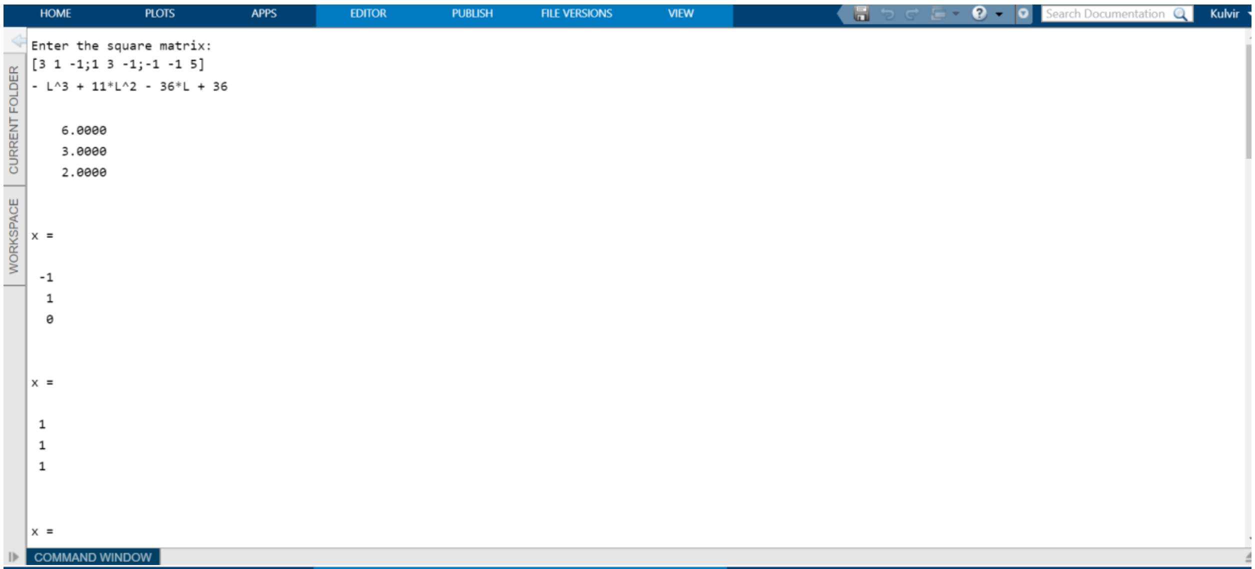
Code Screenshots:

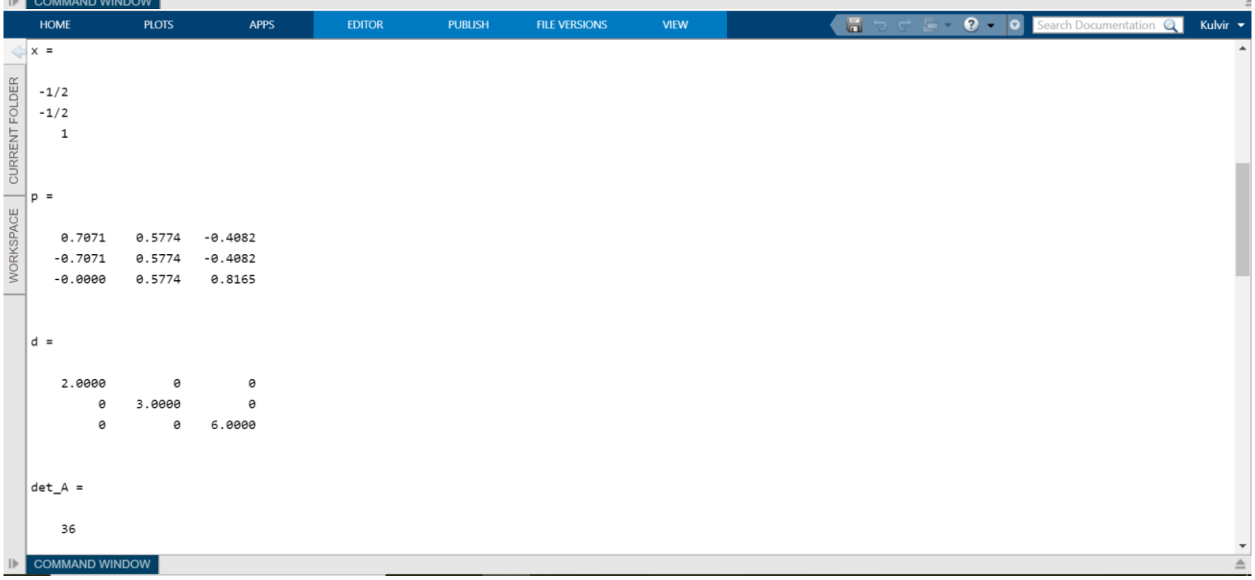
```
clc
clear all
close all
syms L real
A=input("Enter the square matrix:");
ch = det(A-L*eye(length(A)));
disp(ch);
r=solve(ch);
p=poly(A);
r1=roots([p]);
disp(r1);
for i=1:length(A)
    x=null(A-r(i)*eye(length(A)))
end
[p,d]=eig(A)

%%det of A = product of eigen values
det_A = det(A)%determinant of A
prod_A= prod(eig(A))%product of eigen values

%%trace of A = sum of the eigen values
trace_A = trace(A)
sum_A = sum(eig(A))

%%A and A^T have the same eigen values
eig_A=eig(A) %eigen values of A
```

```

%%A and A^T have the same eigen values
eig_A=eig(A) %eigen values of A
eig_AT=eig(transpose(A)) %eigen values of A transpose

%%A-lambda and A^-1 (1/lambda)
inv_A=eig(inv(A))

%A-lambda and A^n -lambda^n
power_A=eig(A^2)

n=length(p);
cht= p(1)*A^(n-1);
for i=2:n
    cht=cht+p(i)*A^(n-i);
end
cht=round(cht);
disp(cht)

%inverse using C H theorem
inv_A=p(1)*A^(n-2);
for i=2:n-1
    inv_A=inv_A+p(i)*A^(n-i-1);
end
inv_A=inv_A/(-p(n))

```

Output Screenshots:



```
Enter the square matrix:
[3 1 -1;1 3 -1;-1 -1 5]
- L^3 + 11*L^2 - 36*L + 36

    6.0000
    3.0000
    2.0000


x =

 -1
  1
  0


x =

 1
 1
 1


x =
```



```
x =

 -1/2
 -1/2
    1


p =

    0.7071    0.5774   -0.4082
   -0.7071    0.5774   -0.4082
   -0.0000    0.5774    0.8165


d =

    2.0000         0         0
         0    3.0000         0
         0         0    6.0000


det_A =

    36
```

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CURRENT FOLDER WORKSPACE

```
prod_A =

   36.0000


trace_A =

    11


sum_A =

   11.0000


eig_A =

    2.0000
    3.0000
    6.0000


eig_AT =
```

COMMAND WINDOW

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CURRENT FOLDER WORKSPACE

```
eig_AT =

    2.0000
    3.0000
    6.0000


inv_A =

    0.1667
    0.3333
    0.5000


power_A =

    4.0000
    9.0000
   36.0000

     6     4    -6
     4     6    -6
    -6    -6    16
```

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CURRENT FOLDER WORKSPACE

```
    0.1667
    0.3333
    0.5000


power_A =

    4.0000
    9.0000
   36.0000

     6     4    -6
     4     6    -6
    -6    -6    16


inv_A =

   1.0e+16 *

    0.6005    0.3002   -0.3002
    0.3002    0.6005   -0.3002
   -0.3002   -0.3002    1.2010

>> |
```

COMMAND WINDOW

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Experiment:2B Diagonalization by similarity transformation, Orthogonal Transformation

Aim:

1. Diagonalize $A=\begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ by similarity transformation.

2. Diagonalize $A=\begin{pmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{pmatrix}$ by orthogonal transformation.

Mathematical Background:

Similarity Transformation

A is said to be **similar** to B if there exist a non-singular matrix P such that

$$B = P^{-1}AP$$

This transformation of A to B is known as **similarity transformation**.

Let $X_1, X_2, \cdots, X_n$ be the n linearly independent eigenvectors of A corresponding to n eigenvalues $\lambda_1, \lambda_2, \cdots, \lambda_n$. $P_{n\times n} = \begin{bmatrix} X_1 & X_2 & \cdots & X_n \end{bmatrix}$ is known as **modal matrix**.

$$\begin{aligned} A \quad P &= A \quad \begin{bmatrix} X_1 & X_2 & \cdots & X_n \end{bmatrix} \\ &= \begin{bmatrix} AX_1 & AX_2 & \cdots & AX_n \end{bmatrix} \\ &= \begin{bmatrix} \lambda_1 X_1 & \lambda_2 X_2 & \cdots & \lambda_n X_n \end{bmatrix} \\ &= \begin{bmatrix} X_1 & X_2 & \cdots & X_n \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & \lambda_n \end{bmatrix} \\ &= P \quad D \end{aligned}$$

Multiplying by P^{-1} on bothsides,

$$P^{-1}AP = (P^{-1}P)D = D$$

where D is the diagonal matrix with eigen values of A as the principal diagonal elements. D is known as **spectral matrix**.

Orthogonal Transformation

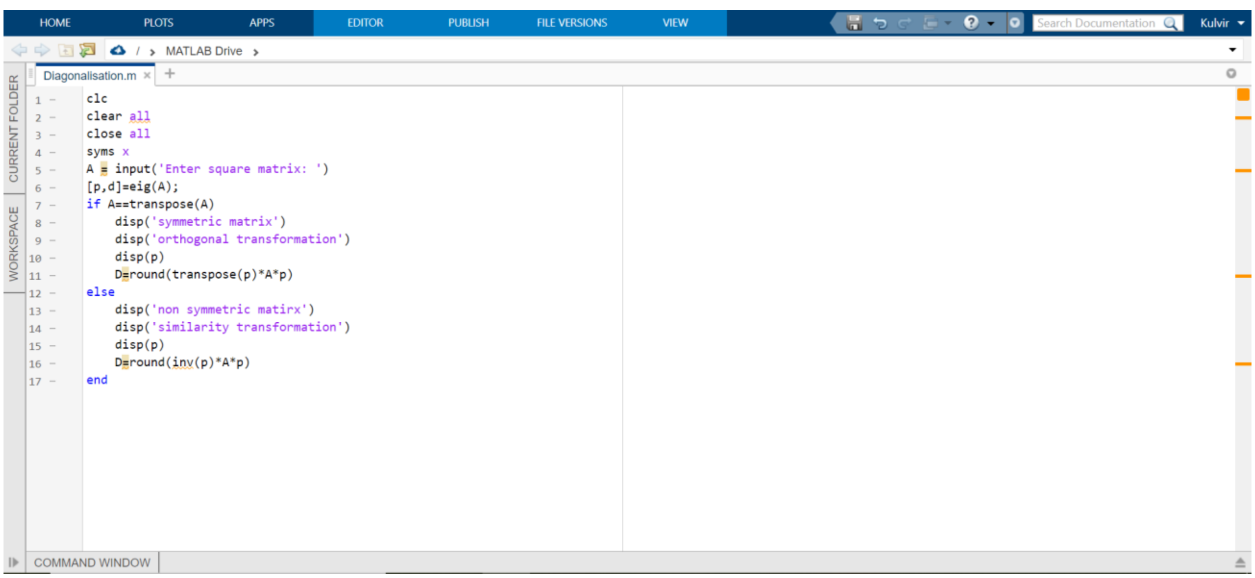
If we normalise each eigen vector and use them to form the normalised modal matrix N then it can be proved that N is an **orthogonal matrix**.

The similarity transformation $P^{-1}AP = D$ takes the form $N^TAN = D$ since $N^{-1} = N^T$ by a property of orthogonal matrix. Transforming A into D by means of the transformation $N^TAN = D$ is called as **orthogonal reduction** or **othogonal transformation**.

Matlab Code:

```
clc
clear all
close all
syms x
A = input('Enter square matrix: ')
[p,d]=eig(A);
if A==transpose(A)
    disp('symmetric matrix')
    disp('orthogonal transformation')
    disp(p)
    D=round(transpose(p)*A*p)
else
    disp('non symmetric matirx')
    disp('similarity transformation')
    disp(p)
    D=round(inv(p)*A*p)
end
```

Code Screenshot:



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Output

1> Similarity Transformation

```
Enter square matrix:
[1 6 1;1 2 0;0 0 3]

A =

     1     6     1
     1     2     0
     0     0     3

non symmetric matirx
similarity transformation
   -0.9487   -0.8944   -0.2357
    0.3162   -0.4472   -0.2357
         0         0    0.9428


D =

    -1     0     0
     0     4     0
     0     0     3

>>
```

2> Orthogonal Transformation

```
Enter square matrix:
[3 2 4;2 0 2;4 2 3]

A =

     3     2     4
     2     0     2
     4     2     3

symmetric matrix
orthogonal transformation
   -0.4862   -0.5649    0.6667
   -0.4834    0.8095    0.3333
    0.7279    0.1602    0.6667


D =

    -1     0     0
     0    -1     0
     0     0     8

>>
```

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