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E-Record 5

Experiment 5A: Z-transforms and their applications for solving Difference Equations

Aim:

1. Solve $y_{n+2} - 4y_{n+1} + 3y_n = 2^n$, $n \geq 0$ $y_0 = 0$, $y_1 = 0$ using Z-transform.Plot the solution

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Mathematical Background:

Z–Transform

If the function u_n is defined for discrete values ($n = 0,1,2,...$) and $u_n = 0$ for $n < 0$, then its Z-transform is defined as

$$Z\{u_n\} = \overline{U}(z) = \sum_{n=0}^{\infty} \frac{u_n}{z^n}$$

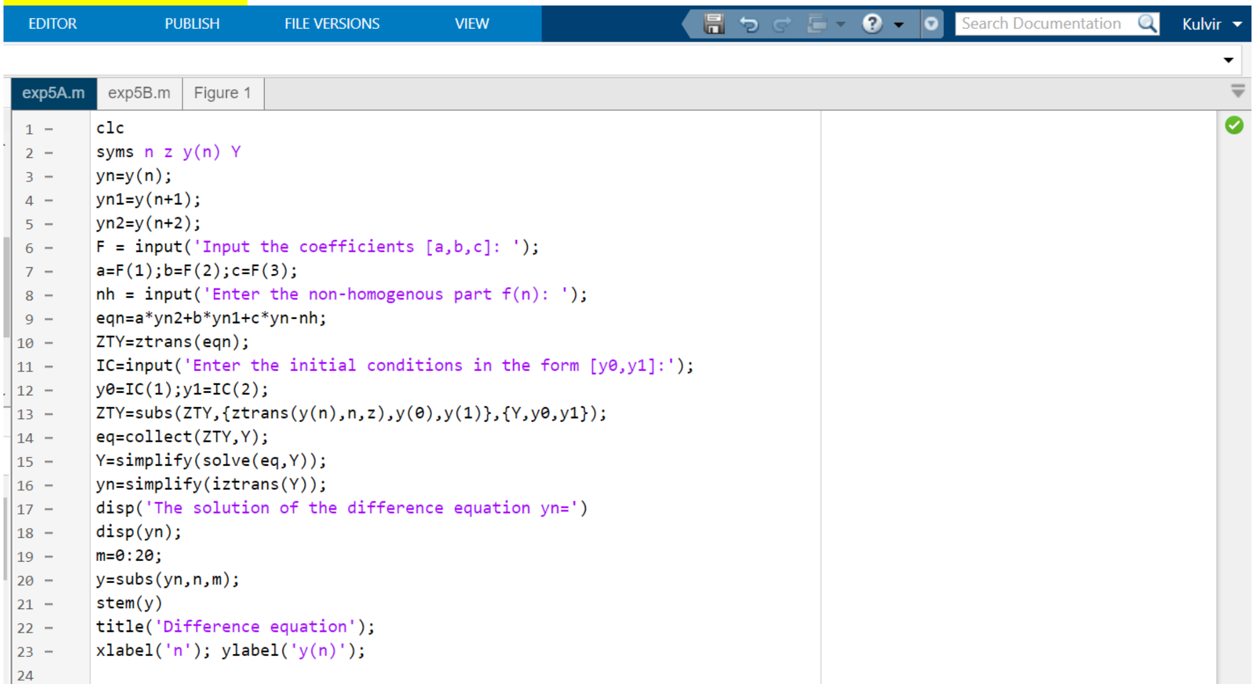
whenever the infinite series converges.
The inverse Z-transform is written as $Z^{-1}\{\overline{U}(z)\} = u_n$.

Matlab Code:

```
clc
syms n z y(n) Y
yn=y(n);
yn1=y(n+1);
yn2=y(n+2);
F = input('Input the coefficients [a,b,c]: ');
a=F(1);b=F(2);c=F(3);
nh = input('Enter the non-homogenous part f(n): ');
eqn=a*yn2+b*yn1+c*yn-nh;
ZTY=ztrans(eqn);
IC=input('Enter the initial conditions in the form [y0,y1]:');
y0=IC(1);y1=IC(2);
ZTY=subs(ZTY,{ztrans(y(n),n,z),y(0),y(1)},{Y,y0,y1});
eq=collect(ZTY,Y);
Y=simplify(solve(eq,Y));
```

```
yn=simplify(iztrans(Y));
disp('The solution of the difference equation yn=')
disp(yn);
m=0:20;
y=subs(yn,n,m);
stem(y)
title('Difference equation');
xlabel('n'); ylabel('y(n)');
```

Code Screenshots:




```
clc
syms n z y(n) Y
yn=y(n);
yn1=y(n+1);
yn2=y(n+2);
F = input('Input the coefficients [a,b,c]: ');
a=F(1);b=F(2);c=F(3);
nh = input('Enter the non-homogenous part f(n): ');
eqn=a*yn2+b*yn1+c*yn-nh;
ZTY=ztrans(eqn);
IC=input('Enter the initial conditions in the form [y0,y1]:');
y0=IC(1);y1=IC(2);
ZTY=subs(ZTY,{ztrans(y(n),n,z),y(0),y(1)},{Y,y0,y1});
eq=collect(ZTY,Y);
Y=simplify(solve(eq,Y));
yn=simplify(iztrans(Y));
disp('The solution of the difference equation yn=')
disp(yn);
m=0:20;
y=subs(yn,n,m);
stem(y)
title('Difference equation');
xlabel('n'); ylabel('y(n)');
```

Output Screenshots:

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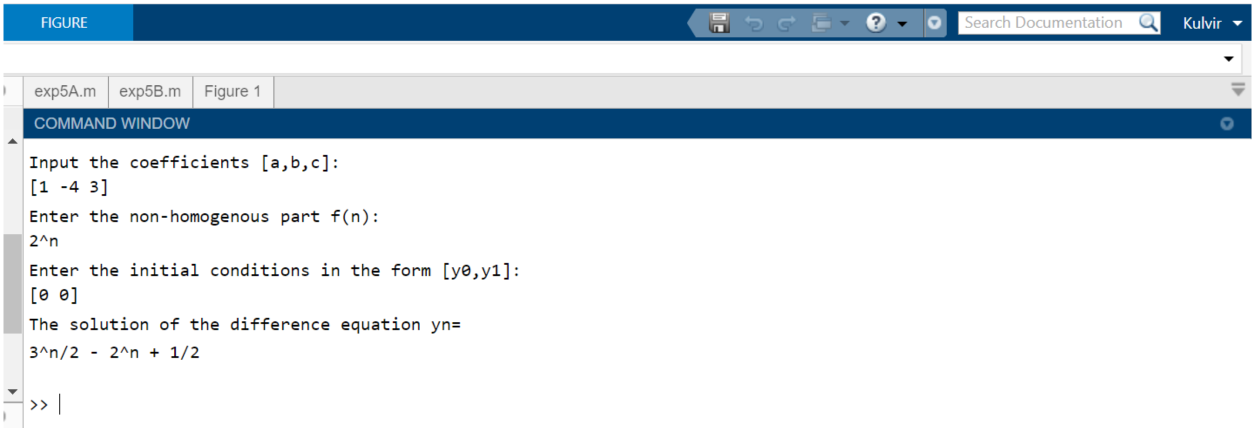

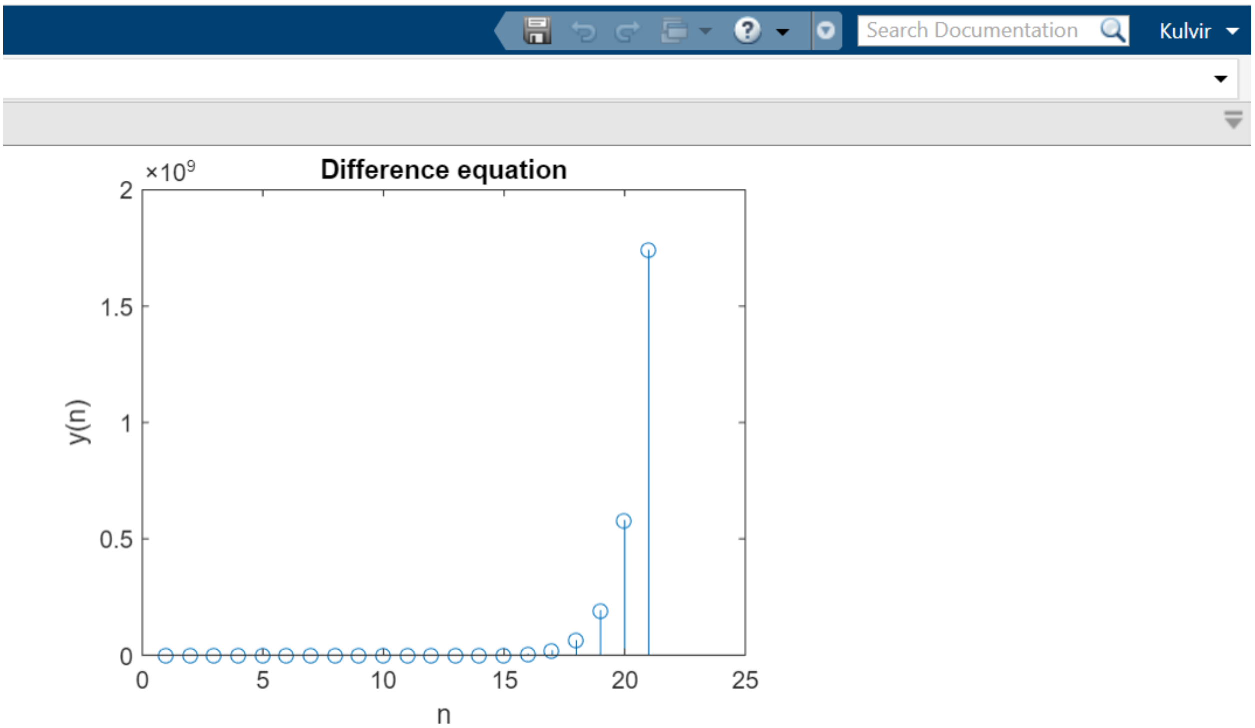

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Experiment 5B: Solution of homogeneous linear Difference Equations

Aim:

1. Solve $2y_{n+2} - 7y_{n+1} + 3y_n = 0$ subject to the given conditions $y_0 = 1$, $y_1 = 1$.Plot the solution.

Mathematical Background:

Consider a second order homogeneous linear difference equation of the form

$$ay_{n+2} + by_{n+1} + cy_n = 0 \tag{1}$$

subject to the initial conditions

$$y_0 = \alpha\ ,\ y_1 = \beta \tag{2}$$

where the a, b, c are constants.

To solve the equation (1) we take the trial solution $y_n = \lambda^n$. Substituting in (1) we get the quadratic equation

$$a\lambda^2 + b\lambda + c = 0. \tag{3}$$

This is called the characteristic equation of (1).

The solutions of (1) are given as per the following cases:

Case 1. If $b^2 - 4ac > 0$, then the roots of (3) are real and different (say λ_1, λ_2).

In this case the solutions of (1) are $y_1 = \lambda_1^n$ and $y_2 = \lambda_2^n$ and hence the general solution of (1) is given by $y_n = k_1y_1 + k_2y_2 = k_1\lambda_1^n + k_2\lambda_2^n$.

Case 2. If $b^2 - 4ac = 0$, then the roots (say $\lambda_1 = \lambda_2$) of (3) are real and equal.

In this case the solutions of (1) are $y_1 = \lambda_1^n$ and $y_2 = n.\lambda_1^n$ and hence the general solution of (1) is given by $y_n = k_1y_1 + k_2y_2 = k_1\lambda_1^n + k_2n.\lambda_1^n$.

Case 3. If $b^2 - 4ac < 0$, then the roots of (3) are complex (say $\lambda_1 = \alpha + i\beta, \lambda_2 = \alpha - i\beta$).

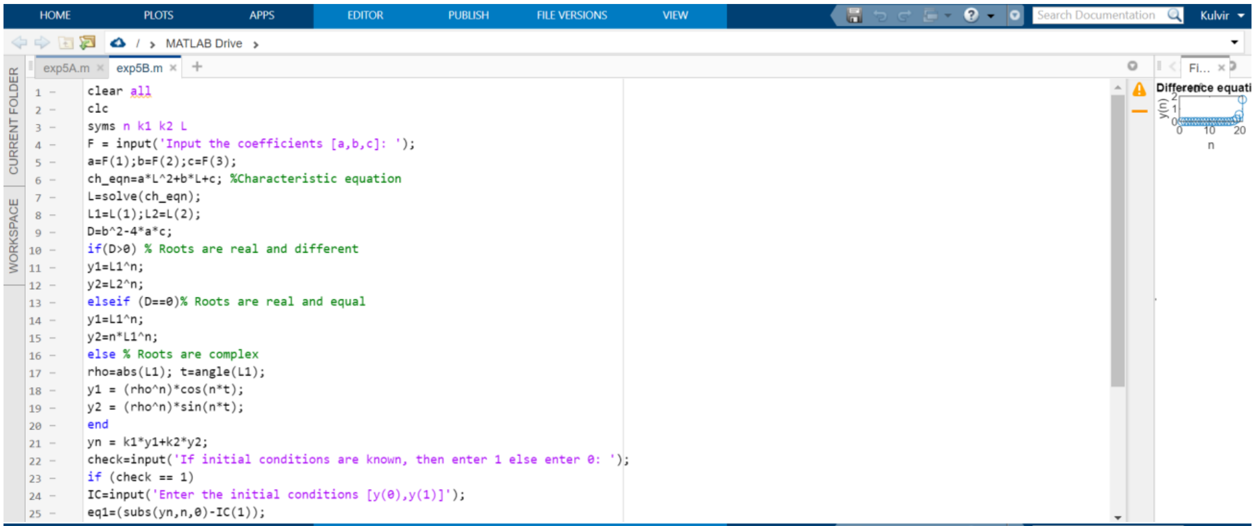
In this case the solutions of (1) are $y_1 = \rho^n \cos n\theta$ and $y_2 = \rho^n \sin n\theta$, where $\rho = \sqrt{\alpha^2 + \beta^2}$, $\theta = \tan^{-1}(\beta/\alpha)$ and hence the general solution of (1) is given by

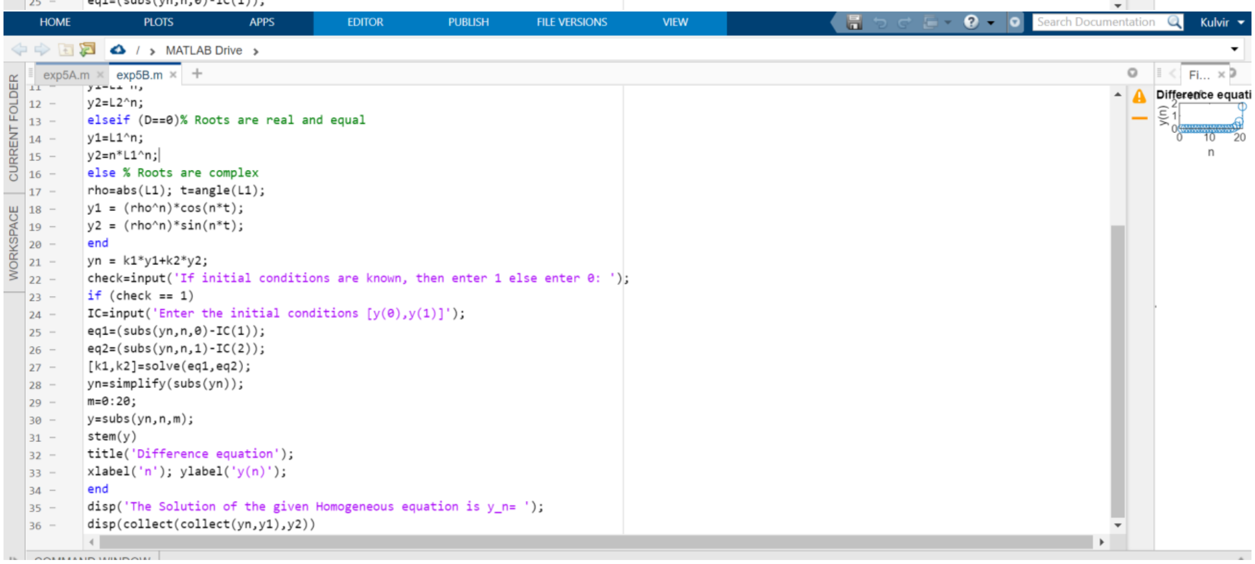
$$y_n = k_1y_1 + k_2y_2 = \rho^n[k_1 \cos n\theta + k_2 \sin n\theta]$$

Matlab Code:

```
clear all
clc
syms n k1 k2 L
F = input('Input the coefficients [a,b,c]: ');
a=F(1);b=F(2);c=F(3);
ch_eqn=a*L^2+b*L+c; %Characteristic equation
L=solve(ch_eqn);
L1=L(1);L2=L(2);
D=b^2-4*a*c;
if(D>0) % Roots are real and different
y1=L1^n;
y2=L2^n;
elseif (D==0)% Roots are real and equal
y1=L1^n;
y2=n*L1^n;
else % Roots are complex
rho=abs(L1); t=angle(L1);
y1 = (rho^n)*cos(n*t);
y2 = (rho^n)*sin(n*t);
end
yn = k1*y1+k2*y2;
check=input('If initial conditions are known, then enter 1 else enter 0: ');
if (check == 1)
IC=input('Enter the initial conditions [y(0),y(1)]');
eq1=(subs(yn,n,0)-IC(1));
eq2=(subs(yn,n,1)-IC(2));
[k1,k2]=solve(eq1,eq2);
yn=simplify(subs(yn));
m=0:20;
y=subs(yn,n,m);
stem(y)
title('Difference equation');
xlabel('n'); ylabel('y(n)');
end
disp('The Solution of the given Homogeneous equation is y_n= ');
disp(collect(collect(yn,y1),y2))
```

Code Screenshots:





Output Screenshots:

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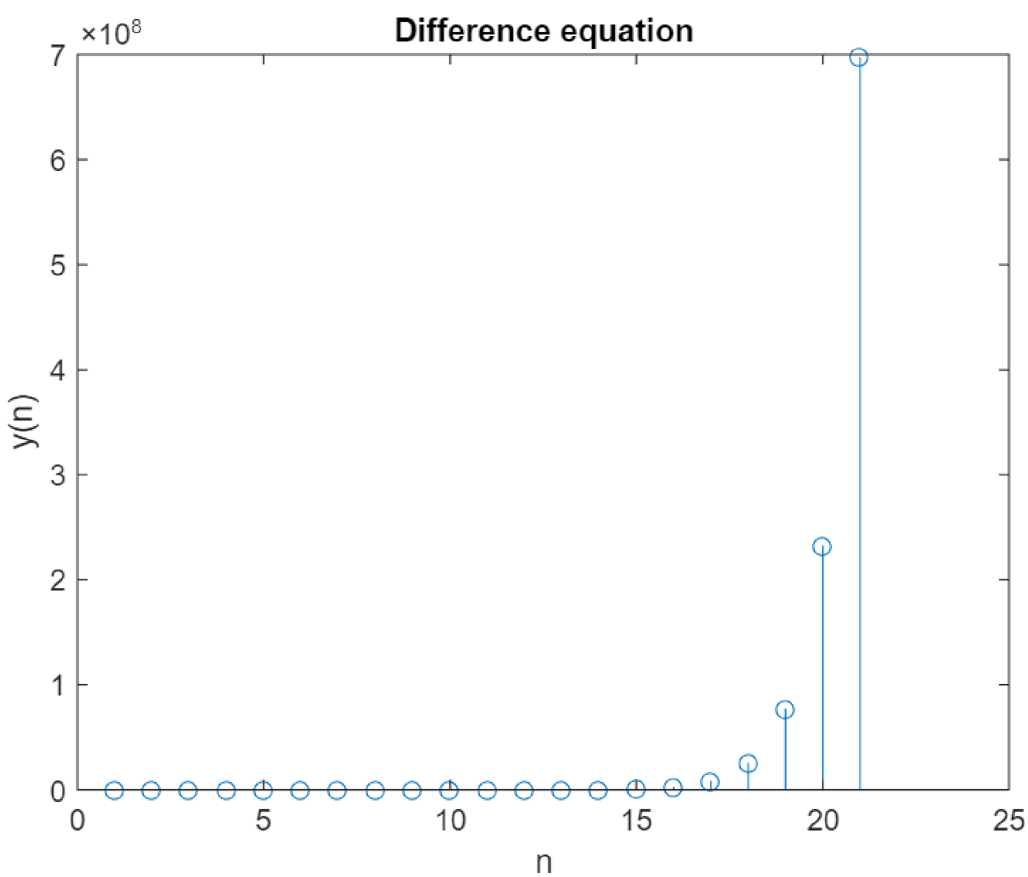
```
Input the coefficients [a,b,c]:
[2 -7 3]
If initial conditions are known, then enter 1 else enter 0:
1
Enter the initial conditions [y(0),y(1)]
[1 1]
The Solution of the given Homogeneous equation is y_n=
(6^n/5 + 4/5)/2^n
```



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