Digital Assignment - 1

Name : KULVIR SINGH
Reg. No : 19BCE2074

Q1)

X	Y	Rank of X	Rank of Y	$d_i = R_x - R_y$	d_i^2
65	68	9	5.5	3.5	12.25
63	66	11	9.5	1.5	2.25
67	68	6.5	5.5	1.0	1
64	65	10	11.5	-1.5	2.25
68	69	4.5	3	1.5	2.25
62	66	12	9.5	2.5	6.25
70	68	2	5.5	-3.5	12.25
66	65	8	11.5	-3.5	12.25
68	71	4.5	1	3.5	12.25
67	67	6.5	8	-1.5	2.25
69	68	3	5.5	-2.5	6.25
71	70	1	2	-1	1

$$\rho = 1 - \frac{6\left[\sum d_i^2 + CF\right]}{n(n^2-1)}$$

$$CF = \frac{m(m^2-1)}{12}$$

4.5 rank is repeated (2x) $CF = \frac{2(2^2-1)}{12} = \frac{1}{2}$

5.5 " " " (4x) $CF = \frac{4(4^2-1)}{12} = 5$

9.5 rank " " 2x $CF = 1/2$

6.5 " " " " $CF = 1/2$

11.5 " " " " $CF = 1/2$

Total correction factor $= 1/2 + 1/2 + 5 + 1/2 + 1/2$
$= 7$

$$\rho = \frac{1-6[72.5+7]}{12(12^2-1)} = 1-\frac{79.5\times 6}{1716}$$

$$= 1-0.2779$$

$$= \boxed{0.7221}$$

Q2a)

x	y	x^2	y^2	xy
25	43	625	1849	1075
28	46	484	2126	1288
35	49	1225	2401	1715
32	41	1024	1681	1312
31	36	961	1296	1116
36	32	1296	1024	1152
29	31	841	961	899
38	30	1444	900	1140
34	33	1156	1089	1122
32	39	1024	1521	1248
320	380	10380	14,828	12067

$$\bar{X} = \frac{1}{n}\Sigma X = \frac{1}{10}\times 320 = 32$$

$$\bar{Y} = \frac{1}{n}\Sigma Y = \frac{1}{10}\times 380 = 38$$

$$\frac{1}{10}\Sigma XY - \frac{1}{10}\Sigma X \times \frac{1}{10}\Sigma Y$$

$$= \frac{1}{10}\times 12067 - 32\times 38$$

$$= -9.3.$$

$$\sigma_x = \sqrt{\frac{1}{n}\Sigma x^2 - \bar{x}^2} = \sqrt{\frac{1}{10}\times 10380-1024} = 3.74$$

$$\sigma_y = \sqrt{\frac{1}{n}\Sigma y^2 - \bar{y}^2} = \sqrt{\frac{1}{10}\times 14838-38^2} = 6.31$$

x on $y \Rightarrow x - \bar{x} = \dfrac{p_{xy}(y - \bar{y})}{\sigma_y^2}$

$\Rightarrow x - 32 = -\dfrac{9.3(y - 38)}{39.8161}$

$\Rightarrow x - 32 = -0.23y + 8.74$

$\Rightarrow \boxed{x = -0.23y + 40.74}$

y on $x \Rightarrow y - \bar{y} = \dfrac{p_{xy}(x - \bar{x})}{\sigma_x^2}$

$\Rightarrow y - 38 = \dfrac{-9.3(x - 32)}{13.99}$

$\Rightarrow \boxed{y = -0.664x + 59.25}$

Q2b)

	x_1	x_2	x_3	x_1^2	x_2^2	x_3^2	$x_1 x_3$	$x_2 x_3$	$x_1 x_2$
	65	56	9	4225	3136	81	585	504	3640
	72	58	11	5184	3364	121	792	638	4176
	54	48	8	2916	2304	64	432	384	2592
	68	61	13	4624	3721	169	884	793	4148
	55	50	10	3025	2500	100	550	500	2750
	59	51	8	3481	2601	64	472	408	3009
	78	55	11	6084	3025	121	858	605	4290
	58	48	10	3364	2304	100	580	480	2784
	57	52	11	3249	2704	121	627	572	2964
	51	42	7	2601	1764	49	357	294	2142
Σ	617	521	98	38753	27423	990	6137	5178	32495

$\bar{x}_1 = \dfrac{617}{n} = \dfrac{617}{10} = 61.7$, $\bar{x}_2 = 52.1$, $\bar{x}_3 = 9.8$

$\sigma_{x_1} = \sqrt{\dfrac{\Sigma x_1^2}{n} - (\bar{x}_1)^2} = \sqrt{68.41} = 8.271$

$\sigma_{x_2} = \sqrt{27.89} = 5.281$

$\sigma_{x_3} = \sqrt{2.96} = 1.7204$

$$\text{cov}(x_1, x_2) = \frac{32495}{10} - (61.7)(52.1)$$

$$= 3249.5 - 3214.57 = 34.93$$

$$\text{cov}(x_2, x_3) = 517.8 - (52.1)(9.8)$$

$$= 7.22$$

$$\text{cov}(x_1, x_3) = 9.04$$

$$r_{x_1 x_2} = \frac{\text{cov}(x_1, x_2)}{\sigma_{x_1} \sigma_{x_2}} = \frac{34.93}{8.271 \times 5.231} = 0.1996$$

$$r_{x_2 x_3} = \frac{\text{cov}(x_2, x_3)}{\sigma_{x_2} \sigma_{x_3}} = 0.79468$$

$$r_{x_1, x_3} = \frac{\text{cov}(x_1, x_3)}{\sigma_{x_1} \sigma_{x_3}} = 0.6353$$

$$(R_{1.23})^2 = \frac{r_{12}^2 + r_{13}^2 - 2 r_{12} r_{13} r_{23}}{1 - r_{23}^2}$$

$$= \frac{0.2356}{0.369}$$

$$= \boxed{0.6384}$$

Q3a) $P = 0.65$, $q = 1 - p = 0.35$

$p \rightarrow$ prob. that a man aged 60 would be alive till 70 years

X = random variable

$$P[X \geq 7] = P[X=7] + P[X=8] + P[X=9] + P[X=10]$$

$$= {}^{10}C_7 (0.65)^7 (0.35)^3 + {}^{10}C_8 (0.65)^8 (0.35)^2 + {}^{10}C_9 (0.65)^9 (0.35) + {}^{10}C_{10} (0.65)^{10} (0.35)$$

$$= 0.252 + 0.175 + 0.0724 + 0.01347$$

$$= 0.5128.$$

Q3 b) $p = 0.001$

$q = 1 - 0.001$

$= 0.999$

$x \Rightarrow$ hitting an aircraft.

$P[x \geq 2] > 0.95$

$1 - P[x < 2] > 0.95$

$P[x < 2] < 0.05$

$P[x = 1] < 0.05$

${}^{n}C_{1} (0.001)(0.999)^{n-1} < 0.05$

$n (0.001)(0.999)^{n-1} < 0.05$

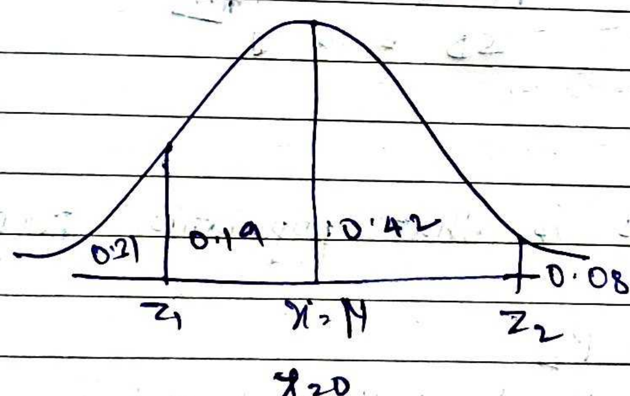
$n (0.999)^{n-1} < 50$

For $n (0.999)^{n-1} < 50$

$\therefore$ $n = 52$ is max value

$\therefore$ No. of shots fired $= \boxed{n \leq 52}$

Q4)



value of $z_1 = -0.5$
for area 0.19

ie, $z_1 = -0.5$

$\Rightarrow \dfrac{x - \mu}{\sigma} = -0.5$

$\Rightarrow \dfrac{45 - \mu}{\sigma} = -0.5$

$\Rightarrow \mu - 0.5\sigma = 45$ — (1)

value of z_2 from area table for area 0.42 $\Big\}$ = 1.4

$$z_2 = 1.4$$

$$\frac{64-\mu}{\sigma} = 1.4$$

$\Rightarrow$ $\mu + 1.4\sigma = 64$ — (2)

Solving (1), (2)

$$\mu + 1.4\sigma = 64$$
$$\underline{\mu - 0.5\sigma = 45}$$
$$1.9\sigma = 19$$
$$\sigma = 10$$

for z_1:

$\mu - 0.5 \times 10 = 45$

$\mu = 50$

SD = 10.

for z_2:

$\mu + 1.4\sigma = 64$

$\mu + 1.4 \times 10 = 64$

$\mu = 64 - 14$

$= 50$

$\mu = 50$ Mean = 50

$\sigma = 10$ SD = 10 Ans.

Q5) Null Hypothesis is that population proportion are equal.

$H_0 : P_1 = P_2$

Alternative Hypothesis : $H_1 : P_1 \neq P_2$

$n_1 = 900$, $P_1 = 20\% = 0.20$

$n_2 = 1600$, $P_2 = 18.5\% = 0.185$

$$P_0 = \frac{n_1 P_1 + n_2 P_2}{n_1 + n_2} = 0.1904.$$

$$z = \frac{p_1 - p_2}{\sqrt{pq\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} = \frac{0.20 - 0.185}{\sqrt{(0.1904)(0.809)\left(\frac{1}{900}+\frac{1}{1600}\right)}}$$

$$= \frac{0.015}{\sqrt{0.154\,(0.001725)}}$$

$$= 0.9259$$

not mention level of significance so we take at 1% level

$$|z| \leq 2.58$$

$\because$ we accept null hypothesis H_0 and conclude that diff. b/w 2 proportion is significant.
ie $P_1 = P_2$.

95% confidence limits for diff. $P_1 - P_2$ are

$$(P_1 - P_2) = \pm 1.96 \frac{\sigma}{\sqrt{n}}$$

$$\Rightarrow P_1 - P_2 = \pm 1.96\,(S.E \text{ of } P_1 - P_2)$$

$$SE \text{ of } P_1 - P_2 = \sqrt{\frac{P_1 Q_1}{n_1} + \frac{P_2 Q_2}{n_2}}$$

$P_1 = 0.20, \quad P_2 = 0.185$

$$= \sqrt{\frac{0.20 \times 0.80}{900} + \frac{0.185 \times 0.815}{1600}}$$

$$= 0.01624$$

Hence 95% confidence limit for $P_1 - P_2$ are $(0.20 - 0.185 \pm 01.96\,(0.016))$

$$= 0.015 \pm 0.0313$$

$$= 0.0463 \text{ or } -0.0163 \;]\rightarrow \text{Ans.}$$