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E-Record 4

Experiment–4A: Solution of homogeneous system of first order and second order differential equations by matrix method(2 Questions)

Question 1

Aim:

1. Solve the system of differential equations $y_1' = 4y_1 + y_2$, $y_2' = 3y_1 + 2y_2$, with the initial conditions $y_1(0) = 2$, $y_2(0) = 0$.

Mathematical Background:

System of First Order Linear Differential Equations

A system of n linear first order differential equations in n unknowns (an $n \times n$ system of linear equations) has the general form:

$$\begin{cases} x_1' = a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n + g_1(t) \\ x_2' = a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n + g_2(t) \\ \vdots = \vdots + \vdots + \vdots + \vdots + \vdots \\ x_n' = a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n + g_n(t) \end{cases} \tag{1}$$

where the coefficients a_{ij}'s are arbitrary constants, and g_i's are arbitrary functions of t. If every term g_i is constant zero, then the system is said to be homogeneous.

The system (1) is most often given in a shorthand format as a matrix-vector equation, in the form:

$$X' = AX + G$$

where $X' = [x_i']_{n\times1}$, $A = [a_{ij}]_{n\times n}$, $X = [x_i]_{n\times1}$, and $G = [g_i(t)]_{n\times1}$.

If the coefficient matrix A has two distinct real eigenvalues λ_1 and λ_2 and their respective eigenvectors are X_1 and X_2, then the 2×2 system

$$X' = AX$$

has a general solution

$$X = C_1X_1e^{\lambda_1 t} + C_2X_2e^{\lambda_2 t}$$

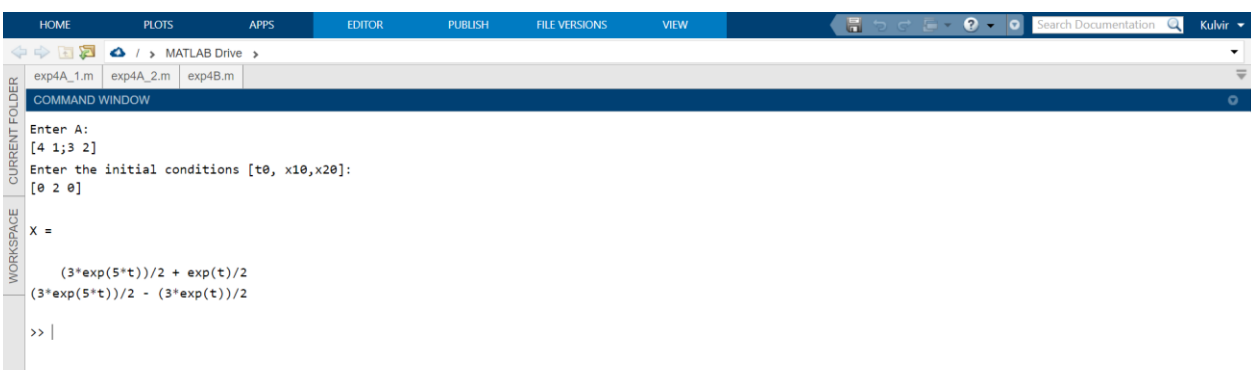
Matlab Code:

```
clc
clear
syms t C1 C2
A=input('Enter A:' );
[P,D]=eig(A);
```

```
L1=D(1);L2=D(4);
y1=C1*exp(L1*t);y2=C2*exp(L2*t);
Y=[y1;y2];
X=P*Y;
Cond=input('Enter the initial conditions [t0, x10,x20]: ');
t0=Cond(1);x10=Cond(2);x20=Cond(3);
eq1=subs(X(1)-x10,t0);eq2=subs(X(2)-x20,t0);
[C1, C2] = solve(eq1,eq2);
X=subs(X)
```

Code Screenshots:

Output Screenshots:



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Question 2

Aim:

2. Solve the system of differential equation $y_1'' = 2y_1 + y_2$, $y_2'' = y_1 + 2y_2$, with the initial conditions $y_1(0) = 0, y_1'(0) = 1,\ y_2(0) = 1, y_2'(0) = 0$.

Mathematical Background:

System of Second Order Linear Differential Equations

Consider the system of second order linear differential equations of the form

$$\begin{cases} x_1'' = a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \\ x_2'' = a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \\ \vdots = \vdots + \vdots + \vdots + \vdots \\ x_n'' = a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n \end{cases} \tag{2}$$

where the coefficients a_{ij}'s are arbitrary constants.

Then, the solution of (2), $X'' = AX$, is

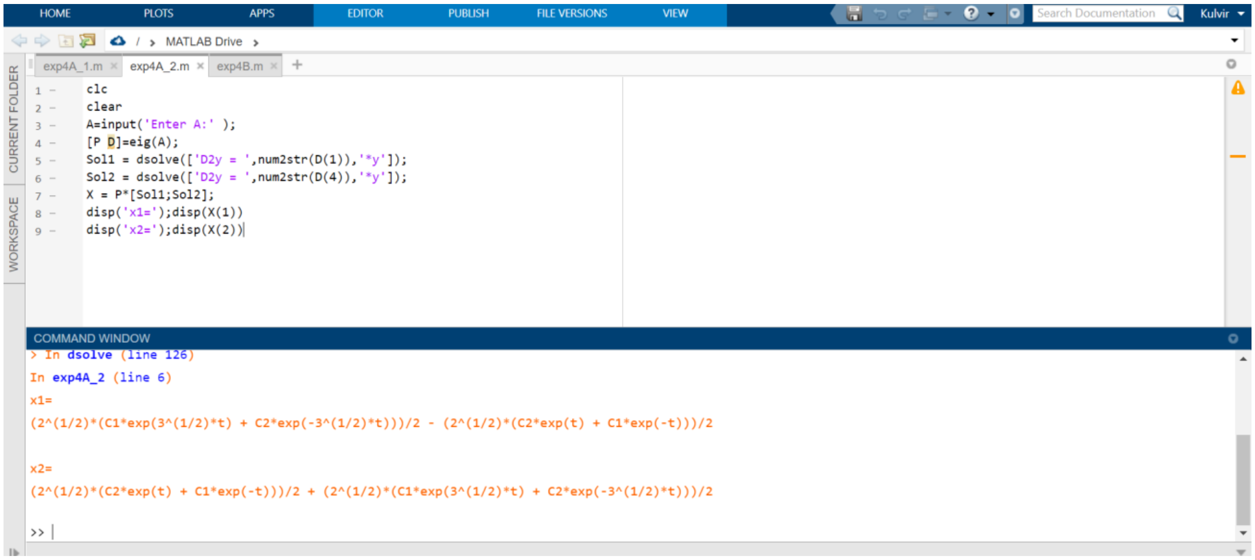
$$X = PY$$

where Y is the solution of $Y'' = DY$, P is the modal matrix of A and D is it's diagonal matrix.

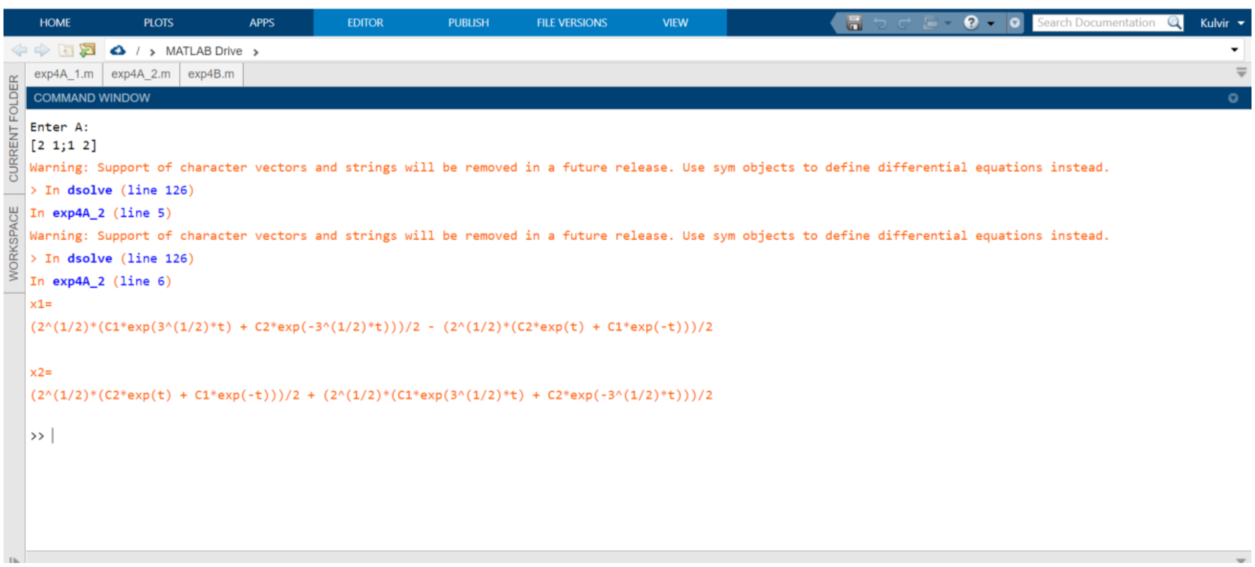
Matlab Code:

```
clc
clear
A=input('Enter A:' );
[P D]=eig(A);
Sol1 = dsolve(['D2y = ',num2str(D(1)),'*y']);
Sol2 = dsolve(['D2y = ',num2str(D(4)),'*y']);
X = P*[Sol1;Sol2];
disp('x1=');disp(X(1))
disp('x2=');disp(X(2))
```

Code Screenshots:



Output Screenshots:



Experiment:4B–Series solutions of ordinary differential equations

Aim:

1. Find the first five terms in the power series solution of the differential equation $y'' + 2y' + y = 0$.

Mathematical Background:

Series Solution when $x = 0$ ia an Ordinary Point of the Equation

$$P_0 \frac{d^2y}{dx^2} + P_1 \frac{dy}{dx} + P_2 y = 0 \tag{1}$$

where P's are polynomial in x and $P_0 \neq 0$ at $x = 0$.

1. Assume its solution to be of the form

$$y = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n + \cdots \tag{2}$$

2. Calculate $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$, from (2) and substitute the values of y, $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ in (1).
3. Equate to zero the coefficients of the various powers of x and determine a_2, a_3, a_4, $\cdots$ in terms of a_0, a_1.
4. Substituting the values of a_2, a_3, a_4, $\cdots$ in (2), we get the desired series solution having a_0, a_1 as its arbitrary constants.

Matlab Code:

```
clc
clear all
close all
syms x a0 a1 a2 a3 a4 a5 A B
a=[a0 a1 a2 a3 a4 a5];
y=sum(a.*x.^[0:5])
dy=diff(y)
d2y=diff(dy)
% de= collect(d2y+y,x)
de= collect(d2y+2*dy+y,x)
coef= coeffs(de,x)
A2=solve(coef(1),a2)
A3=subs(solve(coef(2),a3),a2,A2)
A4=subs(solve(coef(3),a4),{a2,a3},{A2,A3})
A5=subs(solve(coef(4),a5),{a2,a3,a4},{A2,A3,A4})

y=subs(y,{a2,a3,a4,a5},{A2,A3,A4,A5})
soln=coeffs(y,[a1,a0])
gs=A*soln(1)+B*soln(2)
```

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Code Screenshots:

```
clc
clear all
close all
syms x a0 a1 a2 a3 a4 a5 A B
a=[a0 a1 a2 a3 a4 a5];
y=sum(a.*x.^[0:5])
dy=diff(y)
d2y=diff(dy)
% de= collect(d2y+y,x)
de= collect(d2y+2*dy+y,x)
coef= coeffs(de,x)
A2=solve(coef(1),a2)
A3=subs(solve(coef(2),a3),a2,A2)
A4=subs(solve(coef(3),a4),{a2,a3},{A2,A3})
A5=subs(solve(coef(4),a5),{a2,a3,a4},{A2,A3,A4})

y=subs(y,{a2,a3,a4,a5},{A2,A3,A4,A5})
soln=coeffs(y,[a1,a0])
gs=A*soln(1)+B*soln(2)
```

COMMAND WINDOW

```
B*(x^5/24 - x^4/6 + x^3/2 - x^2 + x) + A*(x^5/30 - x^4/8 + x^3/3 - x^2/2 + 1)
```

Output Screenshots:

COMMAND WINDOW

```
y =

a5*x^5 + a4*x^4 + a3*x^3 + a2*x^2 + a1*x + a0


dy =

5*a5*x^4 + 4*a4*x^3 + 3*a3*x^2 + 2*a2*x + a1


d2y =

20*a5*x^3 + 12*a4*x^2 + 6*a3*x + 2*a2


de =

a5*x^5 + (a4 + 10*a5)*x^4 + (a3 + 8*a4 + 20*a5)*x^3 + (a2 + 6*a3 + 12*a4)*x^2 + (a1 + 4*a2 + 6*a3)*x + a0 + 2*a1 + 2*a2
```

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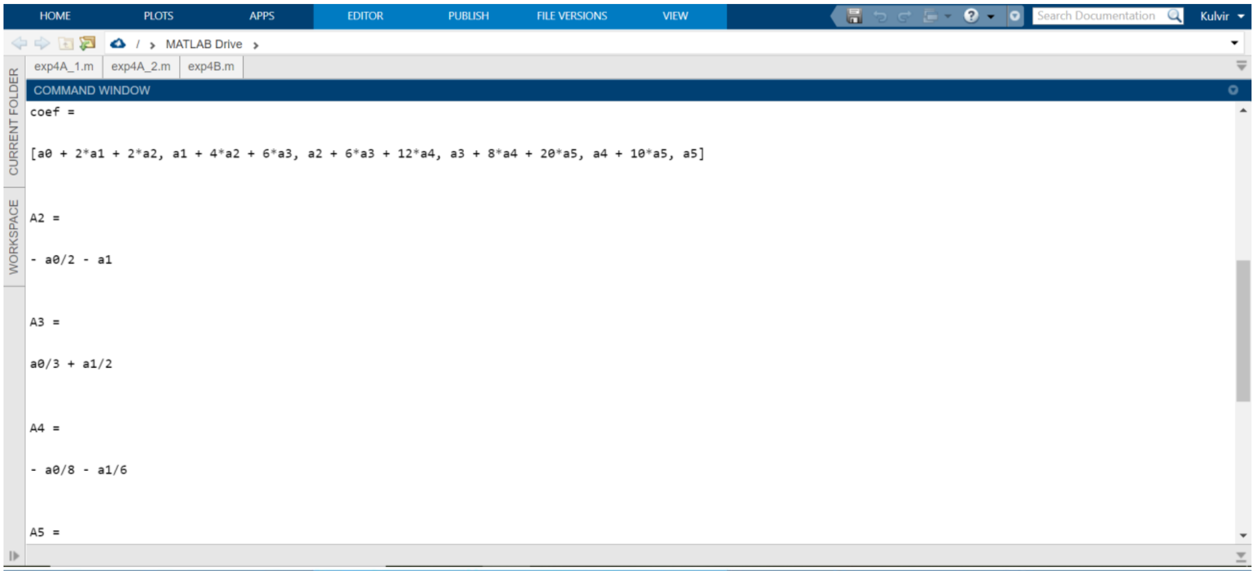

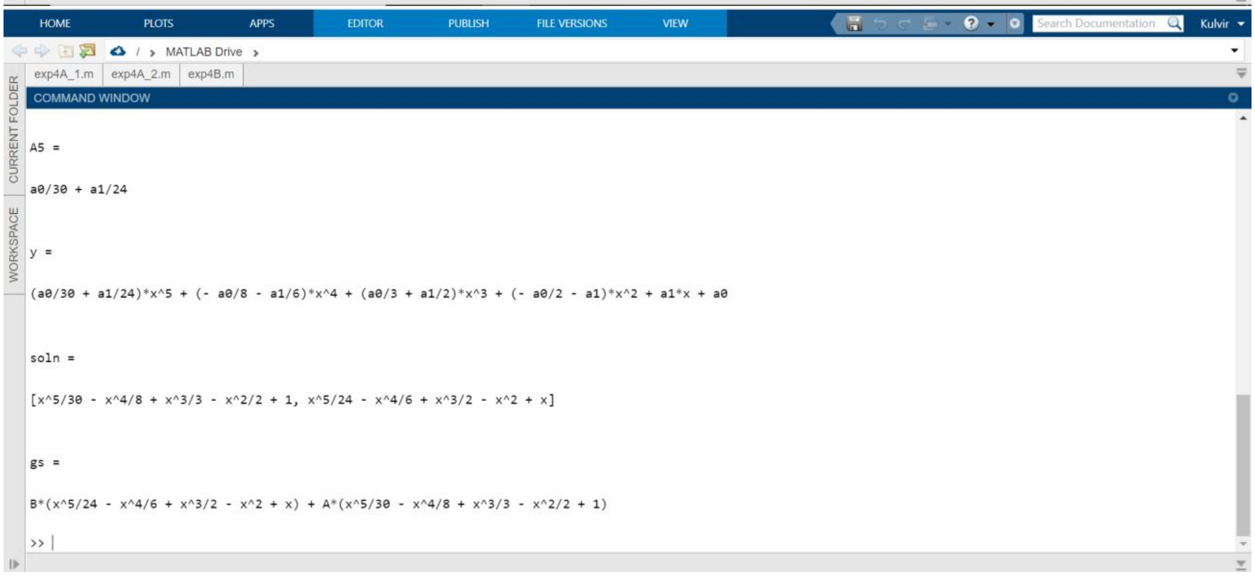

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