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(1)

$$2y + 4z = 2$$
$$x + 2y + 2z = 3$$
$$3x + 4y + 6z = -1$$

Gauss Elimination.

$$\left(\begin{array}{ccc|c} 0 & 2 & 4 & 2 \\ 1 & 2 & 2 & 3 \\ 3 & 4 & 6 & -1 \end{array}\right) = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\boxed{AX = B}$$
$$[A \mid B] = [X]$$

$R_1 \leftrightarrow R_2$

$$\left(\begin{array}{ccc|c} 1 & 2 & 2 & 3 \\ 0 & 2 & 4 & 2 \\ 3 & 4 & 6 & -1 \end{array}\right)$$

$R_3 - 3R_1 \rightarrow R_3$

$$\left(\begin{array}{ccc|c} 1 & 2 & 2 & 3 \\ 0 & 2 & 4 & 2 \\ 0 & -2 & 0 & -10 \end{array}\right)$$

$R_2/2 \longrightarrow R_2$

$$\left(\begin{array}{ccc|c} 1 & 2 & 2 & 3 \\ 0 & 1 & 2 & 1 \\ 0 & -2 & 0 & -10 \end{array}\right)$$

$R_1 + 2R_3 \longrightarrow R_1$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 2 \end{array}\right) \Rightarrow \left.\begin{array}{l} x = -3 \\ y = 5 \\ z = -2 \end{array}\right] \rightarrow \text{Ans.}$$

2) (i) $A^{-1} = ?$, $A = \begin{pmatrix} -4 & -2 \\ 5 & 5 \end{pmatrix}$

Sol^n:

Augment A with I_n, $n = 2$

$$\left[\begin{array}{cc|cc} -4 & -2 & 1 & 0 \\ 5 & 5 & 0 & 1 \end{array}\right]$$

$R_1/-4 \rightarrow R_1$

$$\left[\begin{array}{cc|cc} 1 & 0.5 & -0.25 & 0 \\ 5 & 5 & 0 & 1 \end{array}\right]$$

$R_2 - 5R_1 \rightarrow R_2$

$$\left[\begin{array}{cc|cc} 1 & 0.5 & -0.25 & 0 \\ 0 & 2.5 & 1.25 & 1 \end{array}\right]$$

$R_2/2.5 \rightarrow R_2$

$$\left[\begin{array}{cc|cc} 1 & 0.5 & -0.25 & 0 \\ 0 & 1 & 0.5 & 0.4 \end{array}\right]$$

$R_1 - 0.5R_2 \rightarrow R_1$

$$\left[\begin{array}{cc|cc} 1 & 0 & -0.5 & -0.2 \\ 0 & 1 & 0.5 & 0.4 \end{array}\right]$$

This is reduced into.

$$[\, I_2 \mid B \,]$$

where $B = A^{-1}$

$$\therefore \boxed{A^{-1} = \begin{bmatrix} -0.5 & -0.2 \\ 0.5 & 0.4 \end{bmatrix}}$$ Ans. (i)

2 (ii)

$$\bar{a}_1 = (1, 2, 3)$$
$$\bar{a}_2 = (1, -2, 5)$$
$$\bar{a}_3 = (1, 1, 1)$$

We know that:
Basis can be formed only by linear independent vectors.

$\therefore$

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & -2 & 5 \\ 1 & 1 & 1 \end{bmatrix}$$

$R_2 \to R_2 - R_1$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

$R_3 \to R_3 - R_1$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 2 \\ 0 & -1 & -2 \end{bmatrix}$$

$R_2/2 \to R_2$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & 1 \\ 0 & -1 & 2 \end{bmatrix}$$

$R_3 \to R_3 + R_2$, $R_1 \to R_1 - 2R_2$

$$\begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & -1/2 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_1 \to R_1 - 4R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1/2 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_2 \to R_2 + R_3/2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\therefore$ No. of linearly independent rows = 3

~~the Rank of Matrix~~ = 3.

Ans (ii)

* Since Rank = No. of initial vectors the system of vectors are LI. & form a basis.

3) $x_1 - x_2 = 1$

$-x_1 + 2x_2 - x_3 = 1$

$-x_2 + 2x_3 = 1$

Solⁿ: <u>LU Factorisation</u>

$$L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \qquad U = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

Using <u>Gaussian Elimination</u> Method

$R_2 \leftarrow R_2 - (-R_1)$

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 2 \\ 0 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

on comparing: $l_{21} = -1$

$R_3 \leftarrow R_3 - (-R_2)$

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

$l_{32} = -1$

We get,

$l_{31} = 0$

$u_{11} = 1$

$u_{12} = -1$

$u_{22} = 0$

$u_{13} = 0$

$u_{23} = -2$

$u_{33} = 0$

LU decomposition for A is

$$A = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -2 \\ 0 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

A L U

Now: $AX = B$

$LUX = B$

let $UX = Y$

$\therefore LY = B$

$$\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$y_1 = 1$ $-y_1 + y_2 = 1$, $y_2 + y_3 = 1$

$\Rightarrow y_2 = 2$ $\Rightarrow y_3 = 3.$

$\Rightarrow UX = Y$

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \Rightarrow \begin{aligned} x - y &= 1 \\ y - 2z &= 2 \\ z &= 0 \end{aligned}$$

$\therefore$ $\boxed{\text{Ans: } x = 3,\ y = 2,\ z = 0}$

4) (i) The set P_n of all polynomials in x, with real co-eff. and degree $\le n$, together with 0 polynomial.

$p = a_n x^n + a_{n-1} x^{n-1} + \dots a_1 x^1 + a_0$ be a polynomial $\le n$ and let λ be a real no.

Then $\lambda p = \lambda a_n x^n + \lambda a_{n-1} x^{n-1} + \dots \lambda a_1 x^1 + \lambda a_0$ is also a polynomial of degree $\le n$. Hence $P_n(R)$ is closed under scalar multiplication.

*

Let.

$p = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$

$p' = a'_n x^n + a'_{n-1} x^{n-1} + \dots a'_1 x + a'_0$

Then

$(p+p') = (a_n + a'_n) x^n + (a_{n-1} + a'_{n-1}) x^{n-1} + \dots (a_0 + a'_0)$

is also a polynomial of degree $\le n$.

So $\boxed{P_n \text{ is closed under vector addition}}$

*

Let

$p(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$

$q(x) = b_n x^n + b_{n-1} x^{n-1} + \dots + b_1 x + b_0$

$(p+q)(x) = p(x) + q(x)$

$= (a_n + b_n) x^n + \dots (a_1 + b_1) x + (a_0 + b)$

is also a polynomial of degree $\le n$.

Also $(p+q)(x) = (q+p)(x) \implies \boxed{\text{Commutative}}$

* Similarly

$$(p+q)(x) + w(x) = p(x) + (q+w)(x)$$

$$LHS = \left[(a_n + b_n)x^n + \cdots + (a_0 + b_0)\right] + c_n x^n + c_{n-1} x^{n-1} + \cdots c_0$$

$$= (a_n + b_n + c_n)x^n + \cdots + (a_0 + b_0 + c_0)$$

$$RHS = (a_n x^n + \cdots a_0) + \left[(b_n + c_n)x^n + \cdots (b_0 + c_0)\right]$$

$$= (a_n + b_n + c_n)x^n + \cdots + (a_0 + b_0 + c_0).$$

$$LHS = RHS.$$

$\boxed{\text{Associativity}}$

* $P(x) + -P(x) = 0$

$$a_n x^n + a_{n-1} x^{n-1} + \cdots a_0 + -(a_n x^n + a_{n-1} x^{n-1} + \cdots + a_0)$$

$$= 0.$$

$$\therefore \boxed{LHS = RHS}$$

$\boxed{P(x) + 0 = P(x)}$, $\boxed{1(P(x)) = P(x)}$

multiplicative & Additive inverse exists and is the vector space

Hence $P_n(R)$ is a vector space → Ans

5) $W = \{(a, b, c),\ a^2 + b^2 + c^2 \leq 1\}$

$W = \{(a, b, c),\ a^2 \leq 1 - b^2 - c^2\}$

$= \{(1 - b^2 - c^2), b, c,\ a^2 \leq 1 - b^2 - c^2\}$

<u>basis of W</u>

$= \{b(\cancel{1}-1-0), 1, 0),\ c((1-0-1), 0, 1)\}$

$= \{b(0, 1, 0),\ c(0, 0, 1)\}$

$= (0, 1, 0), (0, 0, 1) \quad \boxed{\in \mathbb{R}^3}$

Hence proved.

5(iii)

$$t^2 + 4t + 3 = a(1+t) + b(1-t) + c(1+t^2)$$

$$\Rightarrow t^2 + 4t + 3 = ct^2 + t(a-b) + (a+b+c)$$

Comparing co-eff.

$c = 1$ —(i)

$4 = a - b$ —(ii)

$a + b + 1 = 3$ ~~(iii)~~

$\Rightarrow a + b = 2$ —(iii)

Adding (ii) & (iii)

$2a = 6$

$\Rightarrow a = 3$

$\therefore b = -1$

$\therefore \boxed{t^2 + 4t + 3 = 3(1+t) - 1(1-t) + 1(1+t^2)}$

Ans