

$$1. a) (\sim P \wedge (\sim Q \wedge R)) \vee ((Q \wedge R) \vee (P \wedge R)) \Leftrightarrow R$$

$$(\sim P \wedge (\sim Q \wedge R)) \vee ((Q \wedge R) \vee (P \wedge R))$$

$$= (\sim P \wedge \sim Q) \wedge R \vee (Q \wedge R) \vee (P \wedge R) \quad \text{Associative}$$

$$= R \wedge (\sim P \wedge \sim Q \vee (Q \wedge R) \vee (P \wedge R))$$

$$= R \wedge (\sim P \wedge \sim Q \vee (Q \vee P) \wedge R) \quad \text{Distributive}$$

$$= R \wedge ((\sim P \vee Q) \wedge (\sim Q \vee Q)) \vee (P \wedge R) \quad \text{Complementarity}$$

$$= R \wedge ((\sim P \vee Q) \wedge T) \vee (P \wedge R) \quad \text{Identity}$$

$$= R \wedge (\sim P \vee Q) \vee (P \wedge R) \quad \text{Identity}$$

$$= R \wedge (\sim P \vee Q \vee P)$$

$$= R \wedge ((\sim P \vee P) \vee Q) \quad \text{Associative}$$

$$= R \wedge (T \vee Q) \quad \text{Identity}$$

$$= R \wedge T \quad \text{Identity}$$

$$= R \quad \text{Identity}$$

$$b) \sim (P \leftrightarrow Q) \Leftrightarrow (P \vee Q) \wedge \sim (P \wedge Q)$$

$$\sim (P \leftrightarrow Q)$$

$$= \sim ((\sim P \wedge \sim Q) \vee (P \wedge Q))$$

$$= \sim (\sim P \wedge \sim Q) \wedge \sim (P \wedge Q) \quad \text{DeMorgan}$$

$$= ((\sim \sim P) \vee (\sim \sim Q)) \wedge \sim (P \wedge Q) \quad \text{DeMorgan}$$

$$= (P \vee Q) \wedge \sim (P \wedge Q) \quad \text{Complementarity.}$$

- 2a) $P \rightarrow ((P \rightarrow Q) \wedge \sim(\sim Q \vee \sim P))$
1. $P \rightarrow ((P \rightarrow Q) \wedge (Q \wedge P))$ DeMorgan's Law
 2. $P \rightarrow ((\sim P \vee Q) \wedge (Q \wedge P))$ $P \rightarrow Q = \sim P \vee Q$
 3. $P \rightarrow ((\sim P \vee Q) \wedge (P \wedge Q))$ Associative
 4. $P \rightarrow ((\sim P \wedge P \wedge Q) \vee (Q \wedge P \wedge Q))$ Distributive
 5. $P \rightarrow ((\sim P \wedge P) \wedge Q) \vee ((Q \wedge Q) \wedge P)$ Associative
 6. $P \rightarrow (F \wedge Q) \vee (Q \wedge P)$ Idempotence, Complementary
 7. $P \rightarrow (F \vee (Q \wedge P))$ Identity
 8. $P \rightarrow (Q \wedge P)$ Identity
 9. $\sim P \vee (P \wedge Q)$ $P \rightarrow Q = \sim P \vee Q$

b) $\sim(P \vee Q) \leftrightarrow (P \wedge Q)$

1. $(\sim(P \vee Q) \wedge (P \wedge Q)) \vee (\sim(\sim(P \vee Q) \wedge \sim(P \vee Q))$ $P \leftrightarrow Q = P \wedge Q \vee \sim P \wedge \sim Q$
2. $(\sim P \wedge \sim Q \wedge (P \wedge Q)) \vee ((P \vee Q) \wedge (\sim P \wedge \sim Q))$ DeMorgan.
3. $(\sim P \wedge P \wedge \sim Q \wedge Q) \vee ((P \vee Q) \wedge (\sim P \wedge \sim Q))$
4. $(F \wedge F) \vee ((P \vee Q) \wedge (\sim P \wedge \sim Q))$ Identity
5. $F \vee ((P \vee Q) \wedge (\sim P \wedge \sim Q))$ Identity
6. $(P \vee Q) \wedge (\sim P \wedge \sim Q)$ Identity
7. $(P \wedge \sim P \wedge \sim Q) \vee (Q \wedge \sim P \wedge \sim Q)$ Distributive

3 a) $P \rightarrow ((P \rightarrow Q) \wedge \sim(\sim Q \vee \sim P))$

1. $P \rightarrow ((P \rightarrow Q) \wedge (Q \wedge P))$

DeMorgan

2. $P \rightarrow ((\sim P \vee Q) \wedge (Q \wedge P))$

$P \rightarrow Q = \sim P \vee Q$

3. $P \rightarrow ((\sim P \wedge Q \wedge P) \vee (Q \wedge Q \wedge P))$

Distributive

4. $P \rightarrow ((F \wedge Q) \vee (Q \wedge P))$

Idempotence, Complementarity

5. $P \rightarrow (Q \vee (Q \wedge P))$

Identity

6. ~~$P \rightarrow \sim P \vee (Q \vee (Q \wedge P))$~~

$P \rightarrow Q = \sim P \vee Q$

~~$\sim P \vee Q$~~

7. $(\sim P \vee Q) \vee (Q \wedge P)$

Associative

8. $(\sim P \vee Q \vee Q) \wedge (\sim P \vee Q \vee P)$

Distributive

9. $(\sim P \vee Q) \wedge (\sim P \vee Q \vee P)$

Idempotence

b) $\sim(P \vee Q) \leftrightarrow (P \wedge Q)$

1. $\sim(P \vee Q) \leftrightarrow (P \wedge Q)$

2. $(\sim(P \vee Q) \wedge (P \wedge Q)) \vee (\sim \sim(P \vee Q) \wedge \sim(P \wedge Q))$

$P \leftrightarrow Q = P \wedge Q \vee P \wedge \sim Q$

3. $(\sim P \wedge \sim Q \wedge P \wedge Q) \vee ((P \vee Q) \wedge (\sim P \vee \sim Q))$

DeMorgan

4. $((\sim P \wedge P) \wedge (\sim Q \wedge Q)) \vee ((P \vee Q) \wedge (\sim P \vee \sim Q))$

Associative

5. $(F \wedge F) \vee ((P \vee Q) \wedge (\sim P \vee \sim Q))$

Complementary

6. $F \vee ((P \vee Q) \wedge (\sim P \vee \sim Q))$

Identity

7. $(P \vee Q) \wedge (\sim P \vee \sim Q)$

Identity

$$4) (P \wedge Q) \vee (\sim P \wedge R) \vee (Q \wedge R)$$

$$5) (P \wedge Q \wedge T) \vee (\sim P \wedge R \wedge T) \vee (Q \wedge R \wedge T)$$

$$6) (P \wedge Q \wedge (R \vee \sim R)) \vee (\sim P \wedge R \wedge (Q \vee \sim Q)) \vee ((P \vee \sim P) \wedge Q \wedge R)$$

$$7) (P \wedge Q \wedge R) \vee (P \wedge Q \wedge \sim R) \vee (\sim P \wedge Q \wedge R) \vee (\sim P \wedge \sim Q \wedge R) \vee$$

$$(P \wedge Q \wedge R) \vee (\sim P \wedge Q \wedge R)$$

$$8) (P \wedge Q \wedge R) \vee (P \wedge Q \wedge \sim R) \vee (\sim P \wedge Q \wedge R) \vee (\sim P \wedge \sim Q \wedge R)$$

$$b) (\sim P \rightarrow R) \wedge (Q \leftrightarrow P)$$

$$\Rightarrow (\sim(\sim P) \vee R) \wedge (\sim P \wedge \sim Q \vee P \wedge Q)$$

$$\Rightarrow (P \vee R) \wedge ((\sim P \wedge \sim Q) \vee (P \wedge Q))$$

$$\Rightarrow ((P \vee R) \wedge (\sim P \wedge \sim Q)) \vee ((P \vee R) \wedge (P \wedge Q))$$

$$\Rightarrow ((P \wedge \sim P \wedge \sim Q) \vee (R \wedge \sim P \wedge \sim Q)) \vee ((P \wedge P \wedge Q) \vee (R \wedge P \wedge Q))$$

$$\Rightarrow ((F \wedge \sim Q) \vee (R \wedge \sim P \wedge \sim Q)) \vee (P \wedge Q) \vee (P \wedge R \wedge Q)$$

$$\Rightarrow F \vee (R \wedge \sim P \wedge \sim Q) \vee (P \wedge Q \wedge T) \vee (P \wedge R \wedge Q)$$

$$\Rightarrow (R \wedge \sim P \wedge \sim Q) \vee (P \wedge Q \wedge (R \vee \sim R)) \vee (P \wedge R \wedge Q)$$

$$\Rightarrow (R \wedge \sim P \wedge \sim Q) \vee (P \wedge Q \wedge K) \vee (P \wedge Q \wedge \sim K) \vee (P \wedge R \wedge Q)$$

$$\Rightarrow (P \wedge \sim R \wedge Q) \vee (P \wedge R \wedge Q) \vee (\sim P \wedge R \wedge \sim Q)$$

$$5a) (\sim P \rightarrow R) \wedge (Q \leftrightarrow P)$$

0 0

$$\Rightarrow (\sim(\sim P) \vee R) \wedge (\sim Q \wedge \sim P) \vee (P \wedge Q)$$

$$\Rightarrow (P \vee R) \wedge (\sim Q \vee (P \wedge Q)) \wedge (\sim P \vee (P \wedge Q))$$

$$\Rightarrow (P \vee F \vee R) \wedge (\sim Q \vee P) \wedge (\sim Q \vee Q) \wedge (\sim P \vee P) \wedge (\sim P \vee Q)$$

$$\Rightarrow (P \vee (Q \wedge \sim Q) \vee R) \wedge (P \vee \sim Q \vee F) \wedge T \wedge T \wedge (\sim P \vee Q \vee F)$$

$$\Rightarrow (P \vee Q \vee R) \wedge (P \vee \sim Q \vee R) \wedge (P \vee \sim Q \vee (R \wedge \sim R)) \wedge (\sim P \vee Q \vee F)$$

$$\Rightarrow (P \vee Q \vee R) \wedge (P \vee \sim Q \vee R) \wedge (P \vee \sim Q \vee R) \wedge (P \vee \sim Q \vee \sim R) \wedge (\sim P \vee \sim Q \vee R) \wedge (\sim P \vee \sim Q \vee \sim R)$$

$$\Rightarrow (P \vee Q \vee R) \wedge (P \vee \sim Q \vee R) \wedge (P \vee \sim Q \vee \sim R) \wedge (\sim P \vee \sim Q \vee R) \wedge (\sim P \vee \sim Q \vee \sim R)$$

$$b) Q \wedge (P \vee \sim Q)$$

$$\Rightarrow (Q) \wedge (P \vee \sim Q)$$

$$- \Rightarrow (P \vee Q) \wedge (P \vee \sim Q)$$

$$\Rightarrow ((P \wedge \sim P) \vee Q) \wedge (P \vee \sim Q)$$

$$\Rightarrow (P \vee Q) \wedge (\sim P \vee Q) \wedge (P \vee \sim Q)$$

$$6a) (Q \wedge \sim R \wedge \sim S) \vee (R \wedge S)$$

$$\Rightarrow (Q \wedge \sim R \wedge \sim S) \vee (T \wedge R \wedge S)$$

$$\Rightarrow (Q \wedge \sim R \wedge \sim S) \vee ((Q \vee \sim Q) \wedge R \wedge S)$$

$$\Rightarrow (Q \wedge \sim R \wedge \sim S) \vee (Q \wedge R \wedge S) \vee (\sim Q \wedge R \wedge S) \rightarrow \boxed{\text{PDNF}}$$

$$(\text{say}) A \Leftrightarrow (Q \wedge \sim R \wedge \sim S) \vee (Q \wedge R \wedge S) \vee (\sim Q \wedge R \wedge S)$$

$$\sim A \Leftrightarrow (\sim Q \wedge \sim R \wedge \sim S) \vee (\sim Q \wedge \sim R \wedge S) \vee (Q \wedge R \wedge \sim S) \vee$$

$$(\sim Q \wedge \sim R \wedge S) \vee (\sim Q \wedge R \wedge \sim S)$$

$$\sim \sim A \Leftrightarrow \sim(\sim Q \wedge \sim R \wedge \sim S) \wedge \sim(\sim Q \wedge \sim R \wedge S) \wedge \sim(Q \wedge R \wedge \sim S) \wedge$$

$$\sim(\sim Q \wedge \sim R \wedge S) \wedge \sim(\sim Q \wedge R \wedge \sim S)$$

$$A \Leftrightarrow (Q \vee R \vee S) \wedge (\sim Q \vee R \vee \sim S) \wedge (\sim Q \vee \sim R \vee S) \wedge$$

$$(Q \vee R \vee \sim S) \wedge (Q \vee \sim R \vee S) \rightarrow \boxed{\text{PCNF}}$$

$$6b) (P \leftrightarrow Q) \leftrightarrow (P \rightarrow Q) \wedge (\sim Q \vee P)$$

$$\Rightarrow (P \leftrightarrow Q) \leftrightarrow ((\sim P \vee Q) \wedge (\sim Q \vee P))$$

$$\Rightarrow \underbrace{((\sim P \wedge \sim Q) \vee (P \wedge Q))}_A \leftrightarrow \underbrace{((\sim P \vee Q) \wedge (\sim Q \vee P))}_B$$

$$\Rightarrow \underbrace{(A \wedge B)}_{F_1} \vee \underbrace{(\sim A \wedge \sim B)}_{F_2}$$

$$\begin{aligned} F_1 &= ((\sim P \wedge \sim Q) \vee (P \wedge Q)) \wedge ((\sim P \vee Q) \wedge (\sim Q \vee P)) \\ &= ((\sim P \vee (P \wedge Q)) \wedge (\sim Q \vee (P \wedge Q))) \wedge ((\sim P \vee Q) \wedge (\sim Q \vee P)) \\ &= ((\sim P \vee P) \wedge (\sim P \vee Q)) \wedge ((\sim Q \vee P) \wedge (\sim Q \vee Q)) \wedge ((\sim P \vee Q) \wedge (\sim Q \vee P)) \\ &= (\sim P \vee Q) \wedge \cancel{(\sim P \vee P)} \wedge (\sim Q \vee P) \wedge \cancel{(\sim Q \vee Q)} \wedge (\sim P \vee Q) \wedge (\sim Q \vee P) \\ &= (\sim P \vee Q) \wedge (\sim Q \vee P) \end{aligned}$$

$$\begin{aligned} F_2 &= \sim ((\sim P \wedge \sim Q) \vee (P \wedge Q)) \wedge \sim ((\sim P \vee Q) \wedge (\sim Q \vee P)) \\ &= (\sim (\sim P \wedge \sim Q) \wedge \sim (P \wedge Q)) \wedge (\sim (\sim P \vee Q) \wedge \sim (\sim Q \vee P)) \\ &= (P \vee Q) \wedge (\sim P \vee \sim Q) \wedge (P \wedge \sim Q) \wedge (Q \wedge \sim P) \\ &= (P \vee Q) \wedge (\sim P \vee \sim Q) \wedge (P \wedge \sim P) \wedge (Q \wedge \sim Q) \\ &= (P \vee Q) \wedge (\sim P \vee \sim Q) \wedge F \wedge F \\ &= F \end{aligned}$$

$$\Rightarrow (\sim P \vee Q) \wedge (\sim Q \vee P) \vee F$$

$$\Rightarrow (\sim P \vee Q) \wedge (\sim Q \vee P) \rightarrow \boxed{PCNF}$$

$$A \Leftrightarrow (\sim P \vee Q) \wedge (\sim Q \vee P)$$

$$\sim A \Leftrightarrow (P \vee Q) \wedge (\sim P \vee \sim Q)$$

$$\sim \sim A \Leftrightarrow \sim (P \vee Q) \vee \sim (\sim P \vee \sim Q)$$

$$\sim \sim A \Leftrightarrow (\sim P \wedge \sim Q) \vee (P \wedge Q) \rightarrow \boxed{PDNF}$$

7a) $P \rightarrow Q, Q \rightarrow R, R \rightarrow S \Rightarrow P \rightarrow S$

Additional premise is P

1. $P \rightarrow Q$ Rule P

2. P Additional Premise

3. Q Modus Ponens (1, 2)

4. $Q \rightarrow R$ Rule P

5. R modus Ponens (3, 4)

6. $R \rightarrow S$ Rule P

7. S modus Ponens (5, 6)

8. $P \rightarrow S$ Rule CP.

7b) $P \vee Q, P \rightarrow R, Q \rightarrow S \Rightarrow \sim S \rightarrow R.$

Additional Premise $\sim S$	
1. $P \vee Q$	Rule P
2. $\sim P \rightarrow Q$	$P \rightarrow Q \Leftrightarrow \sim P \vee Q$
3. $Q \rightarrow S$	Rule P
4. $\sim P \rightarrow S$	Hypothetical Syllogism (2,3)
5. $\sim S$	Additional Premise.
6. P	Modus Tollens (4,5)
7. $P \rightarrow R$	Rule P
8. R	Modus Ponens (6,7)
9. $\sim S \rightarrow R$	Rule CP

Hence Proved.

- 8)
- C : misses many classes through illness
 - S : fails in high school
 - B : reads a lot of books
 - E : uneducated

Premises

- (i) $C \rightarrow S$
- (ii) $S \rightarrow E$
- (iii) $B \rightarrow \sim E$
- (iv) $C \wedge B$

1. $C \rightarrow S$ Rule P, bottom bar
2. $S \rightarrow E$ Rule P
3. $C \rightarrow E$ Hypothetical Syllogism (1,2)
4. $B \rightarrow \sim E$ Rule P
5. $C \wedge B$ Rule P
6. B simplification 5
7. $\sim E$ Modus Ponens (4,6)
8. $\sim C$ Modus Tollens (3,7)
9. $C \wedge \sim C = F$ ~~Prop~~ simplification
 \therefore Inconsistent.

9) Premises :- $A \rightarrow (B \vee C)$

$B \rightarrow \sim A$

$D \rightarrow \sim C$

Conc :- $A \rightarrow \sim D$.

where : A : A works hard

B : enjoys himself

C : " "

D : " "

A to be additional premise.

1. A additional premise

2. $A \rightarrow (B \vee C)$ Rule P

3. $B \vee C$ Modus Ponens (1, 2)

4. $\sim C$ Additional Rule 3.

5. $D \rightarrow \sim C$ Rule P

6. $\sim D$ Modus Tollens (4, 5)

7. $A \rightarrow \sim D$ Rule CP.

Argument has valid conclusion.

$$10a) E \rightarrow S, S \rightarrow H, A \rightarrow \sim H \Rightarrow \sim(E \wedge A)$$

Indirect
Method

Addition Premise be $\sim(\sim(E \wedge A)) \Rightarrow E \wedge A$

1. $E \rightarrow S$ Rule P
2. $S \rightarrow H$ Rule P
3. $E \rightarrow H$ Hypothetical Syllogical (1,2)
4. $E \wedge A$ Additional Premise
5. E Simplification (4)
6. H Modus Ponens (3,5)
7. $A \rightarrow \sim H$ Rule P
8. $\sim A$ Modus Tollens (6,7)
9. A Simplification (4)
10. $A \wedge \sim A \Rightarrow F$

\therefore the initial assumption is invalid
 \therefore the statement is hence proved.

$$b) P \rightarrow (Q \wedge R), (Q \vee S) \rightarrow U, (P \vee S) \Rightarrow U$$

Additional premise be $\sim U$

1. $(Q \vee S) \rightarrow U$ Rule P
2. $\sim U$ Additional Premise
3. $\sim(Q \vee S)$ Modus Tollens (1,2)
4. $P \vee S$ Rule P
5. $\sim P \rightarrow S$ $P \rightarrow S \Rightarrow \sim P \vee S$
6. $\sim Q \wedge \sim S$ Rule T, (3)
7. $\sim S$ Simplification (6)
8. P Modus Tollens (5,7)

$$9. P \rightarrow (Q \wedge R)$$

Rule 9

$$10. Q \wedge R$$

modus ponens (2, 9)

$$11. Q$$

Simplification (10)

$$12. \sim Q$$

(6)

$$13. Q \wedge \sim Q \Rightarrow F$$

↳ Invalid

hence proved.