Homework 10

Due: Friday, Nov 21, 11:59 PM PT

1. The Sushi Express wants to implement a new feature, "order batching." The menu consists of a set of desired food items *F* and a set of combos $c_1, \ldots, c_n$ where each combo is a set of food items along with a price - we represent combo $c_i = (\{f_i^1, f_i^2, \ldots, f_i^k\}, p_i)$, where each f_i^j is a food item in the combo. If a student orders c_i, they receive each of the food items, and they pay the cost p_i. In the new order batching system, the students would input a list of food items that they want, and the order batching system should find the **cheapest** set of combos, which would include **at least one of each desired food** item. Write the decision version of this problem, and show via reduction that this decision problem is NP Hard. (20 points)

   Example:
   Foods F = {Fries, Burger, Pizza, drink, sandwich, cookie, chips}
   C_1 = ({Fries, Pizza, cookie}, 10$)
   C_2 = ({Fries, drink, sandwich}, 15$)
   C_3 = ({drink, Pizza, chips, burger}, 17$)

   Input order: {Fries, drink, pizza}   output: {C_1, C_2} (for 25$) is the cheapest
   Input order: {burger}   output: {C_3} is the cheapest

2. Problem CLIQUE(G,k) asks whether, given a graph G, does G contain a k-clique? A k-clique is defined to be a set of k vertices such that each pair of these vertices shares an edge. Show via reduction that CLIQUE is NP-complete (20 points)

3. Recall the 3-SAT problem: Given a 3-CNF input formula, it tries to find an assignment of variables that satisfies ALL given clauses. Now, we consider a variation - the **partial** satisfiability problem, denoted as 3-Sat(α), for a specified constant α. Here, we are given a collection of *k* clauses, each of which contains exactly three literals, and we are asked to determine whether there is an assignment of true/false values to the literals such that at least αk clauses will be true. Note that for α=1, we have the problem 3-Sat(1) which is exactly the 3-SAT problem as per definition. We want to analyze the problem corresponding to a smaller α, in particular, $\alpha=\frac{15}{16}$ which gives us the problem 3-Sat($\frac{15}{16}$).

   Show that the problem 3-Sat($\frac{15}{16}$) is NP-complete. (20 points)

   Hint: If *x*, *y*, and *z* are variables, there are eight possible clauses containing them: $(x \lor y \lor z)$,$(!x \lor y \lor z)$,$(x \lor !y \lor z)$,$(x \lor y \lor !z)$,$(!x \lor !y \lor z)$,$(!x \lor$

$y \vee !z), (x \vee !y \vee !z), (!x \vee !y \vee !z)$

Ungraded Problems:

1. Given a graph G = (V, E) and two integers k, m, the Dense Subgraph Problem is to find a subgraph G′ = (V′, E′) of G, such that V′ has at most k vertices and E′ has at least m edges. Prove that the Dense Subgraph Problem is NP-Complete. HINT: Use Independent Set for your reduction. (20 points)

2. DOUBLECLIQUE(G, k) asks whether, given a graph G, does G contain 2 vertex-disjoint cliques of size at least k? Show via reduction that DOUBLECLIQUE is NP-Hard. HINT: Use CLIQUE for your reduction. (20 points)

3. DOMINATINGSET(G,k) asks whether given a graph G, is there a set S of at most k vertices such that each vertex in G is either in S or adjacent to at least one vertex in S. Show that DOMINATINGSET is NP-Hard. HINT: Use VERTEXCOVER. (20 points)

4. DELIVERY(S,k,d,m) asks whether, given a set S of major cities, and an arbitrary distance function $d : S \times S \rightarrow R+$, can we find a subset of these major cities of size at most k where we can place distribution points in order to deliver to every city in S in such a way that the distribution center is at most distance m from the destination? Show via reduction that DELIVERY is NP-Hard. NOTE: The distance function does not need to satisfy properties one would ordinarily expect distance functions to satisfy, such as the triangle inequality. HINT: Use a reduction from DOMINATINGSET. (20 points)