Homework 11

Due: Friday, Nov 28, 11:59 PM PT

1) Given an undirected graph G = (V, E), and positive integer k, the max-degree-spanning-tree (MDST) problem asks whether G has a spanning tree whose degree is at most k. The degree of a spanning tree T is defined as the maximum number of neighbors a node has within the tree (i.e., a node may have many edges incident on it in G, but only some of them get included in T). Show that the max-degree-spanning-tree (MDST) problem
   a) is in NP. (4 pts)
   b) is NP-hard. (16 pts)

2) The k-cycle-decomposition problem (for any $k > 1$) is as follows: The input consists of a connected graph G=(V, E) and k positive integers $a_1, \ldots, a_k < |V|$. The goal is to determine if there exist k disjoint cycles of sizes $a_1, \ldots, a_k$ respectively, s.t., each node in V is contained in exactly one cycle. Show that the k-cycle-decomposition problem (for any $k > 1$)
   a) is in NP. (4 pts)
   b) is NP-hard. (16 pts)

3) Given a graph G = (V, E) with an even number of vertices as the input, the HALF-IS problem is to decide if G has an independent set of size $|V| / 2$. Prove that HALF-IS is
   a) is in NP. (4 pts)
   b) is NP-hard. (16 pts)

Ungraded Problems

4) In a certain town, there are many clubs, and every adult belongs to at least one club. The town's people would like to simplify their social life by disbanding as many clubs as possible, but they want to make sure that afterwards everyone will still belong to at least one club. Formally, the Redundant Clubs problem has the following input and output.

INPUT: List P of people; list C of clubs; lists P_i of members of each club i; and number K.

OUTPUT: Yes if there exist a set of K clubs such that, after disbanding all clubs in this set, each person still belongs to at least one club. No otherwise.

Prove that the Redundant Clubs problem

a) is in NP. (4 pts)
b) is NP-hard. (16 pts)

5) You are given a directed graph G=(V,E) with weights on its edges $e \in E$. The weights can be negative or positive. The Zero-Weight-Cycle Problem is to decide if there is a simple cycle in G so that the sum of the edge weights on this cycle is exactly 0. Prove that the Zero-Weight-Cycle problem
a) is in NP. (4 pts)
b) is NP-hard. (16 pts)

6) Suppose we have a variation on the 3-SAT problem called Min-3-SAT, where the literals are never negated. Of course, in this case it is possible to satisfy all clauses by simply setting all literals to true. But, we are additionally given a number k, and are asked to determine if we can satisfy all clauses while setting at most k literals to be true. Prove that Min-3-SAT
a) is in NP. (4 pts)
b) is NP-hard. (16 pts)

7) There are n courses at USC, each of them requires multiple disjoint time intervals. For example, a course may require the time from 9am to 11am and 2pm to 3pm and 4pm to 5pm (you can assume the number of intervals of a course is at least 1, at most n). You cannot choose any two overlapping courses. You want to know, given a number K, if it's possible to take at least K courses. Prove that the Course Choosing problem
a) is in NP. (4 pts)
b) is NP-hard. (16 pts)