Fundamentals of Aerodynamics

Sora Sato (14th Gen. Aero Lead)*

August 29, 2025

GitHub: https://github.com/kuma003 (Univ. account)
https://github.com/kuma-1220 (Personal account)

Contents

Chapter 1

Introduction

1.1 Terminology

This chapter summarizes fundamental terms. Details and derivations are provided in later chapters; refer to them as needed.

1.1.1 Fluid Mechanics

Fluid mechanics is the field that investigates the mechanical behavior of liquids and gases (collectively referred to as **fluids**).

Fluid flows are generally classified as **laminar flow** and **turbulent flow**. Laminar flow is characterized by smooth, orderly motion, while turbulent flow exhibits chaotic, irregular motion. More precisely, the flow regime is determined by the **Reynolds number**, a dimensionless parameter defined as the ratio of inertial to viscous forces:

Def. 1.1 Reynolds Number

The Reynolds number is defined as:

$$Re \equiv \frac{\text{inertial force}}{\text{viscous force}} = \frac{\rho U L}{\mu} = \frac{U L}{\nu} \tag{1.1}$$

where ρ is the fluid density, U is a characteristic velocity, L is a characteristic length, μ is the dynamic viscosity, and ν is the kinematic viscosity. The choice of U and L depends on the specific problem; these values are for reference.

For small Re, viscous forces dominate and the flow remains laminar; for large Re, inertial forces dominate and the flow tends to become turbulent. As a rule of thumb, pipe flow is laminar for $Re < 2000$ and turbulent for $Re > 4000$.

We now turn to the governing equations. For steady, incompressible flow, the motion is described by **Bernoulli's theorem**:

Def. 1.2 Bernoulli's Theorem

In steady, incompressible flow, the following forms are equivalent along a streamline:

$$\begin{aligned} \text{Pressure form}: &\quad p + \frac{1}{2}\rho U^2 + \rho g h = \text{const.} \\ \text{Energy form}: &\quad \frac{p}{\rho} + \frac{1}{2}U^2 + gh = \text{const.} \\ \text{Head form}: &\quad \frac{p}{\rho g} + \frac{U^2}{2g} + h = \text{const.} \end{aligned} \tag{1.2}$$

Here, p is the static pressure, ρ is the fluid density, U is the flow speed, g is the gravitational acceleration, and h is the elevation above a reference level. In the first equation, p is the **static pressure**, $\frac{1}{2}\rho U^2$ is the **dynamic pressure**. In the third, $\frac{p}{\rho g}$ is the **pressure head**, $\frac{U^2}{2g}$ is the **velocity head**, and h is the **elevation head**.

For a body placed in a flow, the velocity at the surface becomes zero (a **stagnation point**). According to Bernoulli's theorem, dynamic pressure is converted to static pressure, so the body experiences a pressure rise of approximately $\frac{1}{2}\rho U^2$, known as the **stagnation pressure**.

For more general cases (unsteady, compressible, or viscous flows), the motion is governed by the **Navier–Stokes equations**:

Def. 1.3 Navier–Stokes Equations

The Navier–Stokes equations, derived from the conservation of mass, momentum, and energy, are given by:

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\frac{1}{\rho}\nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{g} \tag{1.3}$$

where $\mathbf{u}$ is the velocity vector, p is the pressure, ρ is the density, ν is the kinematic viscosity, and $\mathbf{g}$ represents body forces.

Analytical solutions are rarely available due to the nonlinearity of these equations; numerical (computational) methods are standard practice.

1.1.2 Aerodynamics

A vehicle (hereafter, "body") moving through air is subject to aerodynamic forces generated by the surrounding flow. The study of the origins and effects of these forces is called **aerodynamics**. Here, we introduce basic terminology; detailed derivations are provided in later sections.

The principal aerodynamic forces acting on a body are **drag**, which acts parallel to the freestream, and **lift**, which acts perpendicular to it. These can also be decomposed into the body axis (**axial force**) and its normal (**normal force**).

As previously noted, the surface of a body in a flow is subject to stagnation pressure $\frac{1}{2}\rho U^2$. Integrating this pressure over the surface yields the net aerodynamic force. In practice, the force magnitude is typically proportional to the dynamic pressure $\frac{1}{2}\rho U^2$. *

To compare forces across different conditions, it is standard to nondimensionalize them using dynamic pressure and a reference area:

Def. 1.4 Force Coefficient

The **drag coefficient** C_D and **normal force coefficient** C_N are defined as:

$$C_D = \frac{D}{\frac{1}{2}\rho U^2 A}, \quad C_N = \frac{N}{\frac{1}{2}\rho U^2 A} \tag{1.4}$$

where D is drag, N is normal force, ρ is density, U is velocity, and A is a reference area. The same form applies to axial and lift coefficients. A is typically the projected frontal area, or for wings, the planform area.

*Strictly, this is an approximation. According to Bernoulli's theorem, the surface integral of pressure is zero for an inviscid, incompressible, steady flow. This is known as **d'Alembert's paradox**. Resolution requires accounting for viscosity and/or compressibility via the Navier–Stokes equations.

Lift and normal force depend on the body's orientation, especially the **angle of attack** α. For small α, C_N is approximately linear in α:

Def. 1.5 Normal Force Slope

The **normal force coefficient slope** is defined as:

$$C_{N\alpha} = \frac{C_N}{\alpha} \tag{1.5}$$

where α is in radians (dimensionless). Thus, $C_{N\alpha}$ is also dimensionless. For small α, $C_N \approx C_{N\alpha}\,\alpha$.

Moments are similarly nondimensionalized:

Def. 1.6 Moment Coefficient

The moment coefficient is defined as:

$$C_M = \frac{M}{\frac{1}{2}\rho U^2 A L} \tag{1.6}$$

where M is moment and L is a reference length.

In rockets, moments arise from fin misalignment, engine thrust, and especially from the distribution of normal force. The point of application of the resultant normal force is called the **center of pressure** (CP). The relationship between the center of pressure and the center of gravity (CG) determines the pitching moment.

Def. 1.7 Center of Pressure

The center of pressure is defined as:

$$C_{\mathrm{P}} = \frac{\int N(x) x \,\mathrm{d}x}{\int N(x) \,\mathrm{d}x} \tag{1.7}$$

where the integral is along the body axis and $N(x)$ is the normal force distribution. For discrete components i (nose, body, fins, etc.):

$$C_{\mathrm{P}} = \frac{\sum_i N_i C_{\mathrm{P}i}}{\sum_i N_i} \tag{1.8}$$

Rem. 1.1 Center of Gravity

The center of gravity is defined as:

$$C_{\mathrm{G}} = \frac{\int \lambda(x) x \, \mathrm{d}x}{\int \lambda(x) \, \mathrm{d}x} \tag{1.9}$$

where $\lambda(x)$ is the linear mass density.

The **stability margin** is a key parameter for rocket stability:

Def. 1.8 Stability Margin

The stability margin is defined as:

$$\text{Stability Margin} = \frac{C_{\mathrm{P}} - C_{\mathrm{G}}}{L} \tag{1.10}$$

where L is the total length of the vehicle. Both C_{P} and C_{G} are measured from the nose tip.

For static stability, the center of pressure must be aft of the center of gravity (positive margin). Excessive margin increases the *weathercock effect*, making the rocket overly sensitive to crosswinds. A margin of about 10–20% is often recommended.

A moment that opposes angular motion is called a **damping moment**. All previous discussions assumed static effects; damping moments are dynamic and depend on the vehicle's motion history. *

The general form of the damping moment coefficient, as well as typical pitch/yaw and roll damping coefficients for rockets, are as follows:

Def. 1.9 Damping Moment Coefficient

In general, the damping moment coefficient is defined as:

$$C_{m\dot{\theta}} = \frac{4}{\rho v^2 S L} \frac{v}{L} \frac{\partial M}{\partial \dot{\theta}} \tag{1.11}$$

For rockets, the pitch/yaw damping coefficient C_{mq} and roll damping

*This arises from the nonlinearity of the Navier–Stokes equations.

coefficient C_{mp} are:

$$C_{mq} = -4 \sum_i \left(\frac{C_{n\alpha i}}{2} \right) \left(\frac{C_{pi} - C_g}{L} \right)^2, \tag{1.12}$$

$$C_{mp} = -8 \times \frac{(\text{span} + d/2)^4}{\pi L^2 \left(\frac{\pi d^2}{4} \right)}. \tag{1.13}$$

Nonlinear effects can also cause aeroelastic instabilities such as **flutter**, in which the wings oscillate. Above a certain speed, this oscillation can grow and ultimately lead to structural failure.

1.1.3 Numerical Analysis

Chapter 2

Fluid Mechanics

2.1 Navier–Stokes Equations

2.1.1 Sketch Derivation

2.1.2 Bernoulli's Theorem

2.1.3 Boundary Layer and Reynolds Number

2.2 Potential Flow

Chapter 3

Aerodynamics

3.1 Potential Flow, Drag and Lift

3.1.1 Drag and d'Alembert's Paradox

3.1.2 Circulation and Lift

3.2 Barrowman Method

3.2.1 Normal Force

3.2.2 Center of Pressure

3.3 Supersonic Aerodynamics

3.3.1 Speed of Sound and Mach Number

3.3.2 Prandtl–Glauert Transformation

3.3.3 Choked Flow

Chapter 4

Numerical Analysis

4.1 Basics of CFD

4.1.1 Numerical Schemes

4.2 Flight Simulation Fundamentals

4.2.1 Equations of Motion

Appendix A

Mathematical Supplements

A.1 Quaternions

Appendix B

Basics of Strength of Materials

Appendix C

Basics of Homemade Engines

C.1 Hybrid Engine Combustion Theory

C.2 Flow Coefficient

C.3 Nozzle Theory