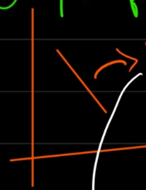
Support Vector classifier indepth Maths

① Eqn of line, plane, hyperplane.



$y = mx + c$

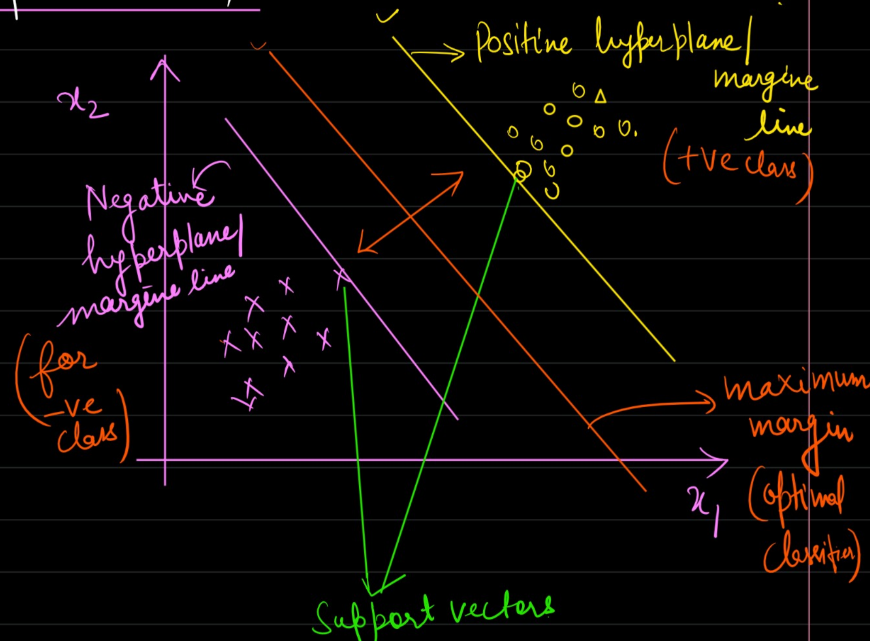
$y = \theta_0 + \theta_1 x_1$

$ax + by + c = 0$

$by = -ax - c$

$y = \boxed{\frac{-a}{b}} x - \boxed{\frac{c}{b}}$

$y = \underset{m}{m}x + \underline{c}$



in more than 3d

$y = \theta_0 + \theta_1 x_1$

$y = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_3 + \cdots \theta_n x_n \longrightarrow$ hyperplane

$y = b + w_1 x_1 + w_2 x_2 + w_3 x_3 + \cdots w_n x_n$

b - bias
w - weights

$y = b + w_1 x_1 + w_2 x_2 + w_3 x_3 + w_4 x_4$

$W = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix} \qquad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$

$W^T = \begin{bmatrix} w_1 & w_2 & w_3 & w_4 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \Rightarrow w_1 x_1 + w_2 x_2 + w_3 x_3 + w_4 x_4 + b$

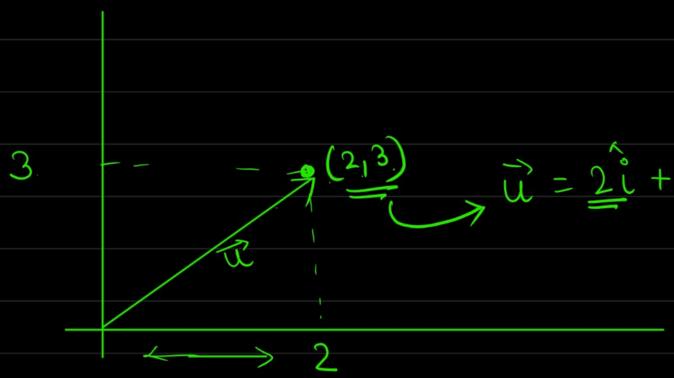
$W^T \qquad X$

$$\boxed{y = W^T x + b} \quad \left(y = mx + c \atop ax + by + c = 0 \right)$$

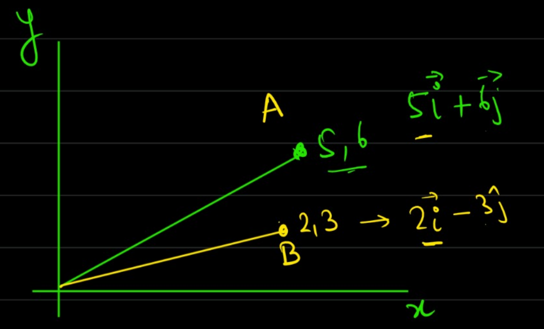
$$\Downarrow$$

$$W^T x + b = 0$$

(2)



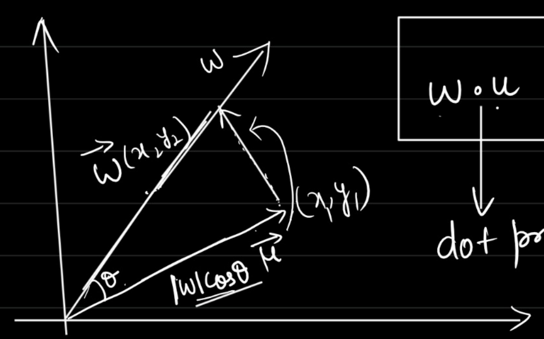
$\vec{u} = 2\hat{i} + 3\hat{j}$



(3) Vector Subtraction $\vec{A} - \vec{B}$

$$(5-2)\vec{i} + (6-3)\vec{j} = 3\vec{i} + 3\vec{j}$$

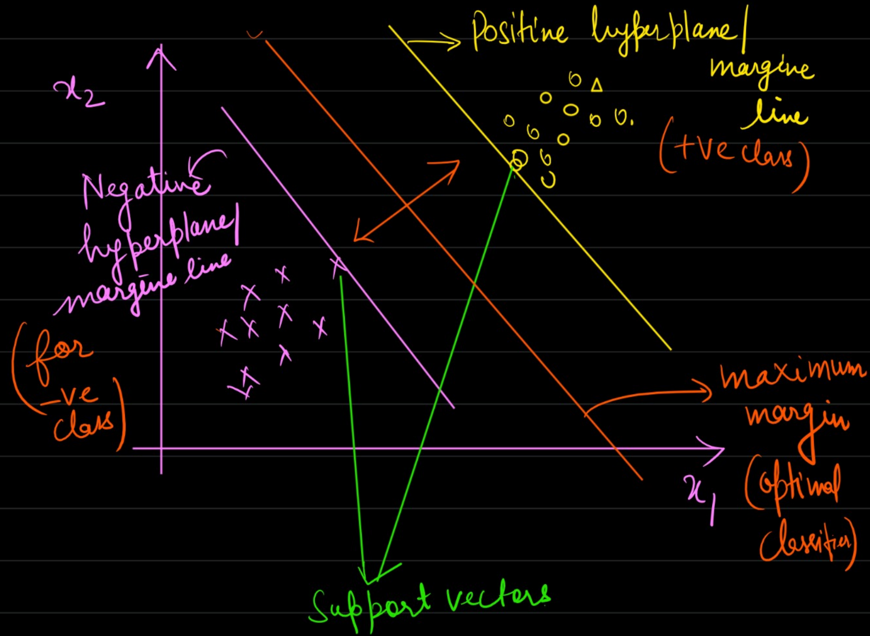
(4) Dot product of vectors.



magnitude of w, magnitude of μ

$$\boxed{w \cdot u = |w| \cos\theta \cdot |\mu|}$$

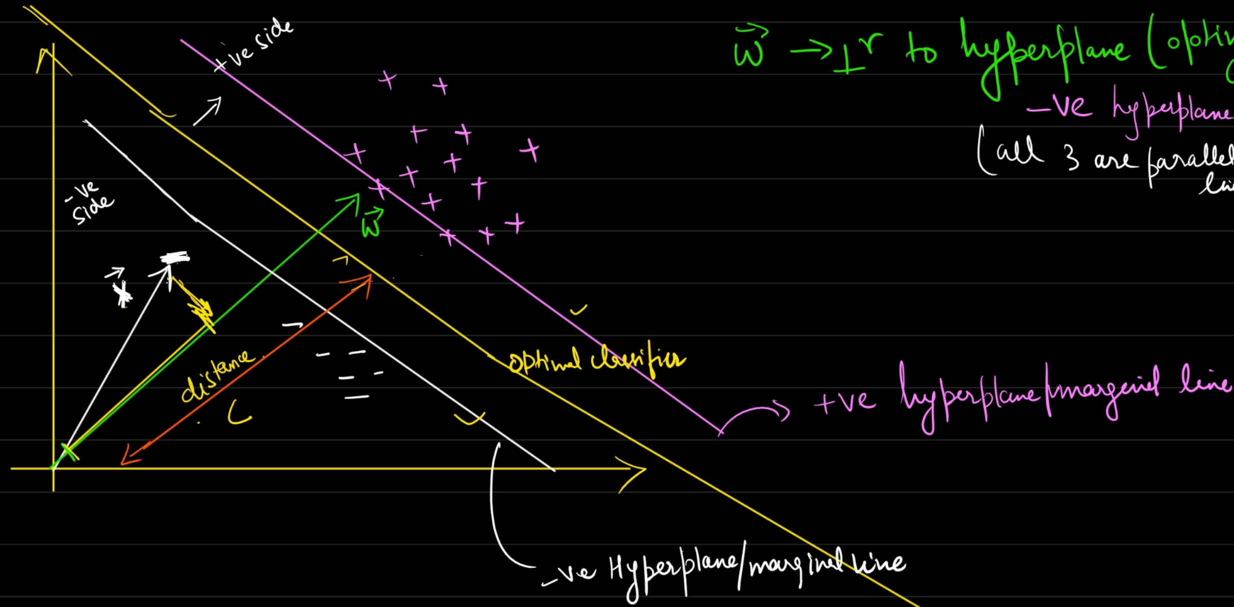
dot product means projection of $\vec{u}$ on $\vec{w}$



$\vec{w} \rightarrow \perp^r$ to hyperplane (optimal classifier)

-ve hyperplane / +ve hyperplane

(all 3 are parallel lines)

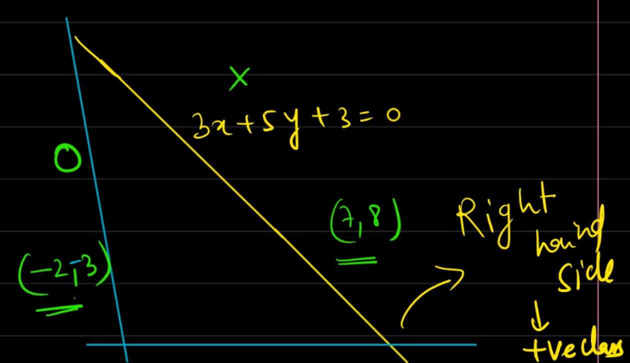


Project is dot product.

$\vec{X} \cdot \vec{w} = (c)$distance (the point lies on decision boundary)

$\vec{X} \cdot \vec{w} > (c)$distance (positive class)

$\vec{X} \cdot \vec{w} < (c)$distance (negative class)



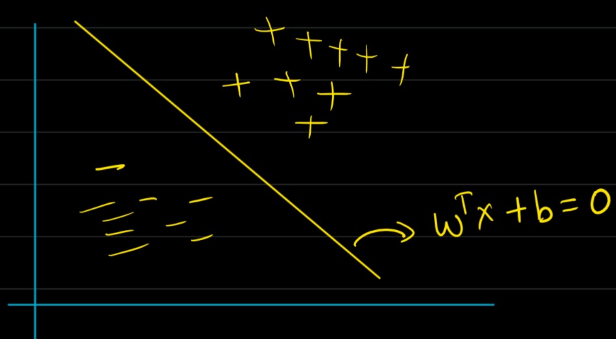
$(7,8) \Rightarrow 3\times7 + 5\times8 + 3 > 0$

$(-2,-3) \Rightarrow 3\times-2 + 5\times-3 + 3 < 0$

↓

-ve side (left side)

-ve class.



$\vec{X} \cdot \vec{w} \geq C$

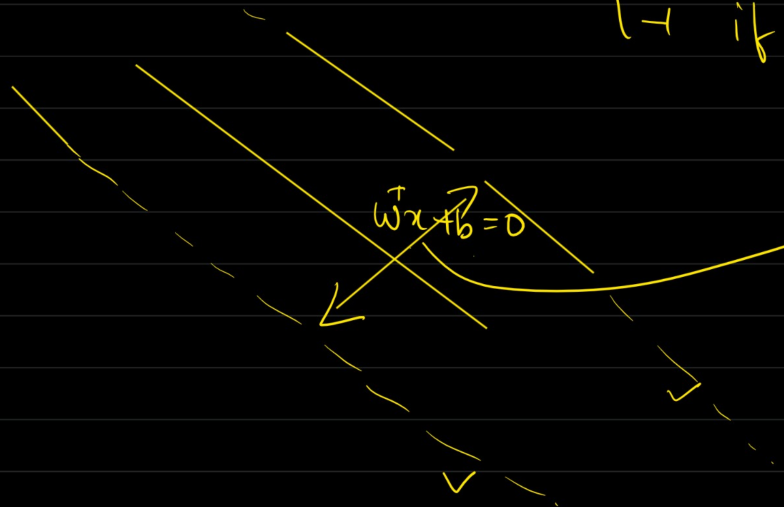
$\vec{X} \cdot \vec{w} - C \geq 0$

$-C$ as b

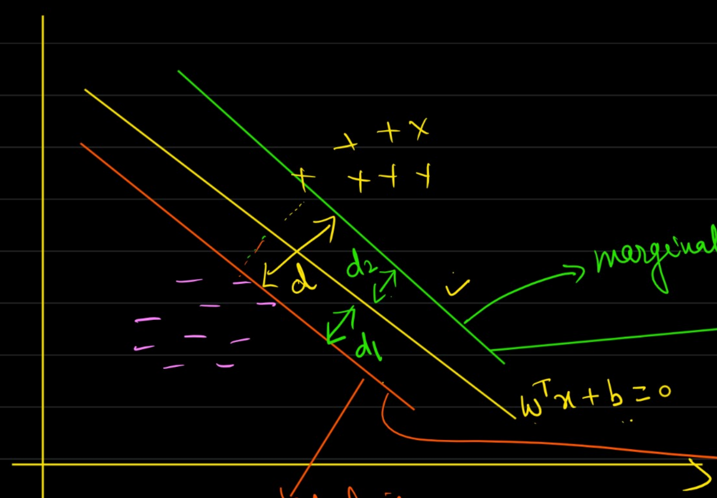
$\vec{X} \cdot \vec{w} + b \geq 0 \longrightarrow$ +ve point

$$\vec{X}\cdot\vec{w} + b \geq 0$$

$$y = \begin{cases} +1 & \text{if } \vec{x}\cdot\vec{w} + b \geq 0 \\ -1 & \text{if } \vec{x}\cdot\vec{w} + b < 0 \end{cases}$$



$\rightarrow$ d is maximum



$\rightarrow$ marginal line for +ve class

$\rightarrow w^T x + b = 1$

$\rightarrow w^T x + b = -1$

marginal line for -ve class

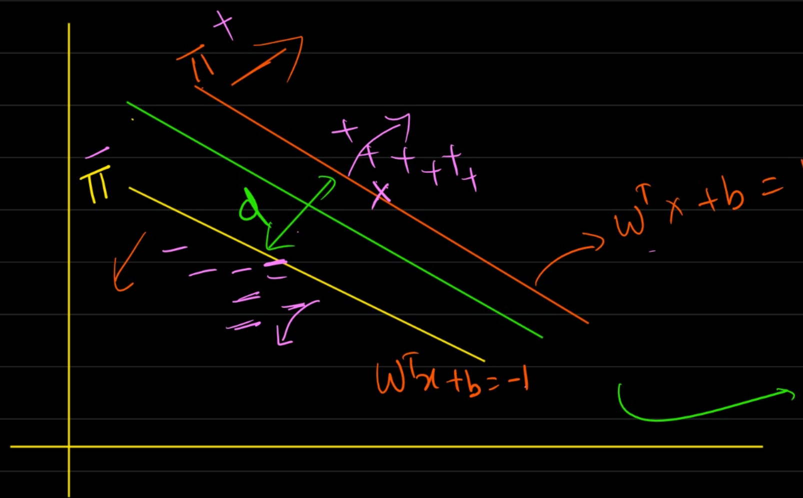
$\begin{cases} \text{for positive class } +1 \\ \text{for negative class } -1 \end{cases}$

* Why equal (both 1)? $\rightarrow$ d_1 & d_2 should be equidistant (optimal line should pass through center of margin.

* Why only 1? $\rightarrow$ It doesn't make a difference

Both will be same line $\leftarrow$ $\begin{cases} 2x + y = 1 \\ 2x + y = 2 \end{cases}$

→ even if we multiply the whole equation with some other number the line doesn't change



→ $w^T x + b = 0$

* We want calculate distance (d) such that no positive or negative point can cross the margin line

$+1$ for +ve class dps: $\vec{w}\cdot\vec{x} + b \geq 1$

-1 for -ve class datapoints: $\vec{w}\cdot\vec{x} + b \leq -1$

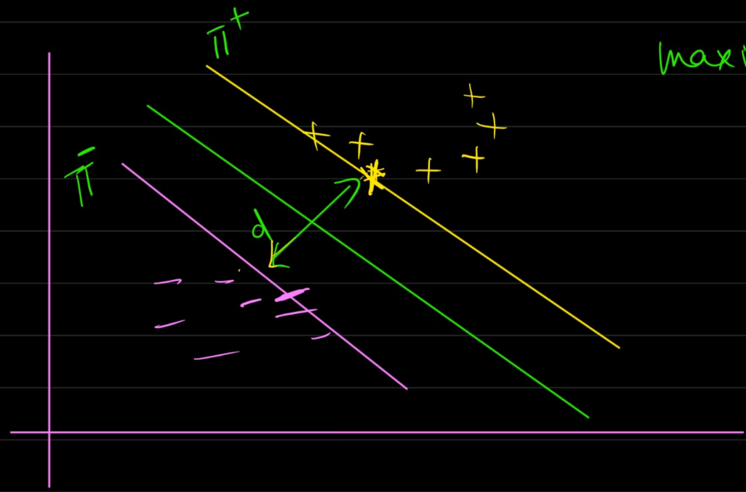
We want to maximize 'd' such that this constraint holds true

for +ve class → $\underset{+1}{y_i}\,(\underbrace{\vec{w}\cdot\vec{x} + b}_{+ve}) \geq 1$

for -ve class = $y_i(\vec{w}\cdot\vec{x} + b) \leq -1$

$= +1\,(\vec{w}\cdot\vec{x} + b) \geq +1$

$= y_i(\vec{w}\cdot\vec{x}_i + b) \geq 1$



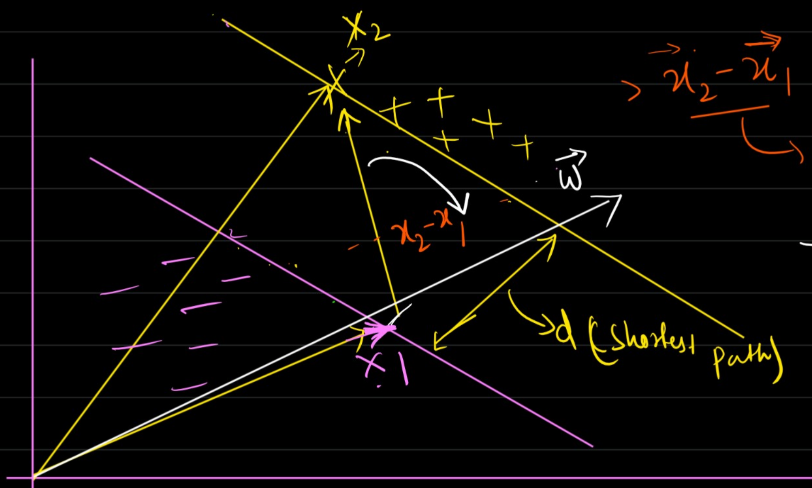
maximise d

$$y_i(\vec{w}\cdot\vec{x}+b) \geq 1$$

for support vector

$$y_i(\vec{w}\cdot\vec{x}+b) = 1$$

equality because Support Vectors falls on marginal hyperplane.



$> \vec{x}_2 - \vec{x}_1$

↳ b/w vectors x_2, x_1

→ To get shortest distance we need a unit vector ⊥r to all marginal hyperplane

→ Projection of $\vec{x}_2 - \vec{x}_1$ on Unit Vector $\vec{w}$ to get d

$$d = (\vec{x}_2 - \vec{x}_1) \cdot \frac{\vec{W}}{\|W\|}$$

$$\frac{x_2 \cdot \vec{w} - \vec{x}_1 \cdot w}{\|W\|} \quad \text{--- (1)}$$

(x_1 & x_2 are support Vectors, they lie on marginal hyperplane)

Since $x_1 x_2$ are Support Vectors, it should follow

$$y_i(\vec{w}\cdot\vec{x}+b) = 1$$

for +ve class = $y_i = 1$ for x_1

$$1 \times (w x_1 + b) = 1$$

$$\vec{w}\vec{x}_1 = 1 - b \quad \text{--- (2)}$$

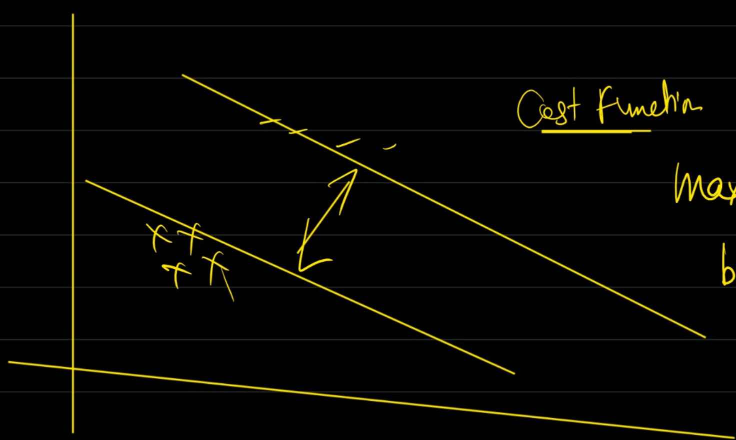
for -ve class $\quad -1(w x_2 + b) = 1$

$$\vec{w}\cdot\vec{x}_2 = -b - 1 \quad \text{--- (3)}$$

putting eqn ② & ③ in ①

$$\frac{(1-b)-(-b-1)}{\|W\|} = \frac{2}{\|W\|} = d$$

$$\underset{(w,b)}{\text{maximise}} \; \frac{2}{\|W\|} \quad \text{such that} \quad y_i(\vec{W}\cdot\vec{X}+b) \geq 1$$



Cost Function

maximise $\frac{2}{\|W\|}$

by changing w, b

(intercept / coeff)

* Modified Cost fn for Hardmargin SVC

$$\text{minimise } \frac{\|W\|}{2} \quad \text{by varying } w \,\&\, b$$

Constraint $y_i(wx_i+b) \geq 1$