Title: 1. Applying the Critical Margin Theory to Citadel's Assets 2. Correction Regarding the Rate Comparison between Critical Margin Decay and Borrow Fee Growth
Author: disoriented_llama
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Linked Post Content:
E: I just realized I didn't actually apply the Critical Margin Theory to Citadel's assets and never meant to so pretend the title says:

>**1. Hypothetical Calculation of Citadel's Annual/Daily Borrow Fees 2. Correction Regarding the Rate Comparison between Critical Margin Decay and Borrow Fee Growth**

I stayed up writing this lol.

This is a continuation of u/Scienceisexy's [*post*](https://www.reddit.com/r/Superstonk/comments/v6cwds/comment/ibeq0cq/?utm_source=share&utm;_medium=web2x&context;=3) *about* u/-einfachman-*'s critical margin theory (found in his* [*Burning Cash DD*](https://www.reddit.com/r/Superstonk/comments/v0zrni/burning_cash/?utm_source=share&utm;_medium=web2x&context;=3)*). Each subsequent Part expands on the previous. Calculations may not include all necessary components but should provide a more accurate representation of the data in question. To be clear, the material starting from Part C1 is not meant to call out OP. I am simply stating what I believe to be incorrect and/or misleading.*

TL;DR Citadel needs a lot of collateral for their shorts and a statistical comparison between critical margin levels and borrow fees indicate a more rapid change in the latter.

Recap:

Simple Interest Formula

>A = P(1 + rt)
>
>A = 100 (1 + (0.01 * 1))
>
>A = $101
>
>*A=Net Liability, P=Initial Short Price, r=Rate of Growth/Decay, t=Time
>
>And from this we know that the maintenance margin has increased $101 - 100 = $1. So I need an additional $1 in assets to keep my position open.

Part A: Total Additional Assets Required to Maintain Citadel's GME Short Position

Citadel manages [$51 billion](https://money.usnews.com/investing/news/articles/2022-06-06/citadels-flagship-portfolio-up-13-far-outpacing-broader-stock-market) in assets (but likely doing [poorly in actuality](https://www.reddit.com/r/Superstonk/comments/v4wxkb/i_spoke_with_a_former_citadel_client_heres_what/?utm_source=share&utm;_medium=web2x&context;=3)). For the sake of simplicity and convenience I will be using $50 billion. Additionally, I will be assuming Citadel has a conservative GME short position of 50 million shares with an annual borrow fee of 1% for this example. First, we'll use the same interest formula, (you end up with the same increase in maintenance margin since the formula uses the same input values):

1. A = P (1 + rt) = $100 (1 + (0.01 * 1)) = $101
2. $101 - $100 = $1

Then we multiply $1 by the number of shares sold short. The total amount required in assets to maintain their GME short position would yield the following: $1 x 50,000,000 = $50MM annually. As a reminder, this is if Citadel only sold 50 million shares short and the borrow fee is 1%. If Citadel were to have a short position of 250 million shares with a 10% borrow fee using the last close price:

1. A = P (1 + rt) = $128.10 (1 + (0.1 * 1)) = $140.91
2. $140.91 - $128.10 = $12.81

$12.81 * 250 million shares = **$3,202,500,000 annually based on 250 million shares sold short with a 10% borrow fee**

Part B: Daily Hard-to-Borrow (HTB) Fee

Ally explains how to calculate the [daily HTB fee](https://www.ally.com/do-it-right/investing/hard-to-borrow-fee-calculation/) as follows:

1. M * 1.02 = C
2. C * n = V
3. V * r = F
4. F / 365 = D
5. $100 (1.02) = $102

**M = Market Prince of Stock, 1.02 = current industry convention (percentage set by securities lending market participants), C = Per Share Collateral Amount, n = Share Quantity, V = Trade Value, F = Annual HTB Fee, D = Daily HTB Fee*

1. $100 * 1.02 = $102
2. $102 * 50,000,000 shares = $5,100,000,000
3. $5,100,000,000 * 0.01 = $51,000,000

$51,000,000 / 365 = $139,726.03 daily. Again, this is based on 50 million shares short with a 1% borrow fee. If Citadel were to have a short position of 250 million shares with a 10% borrow fee using the last close price:

1. $128.10 * 1.02 = $130.66
2. $130.66 * 250,000,000 = $32,665,000,000
3. $32,665,000,000 * 0.1 = $3,266,500,000
4. $3,266,500,000 / 365 = $**8,959,315.07 in daily fees** **based on 250 million shares sold short with a 10% borrow fee**

Part C1: Correction to Comparison between Critical Margin Decay and Borrow Fee Growth

Remember our interest formula?

>A = P(1 + rt)
>
>A = 344.66 (1 + (0.01 * 0.8))
>
>A = $347.42

That formula only finds how much you would pay in total per share, including interest, over the course of

294 calendar days if you started shorting a share on June 8, 2021. In order to compare the rates of critical margin decay and borrow fee growth we need to standardize both results.

Critical Margin Decay

[Original by TiberiusWoodwind](https://preview.redd.it/ujp3jwk386491.jpg?width=864&format;=pjpg&auto;=webp&s;=694d43b22d2b7c3cfaa85cb61758aefdec0fe8dc)

This [chart](https://www.reddit.com/r/Superstonk/comments/v0zrni/burning_cash/?utm_source=share&utm;_medium=web2x&context;=3) should look familiar if you have viewed the aforementioned posts at the top.

ROC = change in quantity 1 / change in quantity 2 = (200% - 320%) / (294 days - 0 days)= -120% / 294 days = **-0.41% critical margin level decay per day**

**ROC = Rate of Change*

Borrow Fee Growth

[Credit to u\Visnisse](https://preview.redd.it/fmwuszaa86491.png?width=6316&format;=png&auto;=webp&s;=f76ef1913975db26b74b1f5d95a60e42369df38b)

For this section I will take the average of the reported borrow fees on March 29, 2022 from the graph above but will have to settle for this [archive](https://web.archive.org/web/20210608141422/https://gme.crazyawesomecompany.com/) for June 8, 2021 (1% borrow fee):

1. average borrow fee = (7.5% + 20% + 22.5% + 22.5% + 25% + 30%) / 6 = 21.25%
2. ROC = (21.25% - 1%) / (294 days - 0 days) = 20.25% / 294 days = **0.06% borrow fee growth per day**

Part C2: Even though we have the two rates of change, we need to standardize them in order to properly compare the two since they used entirely different variables for "quantity 1" (we are comparing apples to oranges when we need two bananas).

Since both rates of change used the same number of days they cancel out, allowing us to focus on the numerator (critical margin decay and borrow fee growth):

Critical Margin Level

1. **Calculate sample standard deviation for each data point.**

[SD = Standard Deviation, xi = Value of Each Data Point, x■ = Mean, N = Number of Data Points](https://preview.redd.it/c0fiuy6o86491.png?width=305&format;=png&auto;=webp&s;=e16879cc3ef6d6146a64fc372e6fcc57205d12eb)

x■ = (320 + 200) / 2 = 260

Data Point	xi - x■	(xi - x■)^(2)
320	320 - 260 = 60	(60)^(2) = 3,600
200	200 - 260 = -60	(-60)^(2) = 3,600

∑ (xi – x■)^(2) = 3,600 + 3,600 = 7,200

7,200 / (2 - 1) = **7,200**

2. **Calculate the Z-score for each data point.**

[z = Z-Score, x = xi, μ = x■, σ = SD](https://preview.redd.it/422gaiia96491.png?width=429&format;=png&auto;=webp&s;=62cacf40a3f963bc8fdf8fba43d2814f82456799)

z = (xi – x■) / SD

Data Point	(xi – x■) / SD
320	(60 / 7,200) = 0.0083
200	(-60 / 7,200) = -0.0083

Borrow Fee

1. **Calculate sample standard deviation for each data point.**

x■ = (7.5 + 20 + 22.5 + 22.5 + 25 + 30) / 6 = 21.25

Data Point	xi - x■	(xi - x■)^(2)
7.5	7.5 - 21.25 = -13.75	(13.75)^(2) = 189.06
20	20 - 21.25 = -1.25	(-1.25)^(2) = 1.56
22.5	22.5 - 21.25 = 1	(1)^(2) = 1
22.5	22.5 - 21.25 = 1	(1)^(2) = 1
25	25 - 21.25 = 3.75	(3.75)^(2) = 14.06
30	30 - 21.25 = 8.75	(8.75)^(2) = 76.56

∑ (xi – x■)^(2) = 189.06 + 1.56 + 1 + 1 + 14.06 + 75.56 = 283.24

283.24 / (6 - 1) = 56.65

2. **Calculate the Z-score for each data point.**

z = (xi – x■) / SD

Data Point	(xi – x■) / SD
7.5	(7.5 - 21.25) / 56.65 = -0.23
20	(20 - 21.25) / 56.65 = 0.35
22.5	(22.5 - 21.25) / 56.65 = 0.40
22.5	(22.5 - 21.25) / 56.65 = 0.40
25	(25 - 21.25) / 56.65 = 0.44
30	(30 - 21.25) / 56.65 = 0.53

Part C3: Comparison between Critical Margin Decay and Borrow Fee Growth

The z-scores for the critical margin levels were 0.0083 and -0.0083 and the z-scores for the borrow fees were -0.23, 0.35, 0.40, 0.40, 0.44, and 0.53. The z-scores for the critical margin levels decreased (went

from positive to negative) and did not deviate far from the mean whereas the z-scores for the borrow fees increased and deviated farther from the mean, indicating a smaller slowdown in the decay of the critical margin level but a larger acceleration in higher borrow fees. **In other words, the critical margin level is decreasing at a slower rate than the rate at which borrow fees are increasing.**

Closing Remarks:

1. Second half of the post seems more like a shitpost than DD:

1a. When solving for "r", both the net liability and initial short price were incorrect.

>A = P(1 + rt)
>
>***quick algebras*
>
>r = ((A/P) -1)/t
>
>r = ((199.41/344.66)-1)/0.8 *<------- incorrect*
>
>r = -0.53***
>
>Holy shit. So the maintenance margin is going up 53% every year…

N*egative borrow fees indicate lenders are* ***paying *borrowers to short the stock so it's in the shorts' interest to borrow. Assuming the above numbers were correct:*

A = $199.41 (1 + (-0.53 * 1)) = $93.72

You would be able to borrow a share for less than half the price of the stock, including interest, for an entire year. Amazing right? So, the initial short price (P) should be $199.41 which would change the net liability (A) appropriately:

A = $199.41 (1 + (0.01 * 1)) = $201.40

r = ((A / P) - 1) / t = ((**$201.40 / $199.41**) - 1) / 0.8 = 0.01 = 1% borrow fee, not maintenance margin (variables must be consistent for calculations to remain valid)

1b. 100x leverage = 10,000%, not 100%.

Yes, options have 100 shares per contract, however, the 95.5% from adding the absolute value of the -42.5% loss and (incorrectly calculated/used) 53% has no bearing on why options contain 100 shares each.

1c. 1% borrow fee on 100 shares ≠ 100%.

>100 shares in a contract, 1% borrow fee per share. Well look at that, 1% * 100 is 100%…

Despite using the same formula (and same values) as the one at the beginning of their post, OP somehow managed to get 100%.

1. A = P (1 + rt) = $100 (1 + (0.01 * 1)) = $101
2. $101 - $100 = $1

$1 * 100 shares = $100 annual fee for borrowing a hundred shares of GME at a price of $100

100 shares x $100 = $10,000

$100 / $10,000 = 0.01 = 1%, not 100%

2. Limitations:

* I used second hand sources for Citadel's holdings but all of their books are probably cooked.
* Most calculations are rudimentary compared to the true complexities behind algos.
* I could only find one source regarding borrow fees from June 8, 2021.
* I used limited resources to substantiate this post, however, they were directly relevant.