DP on Partition

$(1+2)+(3-4)$

$(1+(2+3))-4$

$1+(2+(3-4))$

Ways to solve this problem

effectively

right ways

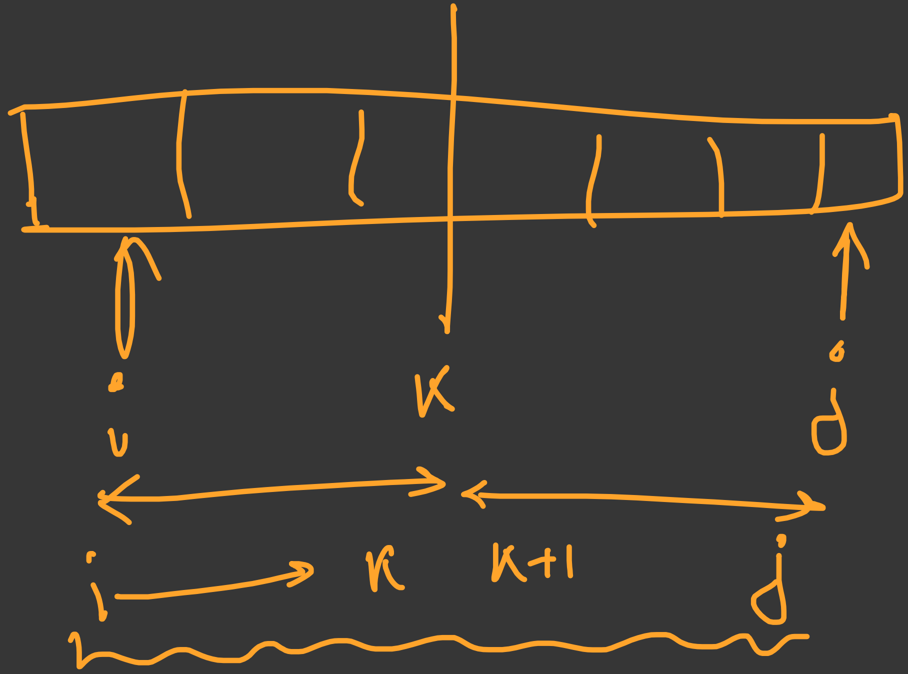
optimal

$1 \mid + \; 2 \mid + \mid 3 \mid - \; 4$

k k k k

$\underline{1} + \underline{(2+3-4)}$

- Whenever we have multiple ways to solve a particular problem we use dp on partition



Problem Statement Suggest Edit

Given a chain of matrices A1, A2, A3,.....An, you have to figure out the most efficient way to multiply these matrices. In other words, determine where to place parentheses to minimize the number of multiplications.

You will be given an array p[] of size n + 1. Dimension of matrix Ai is p[i - 1]*p[i]. You need to find minimum number of multiplications needed to multiply the chain.

$A_1 \quad A_2 \quad A_3 \quad \ldots \quad A_n$

$$[\quad]_{n_1 \times m_1} \times [\quad]_{n_2 \times m_2} \times [\quad]_{n_3 \times n_m} \ldots$$

Ways:

eg: A B C

$A \longrightarrow 10 \times 30$

$B \longrightarrow 30 \times 5$

$C \longrightarrow 5 \times 60$

$\begin{bmatrix} & \end{bmatrix}_{n \times m} \times \begin{bmatrix} & \end{bmatrix}_{m \times a}$

Ways:

$(A \times B) \times C$ } Operation

$(10 \times 30)\ (30 \times 5)$

$\downarrow$

$10 \times 5 \ \times \ 5 \times 60 = 10 \times 60$

AB C

$(10 \times 30 \times 5) + (10 \times 5 \times 60)$

$= 4500$ Operation (minimal)

Q

$n \times a$

$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2 \times 2} \begin{bmatrix} 1 \\ 2 \end{bmatrix}_{2 \times 1}$

$\begin{bmatrix} 1 \times 1 + 2 \times 2 \\ 3 \times 1 + 4 \times 2 \end{bmatrix}$

$2 \times 2 \times 1$

4

arr[] = {10, 20, 30, 40, 50}   N = 5

0 1 2 3 4

A B C D

arr[i-1] × arr[i

$\boxed{A = 10 \times 20}$

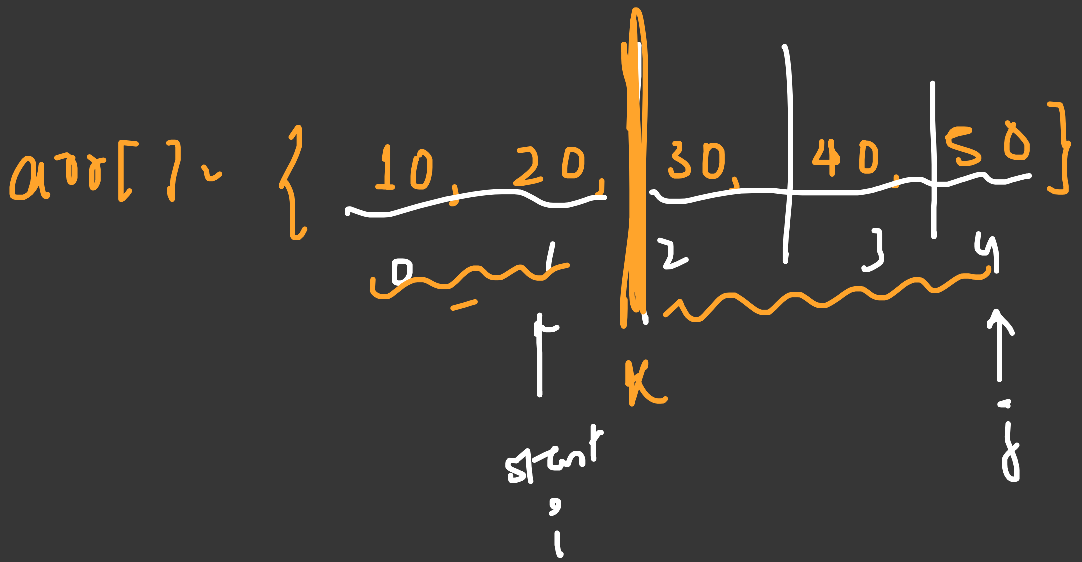
A → 10 × 20
B = 20 × 30
C 30 × 40
D 40 × 50

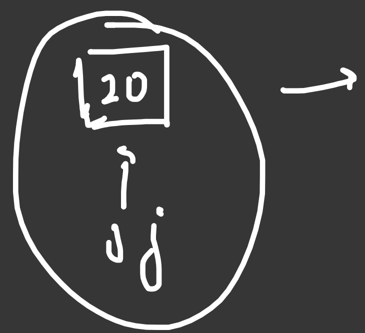
(A × B) × (C × D)

We have multiple ways to solve the problem.

Rules on partition of DP:

- Start with entire array with start index and end index.
- Try all partition $\longrightarrow$ Run a loop to try all partition.
- Return the best possible partition





for(k $\longrightarrow$ i $\longrightarrow$ $j-1$)

$f(i, k), f(k+1, j)$

Recurrence Relation:

f(i, j)

if (i == j) return 0,

for (k = i; k < j; k++)

steps = a[i-1] * arr[k] * arr[j]

+ f(i, k) + f(k+1, j)

mini = min(steps, mini)

Steps = (A) × (BCD)

(10 × 20) * (~~10 × 20~~ 20 × 30 × 30 × 40 × 40 × 50)

(10 × 20)(20 × 50)

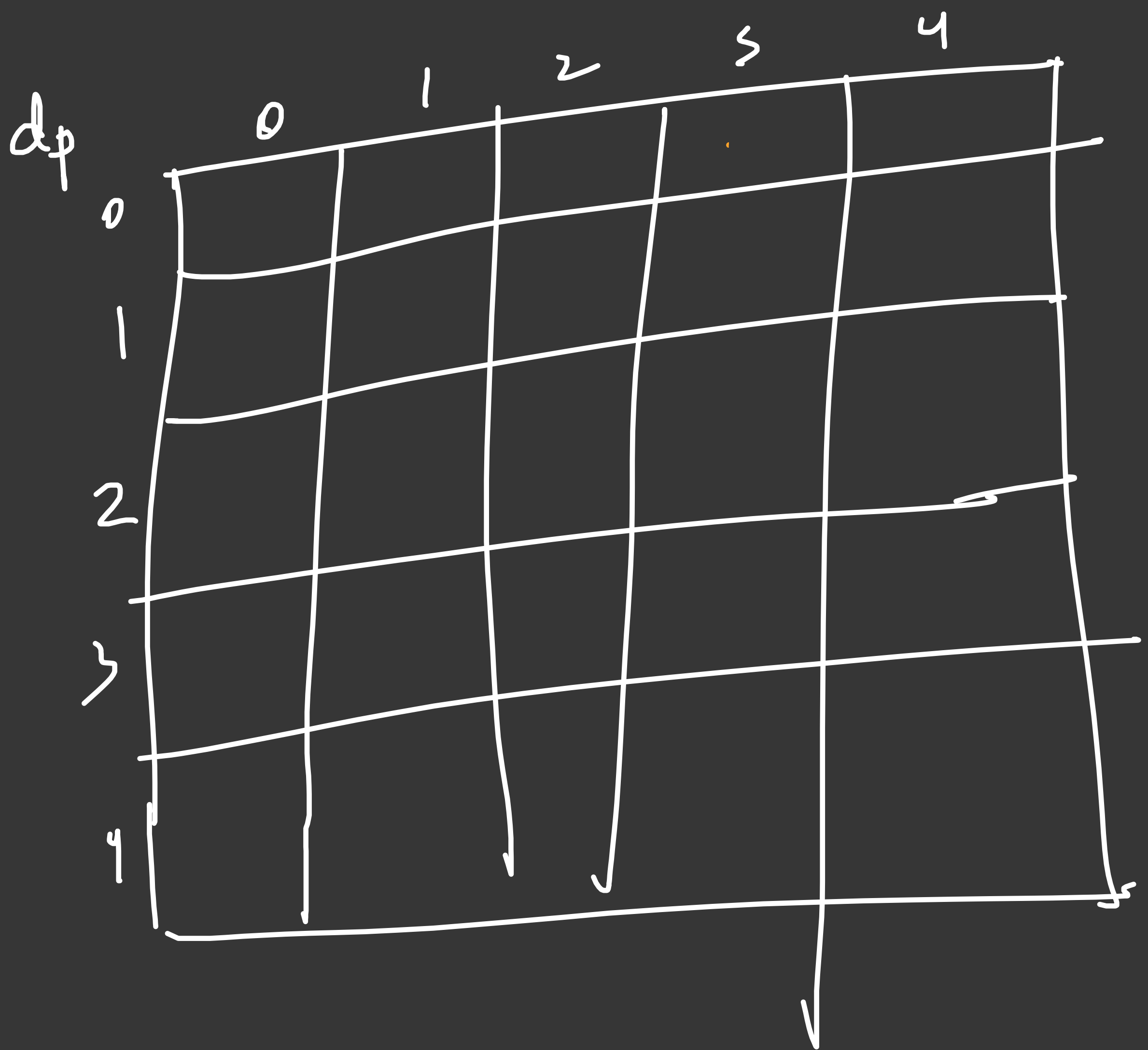
(10 × 50)

i-1   k   j

= 10 × 20 × 50

return mini;

{10, 20, 30, 40, 50}
0 1 2 3 4



f(1, 4)

k = 1

(10 × 20 × 50) + 0

+ 0

mini = 1000

$10 \times 20 \times 30$

$(10 \times 20 \times 30)$

$\{10, 20, 30, 40, 50\}$

$10 \times 20 \times 50 +$

i j

10, 20

$\{20\}$ $\{30, 40, 50\}$

Tabulation:

dp [n] [n] = 0

n2 arrasize → n+1

n+1

base case // . for (i = 1; i < N, i++)

dp[i][i] = 0 , in recursion

i = 1 → n

changing parameter i , j n

j = n-1 → 1

for(n-1 → 0 i--) j is always in

for(j = i+1 ; j < n ; j++) left of i