Exercise 2 Solutions

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1 Question 1

1.1 Solution for part (a)

Since $\mathbf{x} \in \mathbb{R}$,

$f_y(\mathbf{x}) = \frac{1}{\sqrt{2\pi}\sigma} exp - \frac{(\mathbf{x}-\mu_y)^2}{2\sigma^2} P(y)$ (using Bayes Rule)

Taking the logarithm,

$log(f_y(\mathbf{x})) = -\frac{(x-\mu_y)^2}{2\sigma^2} - log(\sqrt{2\pi}\sigma) + log(P(y))$

$log(f_y(\mathbf{x})) = -\frac{\mu_y^2}{(2\sigma^2)} + \frac{\mathbf{x}\mu_y}{(\sigma^2)} - \frac{\mathbf{x}^2}{(2\sigma^2)} - log(\sqrt{2\pi}\sigma) + log(P(y)$

f_y $(\mathbf{x})$ is in the form of $a\mathbf{x}^2 + b\mathbf{x} + c$ where, $a = \frac{-1}{(2\sigma^2)}, b = \frac{\mathbf{x}}{(\sigma^2)}$ and $c = (-\frac{\mathbf{x}^2}{2\sigma^2} - log(\sqrt{2\pi}\sigma) + log(P(y))$

1.2 Solution for part (b)

Since $\mathbf{x} \in \mathbb{R}^d$,

$f_y(\mathbf{x}) = \frac{1}{2\pi^{\frac{d}{2}}|\Sigma|^{-1}} exp(-\frac{1}{2}(\mathbf{x}-\mu_y)^T\Sigma^{-1}(\mathbf{x}-\mu_y))P(y)$ (using Bayes Rule)

Lets denote all constants that are not dependent on $\mathbf{x}$ as C for simplicity purpose.

Taking logarithm,

$log(f_y(\mathbf{x})) = -\frac{1}{2}(x-\mu_y)^T\Sigma^{-1}(x-\mu_y) + log(P(y)) + C$

Calculating the decision boundary by maximizing the posterior for y and making the terms not dependent on y as C, we get,

$log(f_y(\mathbf{x})) = -\frac{1}{2}(\mu_y)^T\Sigma^{-1}(\mu_y) + log(P(y)) + \mathbf{x}^T\Sigma^{-1}\mu_y$ which takes the quadratic form of $x^TAx + b^Tx + C$.

where, $A = \Sigma^{-1}, b = \Sigma^{-1}\mathbf{x}$ and $c = log(P(y)$

2 Question 2

Solution is available at the following URL:
`https://github.com/kumar-shridhar/ML-II-Exercise/blob/master/Exercise%202%20Solutions.ipynb`

3 Question 3

Solution is available at the following URL:
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