(a) In this scenario, there are 100 first-year Harvard Law students. Each student takes two courses: Torts and Contracts. Both courses are held in the same lecture hall, which has 100 seats. The seating for each course is randomly assigned and independent of the other course.

To find the probability that no one has the same seat for both courses, we need to consider all possible scenarios. Let's imagine the students are numbered from 1 to 100.

The first student can sit in any of the 100 seats without any restrictions.

For the second student, there is a 1 in 100 chance of choosing the same seat as the first student in both courses.

For the third student, there are two possibilities:

They choose the seat occupied by the first student in both courses, with a 1 in 100 chance.

They choose the seat occupied by the first student in one course and the seat occupied by the second student in the other course, with a 2 in 100 chance.

We continue this pattern for each student. For the ith student, there are i-1 seats that have already been occupied by the previous i-1 students. Out of these i-1 seats, one seat corresponds to the seat occupied by the first student in both courses. So, the probability of the ith student choosing the same seat in both courses is 1 divided by (i-1).

To find the probability that no one has the same seat for both courses, we subtract the probability of at least one student choosing the same seat from 1 (because we want the complement of that event).

(b) To approximate the probability in a simple but accurate way, we can use a mathematical approximation. For large values, the sum of the reciprocals of the numbers from 2 to 100 can be approximated by ln(100) plus a constant called the Euler-Mascheroni constant, denoted as γ. So the approximation becomes ln(100) + γ.

(c) To approximate the probability that at least two students have the same seat for both courses, we can use a principle called inclusion-exclusion. By considering the first term of the principle, we can simplify the calculation to ln(100) + γ. This approximation assumes a large number of students.

In this scenario, there are 100 passengers lined up to board an airplane with 100 seats. The first passenger decides to sit in a randomly chosen seat. Each subsequent passenger takes their assigned seat if it's available; otherwise, they randomly choose an available seat.

The probability that the last passenger gets to sit in their assigned seat is 1 in 2, which is the same as a fair coin flip. This means there is an equal chance that the last passenger either gets their assigned seat or not