1. A chicken lays n eggs. Each egg independently does or doesn’t hatch, with probability p of

hatching. For each egg that hatches, the chick does or doesn’t survive (independently of the

other eggs), with probability s of survival. Let N ⇠ Bin(n, p) be the number of eggs which

hatch, X be the number of chicks which survive, and Y be the number of chicks which hatch

but don’t survive (so X + Y = N). Find the marginal PMF of X, and the joint PMF of X and Y .

Are they independent?

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To find the marginal probability mass function (PMF) of X, we need to sum over all possible values of Y for each value of X. The marginal PMF of X, denoted as P(X = k), represents the probability that k chicks survive.

The joint PMF of X and Y, denoted as P(X = k, Y = m), represents the probability that k chicks survive and m chicks hatch but don't survive.

To calculate these probabilities, we can use the binomial distribution and the independent probabilities given in the problem.

The probability mass function of N, the number of eggs hatching, is given by the binomial distribution:

P(N = k) = C(n, k) \* p^k \* (1-p)^(n-k)

where C(n, k) is the binomial coefficient, given by C(n, k) = n! / (k! \* (n-k)!).

Using the fact that X + Y = N, we can rewrite the probabilities in terms of X and Y:

P(X = k) = sum(P(X = k, Y = m)) for all m from 0 to n-k.

To find P(X = k, Y = m), we can multiply the probabilities of hatching (p^k \* (1-p)^(n-k)) and the probabilities of survival (s^k \* (1-s)^m):

P(X = k, Y = m) = C(n, k) \* p^k \* (1-p)^(n-k) \* s^k \* (1-s)^m

Now, let's determine if X and Y are independent. Two random variables X and Y are independent if and only if their joint PMF can be expressed as the product of their marginal PMFs:

P(X = k, Y = m) = P(X = k) \* P(Y = m)

Substituting the previously derived expressions for P(X = k, Y = m) and P(X = k), we have:

C(n, k) \* p^k \* (1-p)^(n-k) \* s^k \* (1-s)^m = P(X = k) \* P(Y = m)

From this equation, we can see that the joint PMF of X and Y can be factored into the product of their marginal PMFs. Therefore, X and Y are independent random variables