Optimization Assignment-2

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Problem Statement - Prove that the least perimeter of an isosceles triangle in which a circle of radius r can be inscribed is $6\sqrt{3}r$.

Solution

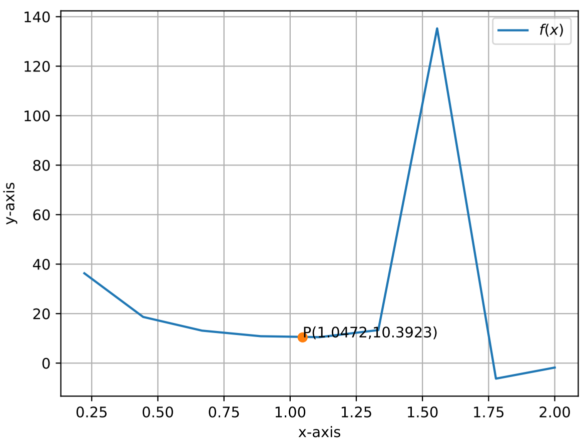

Figure 1: Graph of f(x)

Let x be the length of each of the two equal sides, y be the length of the third side of the $\triangle ABC$ and $\angle B = \angle C$.
Then,

$$x = r\cot\frac{C}{2} + r\cot\frac{A}{2} \quad and \quad y = 2r\cot\frac{C}{2} \tag{1}$$

Now, perimeter of $\triangle ABC$ is,

$$P = 2x + y \tag{2}$$

$$\implies P = 2r[2\cot\frac{C}{2} + \cot\frac{A}{2}] \tag{3}$$

Since, $\frac{A}{2} = \frac{\pi}{2} - C$,

$$\implies P = 2r[2\cot\frac{C}{2} + \tan C] \tag{4}$$

Gradient descent

$$f(x) = 2r[2\cot\frac{x}{2} + \tan x] \tag{5}$$
$$f'(x) = 2r[-cosec^2\frac{x}{2} + \sec^2 x] \tag{6}$$

we have to attain the maximum value of f(x). This can be seen in Figure f(x).Using gradient descent method we can find its minima value.

$$x_{n+1} = x_n - \alpha\nabla f(x_n) \tag{7}$$

$$\implies x_{n+1} = x_n + \alpha(2r[-cosec^2\frac{x}{2} + \sec^2 x]) \tag{8}$$

Taking $x_0 = 0.5, \alpha = 0.001$ and precision = 0.00000001, values obtained using python are:

$$\text{Minima} = 10.3923r \tag{9}$$
$$\implies \boxed{\text{Minima} = 6\sqrt{3}r} \tag{10}$$
$$\boxed{\text{Minima Point} = 1.0471} \tag{11}$$

$\therefore Hence\ \ Proved$