Trinity College o f Engineering and Research, Pune

Computer Graphics

Prof. Prasad I. Bhosle

1. 2D Transformations

- Transformation is a process of modifying and re-positioning the existing graphics.
- 2D Transformations take place in a two dimensional plane.

1. **Translation**
2. **Rotation**
3. **Scaling**
4. **Reflection**
5. **Shear**

1. Translation

In Computer graphics, 2D Translation is a process of moving an object from one position to another in a two dimensional plane.

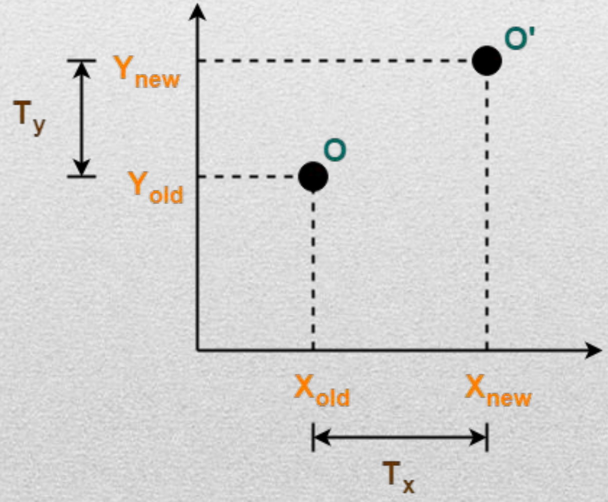
Consider a point object O has to be moved from one position to another in a 2D plane.

Let-

- Initial coordinates of the object O = (X_{old}, Y_{old})
- New coordinates of the object O after translation = (X_{new}, Y_{new})
- Translation vector or Shift vector = (T_x, T_y)

Given a Translation vector (T_x, T_y)-

- T_x defines the distance the X_{old} coordinate has to be moved.
- T_y defines the distance the Y_{old} coordinate has to be moved.



2D Translation in Computer Graphics

1. Translation

This translation is achieved by adding the translation coordinates to the old coordinates of the object as-

- $X_{new} = X_{old} + T_x$
  (This denotes translation towards X axis)
- $Y_{new} = Y_{old} + T_y$
  (This denotes translation towards Y axis)
- In Matrix form, the above translation equations may be represented as in fig.
- The homogeneous coordinates representation of (X, Y) is (X, Y, 1).
- Through this representation, all the transformations can be performed using matrix / vector multiplications.

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix} + \begin{bmatrix} T_x \\ T_y \end{bmatrix}$$

Translation Matrix

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & T_x \\ 0 & 1 & T_y \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ 1 \end{bmatrix}$$

Translation Matrix

(Homogeneous Coordinates Representation)

The above translation matrix may be represented as a 3 x 3 matrix as in fig.

Problem-01:

Given a circle C with radius 10 and center coordinates (1, 4). Apply the translation with distance 5 towards X axis and 1 towards Y axis. Obtain the new coordinates of C without changing its radius.

Solution-

Given-

Old center coordinates of C = $(X_{old}, Y_{old}) = (1, 4)$

Translation vector = $(T_x, T_y) = (5, 1)$

Let the new center coordinates of C = (X_{new}, Y_{new}).

Applying the translation equations, we have-

$X_{new} = X_{old} + T_x = 1 + 5 = 6$

$Y_{new} = Y_{old} + T_y = 4 + 1 = 5$

Thus, New center coordinates of C = (6, 5).

1. Translation

Alternatively,

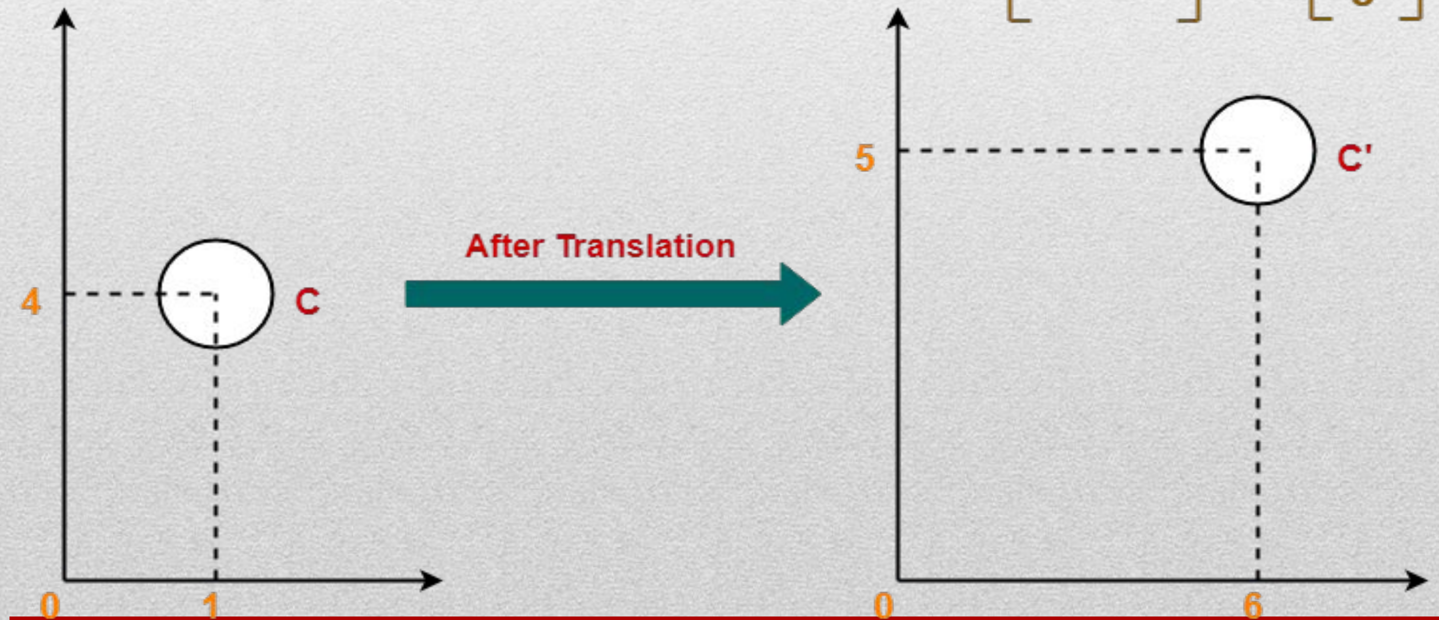
In matrix form, the new center coordinates of C after translation may be obtained as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix} + \begin{bmatrix} T_x \\ T_y \end{bmatrix}$$

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix} + \begin{bmatrix} 5 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} 6 \\ 5 \end{bmatrix}$$

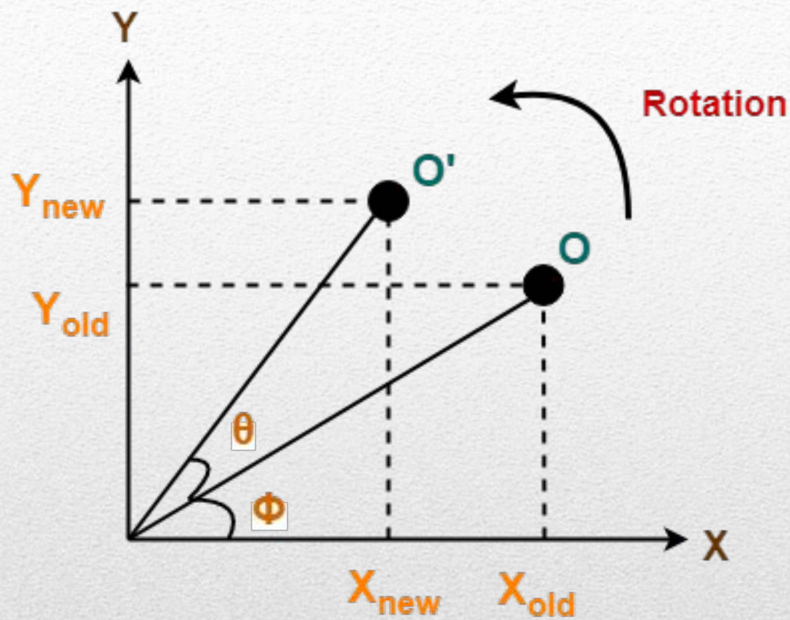
Thus, New center coordinates of C = (6, 5).



2. 2D Rotation

In Computer graphics, 2D Rotation is a process of rotating an object with respect to an angle in a two dimensional plane.

Consider a point object O has to be rotated from one angle to another in a 2D plane.



2D Rotation in Computer Graphics

Let-

- Initial coordinates of the object
  O = (X_{old}, Y_{old})
- Initial angle of the object O with respect to origin = Φ
- Rotation angle = θ
- New coordinates of the object O after rotation = (X_{new}, Y_{new})

This rotation is achieved by using the following rotation equations-

$X_{new} = X_{old} \times \cos\theta - Y_{old} \times \sin\theta$

$Y_{new} = X_{old} \times \sin\theta + Y_{old} \times \cos\theta$

2. 2D Rotation

In Matrix form, the above rotation **(anti**clockwise) equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix}$$

Rotation Matrix

For homogeneous coordinates, the above rotation matrix may be represented as a 3 x 3 matrix as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ 1 \end{bmatrix}$$

Rotation Matrix

(Homogeneous Coordinates Representation)

Problem-01:

Given a line segment with starting point as (0, 0) and ending point as (4, 4). Apply 30 degree rotation anticlockwise direction on the line segment and find out the new coordinates of the line.

Solution-

We rotate a straight line by its end points with the same angle. Then, we re-draw a line between the new end points.

Given-
Old ending coordinates of the line = $(X_{old}, Y_{old}) = (4, 4)$
Rotation angle = $\theta = 30^\circ$

Let new ending coordinates of the line after rotation = (X_{new}, Y_{new}).

Applying the rotation equations, we have-

$X_{new} = X_{old} \times \cos\theta - Y_{old} \times \sin\theta$

$= 4 \times \cos 30^o - 4 \times \sin 30^o$

$= 4 \times (\sqrt{3} / 2) - 4 \times (1 / 2)$

$= 2\sqrt{3} - 2$

$= 2(\sqrt{3} - 1)$

$= 2(1.73 - 1)$

$= 1.46$

$Y_{new} = X_{old} \times \sin\theta + Y_{old} \times \cos\theta$

$= 4 \times \sin 30^o + 4 \times \cos 30^o$

$= 4 \times (1 / 2) + 4 \times (\sqrt{3} / 2)$

$= 2 + 2\sqrt{3}$

$= 2(1 + \sqrt{3})$

$= 2(1 + 1.73)$

$= 5.46$

Thus, New ending coordinates of the line after rotation = (1.46, 5.46).

Alternatively,
In matrix form, the new ending coordinates of the line after rotation may be obtained as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix}$$

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} \cos 30 & -\sin 30 \\ \sin 30 & \cos 30 \end{bmatrix} \times \begin{bmatrix} 4 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} 4 \times \cos 30 - 4 \times \sin 30 \\ 4 \times \sin 30 + 4 \times \cos 30 \end{bmatrix}$$

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} 4 \times \cos 30 - 4 \times \sin 30 \\ 4 \times \sin 30 + 4 \times \cos 30 \end{bmatrix}$$

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} 1.46 \\ 5.46 \end{bmatrix}$$

Thus, New ending coordinates of the line after rotation = (1.46, 5.46).

3. 2D Scaling

In computer graphics, scaling is a process of modifying or altering the size of objects.

- Scaling may be used to increase or reduce the size of object.
- Scaling subjects the coordinate points of the original object to change.
- Scaling factor determines whether the object size is to be increased or reduced.
- If scaling factor > 1, then the object size is increased.
- If scaling factor < 1, then the object size is reduced.

Consider a point object O has to be scaled in a 2D plane.

Let-

- Initial coordinates of the object O = (X_{old}, Y_{old})
- Scaling factor for X-axis = S_x
- Scaling factor for Y-axis = S_y
- New coordinates of the object O after scaling = (X_{new}, Y_{new})

This scaling is achieved by using the following scaling equations-

$X_{new} = X_{old} \times S_x$

$Y_{new} = Y_{old} \times S_y$

3. 2D Scaling

In Matrix form, the above scaling equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix}$$

Scaling Matrix

For homogeneous coordinates, the above scaling matrix may be represented as a 3 x 3 matrix as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ 1 \end{bmatrix}$$

Scaling Matrix

(Homogeneous Coordinates Representation)

Problem-01:

Given a square object with coordinate points A(0, 3), B(3, 3), C(3, 0), D(0, 0). Apply the scaling parameter 2 towards X axis and 3 towards Y axis and obtain the new coordinates of the object.

Solution-

Given-

Old corner coordinates of the square = A (0, 3), B(3, 3), C(3, 0), D(0, 0)

Scaling factor along X axis = 2

Scaling factor along Y axis = 3

For Coordinates A(0, 3)

Let the new coordinates of corner A after scaling = (X_{new}, Y_{new}).

Applying the scaling equations, we have-

$X_{new} = X_{old} \times S_x = 0 \times 2 = 0$

$Y_{new} = Y_{old} \times S_y = 3 \times 3 = 9$

Thus, New coordinates of corner A after scaling = (0, 9).

For Coordinates B(3, 3)

Let the new coordinates of corner B after scaling = (X_{new}, Y_{new}).

Applying the scaling equations, we have-
$X_{new} = X_{old} \times S_x = 3 \times 2 = 6$
$Y_{new} = Y_{old} \times S_y = 3 \times 3 = 9$

Thus, New coordinates of corner B after scaling = (6, 9).

For Coordinates C(3, 0)

Let the new coordinates of corner C after scaling = (X_{new}, Y_{new}).

Applying the scaling equations, we have-
$X_{new} = X_{old} \times S_x = 3 \times 2 = 6$
$Y_{new} = Y_{old} \times S_y = 0 \times 3 = 0$

Thus, New coordinates of corner C after scaling = (6, 0).

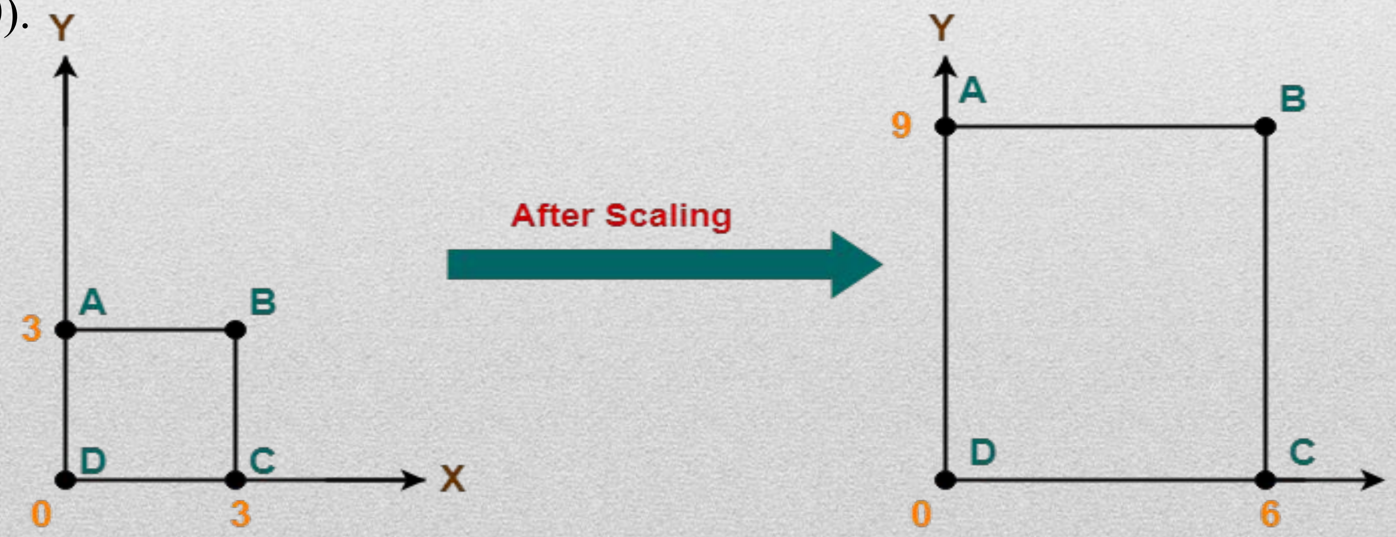
For Coordinates D(0, 0)

Let the new coordinates of corner D after scaling = (X_{new}, Y_{new}).

Applying the scaling equations, we have-
$X_{new} = X_{old} \times S_x = 0 \times 2 = 0$
$Y_{new} = Y_{old} \times S_y = 0 \times 3 = 0$

Thus, New coordinates of corner D after scaling = (0, 0).

Thus, New coordinates of the square after scaling = A (0, 9), B(6, 9), C(6, 0), D(0, 0).



4. 2D Reflection

Reflection is a kind of rotation where the angle of rotation is 180 degree.
The reflected object is always formed on the other side of mirror.
The size of reflected object is same as the size of original object.

Consider a point object O has to be reflected in a 2D plane.
Let-
Initial coordinates of the object O = (X_{old}, Y_{old})
New coordinates of the reflected object O after reflection = (X_{new}, Y_{new})

Reflection On X-Axis:

This reflection is achieved by using the following reflection equations-

$X_{new} = X_{old}$

$Y_{new} = -Y_{old}$

In Matrix form, the above reflection equations may be represented as

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix}$$

Reflection Matrix

(Reflection Along X Axis)

4. 2D Reflection

For homogeneous coordinates, the above reflection matrix may be represented as a 3 x 3 matrix as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ 1 \end{bmatrix}$$

Reflection Matrix

(Reflection Along X Axis)

(Homogeneous Coordinates Representation)

Reflection On Y-Axis:

This reflection is achieved by using the following reflection equations-

$$X_{new} = -X_{old}$$
$$Y_{new} = Y_{old}$$

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix}$$

Reflection Matrix

(Reflection Along Y Axis)

4. 2D Reflection

For homogeneous coordinates, the above reflection matrix may be represented as a 3 x 3 matrix as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ 1 \end{bmatrix}$$

Reflection Matrix

(Reflection Along Y Axis)

(Homogeneous Coordinates Representation)

Problem-01:

Given a triangle with coordinate points A(3, 4), B(6, 4), C(5, 6). Apply the reflection on the X axis and obtain the new coordinates of the object.

Solution-

Given-
Old corner coordinates of the triangle = A (3, 4), B(6, 4), C(5, 6)
Reflection has to be taken on the X axis

For Coordinates A(3, 4)

Let the new coordinates of corner A after reflection = (X_{new}, Y_{new}).

Applying the reflection equations, we have-
$X_{new} = X_{old} = 3$
$Y_{new} = -Y_{old} = -4$

Thus, New coordinates of corner A after reflection = (3, -4).

For Coordinates B(6, 4)

Let the new coordinates of corner B after reflection = (X_{new}, Y_{new}).

Applying the reflection equations, we have-

$X_{new} = X_{old} = 6$

$Y_{new} = -Y_{old} = -4$

Thus, New coordinates of corner B after reflection = (6, -4).

For Coordinates C(5, 6)

Let the new coordinates of corner C after reflection = (X_{new}, Y_{new}).

Applying the reflection equations, we have-

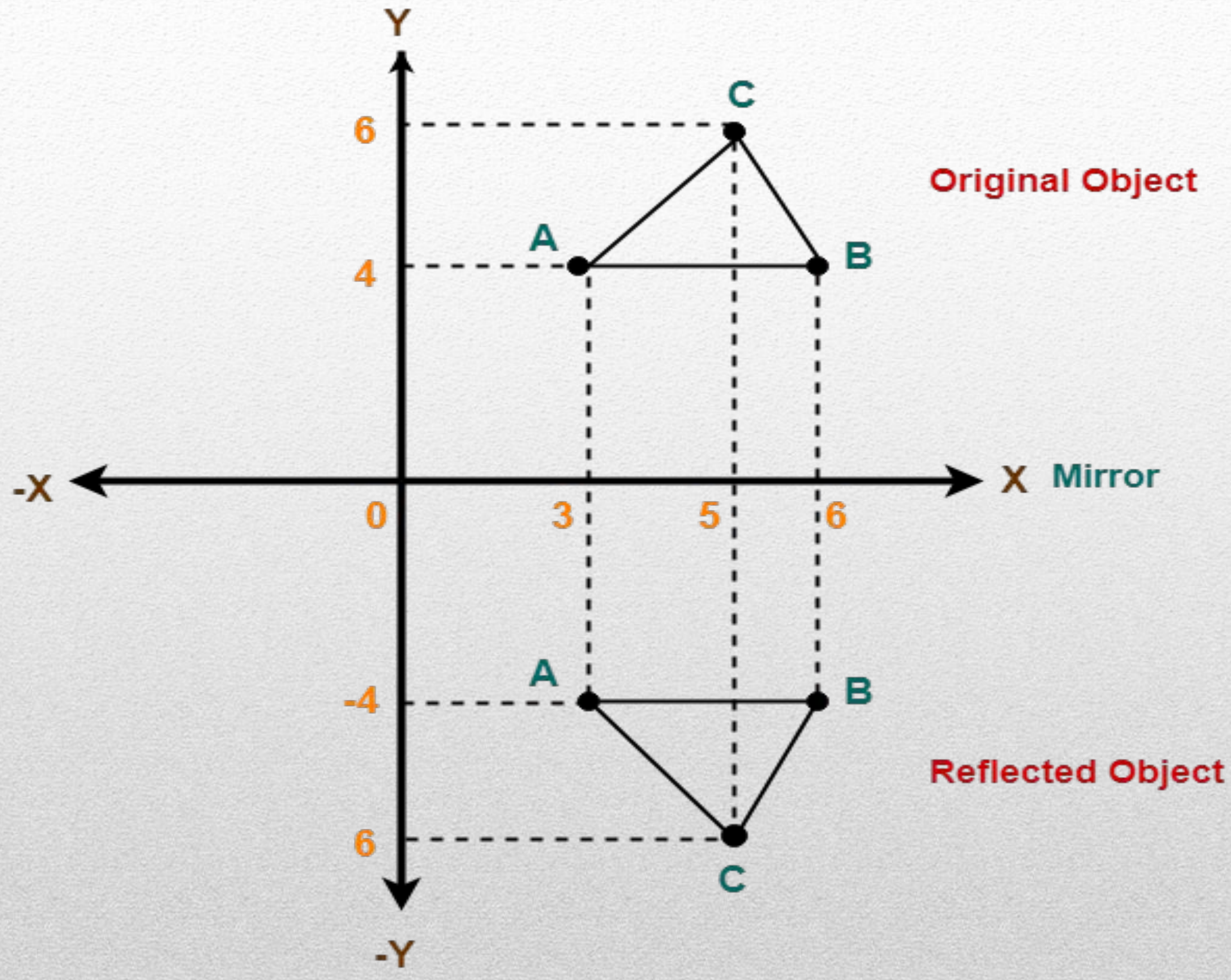
$X_{new} = X_{old} = 5$

$Y_{new} = -Y_{old} = -6$

Thus, New coordinates of corner C after reflection = (5, -6).

Thus, New coordinates of the triangle after reflection = A (3, -4), B(6, -4), C(5, -6).

4. 2D Reflection



5. 2D Shearing

In a two dimensional plane, the object size can be changed along X direction as well as Y direction.

So, there are two versions of shearing-

1. Shearing in X direction
2. Shearing in Y direction

Consider a point object O has to be sheared in a 2D plane.

Let-

Initial coordinates of the object O = (X_{old}, Y_{old})

Shearing parameter towards X direction = Sh_x

Shearing parameter towards Y direction = Sh_y

New coordinates of the object O after shearing = (X_{new}, Y_{new})

Shearing in X Axis-

Shearing in X axis is achieved by using the following shearing equations-

$X_{new} = X_{old} + Sh_x \times Y_{old}$

$Y_{new} = Y_{old}$

5. 2D Shearing

In Matrix form, the above shearing equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} 1 & Sh_x \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix}$$

Shearing Matrix

(In X axis)

For homogeneous coordinates, the above shearing matrix may be represented as a 3 x 3 matrix as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & Sh_x & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ 1 \end{bmatrix}$$

Shearing Matrix

(In X axis)

(Homogeneous Coordinates Representation)

5. 2D Shearing

Shearing in Y Axis-

Shearing in Y axis is achieved by using the following shearing equations-

$X_{new} = X_{old}$

$Y_{new} = Y_{old} + Sh_y \times X_{old}$

In Matrix form, the above shearing equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ Sh_y & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix}$$

Shearing Matrix

(In Y axis)

For homogeneous coordinates, the above shearing matrix may be represented as a 3 x 3 matrix as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ Sh_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ 1 \end{bmatrix}$$

Shearing Matrix

(In Y axis)

(Homogeneous Coordinates Representation)

Problem-01:

Given a triangle with points (1, 1), (0, 0) and (1, 0). Apply shear parameter 2 on X axis and 2 on Y axis and find out the new coordinates of the object.

Solution-

Given-

Old corner coordinates of the triangle = A (1, 1), B(0, 0), C(1, 0)

Shearing parameter towards X direction (Sh_x) = 2

Shearing parameter towards Y direction (Sh_y) = 2

Shearing in X Axis-

For Coordinates A(1, 1)

Let the new coordinates of corner A after shearing = (X_{new}, Y_{new}).

Applying the shearing equations, we have-

$X_{new} = X_{old} + Sh_x \times Y_{old} = 1 + 2 \times 1 = 3$

$Y_{new} = Y_{old} = 1$

Thus, New coordinates of corner A after shearing = (3, 1).

For Coordinates B(0, 0)

Let the new coordinates of corner B after shearing = (X_{new}, Y_{new}).

Applying the shearing equations, we have-
$X_{new} = X_{old} + Sh_x \text{ x } Y_{old} = 0 + 2 \text{ x } 0 = 0$
$Y_{new} = Y_{old} = 0$

Thus, New coordinates of corner B after shearing = (0, 0).

For Coordinates C(1, 0)

Let the new coordinates of corner C after shearing = (X_{new}, Y_{new}).

Applying the shearing equations, we have-
$X_{new} = X_{old} + Sh_x \text{ x } Y_{old} = 1 + 2 \text{ x } 0 = 1$
$Y_{new} = Y_{old} = 0$

Thus, New coordinates of corner C after shearing = (1, 0).
Thus, New coordinates of the triangle after shearing in X axis = A (3, 1), B(0, 0), C(1, 0).

Shearing in Y Axis-

For Coordinates A(1, 1)

Let the new coordinates of corner A after shearing = (X_{new}, Y_{new}).

Applying the shearing equations, we have-

$X_{new} = X_{old} = 1$

$Y_{new} = Y_{old} + Sh_y \times X_{old} = 1 + 2 \times 1 = 3$

Thus, New coordinates of corner A after shearing = (1, 3).

For Coordinates B(0, 0)

Let the new coordinates of corner B after shearing = (X_{new}, Y_{new}).

Applying the shearing equations, we have-

$X_{new} = X_{old} = 0$

$Y_{new} = Y_{old} + Sh_y \times X_{old} = 0 + 2 \times 0 = 0$

Thus, New coordinates of corner B after shearing = (0, 0).

For Coordinates C(1, 0)

Let the new coordinates of corner C after shearing = (X_{new}, Y_{new}).

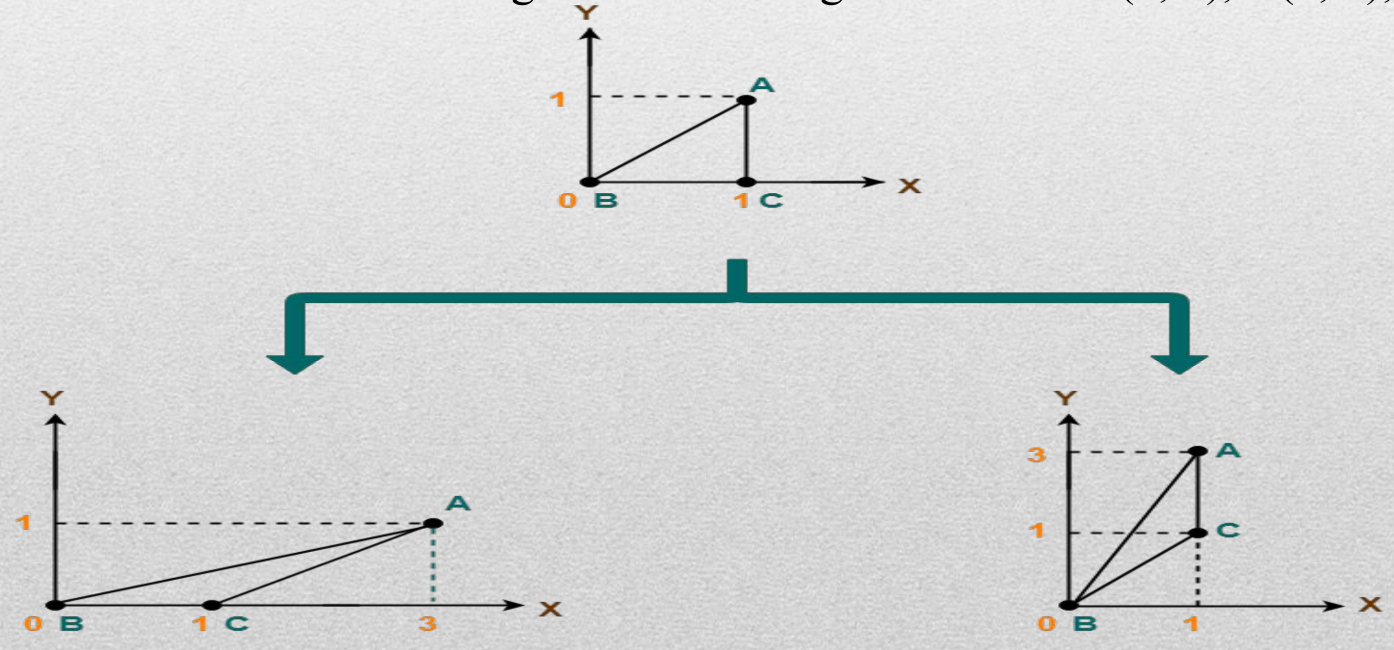
Applying the shearing equations, we have-

$X_{new} = X_{old} = 1$

$Y_{new} = Y_{old} + Sh_y \times X_{old} = 0 + 2 \times 1 = 2$

Thus, New coordinates of corner C after shearing = (1, 2).

Thus, New coordinates of the triangle after shearing in Y axis = A (1, 3), B(0, 0), C(1, 2).



Shearing in X Axis

Shearing in Y Axis

2. 3D Transformations

- Transformation is a process of modifying and re-positioning the existing graphics.
- 3D Transformations take place in a three dimensional plane.

1. **Translation**
2. **Rotation**
3. **Scaling**

1. 3D Translation in Computer Graphics-

In Computer graphics, 3D Translation is a process of moving an object from one position to another in a three dimensional plane.

Consider a point object O has to be moved from one position to another in a 3D plane.

Let-

Initial coordinates of the object O = $(X_{old}, Y_{old}, Z_{old})$

New coordinates of the object O after translation = $(X_{new}, Y_{new}, Z_{old})$

Translation vector or Shift vector = (T_x, T_y, T_z)

Given a Translation vector (T_x, T_y, T_z)-

T_x defines the distance the X_{old} coordinate has to be moved.

T_y defines the distance the Y_{old} coordinate has to be moved.

T_z defines the distance the Z_{old} coordinate has to be moved.

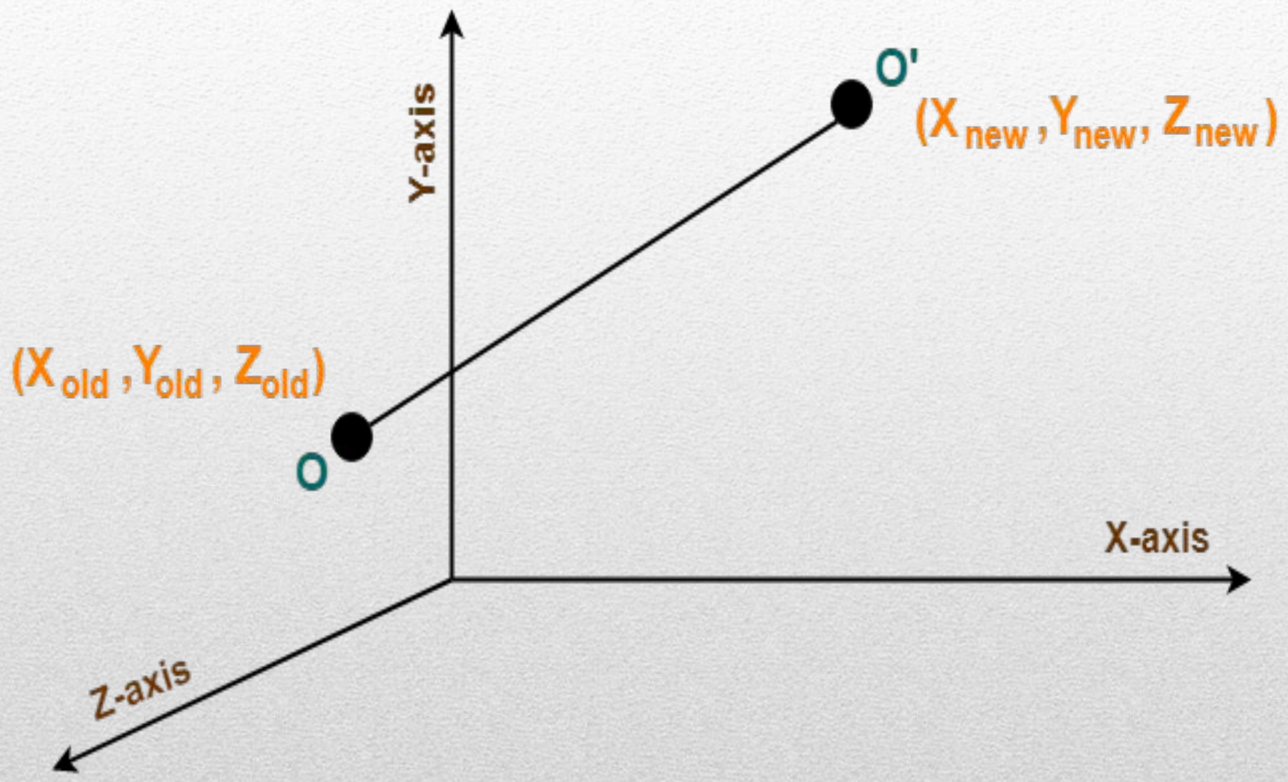
This translation is achieved by adding the translation coordinates to the old coordinates of the object as-

$X_{new} = X_{old} + T_x$ (This denotes translation towards X axis)

$Y_{new} = Y_{old} + T_y$ (This denotes translation towards Y axis)

$Z_{new} = Z_{old} + T_z$ (This denotes translation towards Z axis)

1. 3D Translation in Computer Graphics-



3D Translation in Computer Graphics

1. 3D Translation in Computer Graphics-

In Matrix form, the above translation equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ Z_{old} \\ 1 \end{bmatrix}$$

3D Translation Matrix

1. 3D Translation in Computer Graphics-

Problem-

Given a 3D object with coordinate points A(0, 3, 1), B(3, 3, 2), C(3, 0, 0), D(0, 0, 0). Apply the translation with the distance 1 towards X axis, 1 towards Y axis and 2 towards Z axis and obtain the new coordinates of the object.

Solution-

Given-

Old coordinates of the object = A (0, 3, 1), B(3, 3, 2), C(3, 0, 0), D(0, 0, 0)

Translation vector = $(T_x, T_y, T_z) = (1, 1, 2)$

For Coordinates A(0, 3, 1)

Let the new coordinates of A = $(X_{new}, Y_{new}, Z_{new})$.

Applying the translation equations, we have-

$X_{new} = X_{old} + T_x = 0 + 1 = 1$

$Y_{new} = Y_{old} + T_y = 3 + 1 = 4$

$Z_{new} = Z_{old} + T_z = 1 + 2 = 3$

Thus, New coordinates of A = (1, 4, 3).

1. 3D Translation in Computer Graphics-

For Coordinates B(3, 3, 2)

Let the new coordinates of B = (X_{new}, Y_{new}, Z_{new}).

Applying the translation equations, we have-

$X_{new} = X_{old} + T_x = 3 + 1 = 4$

$Y_{new} = Y_{old} + T_y = 3 + 1 = 4$

$Z_{new} = Z_{old} + T_z = 2 + 2 = 4$

Thus, New coordinates of B = (4, 4, 4).

For Coordinates C(3, 0, 0)

Let the new coordinates of C = (X_{new}, Y_{new}, Z_{new}).

Applying the translation equations, we have-

$X_{new} = X_{old} + T_x = 3 + 1 = 4$

$Y_{new} = Y_{old} + T_y = 0 + 1 = 1$

$Z_{new} = Z_{old} + T_z = 0 + 2 = 2$

Thus, New coordinates of C = (4, 1, 2).

1. 3D Translation in Computer Graphics-

For Coordinates D(0, 0, 0)

Let the new coordinates of D = (X_{new}, Y_{new}, Z_{new}).

Applying the translation equations, we have-

$X_{new} = X_{old} + T_x = 0 + 1 = 1$

$Y_{new} = Y_{old} + T_y = 0 + 1 = 1$

$Z_{new} = Z_{old} + T_z = 0 + 2 = 2$

Thus, New coordinates of D = (1, 1, 2).

Thus, New coordinates of the object = A (1, 4, 3), B(4, 4, 4), C(4, 1, 2), D(1, 1, 2).

2. 3D Rotation in Computer Graphics-

In Computer graphics, 3D Rotation is a process of rotating an object with respect to an angle in a three dimensional plane.

Consider a point object O has to be rotated from one angle to another in a 3D plane.

Let-

Initial coordinates of the object O = (X_{old}, Y_{old}, Z_{old})

Initial angle of the object O with respect to origin = Φ

Rotation angle = θ

New coordinates of the object O after rotation = (X_{new}, Y_{new}, Z_{new})

In 3 dimensions, there are 3 possible types of rotation-

X-axis Rotation

Y-axis Rotation

Z-axis Rotation

2. 3D Rotation in Computer Graphics-

For X-Axis Rotation-

This rotation is achieved by using the following rotation equations-

$X_{new} = X_{old}$
$Y_{new} = Y_{old} \times \cos\theta - Z_{old} \times \sin\theta$
$Z_{new} = Y_{old} \times \sin\theta + Z_{old} \times \cos\theta$

In Matrix form, the above rotation equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ Z_{old} \\ 1 \end{bmatrix}$$

3D Rotation Matrix

(For X-Axis Rotation)

2. 3D Rotation in Computer Graphics-

For Y-Axis Rotation-

This rotation is achieved by using the following rotation equations-

$X_{new} = Z_{old} \times \sin\theta + X_{old} \times \cos\theta$
$Y_{new} = Y_{old}$
$Z_{new} = Y_{old} \times \cos\theta - X_{old} \times \sin\theta$

In Matrix form, the above rotation equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & -\sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ \sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ Z_{old} \\ 1 \end{bmatrix}$$

3D Rotation Matrix

(For Y-Axis Rotation)

2. 3D Rotation in Computer Graphics-

For Z-Axis Rotation-

This rotation is achieved by using the following rotation equations-

$X_{new} = X_{old} \times \cos\theta - Y_{old} \times \sin\theta$

$Y_{new} = X_{old} \times \sin\theta + Y_{old} \times \cos\theta$

$Z_{new} = Z_{old}$

In Matrix form, the above rotation equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ Z_{old} \\ 1 \end{bmatrix}$$

3D Rotation Matrix

(For Z-Axis Rotation)

2. 3D Rotation in Computer Graphics-

Problem-01:

Given a homogeneous point (1, 2, 3). Apply rotation 90 degree towards X, Y and Z axis and find out the new coordinate points.

Solution-

Given-

Old coordinates = $(X_{old}, Y_{old}, Z_{old}) = (1, 2, 3)$

Rotation angle = $\theta = 90^o$

For X-Axis Rotation-

Let the new coordinates after rotation = $(X_{new}, Y_{new}, Z_{new})$.

Applying the rotation equations, we have-

$X_{new} = X_{old} = 1$

$Y_{new} = Y_{old} \times \cos\theta - Z_{old} \times \sin\theta = 2 \times \cos 90° - 3 \times \sin 90° = 2 \times 0 - 3 \times 1 = -3$

$Z_{new} = Y_{old} \times \sin\theta + Z_{old} \times \cos\theta = 2 \times \sin 90° + 3 \times \cos 90° = 2 \times 1 + 3 \times 0 = 2$

Thus, New coordinates after rotation = (1, -3, 2).

2. 3D Rotation in Computer Graphics-

For Y-Axis Rotation-

Let the new coordinates after rotation = (X_{new}, Y_{new}, Z_{new}).

Applying the rotation equations, we have-

$X_{new} = Z_{old} \times \sin\theta + X_{old} \times \cos\theta = 3 \times \sin 90° + 1 \times \cos 90° = 3 \times 1 + 1 \times 0 = 3$

$Y_{new} = Y_{old} = 2$

$Z_{new} = Y_{old} \times \cos\theta - X_{old} \times \sin\theta = 2 \times \cos 90° - 1 \times \sin 90° = 2 \times 0 - 1 \times 1 = -1$

Thus, New coordinates after rotation = (3, 2, -1).

For Z-Axis Rotation-

Let the new coordinates after rotation = (X_{new}, Y_{new}, Z_{new}).

Applying the rotation equations, we have-

$X_{new} = X_{old} \times \cos\theta - Y_{old} \times \sin\theta = 1 \times \cos 90° - 2 \times \sin 90° = 1 \times 0 - 2 \times 1 = -2$

$Y_{new} = X_{old} \times \sin\theta + Y_{old} \times \cos\theta = 1 \times \sin 90° + 2 \times \cos 90° = 1 \times 1 + 2 \times 0 = 1$

$Z_{new} = Z_{old} = 3$

Thus, New coordinates after rotation = (-2, 1, 3).

3. 3D Scaling in Computer Graphics-

- In computer graphics, scaling is a process of modifying or altering the size of objects.
- Scaling may be used to increase or reduce the size of object.
- Scaling subjects the coordinate points of the original object to change.
- Scaling factor determines whether the object size is to be increased or reduced.
- If scaling factor > 1, then the object size is increased.
- If scaling factor < 1, then the object size is reduced.

Consider a point object O has to be scaled in a 3D plane.

Let-

Initial coordinates of the object O = (X_{old}, Y_{old}, Z_{old})

Scaling factor for X-axis = S_x

Scaling factor for Y-axis = S_y

Scaling factor for Z-axis = S_z

New coordinates of the object O after scaling = (X_{new}, Y_{new}, Z_{new})

This scaling is achieved by using the following scaling equations-

$X_{new} = X_{old} \times S_x$

$Y_{new} = Y_{old} \times S_y$

$Z_{new} = Z_{old} \times S_z$

3. 3D Scaling in Computer Graphics-

In Matrix form, the above scaling equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ Z_{old} \\ 1 \end{bmatrix}$$

3D Scaling Matrix

Problem-01:

Given a 3D object with coordinate points A(0, 3, 3), B(3, 3, 6), C(3, 0, 1), D(0, 0, 0). Apply the scaling parameter 2 towards X axis, 3 towards Y axis and 3 towards Z axis and obtain the new coordinates of the object.

Solution-

Given-

Old coordinates of the object = A (0, 3, 3), B(3, 3, 6), C(3, 0, 1), D(0, 0, 0)

Scaling factor along X axis = 2

Scaling factor along Y axis = 3

Scaling factor along Z axis = 3

For Coordinates A(0, 3, 3)

Let the new coordinates of A after scaling = (X_{new}, Y_{new}, Z_{new}).

Applying the scaling equations, we have-

$X_{new} = X_{old} \times S_x = 0 \times 2 = 0$

$Y_{new} = Y_{old} \times S_y = 3 \times 3 = 9$

$Z_{new} = Z_{old} \times S_z = 3 \times 3 = 9$

Thus, New coordinates of corner A after scaling = (0, 9, 9).

For Coordinates B(3, 3, 6)

Let the new coordinates of B after scaling = (X_{new}, Y_{new}, Z_{new}).

Applying the scaling equations, we have-

$X_{new} = X_{old} \times S_x = 3 \times 2 = 6$
$Y_{new} = Y_{old} \times S_y = 3 \times 3 = 9$
$Z_{new} = Z_{old} \times S_z = 6 \times 3 = 18$

Thus, New coordinates of corner B after scaling = (6, 9, 18).

For Coordinates C(3, 0, 1)

Let the new coordinates of C after scaling = (X_{new}, Y_{new}, Z_{new}).

Applying the scaling equations, we have-

$X_{new} = X_{old} \times S_x = 3 \times 2 = 6$
$Y_{new} = Y_{old} \times S_y = 0 \times 3 = 0$
$Z_{new} = Z_{old} \times S_z = 1 \times 3 = 3$

Thus, New coordinates of corner C after scaling = (6, 0, 3).

For Coordinates D(0, 0, 0)

Let the new coordinates of D after scaling = (X_{new}, Y_{new}, Z_{new}).

Applying the scaling equations, we have-

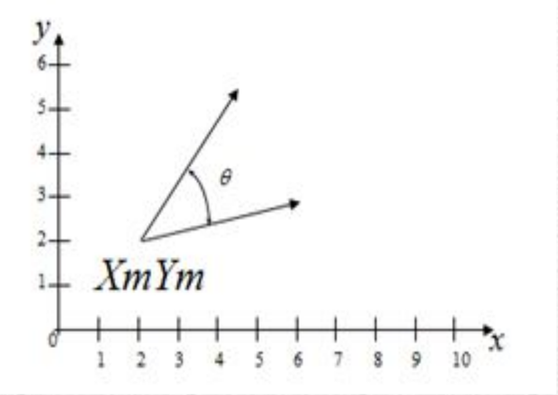
$X_{new} = X_{old} \times S_x = 0 \times 2 = 0$

$Y_{new} = Y_{old} \times S_y = 0 \times 3 = 0$

$Z_{new} = Z_{old} \times S_z = 0 \times 3 = 0$

Thus, New coordinates of corner D after scaling = (0, 0, 0).

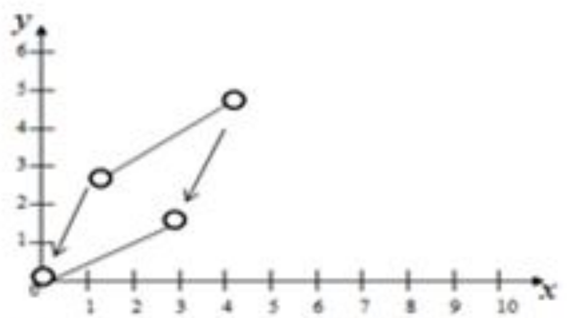
3. Rotation about arbitrary point:



Suppose the reference point of rotation is other than origin, then in that case we have to follow series of transformation.
onsider figure, assume that we have to rotate a point P1 with respect to (Xm, Ym) then we have to perform three steps.

I) **Translation:** First we have to translate the (Xm, Ym) to origin as shown in figure. Translation matrix (T1) will become

$$T1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -Xm & -Ym & 1 \end{bmatrix} \Rightarrow$$

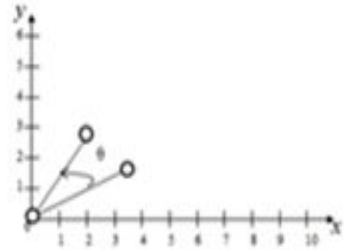


Here transition matrix is Tx=-*Xm* and Ty=-*Ym*

3. Rotation about arbitrary point:

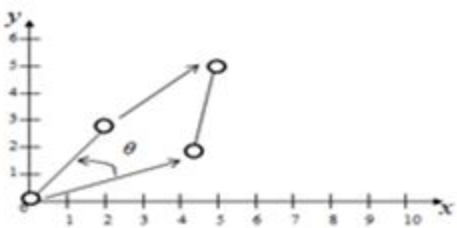
II) Rotation: Rotation it in clockwise or anticlockwise, here we consider the rotation in anticlockwise by angle Θ so our rotation matrix will be as shown in fig.

$$R = \begin{bmatrix} Cos\theta & Sin\theta & 0 \\ -Sin\theta & Cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



III) Transformation: Translate back to original position so the translation matrix T2

$$T2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ Xm & Ym & 1 \end{bmatrix}$$



3. Rotation about arbitrary point:

Lets combine the all matrix

M=Translation*Rotation*Translation

T1*R*T2

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -Xm & -Ym & 1 \end{bmatrix} * \begin{bmatrix} Cos\theta & Sin\theta & 0 \\ -Sin\theta & Cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ Xm & Ym & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -Xm & -Ym & 1 \end{bmatrix} * \begin{bmatrix} Cos\theta & Sin\theta & 0 \\ -Sin\theta & Cos\theta & 0 \\ Xm & Ym & 1 \end{bmatrix}$$

$$= \begin{bmatrix} Cos\theta & Sin\theta & 0 \\ -Sin\theta & Cos\theta & 0 \\ -Xm * Cos\theta + Ym * Sin\theta + Xm & -Xm * Sin\theta - Ym * Cos\theta + Ym & 1 \end{bmatrix}$$

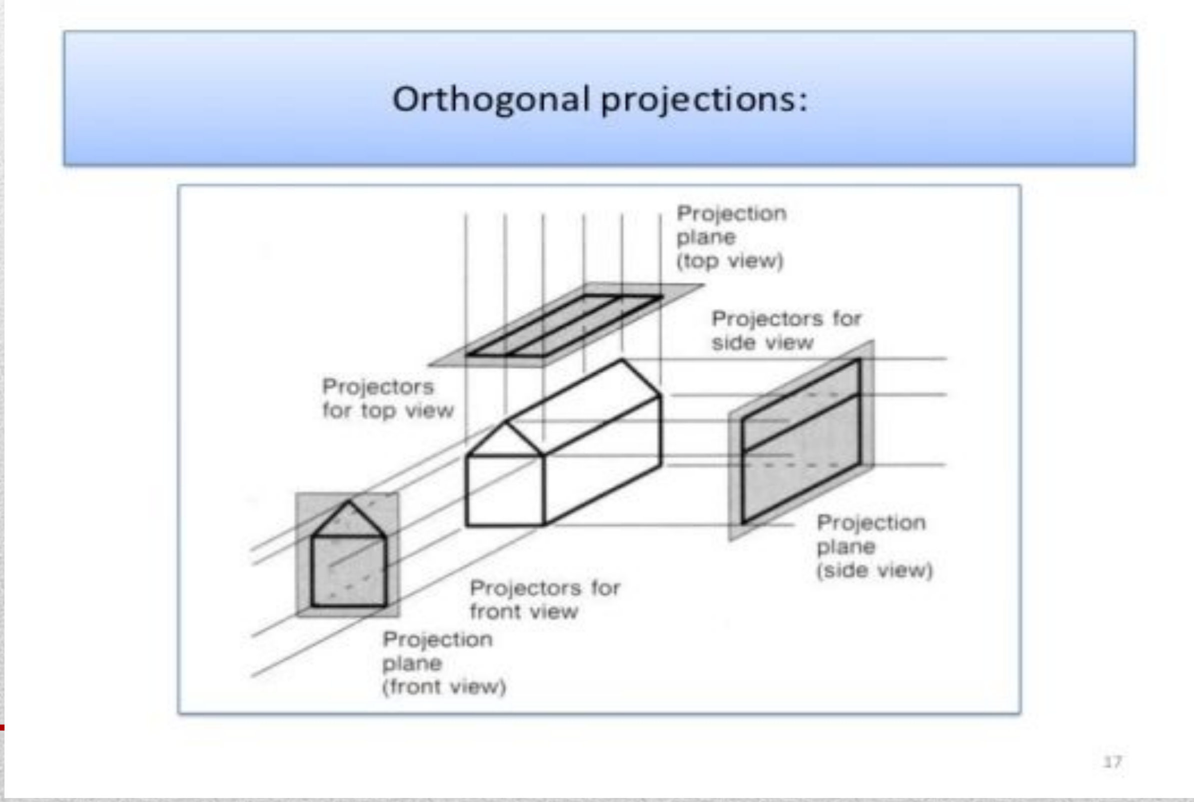
4. Projection

- It is the process of converting a 3D object into a 2D object. .

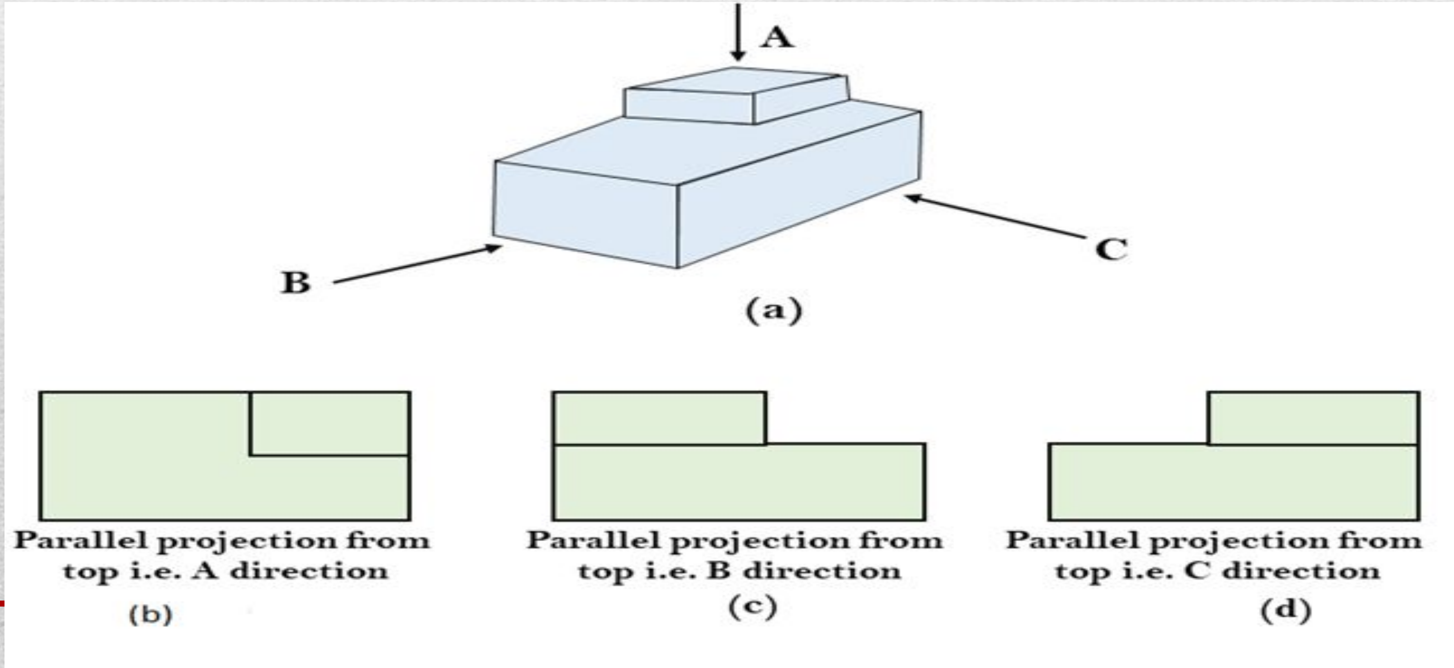
- It is also defined as mapping or transformation of the object in projection plane or view plane. The view plane is displayed surface.

- Types of Projection as follows:

  1. Parallel
  2. Perspective



- **Parallel Projection**
  Parallel Projection use to display picture in its true shape and size.
  When projectors are perpendicular to view plane then is called **orthographic projection**.
- The parallel projection is formed by extending parallel lines from each vertex on the object until they intersect the plane of the screen. The point of intersection is the projection of vertex.
- Parallel projections are used by architects and engineers for creating working drawing of the object, for complete representations require two or more views of an object using different planes.



4.2 Perspective Projection

- In perspective projection farther away object from the viewer, small it appears.
- This property of projection gives an idea about depth. The artist use perspective projection from drawing three-dimensional scenes.
- Two main characteristics of perspective are vanishing points and perspective foreshortening.
- Due to foreshortening object and lengths appear smaller from the center of projection.
- More we increase the distance from the center of projection, smaller will be the object appear.

4.2.1 Vanishing Point

It is the point where all lines will appear to meet. There can be one point, two point, and three point perspectives.

One Point: There is only one vanishing point as shown in fig (a)

Two Points: There are two vanishing points. One is the x-direction and other in the y -direction as shown in fig (b)

Three Points: There are three vanishing points. One is x second in y and third in two directions.

In Perspective projection lines of projection do not remain parallel. The lines converge at a single point called a center of projection. The projected image on the screen is obtained by points of intersection of converging lines with the plane of the screen. The image on the screen is seen as of viewer's eye were located at the centre of projection

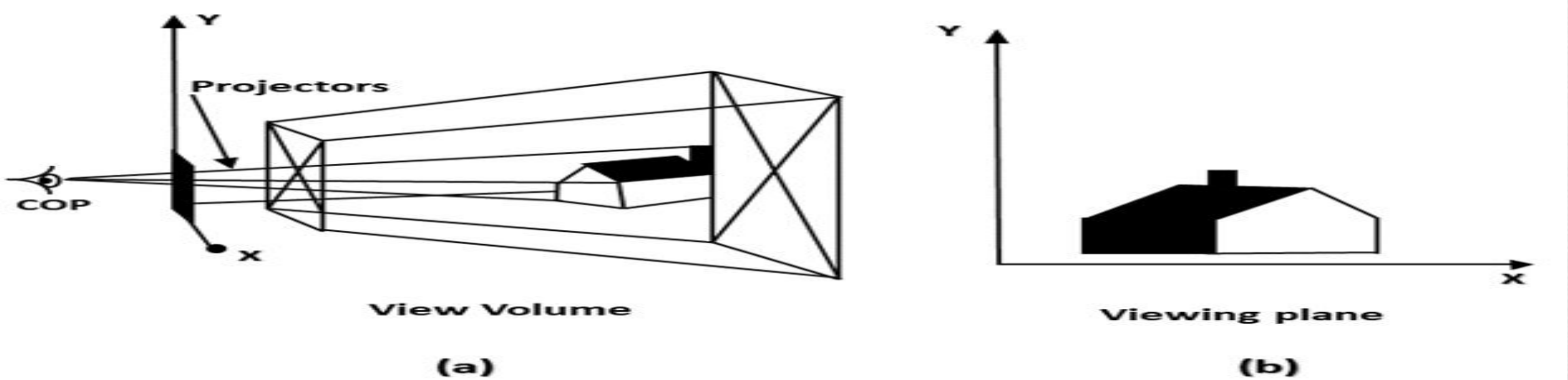
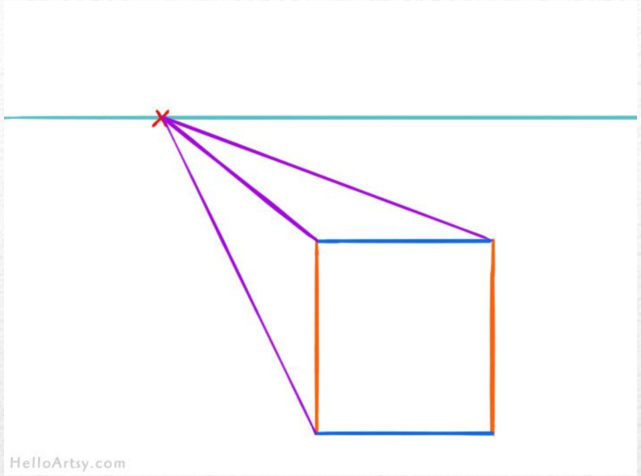
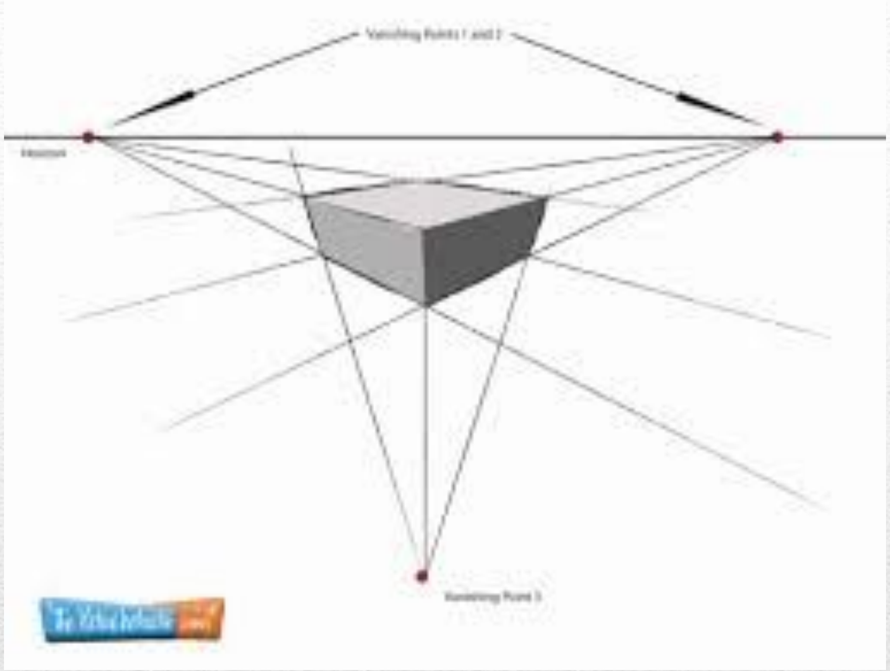


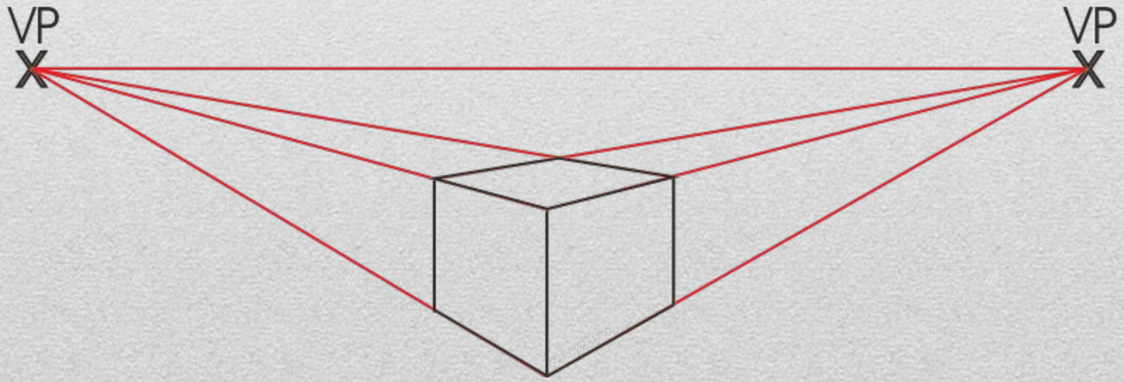
Fig: Prespective Projection



1 Point vanishing



3 Point vanishing



2 Point vanishing