* Matrices:-

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$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

Element member

Principal diagonal element
Or diagonal leading element.

Non-diagonal element

Row(i)

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

Column(j)

Let $A = \begin{bmatrix} -1 & 2 & 3 \\ 4 & 5 & 0 \\ 5 & 0 & -4 \end{bmatrix}$

$a_{11} = -1$, $a_{21} = 4$, $a_{31} = 3$
$a_{12} = 2$, $a_{22} = 5$, $a_{32} = 0$
$a_{13} = 3$, $a_{23} = 0$, $a_{33} = -4$

$\boxed{A = a[ij](m \times n)}$

matrix, Row, Column, Order

m = number of rows
n = no of column

order = $m \times n$

$\begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{bmatrix}$

Q. If $B = [3i+j]$, find b_{32}, b_{23}, b_{13}?

Sol:- $b_{32} = 3\times3+2 = 11$
$b_{23} = 2\times3+3 = 9$
$b_{13} = 1\times3+3 = 6$

Q. If $B = [3i+j]$, find 2×3

$b_{11} = 3\times1+1 = 4$
$b_{12} = 3\times1+2 = 5$
$b_{13} = 3\times1+3 = 6$
$b_{21} = 3\times2+1 = 7$
$b_{22} = 3\times2+2 = 8$
$b_{23} = 3\times2+3 = 9$

Q. If $A = [3i-j]_{3\times4}$

Sol: ~~$a_{11} = 3\times1-1 = 2$~~
~~$a_{12} = 3\times1-2 = 1$~~
~~$a_{13} = 3\times1-3 = 0$~~
~~$a_{21} = 3\times2-1 = 5$~~
~~$a_{22} = 3\times2-2 = 4$~~
~~$a_{23} = 3\times2-3 = 3$~~
~~a_{31}:~~

$a_{11} = 3\times1-1 = 2$
$a_{12} = 3\times1-2 = 1$
$a_{13} = 3\times1-3 = 0$
$a_{14} = 3\times1-4 = -1$

$a_{21} = 3\times2-1 = 5$, $a_{31} = 3\times3-1 = 8$
$a_{22} = 3\times2-2 = 4$, $a_{32} = 3\times3-2 = 7$
$a_{23} = 3\times2-3 = 3$, $a_{33} = 3\times3-3 = 6$
$a_{24} = 3\times2-4 = 2$, $a_{34} = 3\times3-4 = 5$

$$= \begin{bmatrix} 2 & 1 & 0 & -1 \\ 5 & 4 & 3 & 2 \\ 8 & 7 & 6 & 5 \end{bmatrix}$$

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* Types of matrices:-

$\boxed{A = [a_{ij}]_{m\times n}}$

(i) Rectangular matrices. $m \neq n$
(ii) Square matrices $m = n$
(iii) Zero, null/void matrices.

(i) Row matrix.

$[1\ 2\ 3]_{1\times3}$ $[1, 2]_{1\times2}$ $[1, 0, 3, 7, 6]_{1\times5}$

(ii) Column vector:

$A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}_{3\times1}$, $B = \begin{bmatrix} 1 \\ 2 \end{bmatrix}_{2\times1}$

$C = \begin{bmatrix} 0 \\ 1 \\ 3 \\ 4 \end{bmatrix}_{4\times1}$

* Types of Matrix:-

(1) Rectangular Matrix:- $m \neq n$
(ii) Square Matrix:- $m = n$
(3) Zero/ Null/ void matrix :- (O) Capital
(4) Row matrix:- $m=1$, $n=n$, order $(m>1)$

eg: $n = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}_{3\times1}$ eg: $\begin{bmatrix} 2 \\ 1 \end{bmatrix}_{2\times1}$

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Type of square matrix :- $m = n$

(a) Diagonal matrix :- $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}_{3\times3}$

$A = \begin{bmatrix} 2 & 0 \\ 0 & -3 \end{bmatrix}_{2\times2}$

(b) Scalar Matrix :- $A = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}_{3\times3}$

$A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}_{2\times2}$

(c) unit / Identity matrix :- $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3\times3}$

$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times2}$

(d) Triangular matrix :- $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 6 \\ 0 & 0 & 4 \end{bmatrix}_{3\times3}$ upper triangular matrix

$A = \begin{bmatrix} 2 & 0 & 0 \\ & 3 & 0 \\ 5 & & 4 \end{bmatrix}_{3\times3}$ lower triangular matrix.

Notes:

(a) [Principal diagonal element के नीचे शून्य 0 (zero) हो तो upper triangular matrix ~~कहते~~ कहा जाएगा]

(b) [Principal diagonal element के ऊपर शून्य element 0 (zero) हो तो उसे lower triangular matrix कहा जाएगा]

Symmetric matrix :-

$A = \begin{bmatrix} a & b & e \\ b & b & f \\ c & f & c \end{bmatrix}_{3\times3}$

$A = \begin{bmatrix} 1 & 5 & 4 \\ 5 & 2 & 6 \\ 4 & 6 & 3 \end{bmatrix}_{3\times3}$

Skew Symmetric matrix :-

$a_{ij} = -a_{ij}, \; -a_{ij} = a_{ij}$

$A = \begin{bmatrix} 0 & -h & e \\ h & 0 & -f \\ -e & f & 0 \end{bmatrix}_{3\times3}$

$A = \begin{bmatrix} 0 & 5 & -6 \\ -5 & 0 & -4 \\ 6 & 4 & 0 \end{bmatrix}_{3\times3}$

Note :-
$A = [5]_{1\times1}$
- Square matrix
- Row matrix
- Column matrix

~~Zoid~~ Zero / null / void matrix = (O) Capital (O)

$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{2\times3}$

$A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}_{2\times3}$

Note :-

$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times2}$
- Square matrix
- unit / Identity matrix
- Triangular matrix

3/12/24

(*) Algebra of matrix:-

(a) Scalar multiplication.
(b) Addition of matrix.
(c) Subtraction of matrix

(*) Scalar multiplication:-
multiply by 2

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $2A = 2\begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix}$

multiply by 3 [Common = 3]

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $3A = 3\begin{bmatrix} 3 & 6 \\ 9 & 12 \end{bmatrix} = 3\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

(*) Properties of Scalar ~~Matrix~~ multiplication:-

(*)
$$\boxed{\begin{aligned} m(A+B) &= mA+mB \\ (A+B)m &= Am+Bm \\ m(nA) &= n(mA) = mnA \end{aligned}}$$

Q. Find 'X' such that $3X + \begin{bmatrix} 4 & 5 \\ 1 & -3 \end{bmatrix} = \begin{bmatrix} 7 & 11 \\ -8 & 9 \end{bmatrix}$

Sol:- $3X + \begin{bmatrix} 4 & 5 \\ 1 & -3 \end{bmatrix} = \begin{bmatrix} 7 & 11 \\ -8 & 9 \end{bmatrix}$

$= 3X = \begin{bmatrix} 7 & 11 \\ -8 & 9 \end{bmatrix} - \begin{bmatrix} 4 & 5 \\ 1 & -3 \end{bmatrix}$

$3X = \begin{bmatrix} (7-4) & (11-5) \\ (-8-1) & (9-(-3)) \end{bmatrix} = \begin{bmatrix} 3 & 6 \\ -9 & 12 \end{bmatrix}$

$3X = \begin{bmatrix} 3 & 6 \\ -9 & 12 \end{bmatrix}$

$X = \frac{1}{3}\begin{bmatrix} 3 & 6 \\ -9 & 12 \end{bmatrix}$

$X = \frac{1}{\cancel{3}} \times \cancel{3}\begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix}$

$X = \begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix}$

Q. If $A = \begin{bmatrix} -2 & 4 \\ 1 & 3 \end{bmatrix}$, find matrix 'X' such that $A - 2X = \begin{bmatrix} -2 & -8 \\ 3 & -5 \end{bmatrix}$

Sol:- $A - 2X = \begin{bmatrix} -2 & -8 \\ 3 & 5 \end{bmatrix}$

$\begin{bmatrix} -2 & 4 \\ 1 & 3 \end{bmatrix} - 2X = \begin{bmatrix} -2 & -8 \\ 3 & 5 \end{bmatrix}$

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$$-2X = \begin{bmatrix}-2 & -8\\ 3 & -5\end{bmatrix} - \begin{bmatrix}-2 & 4\\ 1 & 3\end{bmatrix}$$

$$-2X = \begin{bmatrix}0 & -12\\ 2 & -8\end{bmatrix}$$

$$X = -\frac{1}{2}\begin{bmatrix}0 & -12\\ 2 & -8\end{bmatrix}$$

$$X = -\frac{1}{\cancel{2}}\cdot\cancel{2}\begin{bmatrix}0 & -6\\ 1 & -4\end{bmatrix}$$

$$X = -1\begin{bmatrix}0 & -6\\ 1 & -4\end{bmatrix}$$

$$X = \begin{bmatrix}0 & 6\\ -1 & 4\end{bmatrix}$$

Multiplication of matrix:-

$$A = \begin{bmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\end{bmatrix}_{m\times n} \quad B = \begin{bmatrix}b_{11} & b_{12}\\ a_{21} & b_{22}\\ b_{31} & a_{32}\end{bmatrix}_{n\times p}$$

$$A = \begin{bmatrix} \end{bmatrix}_{m\times n} \times B = \begin{bmatrix} \end{bmatrix}_{n\times p} = AB = \begin{bmatrix} \end{bmatrix}_{m\times p}$$

$$A = \begin{bmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\end{bmatrix} \quad B = \begin{bmatrix}b_{11} & b_{12}\\ b_{21} & b_{22}\\ b_{31} & b_{32}\end{bmatrix}$$

$$AB = \begin{bmatrix}a_{11}\cdot b_{11} + a_{12}\cdot b_{21} + a_{13}\cdot b_{31} & a_{11}\cdot b_{12} + a_{12}\cdot b_{22} + a_{13}b_{32}\\ a_{21}\cdot b_{11} + a_{22}\cdot b_{21} + a_{23}\cdot b_{31} & a_{21}\cdot b_{12} + a_{22}\cdot b_{22} + a_{23}\cdot b_{32}\end{bmatrix}$$

$$BA = \begin{bmatrix}b_{11} & b_{12}\\ b_{21} & b_{22}\\ b_{31} & b_{32}\end{bmatrix} \times \begin{bmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\end{bmatrix}$$

$$\begin{bmatrix}b_{11}\cdot a_{11} + b_{12}\cdot a_{21} & b_{11}\cdot a_{12} + b_{12}\cdot a_{22} & b_{11}\cdot a_{13} + b_{12}a_{23}\\ b_{21}\cdot a_{11} + b_{22}\cdot a_{21} & b_{21}\cdot a_{12} + b_{22}\cdot a_{22} & b_{21}\cdot a_{13} + b_{22}\cdot a_{23}\\ b_{31}\cdot a_{11} + b_{32}\cdot a_{21} & b_{31}\cdot a_{12} + b_{32}\cdot a_{22} & b_{31}\cdot a_{13} + b_{32}\cdot a_{23}\end{bmatrix}$$

Q. Find $A = \begin{bmatrix}2 & 3\\ 4 & 5\end{bmatrix}$, $B = \begin{bmatrix}2 & 1 & 4\\ 3 & 0 & -1\end{bmatrix}$ find AB and BA?

Sol: $AB = \begin{bmatrix}2 & 3\\ 4 & 5\end{bmatrix} \cdot \begin{bmatrix}2 & 1 & 4\\ 3 & 0 & -1\end{bmatrix}$

$$AB = \begin{bmatrix}2\times2+3\times3 & 2\times1+3\times0 & 2\times4+(3\times-1)\\ 4\times2+5\times3 & 4\times1+5\times0 & 4\times4+(5\times-1)\end{bmatrix}$$

$$AB = \begin{bmatrix}4+9 & 2+0 & 8-3\\ 8+15 & 4+0 & 16-5\end{bmatrix} = \begin{bmatrix}13 & 2 & 5\\ 23 & 4 & 11\end{bmatrix}$$

So, AB does'nt exist to BA

Q If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

find K so, that $A^2 = KA - 2I$?

Sol: $A^2 = A \times A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} \times \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$

$= \begin{bmatrix} 3\times3+(-2\times4) & 3\times-2+(-2\times-2) \\ 4\times3+(-2\times4) & 4\times(-2)+(-2\times-2) \end{bmatrix}$

$= \begin{bmatrix} 9-8 & -6+4 \\ 12-8 & -8+4 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix}$

$2I = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$KA = \begin{bmatrix} 3K & -2K \\ 4K & -4K \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix}$

$\begin{bmatrix} 3K & -2K \\ 4K & -4K \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$= \begin{bmatrix} 3K & -2K \\ 4K & -4K \end{bmatrix} = \begin{bmatrix} 3 & -2+0 \\ 4+0 & -4+2 \end{bmatrix}$

$= \begin{bmatrix} 3K & -2K \\ 4K & -4K \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$

$3K = 3$

$K = \cancel{3}/\cancel{3} = 1$

~~$4K = 2$~~ $\quad 4K = 4$

~~$K = \frac{4}{2}$~~ $\quad K = \frac{4}{4} = 1$

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Determinants

4 / 12 / 24

(*) Evaluate

(1) $\begin{vmatrix} 2 & 5 & 7 \\ 3 & 2 & 8 \\ 6 & 4 & 1 \end{vmatrix}$

Sol: $2\begin{vmatrix} 2 & 8 \\ 4 & 1 \end{vmatrix} - 5\begin{vmatrix} 3 & 8 \\ 6 & 1 \end{vmatrix} + 7\begin{vmatrix} 3 & 2 \\ 6 & 4 \end{vmatrix}$

$2(2-32) - 5(3-48) + 7(12-12)$

$2(-30) - 5(-45) + 7(0)$

$-60 + 225$

$= 165$ Ans

(2) $\begin{vmatrix} -3 & 7 \\ -5 & 4 \end{vmatrix}$

Sol: $-12-(-35) = -12+35 = 23$

(3) Find the value of x $\begin{vmatrix} 2 & 3 & 1 \\ 6 & x & 2 \\ 4 & x & -2 \end{vmatrix}$

Sol: $2\begin{vmatrix} x & 2 \\ x & -2 \end{vmatrix} - 3\begin{vmatrix} 6 & 2 \\ 4 & -2 \end{vmatrix} + 1\begin{vmatrix} 6 & x \\ 4 & x \end{vmatrix} = 0$

$2(-2x-2x) - 3(-12-8) + 1(6x-4x) = 0$

$2(-4x) - 3(-20) + (2x)$

$-8x + 60 + 2x$

$60 = 6x$

~~$+6x = 60$~~

$x = \frac{60}{6}$ 10

$x = 10$ Ans

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(4) $\begin{vmatrix} 1 & -2 & 4 \\ 1 & x & x^2 \\ 4 & 6 & 9 \end{vmatrix} = \begin{vmatrix} 3 & 6 \\ -2 & -4 \end{vmatrix}$

Sol: $1\begin{vmatrix} x & x^2 \\ 6 & 9 \end{vmatrix} + 2\begin{vmatrix} 1 & x^2 \\ 4 & 9 \end{vmatrix} + 4\begin{vmatrix} 1 & x \\ 4 & 6 \end{vmatrix}$

$9x - 6x^2 + 2(9 - 4x^2) + 4(6 - 4x)$

$9x - 6x^2 + 18 - 8x^2 + 24 - 16x$

$-14x^2 - 7x - 42 = 0$

$= -7(2x^2 + x - 6) = 0$

$= 2x^2 + 4x - 3x - 6 = 0$

$2x(x+2) - 3(x+2) = 0$

$(2x-3)(x+2)$

$2x - 3 = 0$

$2x = 3$

$x = 3/2$ or $x + 2 = 0$, $x = -2$

Q. Find x, y and z, if $\begin{bmatrix} 4 & 1 & 2 \\ 0 & 5 & 3 \end{bmatrix} \begin{bmatrix} 3 & 4 & 5 \\ -1 & 0 & -2 \\ 3 & 4 & 7 \end{bmatrix}$

$= \begin{bmatrix} 8x+3y & 6z & 32 \\ 4 & 12 & 26x-5y \end{bmatrix}$

Sol: $\begin{bmatrix} 4 & 1 & 2 \\ 0 & 5 & 3 \end{bmatrix} \cdot \begin{bmatrix} 3 & 4 & 5 \\ -1 & 0 & -2 \\ 3 & 4 & 7 \end{bmatrix}$

$\begin{bmatrix} (4\times3) + (1\times-1) + (2\times3) & 4\times4 + 1\times0 + 2\times4 & 4\times5 + (1\times-2) + 2\times7 \\ (0\times3) + (5\times-1) + (3\times3) & (0\times4) + 5\times0 + 3\times4 & (0\times5) + (5\times-2) + 9\times7 \end{bmatrix}$

$\begin{bmatrix} 12-1+6 & 16+0+8 & 20-2+14 \\ 0-5+9 & 0+0+12 & 0-10+21 \end{bmatrix}$

$= \begin{bmatrix} 17 & 24 & 32 \\ 4 & 12 & 11 \end{bmatrix} = \begin{bmatrix} 8x+3y & 6z & 32 \\ 4 & 12 & 26x-5y \end{bmatrix}$

$8x + 3y = 17$ —— (i)

$26x - 5y = 11$ —— (ii)

$\cancel{6}z = \cancel{24}\,4$

$z = 4$

Now, from (i) and (ii)

$8x + 3y = 17 \times 5$

$26x - 5y = 11 \times 3$

$= 40x + \cancel{15y} = 85$

$78x - \cancel{15y} = 33$

$118x = 118$

$x = \frac{\cancel{118}}{\cancel{118}} = 1$

Now, again from (i)

$8x + 3y = 17$

$8\times1 + 3y = 17$

$3y = 17 - 8$

$\cancel{3}y = \cancel{9}\,3$

$y = 3$

Q. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, find $f(A)$, where $f(x) = x^2 - 5x + 7$

Now put the value of A in place of x

$x^2 - 5x + 7 = A^2 - 5A + 7$

$A^2 = A \times A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \times \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$= \begin{bmatrix} (3\times3)+(1\times-1) & (3\times1)+(1\times2) \\ (-1\times3)+(2\times-1) & (-1\times1)+(2\times2) \end{bmatrix}$

$= \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$

$-5A = -5 \times \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$= \begin{bmatrix} -15 & -5 \\ 5 & -10 \end{bmatrix} \quad \begin{bmatrix} 15 & 5 \\ -5 & 3 \end{bmatrix}$

Now, $A^2 - 5A + 7I$

$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} -15 & -5 \\ 5 & -10 \end{bmatrix} + 7I$

$= \begin{bmatrix} 8-(-15) & 5-(-5) \\ (-5-5) & 3-(-10) \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$= \begin{bmatrix} 23 & -10 \\ -10 & 13 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$= \begin{bmatrix} 23+7 & 10+0 \\ 10+0 & 13+7 \end{bmatrix} = \begin{bmatrix} 30 & 10 \\ 10 & 20 \end{bmatrix}$

Now, $A^2 - 5A + 7I$

$\begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$= \begin{bmatrix} 8-15+7 & 5-5+0 \\ -5+5+0 & 3-10+7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}_{2\times2} = O$

Properties of multiplication:-

Note: Commutative
$AB = BA$

- $A(BC) = (AB)C$
- $AB \neq BA$ (Not in all cases)
- $A(B+C) = AC + AC$
- $(A+B)C = AC + BC$
- $AI = IA = A$

Notes:
Closer law ($3+2=5$)
Commutative law: $A+B = B+A$, $AB = BA$
Associative law: $(A+B)+C = A+(B+C)$, $(AB)C = A(BC)$
Distributive law: $A(B+C) = AB+AC$

Transpose of matrix:-

A' or $A^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}' = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$A = [a_{ij}]_{m\times n}$ then A' or $A^T = [a_{ji}]_{n\times m}$

eg:- $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{2\times3}$

$A' = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}_{3\times2}$

Properties of Transpose of matrix →

$A = [\ \ ]$

- $(A')' = A$
- $(KA)' = (KA)'$ K: constant number
- $(A+B)' = A' + B'$
- $(AB)' = B'A'$ (V.V.I)

Transpose of Matrix (A)

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{2\times3}, \quad A' = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}_{3\times2}$$

Q. $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, find $A+A'$ or $A-A'$

Sol: $A+A'$.

$$A' = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

$$A+A' = \begin{bmatrix} 1+1 & 2+3 \\ 3+2 & 4+4 \end{bmatrix} = \begin{bmatrix} 2 & 5 \\ 5 & 8 \end{bmatrix}$$ Symmetric matrix

Now $A - A'$

$$= \begin{bmatrix} 1-1 & 2-3 \\ 3-2 & 4-4 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$ Skew Symmetric matrix

Symmetric matrix:

A = Square matrix

$\boxed{A' = A}$ Symmetric matrix

eg $A = \begin{bmatrix} 2 & 5 \\ 5 & 8 \end{bmatrix}$

$$A' = \begin{bmatrix} 2 & 5 \\ 5 & 8 \end{bmatrix} = A$$ $(A' = A)$ Symmetric matrix

Skew Symmetric matrix

A = Square matrix.

$\boxed{A' = -A}$ Skew Symmetric matrix

eg:- $\begin{bmatrix} 0 & 5 \\ -5 & 0 \end{bmatrix}$, $A' = \begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}$

$$= -\begin{bmatrix} 0 & 5 \\ -5 & 0 \end{bmatrix} = -A$$ (Skew Symmetric matrix.

Q. Prove that for $A = \begin{bmatrix} 4 & 1 \\ 5 & 8 \end{bmatrix}$, $A+A'$ is a Symmetric matrix, where A' denotes the transpose of A.

Sol: $A' = \begin{bmatrix} 4 & 5 \\ 1 & 8 \end{bmatrix}$

$$A + A' = \begin{bmatrix} 4 & 5 \\ 1 & 8 \end{bmatrix} + \begin{bmatrix} 4 & 1 \\ 5 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} 4+4 & 5+1 \\ 5+1 & 8+8 \end{bmatrix} = \begin{bmatrix} 8 & 6 \\ 6 & 16 \end{bmatrix}$$ Symmetric matrix. Hence Proof

SPS

Q If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ Show that $A'A = I_2$

Sol: $A' = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

~~$A'A = [\cos\theta \cdot \cos\theta + \sin$~~

$A'A = (\cos\theta \cdot \cos\theta - (-\sin\theta \cdot \sin\theta))$
$= \cos^2\theta + \sin^2\theta$
$= 1$

$A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, $A' = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

$A'A = \begin{bmatrix} \cos^2\theta + \sin^2\theta & \cos\theta\sin\theta - \sin\theta\cos\theta \\ \sin\theta\cos\theta - \cos\theta\sin\theta & \sin^2\theta + \cos^2\theta \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ Hence proved

Q If $A = \begin{bmatrix} 3 & -1 \\ 2 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -3 \\ 2 & -1 \end{bmatrix}$, show that $(AB)' = B'A'$

Sol: $(A \times B) = \begin{bmatrix} 3 & -1 \\ 2 & -2 \end{bmatrix} \times \begin{bmatrix} 1 & -3 \\ 2 & -1 \end{bmatrix}$

$= \begin{bmatrix} 3\times1 + (-1\times2) & 3\times-3 + (-1\times-1) \\ 2\times1 + (-2\times2) & 2\times-3 + (-2\times-1) \end{bmatrix}$

$= \begin{bmatrix} 3-2 & -9+1 \\ 2-4 & -6+2 \end{bmatrix} = \begin{bmatrix} 1 & -8 \\ -2 & -4 \end{bmatrix}$

$(AB)' = \begin{bmatrix} 1 & -2 \\ -8 & -4 \end{bmatrix}$

Now $B' = \begin{bmatrix} 1 & 2 \\ -3 & -1 \end{bmatrix}$

and $A' = \begin{bmatrix} 3 & 2 \\ -1 & -2 \end{bmatrix}$

Now $= B' \cdot A' = \begin{bmatrix} 1 & 2 \\ -3 & -1 \end{bmatrix} \cdot \cancel{\begin{bmatrix} 1 & 2 \\ -3 & -1 \end{bmatrix}} \begin{bmatrix} 3 & 2 \\ -1 & -2 \end{bmatrix}$

$= \cancel{\begin{bmatrix} (1\times3)+(2\times-3) & 1\times2+(2\times-1) \\ -3\times1+(-1\times-3) & (-3\times2)+(-1\times-1) \end{bmatrix}} = \begin{bmatrix} 1 & -2 \\ -8 & -4 \end{bmatrix}$

$= \cancel{\begin{bmatrix} 1-6 & 2-2 \\ -3+3 & -6+1 \end{bmatrix}} = \begin{bmatrix} -5 & 0 \\ 0 & 5 \end{bmatrix}$

Q If $A = \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 4 \end{bmatrix}$, Show that $(AB)' = B'A'$?

Sol:- $(AB)' = \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 4 \end{bmatrix}$

$= \begin{bmatrix} 3\times1 & 3\times0 & 3\times4 \\ 5\times1 & 5\times0 & 5\times4 \\ 2\times1 & 2\times0 & 2\times4 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 12 \\ 5 & 0 & 20 \\ 2 & 0 & 8 \end{bmatrix}$

$AB' = \begin{bmatrix} 3 & 5 & 2 \\ 0 & 0 & 0 \\ 12 & 20 & 8 \end{bmatrix}$

$B' = \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix}$ and $A' = \begin{bmatrix} 3 & 5 & 2 \end{bmatrix}$

$B \cdot A' = \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix} \cdot \begin{bmatrix} 3 & 5 & 2 \end{bmatrix}$

$\begin{bmatrix} 1\times3 & 1\times5 & 1\times2 \\ 0\times3 & 0\times5 & 0\times2 \\ 4\times3 & 4\times5 & 4\times2 \end{bmatrix} = \begin{bmatrix} 3 & 5 & 2 \\ 0 & 0 & 0 \\ 12 & 20 & 8 \end{bmatrix}$ Hence proof

* $A =$ any square matrix

A'

$A + A' =$ Symmetric matrix

$A - A' =$ Skew Symmetric matrix

$A = \underbrace{\frac{1}{2}(A+A')}_{\text{Symmetric matrix}} + \underbrace{\frac{1}{2}(A-A')}_{\text{Skew Symmetric matrix}}$

$= \frac{1}{2}A + \cancel{\frac{1}{2}A'} + \frac{1}{2}A - \cancel{\frac{1}{2}A'}$

$= \frac{\cancel{2}A}{\cancel{2}} = A$

Q. Express the matrix $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ as the sum of a symmetric matrix and skew-symmetric matrix.

Sol:- $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, $A' = \begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}$

$A + A' = \begin{bmatrix} 3+3 & -4+1 \\ 1+(-4) & -1+(-1) \end{bmatrix}$

$= \begin{bmatrix} 6 & -3 \\ -3 & -2 \end{bmatrix}$

$A - A' = \begin{bmatrix} 3-3 & -4-1 \\ 1-(-4) & -1-(-1) \end{bmatrix} = \begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}$

$= \frac{1}{2}\begin{bmatrix} 6 & -3 \\ -3 & -2 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}$

$= \begin{bmatrix} 3 & -3/2 \\ -3/2 & -1 \end{bmatrix} + \begin{bmatrix} 0 & -5/2 \\ 5/2 & 0 \end{bmatrix}$

$= \begin{bmatrix} 3+0 & -3/2 + -5/2 \\ -3/2 + 5/2 & -1+0 \end{bmatrix}$

$= \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ Hence proof

* Orthogonal matrix:

$A =$ Square matrix

$A' =$

$\boxed{A \cdot A' = I}$ ∴ Orthogonal matrix

$A + A' =$ Symmetric matrix

$A - A' =$ Skew Symmetric matrix

$A = \frac{1}{2}(A+A') + \frac{1}{2}(A-A')$

Q If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ then, verify that $A \cdot A' = I$.

Sol: $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, $A' = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$A \cdot A' = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \cdot \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

$= \begin{bmatrix} \cos^2\theta + \sin^2\theta & \cos\theta \cdot \sin\theta - \sin\theta \cdot \cos\theta \\ \sin\theta \cdot \cos\theta - \cos\theta \sin\theta & \sin^2\theta + \cos^2\theta \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ~~proo~~ verified.

Q Prove that, $A = \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix}$ is an orthogonal matrix.

Sol: $A = \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix}$

$\cancel{A = \begin{bmatrix} 1/3 & 2/3 & 2/3 \\ 2/3 & 1/3 & -2/3 \\ -2/3 & 2/3 & -1/3 \end{bmatrix}}$

$A' =$

$\cancel{A' = \begin{bmatrix} 1/3 & 2/3 & -2/3 \\ 2/3 & 1/3 & 2/3 \\ 2/3 & -2/3 & -1/3 \end{bmatrix}}$

$\cancel{A \cdot A' = \begin{bmatrix} 1/3 & 2/3 & 2/3 \\ 2/3 & 1/3 & -2/3 \\ -2/3 & 2/3 & -1/3 \end{bmatrix} \cdot \begin{bmatrix} 1/3 & 2/3 & -2/3 \\ 2/3 & 1/3 & 2/3 \\ 2/3 & -2/3 & -1/3 \end{bmatrix}}$

$A' = \frac{1}{3}\begin{bmatrix} 1 & 2 & -2 \\ 2 & 1 & 2 \\ 2 & -2 & -1 \end{bmatrix}$

$A \cdot A' = \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix} \cdot \frac{1}{3}\begin{bmatrix} 1 & 2 & -2 \\ 2 & 1 & 2 \\ 2 & -2 & -1 \end{bmatrix}$

$= \frac{1}{9} \cdot \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & -2 \\ 2 & 1 & 2 \\ 2 & -2 & -1 \end{bmatrix}$

$= \frac{1}{9}\begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ Proved

Q. Using Cramer's rule find $x, y,$ and z
$x+y+z=1$, $2x+3y+z-4=0$, $4x+z+9y=16$?

Sol: $x+y+z=1$
$2x+3y+z=4$
$4x+9y+z=16$

Δ or D $= \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 1 \\ 4 & 9 & 1 \end{vmatrix}$

$1\begin{vmatrix} 3 & 1 \\ 9 & 1 \end{vmatrix} - 1\begin{vmatrix} 2 & 1 \\ 4 & 1 \end{vmatrix} + 1\begin{vmatrix} 2 & 3 \\ 4 & 9 \end{vmatrix}$

$= 1(3-9) - 1(2-4) + 1(18-12)$
$= \cancel{-6} + 2 + \cancel{6}$
$= 2$

$D_x = \begin{vmatrix} 1 & 1 & 1 \\ 4 & 3 & 1 \\ 16 & 9 & 1 \end{vmatrix} = 1\begin{vmatrix} 3 & 1 \\ 9 & 1 \end{vmatrix} - 1\begin{vmatrix} 4 & 1 \\ 16 & 1 \end{vmatrix} + 1\begin{vmatrix} 4 & 3 \\ 16 & 9 \end{vmatrix}$

$1(3-9) - 1(4-16) + 1(36-48)$
$= -6 + \cancel{12} - \cancel{12} = -6$

$D_y = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & 1 \\ 4 & 16 & 1 \end{vmatrix} = 1\begin{vmatrix} 4 & 1 \\ 16 & 1 \end{vmatrix} - 1\begin{vmatrix} 2 & 1 \\ 4 & 1 \end{vmatrix} + 1\begin{vmatrix} 2 & 4 \\ 4 & 16 \end{vmatrix}$

$= 1(4-16) - 1(2-4) + 1(32-16)$
$= -12 + 2 + 16 = 6$

$D_z = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 4 & 9 & 16 \end{vmatrix} = 1\begin{vmatrix} 3 & 4 \\ 9 & 16 \end{vmatrix} - 1\begin{vmatrix} 2 & 4 \\ 4 & 16 \end{vmatrix} + 1\begin{vmatrix} 2 & 3 \\ 4 & 9 \end{vmatrix}$

$= 1(48-36) - 1(32-16) + 1(18-12)$
$= 12 - 16 + 6 = 2$

$x = \frac{D_x}{D} = \frac{-\cancel{6}}{\cancel{2}} = -3$

$y = \frac{D_y}{D} = \frac{\cancel{6}}{\cancel{2}} = 3$

$z = \frac{D_z}{D} = \frac{\cancel{2}}{\cancel{2}} = 1$

Q (v.v.I)
$xy + yz + zx = xyz$
$2xy + 3yz + 2x = 4xyz$
$4xy + 9yz + 2x = 16xyz$

Sol: divided by xyz

$\frac{xy}{xyz} + \frac{yz}{xyz} + \frac{zx}{xyz}$

$= \frac{1}{z} + \frac{1}{x} + \frac{1}{y} = 1$ — (i)

$\frac{2}{z} + \frac{3}{x} + \frac{1}{y} = 4$ — (ii)

$\frac{4}{z} + \frac{9}{x} + \frac{1}{y} = 16$ — (iii)

$\frac{1}{x} = u$, $\frac{1}{y} = v$, $\frac{1}{z} = w$

$w+u+v=1$
$2w+3u+v=4$
$4w+9y+v=16$

Q Solve the following equation by Cramer's rule.
$x-2y=0$
$7x+6y=40$

Sol: Δ or D $\begin{vmatrix} 1 & -2 \\ 7 & 6 \end{vmatrix}$
$= 6+14 = 20 \neq 0$

$D_x = \begin{vmatrix} 0 & -2 \\ 40 & 6 \end{vmatrix} = 0+80 = 80$

$D_y = \begin{vmatrix} 1 & 0 \\ 7 & 40 \end{vmatrix} = 40+0 = 40$

$\frac{D_x}{D} = \cancel{\frac{80}{20}}\ 4$

$\frac{D_y}{D} = \cancel{\frac{40}{20}}\ 2$

Q Solve the following eq by Cramer's rule
$2x-y-z=4$, $-x+2y-z=-5$, $x-y+2z=1$

Sol: Δ $\begin{vmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{vmatrix}$

$2\begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} +1\begin{vmatrix} -1 & 1 \\ 1 & 2 \end{vmatrix} -1\begin{vmatrix} -1 & 2 \\ 1 & -1 \end{vmatrix}$
$(4-1)+1(-2-1)-1(1-2)$

3 -3 -1
$= -1 \neq 0$

$D_x = \begin{vmatrix} 4 & -1 & -1 \\ -5 & 2 & -1 \\ 1 & -1 & 2 \end{vmatrix} = 4\begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} +1\begin{vmatrix} -5 & -1 \\ 1 & 2 \end{vmatrix} -1\begin{vmatrix} -5 & 2 \\ 1 & -1 \end{vmatrix}$
$= 4(4-1)+1(-10+1)-1(5-2)$
$= 12-9-3$
$= 0$

$2x-y-z=4$
$-x+2y-z=-5$
$x-y+2z=1$

$D = \begin{vmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{vmatrix} = 2(4-1)-(-1)(-2+1)-1(1-2)$
$= 6-1+1 = 6 \neq 0$

$D_1 = \begin{vmatrix} 4 & -1 & -1 \\ -5 & 2 & -1 \\ 1 & -1 & 2 \end{vmatrix} = 0$

$D_2 = \begin{vmatrix} 2 & 4 & -1 \\ -1 & -5 & -1 \\ 1 & 1 & 2 \end{vmatrix} = 2\begin{vmatrix} -5 & -1 \\ 1 & 2 \end{vmatrix} - 4\begin{vmatrix} -1 & -1 \\ 1 & 1 \end{vmatrix} \cancel{4}(-1)\begin{vmatrix} -1 & -5 \\ 1 & 1 \end{vmatrix}$
$= -18$

$D_3 = \begin{vmatrix} 2 & -1 & 4 \\ -1 & 2 & -5 \\ 1 & -1 & 1 \end{vmatrix} = 6$

$\frac{D_1}{D} = \frac{0}{6} = 0$, $\frac{D_2}{D} = \frac{-18}{\cancel{6}}\ -3$, $\frac{D_3}{D} = \frac{\cancel{6}}{\cancel{6}} = 1$

$x=0$, $y=-3$, $z=1$

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Q By using determinant, prove the following equations
$x+4y-2z=3$, $3x+y+5z=7$, $2x+3y+z=8$

Sol: Δ or D $= \begin{vmatrix} 1 & 4 & -2 \\ 3 & 1 & 5 \\ 2 & 3 & 1 \end{vmatrix} = 1\begin{vmatrix} 1 & 5 \\ 3 & 1 \end{vmatrix} - 4\begin{vmatrix} 3 & 5 \\ 2 & 1 \end{vmatrix} - 2\begin{vmatrix} 3 & 1 \\ 2 & 3 \end{vmatrix}$

$= 1(1-15)-4(3-10)-2(9-2)$
$= -14+28-14$
$= 14-14 = 0$

$Dx = \begin{vmatrix} 3 & 4 & -2 \\ 7 & 1 & 5 \\ 8 & 3 & 1 \end{vmatrix} = 3\begin{vmatrix} 1 & 5 \\ 3 & 1 \end{vmatrix} - 4\begin{vmatrix} 7 & 5 \\ 8 & 1 \end{vmatrix} - 2\begin{vmatrix} 7 & 1 \\ 8 & 3 \end{vmatrix}$

$= 3(1-15)-4(7-40)-2(21-8)$
$= -42+132-26$
$= 64$

$Dy = \begin{vmatrix} 1 & 3 & -2 \\ 3 & 7 & 5 \\ 2 & 8 & 1 \end{vmatrix} = 1\begin{vmatrix} 7 & 5 \\ 8 & 1 \end{vmatrix} - 3\begin{vmatrix} 3 & 5 \\ \cancel{8} & 1 \end{vmatrix} - 2\begin{vmatrix} 3 & 7 \\ 2 & 8 \end{vmatrix}$

$= 1(7-40)-3(3-10)-2(24-14)$
$= -33+21-20$
$= -12-20$
$= -32$

$Dz = \begin{vmatrix} 1 & 4 & 3 \\ 3 & 1 & 7 \\ 2 & 3 & 8 \end{vmatrix} = 1\begin{vmatrix} 1 & 7 \\ 3 & 8 \end{vmatrix} - 4\begin{vmatrix} 3 & 7 \\ 2 & 8 \end{vmatrix} + 3\begin{vmatrix} 3 & 1 \\ 2 & 9 \end{vmatrix}$

$= 1(8-21)-4(24-14)+3(9-2)$
$-13-40+21$
$= -32$

$x = \frac{Dx}{D} = \frac{0}{64} = 0$, $y = \frac{Dy}{D} = \frac{-32}{0}$, $z = \frac{Dz}{D} = \frac{-32}{0} = 0$

$x=0$, $y=0$, $z=0$

Q By using determinant, prove the following of equation.
$x-y-3z=6$, $x+3y-3z=-4$, $5x+3y+3z=10$.

Sol: Δ or D $= \begin{vmatrix} 1 & -1 & -3 \\ 1 & 3 & -3 \\ 5 & 3 & 3 \end{vmatrix} = 1\begin{vmatrix} 3 & -3 \\ 3 & 3 \end{vmatrix} + 1\begin{vmatrix} 1 & -3 \\ 5 & 3 \end{vmatrix} - 3\begin{vmatrix} 1 & 3 \\ 5 & 3 \end{vmatrix}$

$1(9+9)+1(3+15)-3(3-15)$
$= 18+18-3(-12)$
$= 18+18+36$
$= 36+36 = 72$

$Dx = \begin{vmatrix} 6 & -1 & -3 \\ -4 & 3 & -3 \\ 10 & 3 & 3 \end{vmatrix} = 6\begin{vmatrix} 3 & -3 \\ 3 & 3 \end{vmatrix} + 1\begin{vmatrix} -4 & -3 \\ 10 & 3 \end{vmatrix} - 3\begin{vmatrix} -4 & 3 \\ 10 & 3 \end{vmatrix}$

$6(9+9)+1(-12+30)-3(-12-30)$
$= 108+18+126$
$= 126+126$
252

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Q. $\begin{vmatrix} 1 & -1 & -3 \\ 1 & 3 & -8 \\ 5 & 3 & 5 \end{vmatrix} = 0$

Q1 $\begin{vmatrix} 6 & -1 & -3 \\ -4 & 3 & -3 \\ 10 & 3 & 3 \end{vmatrix}$

(*) Characteristics matrix, characteristic equation and eigen value:-

Characteristic matrix

$[A - \lambda I]$

where

A = Square matrix

λ = Scalar

I = Identity matrix

Characteristic equation:-

$|A - \lambda I| = 0$

$\lambda_1, \lambda_2 \cdots \lambda_n$

eigen value/ characteristic roots/ latent root.

Q. find the eigen value of $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.

Sol:- $\boxed{|A - \lambda I| = 0}$, most imp. point

$= \left| \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| = 0$

$= \left| \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right| = 0$

$= \left| \begin{bmatrix} 1-\lambda & 4 \\ 3 & 2-\lambda \end{bmatrix} \right| = 0$

$(1-\lambda)(2-\lambda) - 12 = 0$

$2 - \lambda - 2\lambda + \lambda^2 - 12 = 0$

$\lambda^2 - 3\lambda - 10 = 0$

$\lambda^2 - 5\lambda + 2\lambda - 10 = 0$

$\lambda(\lambda - 5) + 2(\lambda - 5) = 0$

$(\lambda - 5)(\lambda + 2)$

$\lambda - 5 = 0$

$\lambda = 5$ or $\lambda + 2 = 0$

$\lambda = -2$

Q. find the Characteristic equation of $A = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$.

Sol:- $|A - \lambda I| = 0$

$\Rightarrow \left| \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right|$

$\Rightarrow \left| \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right| = 0$

$\Rightarrow \left| \begin{bmatrix} 2-\lambda & 0 \\ 0 & 3-\lambda \end{bmatrix} \right| = 0$

$= \left| \begin{bmatrix} 2-\lambda & 4 \\ 1 & 3-\lambda \end{bmatrix} \right| = 0$

$(2-\lambda)(3-\lambda)-4=0$

$6-2\lambda-3\lambda+\lambda^2-4=0$

$\lambda^2-5\lambda+2=0$ Ans

Q Find the Characteristic equation of $A=\begin{bmatrix}1&2&3\\4&5&6\\1&0&-1\end{bmatrix}$

Sol: we know that

$|A-\lambda I|=0$

$=\left|\begin{bmatrix}1&2&3\\4&5&6\\1&0&-1\end{bmatrix}-\lambda\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\right|$

$=\left|\begin{bmatrix}1&2&3\\4&5&6\\1&0&-1\end{bmatrix}-\begin{bmatrix}\lambda&0&0\\0&\lambda&0\\0&0&\lambda\end{bmatrix}\right|$

$=\left|\begin{bmatrix}1-\lambda&2&3\\4&5-\lambda&6\\1&0&1+\lambda\end{bmatrix}\right|$

$(1-\lambda)\begin{vmatrix}5-\lambda&6\\0&-1-\lambda\end{vmatrix}-2\begin{vmatrix}4&6\\1&-1-\lambda\end{vmatrix}+3\begin{vmatrix}4&5-\lambda\\1&0\end{vmatrix}=0$

$(1-\lambda)\{(5-\lambda)(-1-\lambda)=0\}-2\{-4-4\lambda-6\}+3\{0-(5-\lambda)\}=0$

$(1-\lambda)\{-5-5\lambda+\lambda+\lambda^2\}+8\lambda+20-15+3\lambda=0$

$(1-\lambda)(\lambda^2-4\lambda-5)+11\lambda+5=0$

$\lambda^2-4\lambda-5-\lambda^3+4\lambda^2+5\lambda+11\lambda+5$

$-\lambda^3+5\lambda^2+12\lambda=0$

$-[\lambda^3-5\lambda^2-12\lambda=0]$ Ans

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$\begin{array}{c} R_1 \\ R_2 \end{array}\begin{vmatrix}2&3\\2&4\end{vmatrix}=8-6=2$

$R_2\to R_2+R_1$

$=\begin{vmatrix}2&3\\(2+2)&(4+3)\end{vmatrix}=\begin{vmatrix}2&3\\4&7\end{vmatrix}=14-12=2$

$R_2\to R_2-R_1$

$=\begin{vmatrix}2&3\\0&1\end{vmatrix}=2-0=2$

$R_2=R_2+3R_1$

$=\begin{vmatrix}2&3\\8&13\end{vmatrix}=$ $26-24=2$

$R_1\to R_1-R_2=\begin{vmatrix}0&-1\\2&4\end{vmatrix}=0+2=2$ Ans

(i) $R_2\to R_2\pm R_1$

(ii) $R_2\to R_2\pm\alpha R_1$

(iii) $R_2\to\alpha R_2+R_1$ (Invalid)

(iv) $C_2\to C_2+C_1$

(v) $C_2\to C_2\pm\alpha C_1$

(vi) $C_2\to\alpha C_2+C_1$

(vii) $R_1\to R_1+2R_2$

(viii) $R_3\to R_3+4R_2$ } valid

(xiii) $R_1\to R_1+2R_2$

$R_3\to R_3-2R_1$ } Invalid

(ix) $R_1\to R_1+2R_2$

$C_1\to C_1+2C_2$ } Invalid

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Q $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$

Sol:- Applying, $R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$$= \begin{vmatrix} 1 & a & a^2 \\ 0 & b-a & b^2-a^2 \\ 0 & c-a & c^2-a^2 \end{vmatrix}$$

expanding along c

$$1 \begin{vmatrix} b-a & b^2-a^2 \\ c-a & c^2-a^2 \end{vmatrix}$$

$$= \begin{vmatrix} b-a & b^2-a^2 \\ c-a & c^2-a^2 \end{vmatrix}$$ (by applying formula $a^2-b^2 = a+b(a-b)$)

$$= b-a \begin{vmatrix} 1 & b+a \\ c-a & c^2-a^2 \end{vmatrix}$$

$$= (b-a)(c-a) \begin{vmatrix} 1 & b+a \\ 1 & c+a \end{vmatrix} = (b-a)(c-a)\{(c+\cancel{a}) - b+\cancel{a}\}$$

$$c-b$$

$$= (b-a)(c-a)(c-b)$$

$$= +(a-b)(c-a)(b-c)$$ proved

Q $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$

Sol: Applying $R_1 \to R_1 - R_3$, $R_2 \to R_2 - R_3$

$$= \begin{vmatrix} 0 & a-c & a^2-c^2 \\ 0 & b-c & b^2-c^2 \\ 1 & c & c^2 \end{vmatrix}$$

Expanding along C_1

$$= \begin{vmatrix} a-c & a^2-c^2 \\ b-c & b^2-c^2 \end{vmatrix}$$

$$= (a-c)(b-c) \begin{vmatrix} 1 & a+c \\ 1 & b+c \end{vmatrix}$$

$$= (a-c)(b-c)\{b+\cancel{c} - (a+\cancel{c})\}$$

$$= (a-c)(b-c)(b-a)$$

$$= -(c-a).(b-c).-(a-b)$$

$$= (a-b)(b-c)(c-a)$$ proved

H/W

Q (i) prove that $\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix} = 0$

(ii) prove that $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix} = 0$

Q(iii) prove that $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$

(iv) prove that $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$

Solution :-

(i) Given determinant

$$= \begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$$

by applying $C_1 \to C_1 + C_2 + C_3$

$= 0$ [$\because C_1$ consist all zeros]

(ii) The given determinant

$$\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$$

$$= \begin{vmatrix} 1 & a & a+b+c \\ 1 & b & a+b+c \\ 1 & c & a+b+c \end{vmatrix}$$

applying $C_3 \to C_3 + C_2$

$= (a+b+c) \cdot \begin{vmatrix} 1 & a & 1 \\ 1 & b & 1 \\ 1 & c & 1 \end{vmatrix}$ [taking common $(a+b+c)$ from C_3

$(a+b+c) \times 0 = 0$

(iii) The given determinant $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$

Applying $R_2 \to R_2 - R_1$
$R_3 \to R_3 - R_1$

$$= \begin{vmatrix} 1 & a & a^2 \\ 0 & b-a & b^2-a^2 \\ 0 & c-a & c^2-a^2 \end{vmatrix}$$

expanding along

$$1 \begin{vmatrix} b-a & b^2-a^2 \\ c-a & c^2-a^2 \end{vmatrix}$$

$$(b-a) \begin{vmatrix} 1 & b+a \\ c-a & c^2-a^2 \end{vmatrix}$$

$(b-a)(c-a) \begin{vmatrix} 1 & b+a \\ 1 & c+a \end{vmatrix} = (b-a)(c-a)\{(c+a) - (b+a)\}$

$= (b-a)(c-a)(c-b)$

$= (a-b)(b-c)(c-a)$ proved

(ii) Applying $R_2 \to R_2 - R_1$
$R_3 \to R_3 - R_1$

$$= \begin{vmatrix} 1 & a & b+c \\ 0 & b-a & c+a-(b+c) \\ 0 & c-a & a+b-(b+c) \end{vmatrix}$$

$$= \begin{vmatrix} 1 & a & b+c \\ 0 & b-a & a-b \\ 0 & c-a & a-c \end{vmatrix}$$

expanding along C_1

$$= \begin{vmatrix} b-a & a-b \\ c-a & a-c \end{vmatrix}$$

$= (b-a)(a-c) - (c-a)(a-b)$
$= ab - bc - a^2 + ca - \{ca - bc - a^2 + ab\}$
$= 0$

(iv) prove that $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$

Sol: ~~Applying $R_2 \to R_2 - R_1$~~
~~$R_3 \to R_3 - R_1$~~

v.v.I

Q prove that $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$

Sol: Applying $C_1 \to C_1 - C_2$
$C_2 \to C_2 - C_3$

$$= \begin{vmatrix} 0 & 0 & 1 \\ a-b & b-c & c \\ a^3-b^3 & b^3-c^3 & c^3 \end{vmatrix}$$

expanding along R_1

$\boxed{(a^3-b^3) = (a-b)(a^2+ab+b^2)}$

$$\begin{vmatrix} a-b & b-c \\ a^3-b^3 & b^3-c^3 \end{vmatrix} = (a-b)(b-c)\begin{vmatrix} 1 & 1 \\ a^2+ab+b^2 & b^2+bc+c^2 \end{vmatrix}$$

$= (a-b)(b-c)\left[\cancel{b^2} + bc + c^2 - \{a^2 + ab + \cancel{b^2}\}\right]$

$= (a-b)(b-c)\{c^2 - a^2 + bc - ab\}$
$= (a-b)(b-c)\{c^2 - a^2 + b(c-a)\}$
$= (a-b)(b-c)(c-a)\{a+b+c\}$

New Lesson:- Co-ordinate Geometry
{Straight light in Space}

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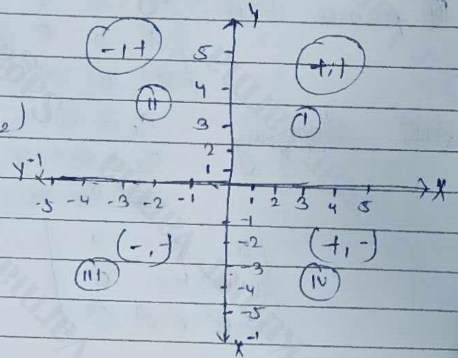
Distance formula



(i) A(x_1, y_1) ——— B(x_2, y_2)

(ii) AB $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ unit

(iii) O $(0,0)$ ——— A(x, y)

(iii) $OA = \sqrt{x^2+y^2}$

Section formula:

$$x = \frac{mx_2 + nx_1}{m+n}, \quad y = \frac{my_2 + ny_1}{m+n}$$

$$P(x,y) = P\left\{\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right\}$$

$$x = \frac{mx_2 - nx_1}{m-n}, \quad y = \frac{my_2 - ny_1}{m-n}$$

Mid-Point

A(x_1, y_1) ——— P(x, y) ——— B(x_2, y_2)

$$x = \frac{x_1 + x_2}{2}, \quad y = \frac{y_1 + y_2}{2}$$

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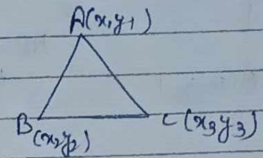
Area of angle.



Area of $\Delta ABC = \frac{1}{2}\left| x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2) \right|$ Sq unit

$$= \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \text{ Sq unit}$$

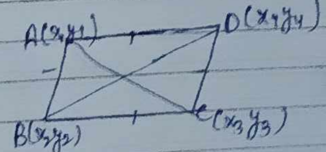
(*) A(x_1, y_1) ——— B(x_2, y_2) ——— C(x_3, y_3)

Collinear

$\therefore$ area of $\Delta ABC = 0$

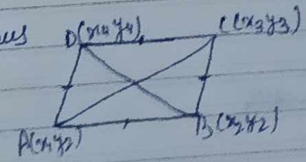
$$= \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

Square



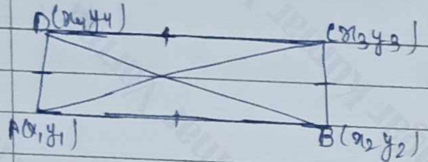
{AB = BC = CD = DA
AC = BD}

Rhombus



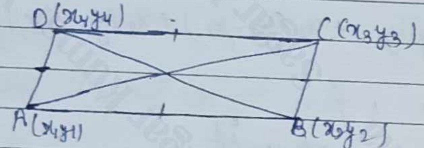
{AB = BC = CD = DA
AC ≠ BD}

#



$\left\{ \begin{matrix} AB = CD \text{ and } DA = BC \\ AC = BD \end{matrix} \right\}$

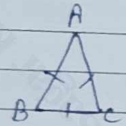
#



$AB = DC$ and ~~BC=BC~~ $AD = BC$

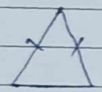
$AC \neq BD$

#



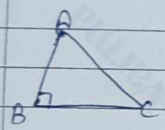
$AB = BC = CA$

equilaterals Δ



$AB = AC$

Isosceles Δ



$AC^2 = AB^2 + BC^2$

Right angle

Point

Ray $\vec{AB}$

$\overleftrightarrow{AB}$ line

line segment.

Straight line

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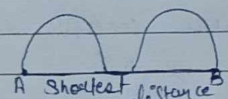
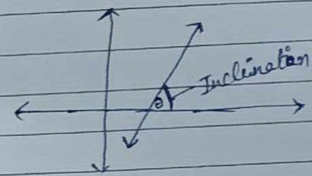


A Shortest distance B

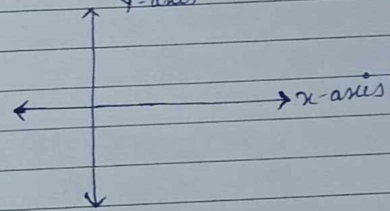
= Displacement

= Straight line

Inclination of a line



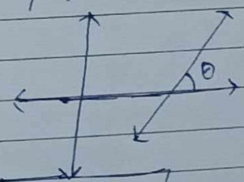
Slope of gradient of a line



$\boxed{0 \leq \theta < 180^\circ}$

Inclination.

slope/gradient of a line



$\boxed{m = \tan\theta}$

slope gradient

$0 \leq \theta < 180$

~~Dee~~ Inclination $\theta = 45^\circ$

$\tan\theta = \tan 45^\circ$

$= 1$

Q. Find the slope of a line whose inclination is (I) 45° (II) 60° (III) 150°

Sol. (I) $\theta = 45$

$m = \tan\theta$

$m = \tan 45°$

$m = 1$

(II) $\theta = 60°$

$m = \tan\theta$

$m = \tan 60°$

$m = \sqrt{3}$

(III) $\theta = 150°$

$m = \tan\theta$

$m = \tan 150°$

$m = \tan(180 - 30)$

$m = -\tan \frac{1}{\sqrt{3}}$

$0 < \theta < 90$, $m = +$

$90 < \theta < 180$ $m = -$

$\theta = 90°$, $m = \infty$

$\theta = 90°$

$m = \tan 90°$

= not defined, ∞

Q. What is the inclination of a line whose slope is (I) zero? (II) positive? (III) negative (IV) not defined?

Sol: (I) $m = 0$ $\quad \theta = \tan\theta$ $\quad (m) = (\text{slope})$

$m = \tan\theta$ $\quad \tan\theta = 0$

$m = \tan 0°$ $\quad \tan 0 = \tan\theta$

$= 0$ $\quad \theta = 0$

(II) Positive $(m = +)$

$m = +$

$\theta = ?$

$m = \tan\theta$

$m > 0$

θ is lies below 0 and 90°

θ is acute angle

(III) $m = -$ (negative)

$\theta = ?$

$m < 0$

θ lies in 90 and 180°

θ is obtuse angle.

~~m = not defined~~

(IV) $m =$ not defined

$\theta = \frac{\pi}{2}$

A (x_1, y_1) —————— B (x_2, y_2)

Slope $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

or $m = \tan\theta$

Q Find the slope of a line which passes through the points (i) (0,-3), (2,1) (ii) (2,5) and (-4,-4)

Sol: (i) $x_1 = 0$, $y_1 = -3$
$x_2 = 2$, $y_2 = 1$

slope $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1+3}{2-0} = \frac{\cancel{4}}{\cancel{2}}\ 2$

(ii) $x_1 = 2$, $y_1 = 5$
$x_2 = -4$, $y_2 = -4$

Sol: slope: $\frac{y_2 - y_1}{x_2 - x_1} = \frac{-4-5}{-4-2} = \frac{-\cancel{9}\ 3}{-\cancel{6}\ 2} = 3/2$

SPS

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Q If the slope of the line passing through points (2,5) and (x,3) is 2 find the value of x.

Sol:- Let A(2,5) and B(x,3)

Slope of AB = 2

$\frac{y_2 - y_1}{x_2 - x_1} = \frac{3-5}{x-2} = 2$

$= 3-5 = 2x-4$
$-2 = 2x-4$
$2x = 4-2$
$\cancel{2}x = \cancel{2}\ 1$
$x = 1$ Proof

Q Find the value of x, so that the inclination of the line passing through the point (x,-3) and (2,5) is 135°

Sol: $m = \tan\theta$
$m = \tan 135°$
$m = \tan(90+45)$
$m = \tan 45°$
$m = -1$

$\frac{y_2 - y_1}{x_2 - x_1} = \frac{5+3}{2-x} = -1$

$8 = -2 + x$
$x = \cancel{6}\ 10°$ 8+2
$x = 10°$

SPS

Q. Find the angle between the x-axis and the line joining the point (3, -1) and (4, -2)

Sol. $x_1 = 3,\ y_1 = -1$

$x_2 = 4,\ y_2 = -2$

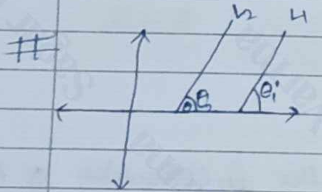
$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{-2+1}{4-3} = \frac{-1}{1} = -1$$

$\tan\theta = -1$

$\tan\theta = \tan 135^\circ$

$\theta = 135^\circ$

#



$L_1 \parallel L_2 = m_1 = m_2$

Q. Show that the line joining the point (2, -3) and (-5, 1) is parallel to the line joining the point (7, -1) and (0, 3).

Sol:- Let $A(2, -3)$ $B(-5, 1)$

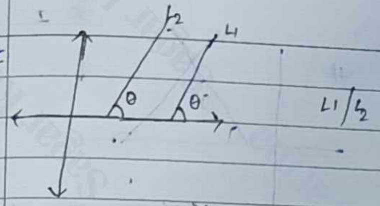
Slope of AB $(m_1) = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-(-3)}{-5-2} = \frac{4}{-7}$

again

Slope of PQ $(m_2) = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-(-1)}{0-7} = \frac{4}{-7}$

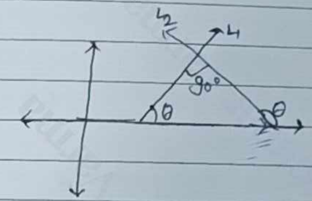
Hence, $m_1 = m_2$ so $L_1 \parallel L_2$ Proved

#



$L_1 \parallel L_2 \iff m_1 = m_2$

#



$L_1 \perp L_2 \iff m_1 \cdot m_2 = -1$

Q. Show that the line joining the point (2, -5) and (-2, 5) is perpendicular to the line segment joining the point (6, 3) and (1, 1)

Sol: $A(2, -5)$ $B(-2, 5)$

$x_1 = 2,\ y_1 = -5$

$x_2 = -2,\ y_2 = 5$

$$= \frac{5+5}{-2-2} = \frac{\cancel{10}\ 5}{-\cancel{4}\ 2}$$

Let $C(6, 3)$ and $D(1, 1)$

$$m_2 = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{1-3}{1-6} = \frac{-2}{-5} = \frac{2}{5}$$

$\therefore m_1 \cdot m_2 = -\frac{\cancel{5}}{\cancel{2}} \times \frac{\cancel{2}}{\cancel{5}}$

$m_1 \cdot m_2 = -1$

So, $L_1 \perp L_2$ proved

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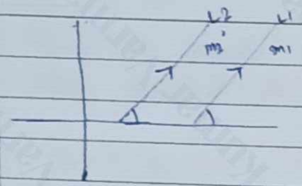
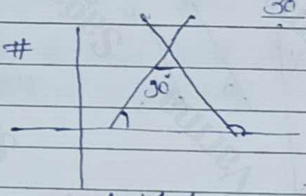
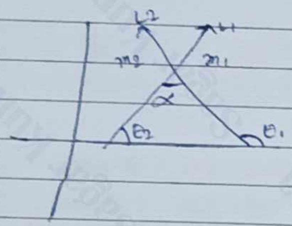




$L_1 \parallel L_2 \iff m_1 = m_2$

$L_1 \perp L_2 \iff m_1 \cdot m_2 = -1$



Q. Find the angle between the lines whose slopes are $\frac{1}{2}$ and 3. $\tan\alpha = \left|\frac{m_2 - m_1}{1 + m_1 \cdot m_2}\right|$

Sol:- Let $m_1 = \frac{1}{2}$ and $m_2 = 3$

we know that,

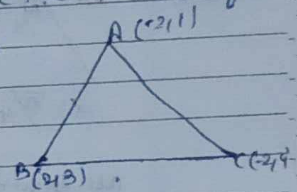
$= \tan\alpha = \left|\frac{m_2 - m_1}{1 + m_1 \cdot m_2}\right|$

$= \cancel{\tan\alpha} \left|\frac{3 - \frac{1}{2}}{1 + 3 \times \frac{1}{2}}\right| = \frac{\cancel{5/2}}{\cancel{5/2}} = 1$

$\tan\alpha = 1 = \tan 45°$

$\alpha = 45°$ Ans

Q. If $A(-2,1)$, $B(2,3)$ and $C(-2,4)$ be the vertices of a $\triangle ABC$ show that $\tan B = \frac{2}{3}$.



Sol:- Let,

slope of $m_1 = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{3 - 1}{2 - (-2)} = \frac{\cancel{2}}{\cancel{4}}\,\frac{1}{2}$

slope of $m_2 = \frac{-4 \; 3}{-2-2} = \frac{+7}{+4}$

we know that,

$\tan\alpha = \left|\frac{m_2 - m_1}{1 + m_1 \cdot m_2}\right|$

$\tan B = \left|\frac{\frac{7}{4} - \frac{1}{2}}{1 + \frac{7}{4} \cdot \frac{1}{2}}\right|$

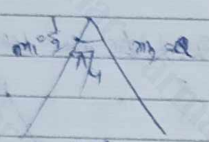
$\tan B = \left|\frac{5/4}{15/8}\right|$

$\tan B = \frac{\cancel{5}}{\cancel{4}_2} \times \frac{\cancel{8}}{\cancel{15}\,3}$

$\cancel{\tan B = 2/3}$ $\tan B = 2/3$ Ans

Q. Find the angle between two lines is $\frac{\pi}{4}$ and the slope of the line is $\frac{1}{2}$ find the slope of the other line.

Sol:



$\alpha = \pi/4$

we know that

$\tan\alpha = \left|\frac{m_2 - m_1}{1 + m_1 \cdot m_2}\right|$

$\tan\frac{\pi}{4} = \left|\frac{m - \frac{1}{2}}{1 + \frac{1}{2}\cdot m_2}\right|$

$1 = \left|\frac{\frac{2m_2 - 1}{2}}{\frac{2 + m_2}{2}}\right| = \frac{2m_2 - 1}{2 + m_2}$

$2 + m_2 = 2m_2 - 1$

$m_2 = 3$

Q Show that the points (5,1) (1,-1) and (11,4) are collinear.

Sol:- Let A(5,1) B(1,-1) and C(11,4)

Slope of AB $= \frac{-1-1}{1-5} = \frac{\cancel{-2}}{\cancel{-4}} = \frac{1}{2}$

Slope of BC $= \frac{4-(-1)}{11-1} = \frac{\cancel{5}}{\cancel{10}} = \frac{1}{2}$

Slope of AB = Slope of BC $= \frac{1}{2}$
So, A, B, and C are collinear

Important

Q If the points (h,0), (a,b) and (0,k) lie on a line. Show that $\frac{a}{h} + \frac{b}{k} = 1$

Sol:- Let A(h,0) B(a,b) and C(0,k)

Slope of AB $= \frac{b-0}{a-h} = \frac{b}{a-h}$

Slope of BC $= \frac{k-b}{0-a} = \frac{k-b}{-a}$

A/Q

Slope of AB = Slope of BC

$\frac{b}{a-h} = \frac{k-b}{-a}$

$(a-h)(k-b) = -ab$

$ak - \cancel{ab} - hk + hb = -\cancel{ab}$

$ak - hk + hb = 0$

$ak + hb = hk$

$\frac{ak + hb}{hk} = \frac{hk}{hk}$

$\frac{a}{h} + \frac{b}{k} = 1$ P

SPS

SPS

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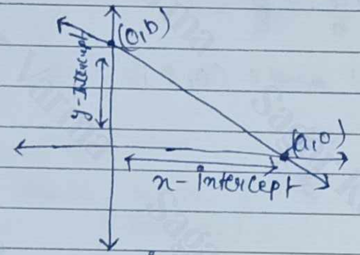
Intercept :-



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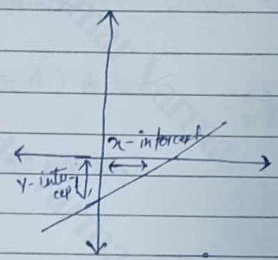

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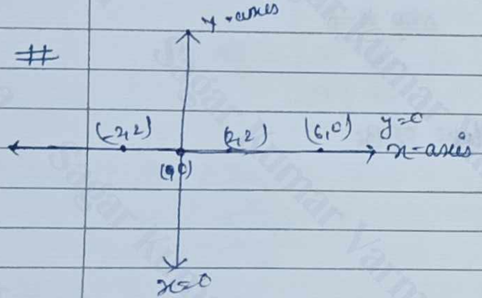
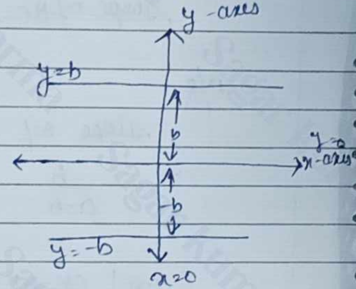
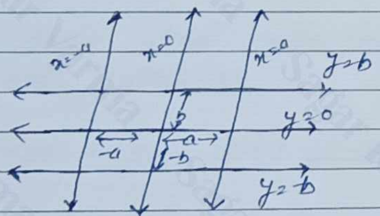




equation of line ∥ to x-axis $y = \pm b$

#



equation of x-axis, $\boxed{y=0}$

equation of y-axis $x=0$

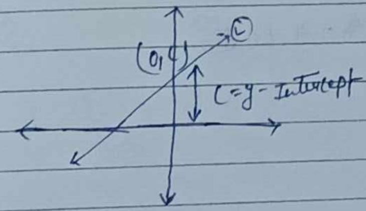
eq of line ∥ to x-axis $= y \pm b$

eq of line ∥ to y-axis $: x \pm a$

equation of line

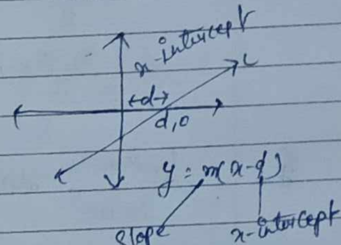
1. Slope Intercept form, $y = mx+c$
2. point slope form $y-y_1 = m(x-x_1)$
3. Two point form $y-y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$
4. Two intercept form $\frac{x}{a}+\frac{y}{b}=1$
5. perpendicular/Normal form : $x\cos\alpha + y\sin\alpha = p$
6. General form, $ax+by+c=0$

(1) slope — Intercept form.



$y = mx + c$

slope, y - Intercept



$y = m(x-d)$

slope, x-intercept

Q. find the equation of a line whose slope is $1/2$ and Y-intercept equal to $-5/4$.

Sol:- Slope $= m = 1/2$

Y-Intercept $c = -5/4$

we know that,

$y = mx + c$

$y = \frac{1}{2}x - \frac{5}{4} = \frac{2x-5}{4}$

$4y = 2x - 5$

$2x - 4y - 5 = 0$ Ans

Q Evaluate $\int x \tan^{-1}x \, dx$

Sol:- $\tan^{-1}x \int x\,dx - \int \left[\frac{d}{dx}(\tan^{-1}x) \int x\,dx \right]$

$\tan^{-1}x \cdot \frac{x^2}{2} - \int \frac{1}{1+x^2} \cdot \frac{x^2}{2}\,dx$

$\frac{x^2 \tan^{-1}x}{2} - \frac{1}{2}\int \left(1 - \frac{1}{1+x^2}\right)$

$\frac{x^2 \tan^{-1}x}{2} - \frac{1}{2}\int dx + \frac{1}{2}\int \frac{1}{1+x^2}\,dx$

$\frac{x^2 \tan^{-1}x}{2} - \frac{x}{2} + \frac{1}{2}\tan^{-1}x + c$

$= \frac{1}{2}(1+x^2)\tan^{-1}x - \frac{x}{2} + c$

Q $\int \frac{e^{\tan^{-1}x}}{1+x^2}\,dx$

Sol: $\int \frac{e^{\tan^{-1}x}}{1+x^2}\,dx$

$= \tan^{-1}x = t$

$= \frac{1}{1+x^2}\,dx = dt$

$= \int e^t\,dt$

$= e^{\tan^{-1}x} + c$

Q Prove that $\cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ = \frac{1}{16}$

Sol: we have,

$\cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ$

$= \frac{1}{2}\cos 60^\circ \cos 40^\circ (2\cos 80^\circ \cos 20^\circ)$

$= \frac{1}{2} \times \frac{1}{2}\cos 40^\circ \{\cos(80^\circ+20^\circ) + \cos(80^\circ-20^\circ)\}$

$= \frac{1}{4}\cos 40^\circ (\cos 100^\circ + \cos 60^\circ$

$= \frac{1}{4}\cos 40^\circ (\cos 100^\circ + \frac{1}{2})$

$= \frac{1}{8}(2\cos 100^\circ \cos 40^\circ) + \frac{1}{8}\cos 40^\circ$

$= \frac{1}{8}\cos(100^\circ+40^\circ) + \cos(100^\circ-40^\circ) + \frac{1}{8}\cos 40^\circ$

$= \frac{1}{8}(\cos 140^\circ + \cos 60^\circ) + \frac{1}{8}\cos 40^\circ$

$= \frac{1}{8}\cos 140^\circ + \frac{1}{16} + \frac{1}{8}\cos 40^\circ$

$= \frac{1}{8}\cos(180 - 40^\circ) + \frac{1}{16} + \frac{1}{8}\cos 40^\circ$

$= \cancel{-\frac{1}{8}\cos 40^\circ} + \frac{1}{16} + \cancel{\frac{1}{8}\cos 40^\circ}$

$= \frac{1}{16} = RHS =$

Q $\sin 10^\circ \sin 30^\circ \sin 50^\circ \sin 70^\circ = \frac{1}{16}$.

Sol:- $\frac{1}{2}(2\sin 70^\circ \sin 10^\circ)\sin 50^\circ \frac{1}{2}$

$= \frac{1}{4}\{\cos(70^\circ-10^\circ) - \cos(70+10)\}\sin 50^\circ$

$= \frac{1}{4}\cos 60^\circ \sin 50^\circ - \cos 80 \sin 50^\circ$

$= \frac{1}{4}\left(\frac{1}{2}\sin 50^\circ - \cos 80^\circ \sin 50^\circ\right)$

$= \frac{1}{8}\left(\sin 50^\circ - \cos 80^\circ \sin 50^\circ\right)$

$= \frac{1}{8}\left(\sin 50^\circ - 2\cos 80^\circ \sin 50^\circ\right)$

$= \frac{1}{8}\left(\sin 50^\circ - \sin(80+50) - \sin(80-50)\right)$

$= \frac{1}{8}\left(\sin 50^\circ - \sin 130^\circ + \sin 30^\circ\right)$

$= \frac{1}{8}\left(\sin 50^\circ - \sin 130^\circ + \frac{1}{2}\right)$

$= \frac{1}{8}\left(\sin 50 - \sin(180-50) + \frac{1}{2}\right)$

$= \frac{1}{8}\ \sin 50 - \sin(180-50) + \frac{1}{2}$

$= \frac{1}{16}\ \cancel{\sin 50} - \cancel{\sin 50}$

$= \frac{1}{16}$

Ex(15) Find angle between lines whose slopes $\frac{1}{2}$ and 3

sol: $m_1 = \frac{1}{2}$ and $m_2 = 3$

$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right| =$

$= \left(\frac{3 - \frac{1}{2}}{1 + 3 \times \frac{1}{2}}\right) = \left(\frac{5/2}{5/2}\right) = 1$

Here $\theta = 45^\circ$.

Q If $\tan 5/6$ and $\tan B = \frac{1}{11}$ Prove $(A+B) = 45^\circ$

Sol:- $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$

$\tan(A+B) = \frac{\frac{5}{6} + \frac{1}{11}}{1 - 5/6 \times \frac{1}{11}}$

$= \frac{\frac{55+6}{66}}{\frac{66-5}{66}} = \frac{\frac{61}{66}}{\frac{61}{66}} = 1$

$\tan(A+B) = 1$

$\cancel{\tan}(A+B) = \cancel{\tan} 45^\circ$

$(A+B) = 45^\circ$

Q $\int_0^{\pi/2} \log(\sin x)\, dx$

Sol: $\int_0^{\pi/2} \log(\sin x)\, dx$ —— (1)

$I_1 = \int_0^{\pi/2} \log\left(\sin\left(\frac{\pi}{2} - x\right)\right) dx$

$= \int_0^{\pi/2} \log(\cos x)\, dx$ —— (11)

add (1) and (11)

$I_1 + I_1 = \int_0^{\pi/2} \log(\sin x)\, dx + \int_0^{\pi/2} \log(\cos x)\, dx$

$= 2I \int_0^{\pi/2} \log(\sin x \cos x)\, dx$

$$2I = \int_0^{\pi/2} \log\left(\frac{2\sin x\cos x}{2}\right) dx$$

$$2I = \int_0^{\pi/2} \log\left[\frac{2\sin x\cos x}{2}\right] - \log 2x \,/\, dx$$

$$2I \int_0^{\pi/2} \log(\sin 2x) - \log 2x \,]\, dx$$

$$2I = \int_0^{\pi/2} \log(\sin 2x) - \int_0^{\pi/2} \log 2\, dx \quad \text{--- (3)}$$

$$= \frac{\pi}{2} \log 2$$

Q $y = \tan^{-1}\frac{(\sin x)}{(1+\cos x)}$

Sol: $y \tan^{-1}\left(\frac{\cancel{2}\sin x/2 \cdot \cos x/2}{\cancel{2}\cos^2 x/2}\right)$

$$= \tan^{-1}\left(\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}\right)$$

$$= \tan^{-1}\left(\tan\frac{x}{2}\right)$$

$$= \frac{x}{2}\ \&$$

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(*)

	0°	30°	45°	60	90°
sinθ	0	1/2	$\frac{1}{\sqrt2}$	$\sqrt3/2$	1
cosθ	1	$\sqrt3/2$	$\frac{1}{\sqrt2}$	1/2	0
tanθ	0	$1/\sqrt3$	1	$\sqrt3$	∞
cosecθ	∞	2	$\sqrt2$	$2/\sqrt3$	1
secθ	1	$2/\sqrt3$	$\sqrt2$	2	∞
cotθ	∞	$\sqrt3$	1	$1/\sqrt3$	0

$90 = \pi/2$

$90+\theta = \frac{\pi}{2}+\theta$

$180-\theta = \pi-\theta$

sin/cosec

$\frac{\pi}{2}-\theta = 90°-\theta$

$0°+\theta = \theta$

All (+)

180°=π ⟶ 0°/360/2π

$180+\theta = \pi+\theta$

$270-\theta = 3\pi/2-\theta$

tan/cot

$270+\theta = \frac{3\pi}{2}+\theta$

$360-\theta = 2\pi-\theta$

$0-\theta = -\theta$

cos/sec

(i) $n \times \frac{\pi}{2} \pm \theta$

n = odd

I = change
- sin ⟷ cos
- sec ⟷ cosec
- tan ⟷ cot

(ii) Angle (quadrant)

n = even I no change

II quadrant

#1 $\sin(\frac{\pi}{2}-\theta) = \cos\theta$

$\cos(\frac{\pi}{2}-\theta) = \sin\theta$

$\tan(\frac{\pi}{2}-\theta) = \cot\theta$

$\text{cosec}(\pi/2-\theta) = \sec\theta$

$\sec\theta = (\frac{\pi}{2}-\theta) = \text{cosec}\theta$

$\cot\theta = (\frac{\pi}{2}-\theta) = \tan\theta$

#2 $\sin(\frac{\pi}{2}+\theta) = \cos\theta$

$\cos(\frac{\pi}{2}+\theta) = -\sin\theta$

$\tan\theta(\frac{\pi}{2}+\theta) = -\cot$

$\text{cosec}(\frac{\pi}{2}+\theta) = \sec\theta$

$\sec(\frac{\pi}{2}+\theta) = -\text{cosec}\theta$

$\cot(\frac{\pi}{2}+\theta) = -\tan\theta$

#3 $\sin(\pi-\theta) = \sin\theta$

$\cos(\pi-\theta) = -\cos\theta$

$\tan(\pi-\theta) = -\tan\theta$

$\cot(\pi-\theta) = -\cot\theta$

$\sec(\pi-\theta) = -\sec\theta$

$\text{cosec}(\pi-\theta) = \text{cosec}\,\theta$

#4 $\sin(\pi+\theta) = -\cancel{\cos\theta}\,\sin\theta$

$\cos(\pi+\theta) = -\cos\theta$

$\tan(\pi+\theta) = \tan\theta$

$\cot(\pi+\theta) = \cot\theta$

$\sec(\pi+\theta) = -\sec\theta$

$\text{cosec}(\pi+\theta) = -\text{cosec}\,\theta$

#5 $\sin(\frac{3\pi}{2}-\theta) = -\cos\theta$

$\cos(\frac{3\pi}{2}-\theta) = -\sin\theta$

$\tan(\frac{3\pi}{2}-\theta) = \cot\theta$

$\text{cosec}(\frac{3\pi}{2}-\theta) = -\sec\theta$

$\sec(\frac{3\pi}{2}-\theta) = -\text{cosec}\theta$

$\cot(\frac{3\pi}{2}-\theta) = \tan\theta$

#6 $\sin(\frac{3\pi}{2}+\theta) = -\cos\theta$

$\cos(\frac{3\pi}{2}+\theta) = \sin\theta$

$\tan(\frac{3\pi}{2}+\theta) = -\cot\theta$

$\cot(\frac{3\pi}{2}+\theta) = -\tan\theta$

$\sec(\frac{3\pi}{2}+\theta) = \text{cosec}\theta$

$\text{cosec}(\frac{3\pi}{2}+\theta) = -\sec\theta$

#7 $\sin(2\pi-\theta) = -\sin\theta$

$\cos(2\pi-\theta) = \cos\theta$

$\tan(2\pi-\theta) = -\tan\theta$

$\cot(2\pi-\theta) = -\cot\theta$

$\sec(2\pi-\theta) = \sec\theta$

$\text{cosec}(2\pi-\theta) = -\text{cosec}\theta$

#8 $\sin(-\theta) = -\sin\theta$

$\cos(-\theta) = \cos\theta$

$\tan(-\theta) = -\tan\theta$

$\cot(-\theta) = -\cot\theta$

$\sec(-\theta) = \sec\theta$

$\text{cosec}(-\theta) = -\text{cosec}\theta$

SPS SPS

15 | 1 | 25

Q If $\sec\theta = \frac{-13}{12}$ and θ lies in the second quadrant find the value of all Quadrant

Sol: Here, $\sec\theta = \frac{H}{B} = \frac{-13}{12}$

$P^2 = \sqrt{b^2 - h^2}$

$P^2 = \sqrt{(13)^2 - (12)^2}$

$P^2 = \sqrt{169 - 144}$

$P^2 = \sqrt{25}$

$P = 5$

$\sin\theta = \frac{5}{13}$, $\cos\theta = -\frac{12}{13}$, $\tan\theta = -\frac{5}{12}$

$\operatorname{cosec}\theta = \frac{13}{5}$, $\sec\theta = \frac{-13}{12}$, $\cot\theta = -\frac{12}{5}$

Q $\sin(240^\circ)$

Sol: $\sin(180^\circ + 60^\circ)$

$\sin 60^\circ$

$= -\frac{\sqrt{3}}{2}$

Q $\cos 315^\circ$

Sol: $\cos(360 - 45^\circ)$

$\cos 45^\circ$

$= \frac{1}{\sqrt{2}}$

Q $\sin 420^\circ$

Sol: $\sin(360^\circ + 60^\circ)$

$\sin 60^\circ$

$= \frac{\sqrt{3}}{2}$

H/W

(i) $\sin 765^\circ$

(ii) $\operatorname{cosec}(-1110^\circ)$

(iii) $\cot(-600^\circ)$

Q $\tan 315^\circ$

Sol: $\tan(360^\circ - 45^\circ)$

$\tan 45^\circ = -1$

(i) $\sin 765^\circ$

$\sin(2 \times 360 + 45^\circ)$

$\sin 45^\circ$

$= \frac{1}{\sqrt{2}}$

(ii) $\operatorname{cosec}(-1110^\circ)$

Sol: $\operatorname{cosec}(3 \times 360 + 30^\circ)$

$\operatorname{cosec} 30^\circ$

$= -2$

(iii) $\cot(-600^\circ)$

Sol: $-\cot 600^\circ$

$-\cot(6 \times 90 + 60^\circ)$

$-\cot 60^\circ$

$= -\frac{1}{\sqrt{3}}$

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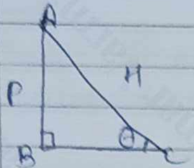

(i) $\sin\theta = \frac{P}{H}$, $\operatorname{cosec}\theta = \frac{H}{P}$

(2) $\cos\theta = \frac{B}{H}$, $\sec\theta = \frac{H}{B}$

(3) $\tan\theta = \frac{P}{B}$, $\cot\theta = \frac{B}{P}$

(*) $\sin\theta \cdot \operatorname{cosec}\theta = 1$

$\sin\theta = \frac{1}{\operatorname{cosec}\theta}$

$\operatorname{cosec}\theta = \frac{1}{\sin\theta}$

(ii) $\cos\theta \cdot \sec\theta = 1$

$\cos\theta = \frac{1}{\sec\theta}$

$\sec\theta = \frac{1}{\cos\theta}$

(iii) $\tan\theta \cdot \cot\theta = 1$

$\tan\theta = \frac{1}{\cot\theta}$

$\cot\theta = \frac{1}{\tan\theta}$

(iv) $\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

$\sin\theta = \sqrt{1-\cos^2\theta}$

$\cos^2\theta = 1 - \sin^2\theta$

$\cos\theta = \sqrt{1-\sin^2\theta}$

(v) $\sec^2\theta - \tan^2\theta = 1$

$\sec^2\theta = 1 + \tan^2\theta$

$\sec\theta = \sqrt{1+\tan^2\theta}$

$\tan^2\theta = \sec^2\theta - 1$

$\tan\theta = \sqrt{\sec^2\theta - 1}$

(vi) $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

$\operatorname{cosec}^2\theta = 1 + \cot^2\theta$

$\operatorname{cosec}\theta = \sqrt{1+\cot^2\theta}$

$\cot^2\theta = \operatorname{cosec}^2\theta - 1$

$\cot\theta = \sqrt{\operatorname{cosec}^2\theta - 1}$

SPS

(i) $\sin 2A = 2\sin A \cdot \cos A$, $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= 1 - \sin^2\theta - \sin^2\theta$

(ii) $\cos 2A = \cos^2 A - \sin^2 A$

$= 1 - 2\sin^2 A$

$= 2\cos^2 A - 1$

$\cos 2\theta = 1 - 2\sin^2\theta$

$2\sin^2\theta = 1 - \cos 2\theta$

$1 - \cos 2\theta = 2\sin^2\theta$

$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$

$\cos 2\theta = \cos^2\theta - (1-\cos^2\theta)$

$\cos 2\theta = 2\cos^2\theta - 1$

$1 + \cos 2\theta = 2\cos^2\theta$

$\frac{2\tan A}{1+\tan^2 A} = \frac{1-\tan^2 A}{1+\tan^2 A}$

(*) $\sin 3A = 3\sin A - 4\sin^3 A$

$\cos 3A = 4\cos^3 A - 3\cos A$

$\tan 3A = \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$

(*) $\sin(A+B) = \sin A \cdot \cos B + \cos A \cdot \sin B$

$\sin(A-B) = \sin A \cdot \cos B - \cos B \cdot \sin B$

$\cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B$

$\cos(A-B) = \cos A \cdot \cos B + \sin A \cdot \sin B$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$

$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \cdot \tan B}$

$2\sin A \cdot \cos B = \sin(A+B) + \sin(A-B)$

$2\cos A \cdot \sin B = \sin(A+B) - \sin(A-B)$

$2\cos A \cdot \cos B = \cos(A+B) + \cos(A-B)$

$-2\sin A \cdot \sin B = \cos(A+B) - \cos(A-B)$

or $2\sin A \cdot \sin B = \cos(A-B) - \cos(A+B)$

SPS

$\sin C + \sin D = 2\sin\frac{(C+D)}{2} \cdot \cos\frac{(C-D)}{2}$

$\sin C - \sin D = 2\cos\frac{(C+D)}{2} \cdot \sin\frac{(C-D)}{2}$

$\cos C + \cos D = 2\cos\frac{(C+D)}{2} \cdot \cos\frac{(C-D)}{2}$

$\cos C - \cos D = -2\sin\frac{(C+D)}{2} \cdot \sin\frac{(C-D)}{2}$

Q. Show that $\sin 105^\circ + \cos 105^\circ = \frac{1}{\sqrt{2}}$

Sol:- ~~$\sin(60+45^\circ) + \cos(60+45^\circ) = \frac{1}{\sqrt{2}}$~~

~~$\sin 45^\circ + \cos 45^\circ$~~

~~$\sin + \cos(45^\circ)$~~

~~$= \sin + \cos(45^\circ) = \frac{1}{\sqrt{2}}$~~

$\sin(60^\circ + 45^\circ) + \cos(60^\circ + 45^\circ)$

$\sin 60^\circ \cdot \cos 45^\circ + \cos 60^\circ \sin 45^\circ + \cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ$

$= \frac{1}{2} \times \frac{1}{\sqrt{2}} + \frac{1}{2} \times \frac{1}{\sqrt{2}}$

$= \frac{1}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{1+1}{2\sqrt{2}} = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}}$ proof

Q. Calculate the value of

(i) $\sin 15^\circ$

$\sin(45^\circ - 30^\circ)$

$\sin 30^\circ$

$= \sin 45^\circ \cdot \cos 30^\circ - \cos 45^\circ \cdot \sin 30^\circ$

$\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$

$\frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$

$= \frac{\sqrt{3}-1}{2\sqrt{2}}$

~~(i) $\sin(45^\circ + 30^\circ)$ $\sin 30^\circ$~~

$= \frac{1}{2}$

~~(iii) $\tan 75^\circ$~~

~~$\tan(45^\circ + 30^\circ)$~~

~~$\tan 30^\circ$~~

$= \frac{1}{\sqrt{3}}$

$\sin(45 + 30)$

(ii) $\sin 75^\circ$

Sol: $\sin(45^\circ + 30^\circ)$

$\Rightarrow \sin 45^\circ \cdot \cos 30^\circ + \cos 45^\circ \sin 30^\circ$

$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$

$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$

$= \frac{\sqrt{3}+1}{2\sqrt{2}}$

(iii) $\tan 75^\circ$

$\tan(45^\circ + 30^\circ)$

$\frac{\tan(A+B)}{1 - \tan A \cdot \tan B}$

$\frac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \cdot \tan 30^\circ}$

$$\frac{1+\frac{1}{\sqrt3}}{1-\frac{1}{\sqrt3}}$$

$$= \frac{\frac{\sqrt3+1}{\cancel{\sqrt3}}}{\frac{\sqrt3-1}{\cancel{\sqrt3}}}$$

$$= \frac{\sqrt3+1}{\sqrt3-1}$$

$$= \frac{\sqrt3+1}{\sqrt3-1} \times \frac{\sqrt3+1}{\sqrt3+1}$$

$$= \frac{3+1+2\sqrt3}{3-1}$$

$$= \frac{4+2\sqrt3}{2}$$

$$= \frac{\cancel{2}(2+\sqrt3)}{\cancel{2}}$$

$$= (2+\sqrt3)$$

Q. Prove that $\cos\left(\frac{3\pi}{2}+\theta\right)\cdot\cos(2\pi+\theta)\left[\cot\left(\frac{3\pi}{2}-\theta\right)+\cot(2\pi+\theta)\right]$

Sol.

$\sin\theta\cdot\cos\theta\,[\tan\theta+\cot\theta]$

$\sin\theta\cdot\cos\theta\left[\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}\right]$

$\cancel{\sin\theta\cos\theta} \times \frac{\sin^2\theta+\cos^2\theta}{\cancel{\sin\theta\cdot\cos\theta}}$

$= 1$ [proved] L.H.S = R.H.S

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Q. Prove that $\frac{\sec 8\theta-1}{\sec 4\theta-1} = \frac{\tan 8\theta}{\tan 2\theta}$

Sol:- L.H.S $= \frac{\sec 8\theta-1}{\sec 4\theta-1}$

$$= \frac{\frac{1}{\cos 8\theta}-1}{\frac{1}{\cos 4\theta}-1}$$

$$= \frac{1-\cos 8\theta}{\cos 8\theta} \times \frac{\cos 4\theta}{1-\cos 4\theta}$$

$$= \frac{1-\cos^2 4\theta}{\cos 8\theta} \times \frac{\cos 4\theta}{1-\cos^2 2\theta}$$

$$= \frac{2\sin^2 4\theta}{\cos 8\theta} \times \frac{\cos 4\theta}{2\sin^2 2\theta}$$

$$= \frac{2\sin 4\theta\cdot\sin 4\theta\cdot\cos 4\theta}{2\sin 2\theta\cdot\sin 2\theta\cdot\cos 8\theta}$$

$$= \frac{\sin 8\theta\cdot\sin 4\theta}{2\sin 2\theta\cdot\sin 2\theta\cos 8\theta}$$

$$= \frac{\tan 8\theta\cdot\sin 2\cdot 2\theta}{2\sin 2\theta\cdot\sin 2\theta}$$

$$= \frac{\tan 8\theta\cdot\cancel{2\sin 2\theta}\cos 2\theta}{\cancel{2\sin 2\theta}\cdot\sin 2\theta}$$

$$= \frac{\tan 8\theta}{\frac{\sin 2\theta}{\cos 2\theta}}$$

$$= \frac{\tan 8\theta}{\tan 2\theta}.$$ Proved

Q Find the value of

(i) $\sin 18^\circ$ (ii) $\cos 18^\circ$ (iii) $\cos 36^\circ$ (iv) $\sin 36^\circ$
(v) $\sin 72^\circ$ (vi) $\cos 72^\circ$ (vii) $\sin 54^\circ$ (viii) $\cos 54^\circ$

(I) $\sin 18^\circ$

Sol: Let $\theta = 18$

$5\theta = 90$

$5\theta - 3\theta = 90 - 3\theta$

$2\theta = 90 - 3\theta$

$\sin 2\theta = \sin(90 - 3\theta)$

$2\sin\theta\cos\theta = \cos 3\theta - 4\cos^3\theta$

$= 2\sin\theta\cdot\cos\theta - 4\cos^3\theta + 3\cos\theta = 0$

$= \cos\theta(2\sin\theta - 4\cos^2\theta + 3) = 0$

$= 2\sin\theta - 4(1 - \sin 2\theta) + 3 = 0$

$= 2\sin\theta - 4 + 4\sin^2\theta + 3 = 0$

$\boxed{4\sin^2\theta + 2\sin\theta - 1} = 0$

$\sin\theta = \dfrac{-2 \pm \sqrt{2^2 - 4\cdot 4(-1)}}{2\times 4}$

$= \dfrac{-2 \pm \sqrt{4+16}}{8} = \dfrac{-2 \pm \sqrt{20}}{8}$

$= \dfrac{-\cancel{2} \pm \cancel{2}\sqrt{5}}{\cancel{8}4}$

$= \dfrac{-1 \pm \sqrt{5}}{4}$ Ans

Q Prove that $\tan 56^\circ = \dfrac{\cos 11 + \sin 11}{\cos 11 - \sin 11}$

Sol: L.H.S $\tan(45 + 11)$

$\tan\theta^\circ = \dfrac{\tan 45 + \tan 11}{1 - \tan 45\cdot\tan 11}$

$= \dfrac{1 + \tan 11}{1 - 1\times\tan 11}$

$= \dfrac{1 + \dfrac{\sin 11^\circ}{\cos 11^\circ}}{1 - \dfrac{\sin 11}{\cos 11}}$

$= \dfrac{\dfrac{\cos 11^\circ + \sin 11}{\cancel{\cos 11^\circ}}}{\dfrac{\cos 11 - \sin 11^\circ}{\cancel{\cos 11^\circ}}}$

$= \dfrac{\cos 11^\circ + \sin 11}{\cos 11 - \sin 11}$ R.H.S

So, L.H.S = R.H.S

V.V.I

Q Prove that $\tan 70^\circ = \tan 20^\circ + 2\tan 50^\circ$

Sol: $\tan 70^\circ = \tan(50 + 20)$

$\tan 70^\circ = \dfrac{\tan 50 + \tan 20}{1 - \tan 50\tan 20}$

$\Rightarrow \tan 70 - \tan 70 \cdot \tan 50 \tan 20 = \tan 50 + \tan 50$

$\Rightarrow \tan 70^\circ - \cot 20^\circ \cdot \tan 50 \cdot \tan 20 = \tan 50 + \tan 20^\circ$

$\Rightarrow \tan 70^\circ - \tan 50^\circ = \tan 50 + \tan 20$

$\Rightarrow \tan 70^\circ = \tan 20^\circ + 2 \tan 50$ proved

Q. Prove that $\frac{\cos x + \sin x}{\cos x - \sin x} - \frac{\cos x - \sin x}{\cos x + \sin x} = 2 \tan 2x$

Sol:- L.H.S $= \frac{\cos x + \sin x}{\cos x - \sin x} - \frac{\cos x - \sin x}{\cos x + \sin x}$

$= \frac{(\cos x + \sin x)^2 - (\cos x - \sin x)^2}{(\cos x - \sin x)(\cos x + \sin x)}$

$= \frac{4 \cos x \cdot \sin x}{\cos^2 x - \sin^2 x} = \frac{2 \cdot 2 \sin x \cdot \cos x}{\cos 2x}$

$= \frac{2 \sin 2x}{\cos 2x} = 2 \tan 2x$ proved

Q. Prove that $\frac{1 + \sin 2x - \cos 2x}{1 + \sin 2x + \cos 2x} = \tan x$

Sol:

$\frac{1 + \sin 2x - \cos 2x}{1 + \sin 2x + \cos 2x}$

$= \frac{1 - \cos 2x + \sin 2x}{1 + \cos 2x + \sin 2x}$

$= \frac{2 \sin^2 x + 2 \sin x \cdot \cos x}{2 \cos^2 x + 2 \sin x \cdot \cos x}$

$\frac{2 \sin x (\sin x + \cos x)}{2 \cos x (\sin x + \cos x)}$

$= \frac{\sin x}{\cos x}$

$= \tan x$ proved

Q. Prove that: $\cos 4x = 1 - 8 \sin^2 x \cdot \cos^2 x$

Sol:- $\cos 2 \cdot (2x)$

$= 1 - 2 \cdot \sin^2 (2x)$

$= 1 - 2 (2 \sin x \cos x)^2$

$= 1 - 2 \cdot 4 \sin^2 x \cdot \cos^2 x$

$1 - 8 \sin^2 x \cdot \cos^2 x$

v.v.I

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Q. Prove that:- $\tan 40 + 2\tan 10^\circ = \tan 50^\circ$

Sol:- $\tan(50-40) = \tan 10$

$\Rightarrow \dfrac{\tan 50 - \tan 40}{1+\tan 50 \tan 40} = \tan 10$

$\Rightarrow \tan 50 - \tan 40 = \tan 10 + \tan 50 \cdot \tan 40 \cdot \tan 10$

$\Rightarrow \tan 50 - \tan 40 = \tan 10 + \tan(90-40) \cdot \tan 40 \cdot \tan 10$

$\Rightarrow \tan 50 - \tan 40 = \tan 10 + \cot 40 \cdot \tan 40 \ \tan 10$

$\Rightarrow \tan 50 - \tan 40 = \tan 10 + \tan 10^\circ$

$\Rightarrow \tan 50 = \tan 40 + 2\tan 10^\circ$

SPS

v.v.I

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$\sin 2A = 2\sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$= 1 - 2\sin^2 A$

$= 2\cos^2 A - 1$

$1+\cos 2A = 2\cos^2 A$

$1-\cos 2A = 2\sin^2 A$

$\tan 3A = \dfrac{3\tan A - \tan^3 A}{1-3\tan^2 A}$

$\sin A = 3\sin \dfrac{A}{3} - 4\sin^3 \dfrac{A}{3}$

$\cos A = 4\cos^3 \dfrac{A}{3} - 3\cos \dfrac{A}{3}$

$\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$

$\tan A = \dfrac{3\tan \frac{A}{3} - \tan^3 \frac{A}{3}}{1 - 3\tan^2 \frac{A}{3}}$

$\sin A = 2\sin \dfrac{A}{2} \cos \dfrac{A}{2}$

$\cos A = \cos^2 \dfrac{A}{2} - \sin^2 \dfrac{A}{2}$

$1 - 2\sin^2 \dfrac{A}{2}$

$2\cos^2 \dfrac{A}{2} - 1$

$1+\cos A = 2\cos^2 \dfrac{A}{2}$

$1-\cos A = 2\sin^2 \dfrac{A}{2}$

$\tan A = \dfrac{2\tan \frac{A}{2}}{1-\tan^2 \frac{A}{2}}$

$\sin 3A = 3\sin A - 4\sin^3 A$

$\sin^3 A = \dfrac{3\sin A - \sin 3A}{4}$

$\cos 3A = 4\cos^3 A - 3\cos A$

$\cos^3 A = \dfrac{3\cos A + \cos^3 A}{4}$

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Q Prove that :- $\frac{1+\cos 4x}{\cot x - \tan x} = \frac{1}{2}\sin 4x$.

Sol :- L.H.S $= \frac{1+\cos 4x}{\cot x - \tan x}$

$= \frac{1+\cos 2\cdot 2x}{\frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}}$

$= \frac{2\cos^2 2x}{\frac{\cos^2 x - \sin^2 x}{\sin x \cdot \cos x}}$

$= \frac{2\cos^2 2x \cdot \sin x \cdot \cos x}{\cos^2 x - \sin^2 x}$

$= \frac{\cos^{\cancel{2}} 2x \cdot 2\sin x \cdot \cos x}{\cancel{\cos 2x}}$

$= \frac{2}{2}\cos 2x \sin 2x$

$= \frac{1}{2}\sin 2\cdot 2x$

$= \frac{1}{2}\sin 4x$

So, L.H.S = R.H.S

Q Prove that :- $\frac{\sin x}{1+\cos x} = \tan\frac{x}{2}$

L.H.S $= \frac{\sin x}{1+\cos x}$

$= \frac{\cancel{2}\sin\frac{x}{2}\cancel{\cos\frac{x}{2}}}{\cancel{2}\cos^{\cancel{2}}\frac{x}{2}}$

$= \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}$

$= \tan\frac{x}{2}$ = R.H.S

So, L.H.S = R.H.S

Q Prove that : $\frac{\sin x}{1-\cos x} = \tan\left(\frac{\pi}{4}+\frac{x}{2}\right)$

Sol: L.H.S $= \frac{\cos x}{1-\sin x}$

$= \frac{\cos^2\frac{x}{2} - \sin^2\frac{x}{2}}{\cos^2\frac{x}{2} + \sin^2\frac{x}{2} - 2\sin\frac{x}{2}\cdot\cos\frac{x}{2}}$

$= \frac{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)\cancel{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)}}{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)^{\cancel{2}}}$

$= \frac{\cos\frac{x}{2}+\sin\frac{x}{2}}{\cos\frac{x}{2}-\sin\frac{x}{2}}$

$$= \frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}$$

$$= \frac{\tan\frac{\pi}{4} + \tan\frac{x}{2}}{1 - \tan\frac{\pi}{4}\cdot\tan\frac{x}{2}}$$

$$= \tan\left(\frac{\pi}{4}+\frac{x}{2}\right) \text{ R.H.S}$$

(Imp) Q. $\sqrt{\frac{1+\sin x}{1-\sin x}} = \tan\left(\frac{\pi}{4}+\frac{x}{2}\right)$

Sol: L.H.S $= \sqrt{\frac{1+\sin x}{1-\sin x}}$

$$= \frac{\sqrt{1+\sin x}}{\sqrt{1-\sin x}}$$

$$= \frac{\sqrt{1+\sin x}}{\sqrt{1-\sin x}} \times \frac{\sqrt{1+\sin x}}{\sqrt{1+\sin x}}$$

$$= \frac{\sqrt{1-\sin^2 x}}{1-\sin x} \quad = \frac{1+\sin x}{\sqrt{1-\sin^2 x}}$$

$$= \frac{\cos x}{1-\sin x} \quad = \frac{1+\sin x}{\cos x}$$

$$= \frac{-\cos\left(\frac{\pi}{2}+x\right)}{\sin\left(\frac{\pi}{2}+x\right)}$$

$$= \frac{2\sin^2\left(\frac{\pi}{4}+\frac{x}{2}\right)}{2\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)} = \cos\left(\frac{\pi}{4}+\frac{x}{2}\right)$$

$$= \frac{\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)}{\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)}$$

$$= \tan\left(\frac{\pi}{4}+\frac{x}{2}\right) = \text{R.H.S}$$

So, L.H.S = R.H.S proof

2nd method,

L.H.S $= \sqrt{\frac{1+\sin x}{1-\sin x}}$

$$= \frac{\sqrt{1+\sin x}}{\sqrt{1-\sin x}} \times \frac{\sqrt{1-\sin x}}{\sqrt{1-\sin x}}$$

$$= \frac{\sqrt{1-\sin^2 x}}{1-\sin x}$$

$$= \frac{\sin\left(\frac{\pi}{2}+x\right)}{1+\cos\left(\frac{\pi}{2}+x\right)}$$

$$= \frac{\cancel{2}\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)\cdot\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)}{\cancel{2}\cos^2\left(\frac{\pi}{4}+\frac{x}{2}\right)}$$

$$= \frac{\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)}{\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)}$$

$$\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)$$

L.H.S = R.H.S proof

Level I

Q. If $A+B+C = \pi$ prove that $\sin 2A + \sin 2B + \sin 2C = 4\sin A\cdot\sin B\sin C$

Sol: $\sin 2A + \sin 2B + \sin 2C$ (CD formula)

$2\sin\left(\frac{2A+2B}{2}\right)\cdot\cos\left(\frac{2A-2B}{2}\right) + 2\sin C\cdot\cos C$

$= 2\sin(A+B)\cdot\cos(A-B) + 2\sin C\cdot\cos C$

$= 2\sin C\cdot\cos(A-B) + 2\sin C\cdot\cos C$

$= 2\sin C\,\{\cos(A-B) + \cos C\}$

$= 2\sin C\,\{\cos(A-B) - \cos(A+B)\}$ $[\because \cos C = -\cos(A+B)]$

$= 2\sin C\cdot 2\sin A\sin B$

$= 4\sin A\cdot\sin B\cdot\sin C$

So, L.H.S = R.H.S proved

V.Imp

Q. If $A+B+C = \pi$ prove that $\sin 2A = \sin 2B + \sin 2C = 4\cos A\cdot\cos B\cdot\cos C$

Sol: L.H.S $= \sin 2A = \sin 2B + \sin 2C$

$= 2\cos\left(\frac{2A+2B}{2}\right)\cdot\sin\left(\frac{2A-2B}{2}\right) + 2\sin C\cdot\cos 2C$ {C.D}

$= 2\cos(A+B)\cdot\sin(A-B) + 2\sin C\cdot\cos C$

$= -2\cos C\cdot\sin(A-B) + 2\sin C\cdot\cos C$ $\{A+B = \pi - C,\ \cos(A+B) = \cos(\pi - C)\}$

$= 2\cos C\,\{\sin C - \sin(A-B)\}$

$= 2\cos C\,\{\sin\{\pi-(A+B)\} - \sin(A-B)\}$ $\{A+B+C=\pi,\ C = \pi-(A+B)\}$

$= 2\cos C\,\{\sin(A+B) - \sin(A-B)\}$

$= 2\cos C\cdot 2\cos A\cdot\cos B$

$= 4\cos A\cdot\cos B\cdot\cos C =$ R.H.S

So, L.H.S = R.H.S proved

Q. If $A+B+C = \pi$ prove that $\cos 2A + \cos 2B + \cos 2C = 1 - 4\cos A\cdot\cos B\cdot\cos C$.

Sol:- L.H.S $= \cos 2A + \cos 2B + \cos 2C$

$= 2\cos\left(\frac{2A+2B}{2}\right)\cdot\cos\left(\frac{2A-2B}{2}\right) + 2\cos^2 C - 1$

$= 2\cos(A+B)\cdot\cos(A-B) + 2\cos^2 C - 1$

$= -2\cos C\cdot\cos(A-B) + 2\cos^2 C - 1$

$= 2\cos C\,\{\cos C - \cos(A-B)\} - 1$

$= 2\cos C\,[\cos\{\pi-(A+B)\} - \cos(A-B)] - 1$

$= 2\cos C\,[-\cos(A+B) - \cos(A-B)] - 1$

$= -2\cos C\,[\cos(A+B) + \cos(A-B)] - 1$

$= -2\cos C\cdot 2\cos A\cdot\cos B - 1$

$= -4\cos A\cdot\cos B\cdot\cos C - 1$

$= -1 - 4\cos A\cdot\cos B\cdot\cos C$.

Q. If $A+B+C = \pi$ prove that $\cos 2A + \cos 2B - \cos 2C = 1 - 4\sin A\cdot\sin B\cdot\sin C$.

Sol:-

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Differentiation $\frac{d}{dx}$ (function)

1. $\frac{d}{dx} x^n = nx^{n-1}$
2. $\frac{d}{dx} x = 1$
3. $\frac{d}{dx} \sqrt{x} = \frac{1}{2\sqrt{x}}$
4. $\frac{d}{dx} \frac{1}{x^n} = \frac{-n}{x^{n+1}}$
5. $\frac{d}{dx} \frac{1}{x} = \frac{-1}{x^2}$
6. $\frac{d}{dx} e^x = e^x$
7. $\frac{d}{dx} a^x = a^x \log a$
8. $\frac{d}{dx}$ (Constant) $= 0$
9. $\frac{d}{dx} \log x = \frac{1}{x}$
10. $\frac{d}{dx} \sin x = \cos x$
11. $\frac{d}{dx} \cos x = -\sin x$
12. $\frac{d}{dx} \tan x = \sec^2 x$
13. $\frac{d}{dx} \cot x = -\operatorname{cosec}^2 x$
14. $\frac{d}{dx} \sec x = \sec x \cdot \tan x$
15. $\frac{d}{dx} \operatorname{cosec} x = -\operatorname{cosec} x \cdot \cot x$
16. $\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$
17. $\frac{d}{dx} \cos^{-1} x = \frac{-1}{\sqrt{1-x^2}}$
18. $\frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2}$
19. $\frac{d}{dx} \cot^{-1} x = \frac{-1}{1+x^2}$
20. $\frac{d}{dx} \sec^{-1} x = \frac{1}{x\sqrt{x^2-1}}$
21. $\frac{d}{dx} \operatorname{cosec}^{-1} x = \frac{-1}{x\sqrt{x^2-1}}$
22. $\frac{d}{dx} c = 0$
23. $\frac{d}{dx} cx = c\frac{d}{dx} x$
24. $\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$
25. $\frac{d}{dx}(u \cdot v) = u\frac{dv}{dx} + v\frac{du}{dx}$
26. $\frac{d}{dx}\frac{u}{v} = \frac{v\frac{du}{dx} \cdot u\frac{dv}{dx}}{v^2}$

Q. $\frac{d}{dx}(2x^5 + \sin x + 2\cos^{-1} x)$

Sol: $\frac{d}{dx} 2x^5 + \frac{d}{dx}(\sin x) + \frac{d}{dx}(2\cos^{-1} x)$

$2\frac{d}{dx} x^5 + \frac{d}{dx} \sin x + \frac{d}{dx} 2\cos^{-1} x$

$2 \times 5x^4 + \cos x + 2\frac{-1}{\sqrt{1-x^2}}$

$= 10x^4 + \cos x - \frac{2}{\sqrt{1-x^2}}$

Q. $\frac{d}{dx}(x^2 - 5x + 6)$

Sol: $\frac{d}{dx}(x^2) - \frac{d}{dx}(5x) + \frac{d}{dx}(6)$

$2x - 5 + 0$

$= 2x - 5$

(i) $\frac{d}{dx} x^n = nx^{n-1}$

(ii) $\frac{d}{dx} x = 1$

(iii) $\frac{d}{dx} \sqrt{x} = \frac{1}{2\sqrt{x}}$

(iv) $\frac{d}{dx} \frac{1}{x^n} = \frac{-n}{x^{n+1}}$

(v) $\frac{d}{dx} \frac{1}{x} = \frac{-1}{x^2}$

(vi) $\frac{d}{dx} e^x = e^x$

(vii) $\frac{d}{dx} a^x = a^x \log a$

(viii) $\frac{d}{dx} \log x = \frac{1}{x}$

(ix) $\frac{d}{dx}(\text{constant}) = 0$

(x) $\frac{d}{dx} \sin x = \cos x$

(xi) $\frac{d}{dx} \cos x = -\sin x$

(xii) $\frac{d}{dx} \tan x = \sec^2 x$

(xiii) $\frac{d}{dx} \cot x = -\sec^2 x$

(xiv) $\frac{d}{dx} \sec x = \sec x \cdot \tan x$

(xv) $\frac{d}{dx} \operatorname{cosec} x = -\operatorname{cosec} x \cdot \cot x$

(xvi) $\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$

(xvii) $\frac{d}{dx} \cos^{-1} x = \frac{-1}{\sqrt{1-x^2}}$

(xviii) $\frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2}$

(xix) $\frac{d}{dx} \cot^{-1} x = -\frac{1}{1+x^2}$

(xx) $\frac{d}{dx} \sec^{-1} x = \frac{1}{x\sqrt{1-x^2}}$

(xxi) $\frac{d}{dx} \operatorname{cosec}^{-1} x = -\frac{1}{x\sqrt{x^2-1}}$

For constant:

(i) $\frac{d}{dx} c = 0$

(ii) $\frac{d}{dx} cx = c \cdot \frac{dx}{dx}$

(iii) $\frac{d}{dx} \frac{x}{c} = \frac{1}{c} \frac{dx}{dx}$

(iv) $\frac{d}{dx}(u \pm v) = \frac{du}{dx} + \frac{dv}{dx}$

(v) $\frac{d}{dx}(u \cdot v) = u\frac{dv}{dx} + v\frac{du}{dx}$

(vi) $\frac{d}{dx} \frac{u}{v} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Division or quotient

Solve

(i) $\frac{d}{dx}(e^x - \sin x + \sec^{-1} x)$

(ii) $\frac{d}{dx}(e^x \cdot \sin x)$

(iii) $\frac{d}{dx}\left(\frac{e^x}{\sin}\right)$

Sol^n

(i) $\frac{d}{dx} e^x - \frac{d}{dx} \sin x + \frac{d}{dx} \sec^{-1} x$

$e^x - \cos x + \frac{1}{x\sqrt{x^2-1}}$

(ii) $\frac{d}{dx}(e^x \sin x)$

$e^x \frac{d}{dx} \sin x + \sin x \frac{d}{dx} e^x$

$e^x \cos x + \cancel{\tan x} \sin e^x$

$e^x(\cos x + \sin x)$

(iii) $\frac{d}{dx}\left(\frac{e^x}{\sin x}\right)$

$\frac{\sin x \cdot \frac{d}{dx} e^x - e^x \frac{d}{dx}(\sin x)}{\sin^2 x}$

$\frac{\sin e^x - e^x \cos x}{\sin^2 x}$

$e^x\left(\frac{\sin x - \cos x}{\sin^2 x}\right)$

① Addition Rule:

$$\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$$

② Subtraction Rule:

$$\frac{d}{dx}(u-v) = \frac{du}{dx} - \frac{dv}{dx}$$

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③ Multiplication / Product Rule:

$$\frac{d}{dx}(u \cdot v) = u\frac{dv}{dx} + v\frac{du}{dx}$$

④ Division / Quotient Rule

$$\frac{d}{dx}\frac{u}{v} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Q. Solve: $\frac{d}{dx}(\sin x \cdot \log x)$

$\sin x \frac{d}{dx}(\log x) + \log x \frac{d}{dx} \sin x$

$\sin x \frac{1}{x} + \log x \cos x$

$\frac{\sin x}{x} + \log \cos x \log x$

(ii) $\frac{d}{dx} \frac{\log x}{e^x}$

Sol: $\frac{d}{dx} \frac{d}{dx} \frac{u}{v} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$$\frac{d}{dx} = \frac{e^x \frac{d}{dx}\log x - \log x \frac{d}{dx} e^x}{e^x}$$

$$\frac{d}{dx} = \frac{e^x \frac{1}{x} - \log x \, e^x}{e^x}$$

$$= \frac{\frac{e^x}{x} - \log e^x}{e^{2x}} = \frac{e^x\left(\frac{1}{x} - \log x\right)}{e^{2x}}$$

$$= \frac{\left(\frac{1}{x} - \log x\right)}{e^x}$$

$$= \frac{1 - x\log x}{x \cdot e^x}$$

#. $\frac{d}{dx}(\sin x \cdot \cos x)$

Sol:- $\sin x \frac{d}{dx}\cos x + \cos \frac{d}{dx} \sin x$

$= \sin x(-\sin x) + \cos x \cdot \cos x$

$= -\sin^2 x + \cos^2 x$

$= \cos^2 x - \sin^2 x$

$= \cos 2x$

Q. $\frac{d}{dx}(x^2 \cdot \sin x)$

Sol: $x^2 \cdot \frac{d}{dx} \sin x + \sin x \frac{d}{dx} x^2$

$x^2 \cdot \cos x + \sin x \, 2x$

$x^2 \cos x + 2x \sin x.$

Q $\frac{d}{dx}\left(\frac{x^2}{\sin x}\right)$

Sol: $\frac{\sin x \frac{d}{dx} \cdot x^2 - x^2 \cdot \frac{d}{dx} \sin x}{\sin^2 x}$

$\frac{\sin x \cdot 2x - x^2 \cdot \cos x}{\sin^2 x}$

$\frac{2x \sin x - x^2 \cos x}{\sin^2 x}$ Ans

Q $\frac{d(\tan 2x)}{dx}$

Sol: $\frac{d(\tan 2x)}{d(2x)} \times \frac{d(2x)}{dx}$

$\sec^2 2x \times 2 \frac{dx}{dx}$

$\sec^2 2x \times 2$

$2 \sec^2 2x$ Ans

Q $\frac{d}{dx} \sec 5x$

Sol: $\frac{d(\sec 5x)}{d(5x)} \times \frac{d(5x)}{dx}$

~~sec5x·tan x × 5~~ $\sec 5x \cdot \tan 5x \times 5$

$= 5 \sec 5x \cdot \tan 5x$

Q $\frac{d}{dx}\sqrt{\sin 5x}$

Sol: $\frac{d\sqrt{\sin 5x}}{d(\sin 5x)} \times \frac{d \sin 5x}{d(5x)} \times \frac{d(5x)}{dx}$

$= \frac{1}{2\sqrt{\sin 5x}} \times \cos 5x \times 5$

$= \frac{5 \cos 5x}{2\sqrt{\sin 5x}}$ Ans