Unit - 1

Units & Measurement

Measurement

In physics, we required to measure the physical quantities because we need the accurate measurement.

Hence, measurement consist of the comparison of an unknown quantity with a known fixed quantity. It is compulsory part of development technology.

The main purpose of measurement in engineering and science is to determine whether a job has been manufactured to the requirement of specification.

Accuracy of measurement depends on

- Method of measurement
- Measuring instrument.

Unit of a physical quantity.

The standard used for measurement of a physical quantity is called unit of that quantity. It is the reference standard used to measure that physical quantities.

Concept of Length, mass & time

Length of an object may be defined as the distance of separation between any two points at the extreme ends of the object.

Mass of an object may be defined as the quantity of matter in the object, which can never be zero.

The concept of time occurred first time from the motion of the moon across the sky, the formation of day and night as a result of rotation of the earth around the axis. A/c to Einstein, time is what clock reads.

Word done (w) is defined as the dot product of force f and displacement s.

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Work done (w) is defined as the dot Product of force (f) and displacement S.

$$w = f \cdot s$$

Heat :- Heat is form of energy. It is ~~Scensi e Sentasive~~ extensives porperty. Heat is S.I unit of Joule

Work done (w) is defined as the dot product of force and displacement. S

$$\omega = \vec{F} \cdot \vec{S}$$

Heat is form of energy. It is extensive porperty. Heat is S.I Unit Joule.

Requirement of standard unit

- It should be universally accepted.
- It should definite and well defined.
- It should be invariable (fixed) with time and place.
- It should be easily reproducible and non-perisable.
- It should be easily comparable with other similar unit.
- It should be readily available.

Units can be classified into two groups.

(I) Fundamental units — is the units of fundamental quantities, which doesnot depend on any other unit.

(II) Derived unit — is the units used to measure derived quantities, which depends on fundamental units for their measurement.

System of Units

FPS System — In this system, unit measured for length is foot, mass in pound and time in second.

CGS System — In this system, the unit of length, mass and time are centimeter, gram and second respectively.

MKS System — In this system, the unit of length, mass and time are meters, kilogram and second respectively.

SI unit system — This system is widely used in all ~~measure-ments~~ measurements throughout the world. The system is based on seven basic units and two supplementary units.

Fundamental unit	S.I		d.S.
Mass	Kilogram	K/g	M
length	Mettre	m	L
Time	Second	s	T
Temperature	Kalivn	K	K
Current	Ampere	A	A
Louminious intensity	Candala	cd	cd
Amount of sustance	Mole	mol	mol

Force = Mass x accelpration

$$\text{Mass} \times \frac{\text{Velocity}}{\text{Tim}} \qquad \text{Speed} = \frac{\text{dist}}{\text{Tim}}$$

$$\text{Mass} \times \frac{\frac{\text{distan}}{\text{Tin}}}{\text{Time}}$$

$$\cancel{\times} \frac{\text{distance}}{\text{Tix} \cdot \text{Tix}}$$

$$M \cancel{\times} \frac{L}{T \times T}$$

$$\times M \cancel{\times} \frac{L}{T^2}$$

$$F = M^1 \times L^1 \times T^{-2}$$

$$[M^1 L^1 T^{-2}]$$

hysical Quantities

hysical quantities are the quantities which are used to describe he property of a physical phenomenon. It measures the properties of material or substance.

hysical Quantities are classified into two types:-

1) Fundamental or base quantities — is the physical quantities which cannot be expressed in terms of any other physical quantities. It donot depend on any other physical quantity for its measurement.

There are 7 fundamental quantity and 2 Supplementary quantity.

Sl. No.	Fundamental P.Q.	Symbol	Fundamental unit	Symbol of units
1.	Mass	m	Kilogram	kg
2.	Length	l	Metre	m
3.	Time	t	Second	s
4.	Temperature	T	Kelvin	K
5.	Electric Current	I	Ampere	A
6.	Luminous Intensity	L	Candela	cd
7.	Amount of Substance	n	Mole	mol

No.	Supplementary P.Q	Symbol	Fundamental unit	Symbol of units
1.	Plane Angle		Radian	rad
2.	Solid Angle		Steradian	sr

Derived Quantities — is the physical quantities that can be expressed in terms of fundamental quantities. It depends upon more than one fundamental quantity for ~~its~~ its measurement.

Power :- The rate at which work is done is called power and is defined as.

$$P = \frac{W}{\Delta t}$$

= The rate at which work is done is called power and is defined as.

$$P = \frac{W}{t}$$

Work :- Work done is defined as the dot product of force and displacement. S

$$W = F \cdot S$$

Powr :- The rate at which is done

The rate at which is work is done is called power

$$P = \frac{W}{t}$$

$F =$
$d =$
$t =$

$$\text{Power} = \frac{W}{t} = \frac{F \cdot d}{t} = c$$

Desired units on SI

Sl. No.	Derived Physical Quantity	Symbol	Fundamental Unit	Symbol
1.	Area	A	Square metre	m^2
2.	Volume	V	Cubic metre	m^3
3.	Velocity	v	metre/second	m/s
4.	Acceleration	a	metre/Sq. second	m/s^2
5.	Force	F	Newton	N
6.	Pressure	P	Newton/Sq. metre	N/m^2
7.	Density	ρ	Kg / cubic met.	Kg/m^3
8.	Work	W	Joule	J
9.	Speed	S	Metre/second	M/s

Some practical units

1 Macro-cosm measurement (very large distance)

(i) Astronomical Unit (AU) – is an average distance of the centre of the sun from the centre of the earth.

$$1AU = 1.496 \times 10^{11} m \simeq 1.5 \times 10^{11} m.$$

(ii) Light Year (LY) — is the distance travelled by light in vacuum in one year. The velocity of light in vacuum (c) = 3×10^8 m/s.

$$1 year = 365 \times 24 \times 60 \times 60 \text{ sec.}$$
$$1 \text{ Light year} = 3 \times 10^8 \times 365 \times 24 \times 60 \times 60 \text{ m}$$
$$1LY = 9.46 \times 10^{15} \text{ m.}$$

(iii) Par Sec — is the radius of a circle at the centre of which an arc of the circle, 1AU long subtends an angle 1"

$$1 \text{ Par Sec} = 3.1 \times 10^{16} = 3.276955 \times \text{Light Year.}$$

Micro-cosm measurement (very small distance)

(i) 1 micron (1 μ) = 10^{-6} m
(ii) 1 nanometer (1 nm) = 10^{-9} m
(iii) 1 Angstorm (1 Å) = 10^{-10} m
(iv) 1 fermi (1 fm) = 10^{-15} m

Measuring heavy masses.

(i) 1 tonne or 1 ton = 10^3 kg
(ii) 1 Quintal = 100 kg
(iii) 1 slug = 14.57 Kg

Lunar month – is the time taken by the moon to complete one revolution around the earth in its orbit.

1 Lunar month = 27.3 days.

Shake – is the smallest unit of time.

1 Shake = 10^{-8} sec.

Some useful units of length.

(i) 1 inch = 0.0254 m
(ii) 1 foot = 0.3048 m
iii) 1 yard = 0.9144 m
iv) 1 mile = 1.609 Km
v) 1 Nautical mile = 1.852 KM
vi) 1 Newton = 10^5 Dyne = 10^7 erg

An erg – is the amount of work done by exerting a force of one dyne across a 1 cm distance. It will be gm-cm^2/s^2 throughout CGS base unit.

SI Prefixes.

The physical quantities whose magnitude is either too Large or too small can be expressed more compactly by used of certain prefixes. The prefixes commonly used for the powers of 10 as listed below:-

SL. No.	Power of 10	Prefixes	Symbol
1.	10^{-18}	atto	a
2.	10^{-15}	femto	f
3.	10^{-12}	pico	p
4.	10^{-9}	nano	n
5.	10^{-6}	micro	μ
6.	10^{-3}	milli	m
7.	10^{-2}	centi	c
8.	10^{-1}	deci	d
9.	10^{1}	deca	da
10.	10^{2}	hecto	h
11.	10^{3}	KiLo	K
12.	10^{6}	mega	M
13.	10^{9}	giga	G
14.	10^{12}	tera	T
15.	10^{15}	peta	P
16.	10^{18}	exa	E

Dimensions

The dimensional formula of any physical quantity is the formula that tells which of the fundamental units has been used for the measurement of that physical quantity. It is the powers to which the units of base quantities are raised to represent a derived unit of that quantity.

The relation between the physical quantity and fundamental quantity expressed in the form of equation is called dimensional equation.

In mechanics, all physical quantity is expressed by the symbol of dimensions of [M], [L] & [T].

Following table shows the dimensional formula and dimensional units of physical quantities

SL.No	Physical Quantity	Relation with other quantity	Dimensional formula	SI unit
1.	Volume	Length x Breadth x Height	$M^0 L^3 T^0$	m^3
2.	Acceleration	$\frac{\text{change in velocity}}{\text{Time Taken}}$	$M^0 L^1 T^{-2}$	m/s^2
3.	Velocity	$\frac{\text{change in displacement}}{\text{Time Taken}}$	$M^0 L^1 T^{-1}$	m/s
4.	Force	mass x acceleration	$M^1 L^1 T^{-2}$	Newton (N)
5.	Pressure	$\frac{\text{Force}}{\text{Area}}$	$M^1 L^{-1} T^{-2}$	N/m^2
6.	Impulse	Force x time	$M^1 L^1 T^{-1}$	NS
7.	Work	Force x Distance	$M^1 L^2 T^{-2}$	Joule (J)
8.	Density	$\frac{\text{Mass}}{\text{Volume}}$	$M^1 L^{-3} T^0$	Kg/m^3
9.	Kinetic Energy	$\frac{1}{2}.mv^2$	$M^1 L^2 T^{-2}$	Joule (J)
10.	Potential Energy	Mass x acceleration x length	$M^1 L^2 T^{-2}$	Joule (J)
11.	Power	$\frac{\text{Work}}{\text{Time}}$	$M^1 L^2 T^{-3}$	Watt (W)

Sl. No.	Physical Quantity	Relation with other quantity	Dimensional formula	SI unit
12.	Moment of force	Force × Distance	$M^1 L^2 T^{-2}$	N-m
13.	Torque	Moment of Inertia × Angular Acceleration	$M^1 L^2 T^{-2}$	N-m
14.	Heat (Q)	Energy	$M^1 L^2 T^{-2}$	Joule
15.	Surface Tension	$\frac{\text{Force}}{\text{Length}}$	$M^1 L^0 T^{-2}$	N/m
16.	Surface Energy	Energy of free space	$M^1 L^2 T^{-2}$	Joule
17.	Stress	$\frac{\text{Force}}{\text{Area}}$	$M^1 L^{-1} T^{-2}$	N/m^2
18.	Strain	$\frac{\text{Change in dimension}}{\text{original dimension}}$	$M^0 L^0 T^0$	No unit
19.	Co-efficient of elasticity	$\frac{\text{Stress}}{\text{Strain}}$	$M^1 L^{-1} T^{-2}$	N/m^2
20.	Moment of Inertia (I)	Mass × (radius of gyration)^2	$M^1 L^2 T^0$	$Kg\ m^2$
21.	Universal constant of Gravitation	$G = \frac{Fr^2}{m_1 m_2}$	$M^{-1} L^3 T^{-2}$	$Nm^2 Kg^{-2}$
22.	Radius of Gyration	Distance	$M^0 L^1 T^0$	m
23.	Angle	$\frac{\text{Length}}{\text{Radius}}$	$M^0 L^0 T^0$	Radian
24.	Frequency	No. of vibration / sec.	$\frac{1}{T} = M^0 L^0 T^{-1}$	Hertz (Hz) or s^{-1}
25.	Planck's constant (h)	$\frac{\text{Energy (E)}}{\text{Frequency } (\nu)}$	$M^1 L^2 T^{-1}$	J.s
26.	Latent heat (L)	$\frac{\text{Heat (Q)}}{\text{Mass (m)}}$	$M^0 L^2 T^{-2}$	J/kg
27.	Temperature (T)	———	$M^0 L^0 T^0 K^1$	K
28.	Wavelength	Length of one wave	$M^0 L^1 T^0$	m
29.	Electric current (I)	Ampere is the fundamental unit	$M^0 L^0 T^0 A^1$	A
30.	Momentum	Mass × Velocity	$M^1 L^1 T^{-1}$	kg m/s
31.	Angular velocity	$\frac{\text{Angle}}{\text{Time}}$	$M^0 L^0 T^{-1}$	radian/sec
32.	Resistance or Impedance	$\frac{\text{Voltage}}{\text{current}}$	$M^1 L^2 T^{-3} I^{-2}$	Ohm
33.				

Characteristics of Dimensions

Dimensions donot depend on the system of units.
Quantities with similar dimensions can be added or subtracted from each other.
Dimensions can be obtained from the units of the physical quantities and vice versa.
Two different quantities can have the same direction.

Uses of dimensional equations

To check the dimensional correctness of given physical equation.
To convert a physical quantity from one system of units to another system
To establish relation among various physical quantities.

Limitations of Dimensions.

The constant of proportionality cannot be determined
The method of dimensions gives us no such information about the dimensionless constant in the formula.
We cannot determine or derive the formulae containing trigonometric functions, large functions, etc, which has no dimensions.
This method cannot be used to derive equations involving addition and subtraction of physical quantities.

Principle of Homogeneity of Dimensions

According to principle of homogeneity, the dimensions of all the terms in the physical equation must be same. For an equation to be dimensionally correct, the dimensions of each term on LHS must be equal to the dimensions of each term on RHS.

Defects of Dimensional Analysis.

(I) While deriving the formula, the proportionality constant cannot be found.
(II) Its physical quantity that depends on more than three independent physical quantity cannot be deducted.
(III) This method cannot be used if the physical quantity depends on more parameters than the number of fundamental quantities.

Note :-

(I) Numbers are dimensionless
(II) Mathematical constant like π, e, etc are dimensionless.

Accuracy, Precision & Error.

Accuracy is the agreement of the result of a measurement with the true value of the measured quantity. It measures the closed measured value is the true of quantity. Accuracy of measurement depends upon human limitation, ie, personal error, random error. It also depends on least count and range of instrument.

Precision is defined as the repeatability of a measuring process. It describes the limitation of the measuring instrument. It is determined by the Least count of the measuring instrument. It depends on quality of instrument, human limitation & conditions of Surrounding

An error is a fault which may occur even in the most careful observation. It arises due to human limitations and instrumental limitations. The difference between the true value and measured value of a quantity is called error.

There are three types of error:-

(i) Instrumental or constant error – is caused due to faulty instrument

(ii) Systematic error – is caused due to defective Settings or adjustment or unsystematicness of the experimenter.

(iii) Random error – is caused due to changed due to changed in experimental conditions, human limitations.

Estimation of errors.

Any Statistical process which seeks to approximate an unknown value, such as distribution is known as the estimation of errors.

There are three types of error.

(1) Absolute Error

The difference between the measured value of a quantity and its actual value gives the absolute error. It is the variation between the actual values and measured values. It is given by

Absolute Error = | Actual value − Absolute ~~Error~~ value |

(2) Relative Error

The ratio of the absolute error to the accepted measurement gives the relative error. It is given by the formula

$$\text{Relative Error} = \frac{\text{Absolute Error}}{\text{Actual value.}}$$

(3) Percentage Error

It is another way of expressing the error in measurement. This calculation allows us to gauge how accurate a measured value is with respect to the true value. It is given by the formula

$$\text{Percentage error} = \text{Relative Error} \times 100$$

Significant Figures

A significant figure is defined as a figure in any place (in number) which is reasonably trustworthy (meaningful). It measures accuracy of particular measurement of a physical quantity.
Significant digits are the digits that are meaningful in assigning a true value or realistic value to a result.

Rules to determine Significant figure :-

1. All non-zero digits are significant.
2. All zeros occurring between two non-zero digits are significant.
3. All zeroes to the right of the last non-zero digit are not significant. Its value signifies when they come from measurement.
4. All zeros to the right of a decimal place and to the left of a non-zero digit are not significant.

Examples :-

Sl. No.	Value	Number of Significant figures.
1.	1.080	4
2.	0.0018	2
3.	0.00180	3
4.	0.01800	4
5.	4.38×10^{11}	3
6.	$\pi = 3.14159265 4$ $\simeq 3.14$	3

Note :- For ideal measurement, it should be accurate as well as precise

UNIT - 2

MECHANICS.

2.1 Motion along a Straight line & Force

Concept of Scalar & Vector quantities

On the basis of magnitude and direction, all the physical quantities are classified into two groups as scalars and vectors.

A scalar quantity is a physical quantity which have only magnitudes and no direction. Such physical quantities can be described just by their numerical value without directions. Examples are Mass, speed, volume, density, time, energy, etc.

On the other hand, a vector quantity is a physical quantity that has both, magnitude and directions. The term also denotes the mathematical or geometrical representation of such a quantity. It have many real-life applications including situations involving force or velocity. Examples are Displacement, force, momentum, torque, weight, electromagnetic field, etc.

Equation of motion with constant ~~motion~~ acceleration (Derivation not required)

Motion is the state of change in the position of an object over time. It is described in terms of displacement, distance, velocity, acceleration, time and Speed. The relations between these quantities are known as the equations of motions.

Consider an object moving along a straight line path with initial velocity (u), after some ~~time~~ time (t), its final velocity (v), uniform acceleration (a) and distance travelled by the object in time (t) is S, then the following relations are obtained, which are called equations of one dimensional motion.

There are three equations of motion which are also known as the laws of constant acceleration.

(i) $v = u + at$

(ii) $s = ut + \frac{1}{2}at^2$

(iii) $v^2 = u^2 + 2as$

These equations are referred as SUVAT equations where SUVAT stands for displacement (s), initial velocity (u), final velocity (v), acceleration (a) and time (t).

Derivation :-

$$a = \frac{v-u}{t}$$

$$v - u = at$$

$$v = u + at$$

$\because$ Displacement = Avg. velocity x Time

$$\text{Avg. velocity} = \frac{u+v}{2}$$

$$\therefore \quad s = \frac{u+v}{2} \times t$$

$$2s = ut + vt$$

$$2s = ut + (u+at)t = ut + ut + at^2$$

$$s = ut + \frac{1}{2}at^2$$

we have, $v = u + at \Rightarrow t = \frac{v-u}{a}$

$$\cancel{s = ut + \frac{1}{2}at^2}$$

$$s = \left(\frac{v+u}{2}\right)(t)$$

$$s = \frac{v+u}{2} \times \frac{v-u}{a}$$

$$s = \frac{v^2-u^2}{2a}$$

$$v^2 - u^2 = 2as.$$

Equation of motion of falling body under gravity

For a freely falling body,

Acceleration (a) = Acceleration due to gravity (g)
Distance travelled (s) = Height above the surface of the earth (h)

Thus, the equations of motion for a freely falling body is

	Body moves upward direction
① $v = u + gt$	$v = u - gt$
② $h = ut + \frac{1}{2} gt^2$	$h = ut - \frac{1}{2} gt^2$
③ $v^2 = u^2 + 2gh$	$v^2 = u^2 - 2gh$

In the above equation, replacing + by - if the body is thrown upwards.

Maximum Height Attained.

Let a body be projected vertically upwards with an initial velocity u. As it moves upwards, with its acceleration is taken as $-g$. As the body goes up, its velocity decreases and finally becomes zero ($v=0$), when its reaches maximum height. Now, the above equation ③ becomes

$$0 = +u^2 - 2gh$$
$$u^2 = 2gh$$
$$h = \frac{u^2}{2g} \quad \text{——} \quad (4)$$

Time of Ascent (t_1)

The time taken by a body thrown up to reach maximum height is called its time of ascent. Let t_1 be the time of ascent. At the maximum height, its velocity $v = 0$, then the equation becomes,

$$0 = u - gt_1$$
$$u = gt_1$$
$$t_1 = \frac{u}{g} \quad \text{--- (5)}$$

Time of descent (t_2)

After reaching the maximum height, the body begins to travel downwards like a freely falling body. The time taken by a freely falling body to reach the ground is called the time of descent (t_2). In this case, $u = 0$ and g is positive. Then, equation 2 becomes

$$h = 0 \times t_2 + \frac{1}{2} \times gt_2^2$$
$$h = \frac{1}{2} gt_2^2$$
$$t_2^2 = \frac{2h}{g}$$
$$t_2 = \sqrt{\frac{2h}{g}} \quad \text{--- (6)}$$

By equation (4)

$$h = \frac{u^2}{2g}$$
$$t_2 = \sqrt{\frac{2}{g} \times \frac{u^2}{2g}} = \sqrt{\frac{u^2}{g^2}}$$
$$t_2 = \frac{u}{g} \quad \text{--- (7)}$$

The above discussion makes it clear that time of ascent is equal to the time of descent in the case of bodies moving under gravity.

$$\therefore \; t_1 = t_2$$

Time of flight (t_f)

The time of flight is the time taken by a body to remain in air and is given by the sum of the time of ascent (t_1) and the time of descent (t_2).

$$t_f = t_1 + t_2$$

$$t_f = \frac{u}{g} + \frac{u}{g}$$

$$t_f = \frac{2u}{g}$$

velocity of a body dropped from a height

when a body is dropped from a height h, its initial velocity (u) is zero. Let the final velocity on reaching a ground is v, then equation 3 become

$$v^2 = 2gh$$

$$v = \sqrt{2gh}$$

At the same time, from equation (4), we note that

$$u = \sqrt{2gh}$$

This means that velocity of the body falling from a height (h) on reaching the ground is equal to the velocity with which it is projected vertically upwards to reach the same height (h). Hence, the upward velocity at any point in its flight is the same as its downward velocity at that point.

Note :– A motion under gravity is one that occurs when an item moves as a result of the application of gravity. When an object is raised or moved away from the ground, a downward force is applied to the object. This force is referred to as gravity. The earth's gravity pushes the object towards itself with an acceleration due to gravity. This force is a result of the earth's gravitational attraction and is related to the idea of gravitational forces.

Newton's Law of Motion

Newton's Laws are essential because they relate to everything we do or see in everyday life. These laws tell us how things move or stay still, it imply the relationship between an object's motion and the forces acting on it.

Newton's first Law of Motion

Any object remains in the state of rest or in a uniform motion along a straight line, it is compelled to change the State by applying an external force. This law as also known as Law of inertia.

It defines inertia, force and inertial frame of reference.

Newton's Second Law of Motion

The rate of change of linear momentum of a body is directly proportional to the external force applied on it and this change in momentum takes place in the direction of the applied force.

$$\text{Force}, F \propto \frac{\text{change in momentum}}{\text{Time}}$$

$$F = k \cdot \frac{d\vec{p}}{dt}$$

$$F = k \cdot m \frac{d\vec{v}}{dt}$$

$$F = km\vec{a}$$

where k = constant of proportionality.

Newton's third Law of Motion

To every action, there is equal and opposite reaction force. When a body A exerts a force on another body B, B exerts an equal and opposite force on A. This law is also known as Law of action and reaction.

Some daily life examples of Newton's 1st, 2nd & 3rd laws of motion.

(1) The motion of a ball falling through the atmosphere or a model rocket being launched up into the atmosphere are both, excellent examples of Newton's 1st law.

(2) Riding a bicycle is an excellent example of 2nd law of Newton. Here, the bicycle is a mass and leg muscle pushing on the pedals of the bicycle is the force.

(3) When you hit a wall with a certain amount of force, and the wall returns that same amount of force. This is an excellent example of Newton's 3rd law.

FORCE.

Force is a physical cause that changes or may tend to change the state of rest or the state of motion of an object and also the shape of the object. It is an influence that can change the motion of an object.

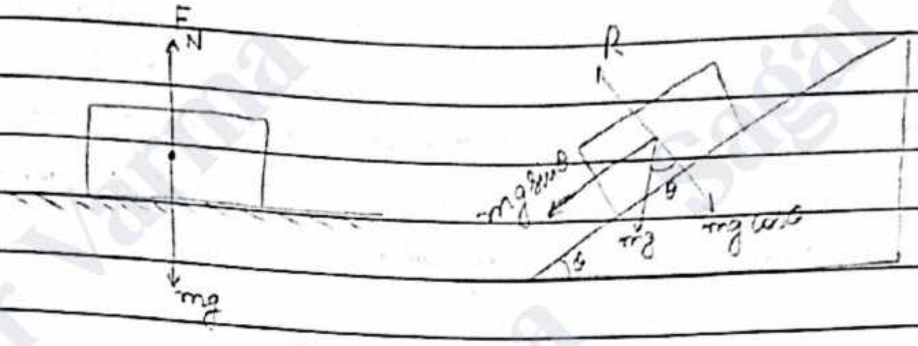
A force can cause an object with mass to change its velocity, i.e., to accelerate. It can also be described intuitively as a push or a pull. It has both magnitude and direction.

It is measured in the SI unit of Newton (N).

Common Forces in Mechanics

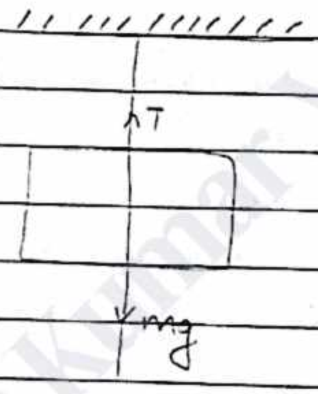
Weight – Weight of a body is the force with which earth attracts it. Its magnitude is given by mg. It is also k/a gravitational force.

2. Reaction or Normal Force – When a body is placed on a rigid surface, then the body experiences a force which is perpendicular to the surface in contact. This force is called reaction or normal force.



$$R = mg\cos\theta$$

3. Tension – The force exerted by the end of string, rope or chain against pulling (applied) force is called the tension. Its direction is taken along the string and away from the body under consideration.

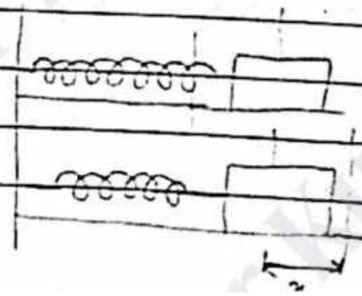


4. Spring Force – Every spring resists any change in its length. This resistive force increases with change in length. It is given by — $F = -kx$

Here,

x = change in length

K = Spring constant.



Impulse

Impulse is defined as the product of force and the small time interval for which it acts. Forces acting for short duration are called impulsive forces. It is given by

$$J = \int F \cdot dt$$

when a tennis ball hits the racket, it is supplied with a high magnitude and short duration force that helps to change the direction of motion of the ball

It is a vector quantity and SI unit is N·s
If force of an impulse is changing with time, then the impulse is measured by finding the area bound by force-time graph for that force.

Linear Momentum

The linear momentum of a body is defined as the product of the mass of the body and its velocity.

$$\text{Linear Momentum} = \text{mass} \times \text{velocity}$$

$$\vec{p} = m\vec{v}$$

Law of Conservation of Momentum

According to the law of conservation of linear momentum, if the net external force acting on a ~~body~~ system of bodies is zero, then the momentum of the system remains constant. If

$$F_{ext} = 0$$

$$\therefore \sum_{i=1}^{n} P_i = \text{Constant} \quad \text{or} \quad P = \text{Constant}$$

Example - Recoiling of a gun, Flight of rockets, Jet Planes.

$$M_1 U_1 + M_2 U_2 = M_1 V_1 + M_2 V_2$$

2.2 Kinetics

Application of Law of Motion.

Recoil of Gun :-

When a bullet is fired from the gun, due to the reaction force, the gun recoils in the direction opposite to the motion of bullet. The velocity with which it recoils is called Recoil Velocity.

Derivation for recoil velocity of Gun.

The initial velocity of gun & bullet are at rest. Hence, u_1 and u_2 = 0.

Let m and M be the mass of bullet and gun respectively.

The sum of momentum before firing bullet is zero. Therefore,

$$mU_1 + MU_2 = 0.$$

If V_1 and V_2 are the final velocity of bullet and gun receptively after firing.

Then, according to Law of conservation of momentum,

Total momentum before recoil = Total momentum after recoil

$$\therefore \quad 0 = mV_1 + MV_2$$

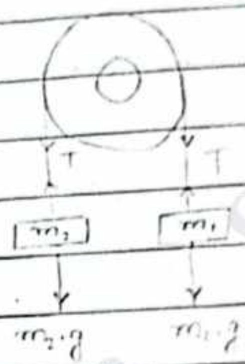
$$\Rightarrow \quad MV_2 = -mV_1$$

$$V_2 = \frac{-mV_1}{M}$$

This is the recoil velocity of gun. Here negative sign indicates that the direction of the velocity of the gun is opposite to the direction of velocity of the bullet. Recoil velocity of gun is much smaller than velocity of bullet.

Hence, Recoiling is the backward momentum of a gun to balance the forward momentum of the bullet, therefore, it follows under third law of motion. This law of motion is primarily involved during the gun recoil.

Motion of two connected bodies by light inextensible string over smooth pulley.



Consider a light inextensible string passing over a smooth pulley as shown in figure, so that the tension (T) in both the strings is same.
Let mass (m_1) is greater than mass (m_2).
Since, the string is inextensible, the upward direction of acceleration of mass (m_2), will be equal to the downward acceleration of mass (m_1).
Therefore, the acceleration is given by

$$\boxed{a = \frac{g\,(m_1 - m_2)}{m_1 + m_2}}\ m/s^2$$

and tension in the string

$$\boxed{T = \frac{2\,m_1 \cdot m_2 \cdot g}{m_1 + m_2}}\ N$$

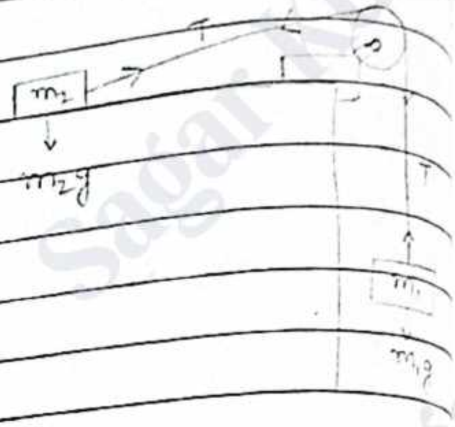
Impulse-Momentum Theorem helps us to establish the relation between two concept which states the change seen momentum of an object is equivalent to the amount of impulse exerted on it. Therefore,

$$\vec{J} = \vec{P_2} - \vec{P_1}$$

P_1 & P_2 is Initial & Final momentum.

Now, let us consider the following cases of motion of two bodies connected by a string.

1. Case -1 :- Let us consider the motion of two bodies connected by an ~~ext~~ inextensible string, one of which is hanging freely and the other is lying on a smooth horizontal plane as shown in figure.



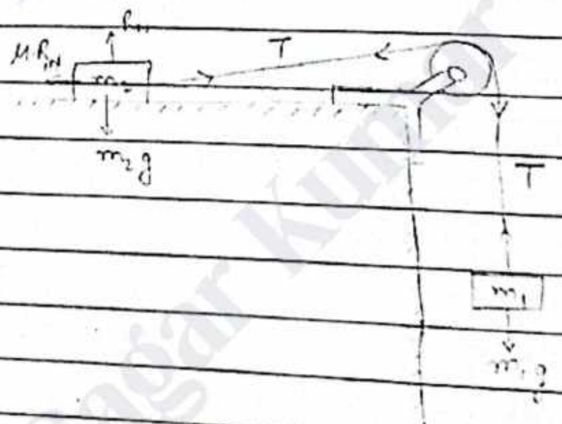
Since, the string is inextensible, the tension (T) in both the strings will be equal. The acceleration of the system is given by

$$a = \frac{m_1 \cdot g}{m_1 + m_2} \quad m/s^2$$

and the tension in the string

$$T = m_2 a = \frac{m_1 \cdot m_2 \cdot g}{m_1 + m_2} \quad N.$$

2. Case 2 :- Here, instead of smooth plane, it is a rough horizontal plane as shown in figure below.



Then, the frictional force is equal to

$$\mu R_N = \mu \, m_2 g$$

where, μ = co-efficient of friction.

The acceleration of the system are

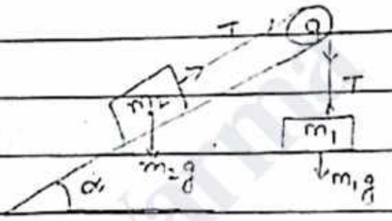
$$a = \frac{g(m_1 - \mu m_2)}{m_1 + m_2} \; m/s^2$$

and the tension (T) of the string is

$$T = \frac{m_1 m_2 g (1+\mu)}{m_1 + m_2} \; N$$

Case 3:- When the plane is a smooth inclined plane, as shown in figure, then

The acceleration of system is



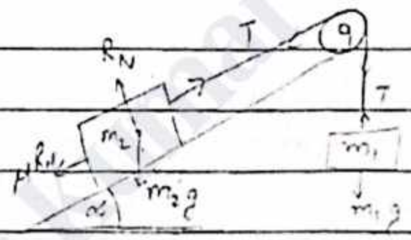
$$a = \frac{g(m_1 - m_2 \sin\alpha)}{m_1 + m_2} \; m/s^2$$

and the tension (T) of the string is

$$T = \frac{m_1 m_2 g (1 + \sin\alpha)}{m_1 + m_2} \; N.$$

Case 4:- When the plane is a rough inclined plane as shown in figure, then

The acceleration of system is



$$a = \frac{g(m_1 - m_2 \sin\alpha - \mu m_2 \cos\alpha)}{m_1 + m_2} \; m/s^2$$

and the tension (T) of the string are

$$T = \frac{m_1 m_2 \cdot g \cdot (1 + \sin\alpha + \mu \cos\alpha)}{m_1 + m_2} \; N$$

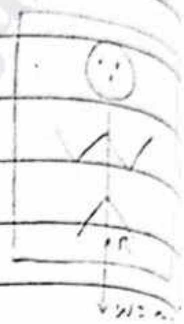
$\therefore$ Apparent weight (w')

$$w' = R = mg + ma$$

Note :- Apparent wt (w') > Actual wt (mg)

Case C :- If the lift is accelerated downward

Here, the weight 'mg' acts downwards, while the reaction R acts upwards.



Apparent weight $w' = R$

$$\Rightarrow mg - R = ma$$

$$R = mg - ma$$

$$R = m(g - a)$$

Note :- (i) Apparent wt (w') < Actual wt (w)

(ii) If $g = a$, then $w' = 0$.

Thus, in a freely falling lift, the man will experience a [feeling] of weightlessness.

Case D :- If the lift is accelerated downwards such that $a >$ [g]

In this case, ma is greater than the weight mg. Th[us]

$$w' = m(g - a) = \text{negative}$$

There is no meaning of negative apparent weight, there[fore it] will be zero.

So, the man will be accelerated upward and will stay [at the] ceiling of the lift.

Note :-

Apparent weight is a property of objects that corresponds to [how] heavy an object is. It will differ from the weight of an [object] whenever the force of gravity acting on the object is not b[alanced] by an equal but opposite normal force.

Motion of Lift.

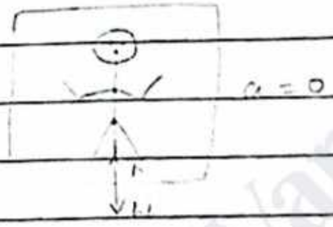
If a body of mass m is carried by a lift with an upward acceleration a, then the forces acting on a body are
(i) the reaction R on the floor of a lift upwards
(ii) the weight mg of the body acting vertically downwards.
(iii) Tension T in cable supporting the lift, also called Reaction of the lift.

Motion in a lift.

Weight :
The pull on earth on any body under its gravitational influence is called the weight of the body. This force is directed towards the centre of the earth, which produces acceleration. If W is the weight of body of mass m, then

$$\vec{W} = m\vec{g} = mg$$

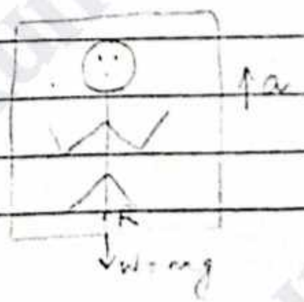
Case A :- If the lift is unaccelerated (when $\vec{v} = 0$ or constant)
In this case, there is a reaction



$R = mg$

Hence,
Apparent weight = Actual weight
$W' = \text{Actual wt} = mg$.

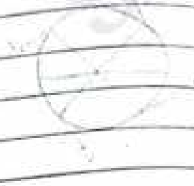
Case B :- If the lift is accelerated upward (when $\vec{a}$ = constant)
The net forces acting on the man are



$R - mg = ma$
$R = ma + mg$

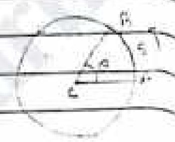
2.3 Angular Motion

If the motion of a body takes place along the circumference of a circle, then it is called as angular motion or circular motion.



When a particle moves along a circular path, then its linear velocity is directed along the tangent to a circle at any given instant. Thus velocity is called as instantaneous linear velocity of a particle. As the particle moves along a circle, the direction of instantaneous linear velocity change continuously. When the magnitude of instantaneous linear velocity remains constant, then the motion is called as a uniform circular motion.

Angular Displacement (θ)



Consider a particle moving along a circle and let it moves from A to B in short time.

A = First position

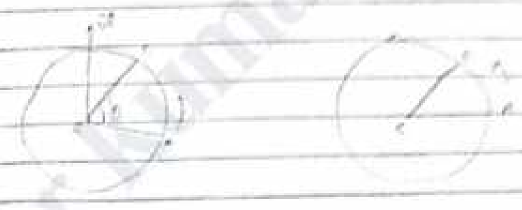
B = Second position

θ = Angular displacement.

The line joining the centre and position of a particle is called the radius vector. Thus, CA and CB are the radius vector. The angle through which the radius vector turns is called as angular displacement. It can also be defined as the angle subtended at the centre of a circle by the path travelled.

SI unit – Radian (rad). Direction – Perpendicular to the plane of the circle given by right-handed Screw rule

Angular Velocity (ω)



The rate of change of angular displacement with respect to time is called angular velocity (ω) of a particle

$$\omega = \frac{d\theta}{dt}$$

SI unit – rad/sec

Direction – perpendicular to the plane of a circle given by the right-handed screw rule

Angular Acceleration (α)

The rate of change of angular velocity with respect to time is k/a angular acceleration.

$$\alpha = \frac{d\omega}{dt} = \frac{\omega_2 - \omega_1}{t}$$

SI unit – rad/sec^2

Direction – Perpendicular to the plane of a circle given by the right-handed screw rule.

Period of Revolution

When a body rotating about an axis completes one revolution in time T, called period of revolution. Thus, the angular velocity is

$$\omega = \frac{2\pi}{T} \qquad [\because \theta = 2\pi \text{ radian for one complete revolution}]$$

Frequency of Revolution

When the number of revolutions completed by body in one second is n, called frequency of revolution. Then

$$n = \frac{1}{T}$$

$$\therefore \omega = 2\pi n$$

If the no. of complete revolutions N in time t, then

$$\omega = \frac{2\pi N}{t}$$

Uniform & Non-Uniform Angular Velocity

Angular velocity of a rotating body is said to be uniform if it describes equal angles in equal intervals of time, however small the intervals may be. Otherwise, it is said to be non-uniform.

Uniform & Non-uniform Angular Acceleration.

Angular acceleration of a rotating body is said to be uniform if equal changes in angular velocity takes place in equal intervals of time, however small the time intervals may be. Otherwise it is said to be non-uniform.

Relation between linear velocity and angular velocity

Consider a particle undergoing uniform circular motion. It moves from point A to point B in time t

Let, S = Linear displacement

θ = Angular displacement

v = Linear velocity

r = Radius of a circle

ω = Angular velocity

We know that,

$$v = \frac{S}{t}$$

$$v = \frac{r\theta}{t} \qquad [\because S = r\theta]$$

$$v = r \times \frac{\theta}{t}$$

$$v = r\omega \quad \text{--- (1)} \qquad \left[\because \omega = \frac{\theta}{t}\right]$$

In vector notation, $\vec{v} = \vec{r}\vec{\omega}$

Thus, linear velocity is radius times the angular velocity.

Relation between linear acceleration and angular acceleration

Differentiate eq. (1) w.r.t. time t, we get

$$\frac{dv}{dt} = \frac{d}{dt}(\omega r) = \frac{d\omega}{dt} \times r$$

$$a = \alpha \cdot r \qquad \left[\because a = \frac{dv}{dt} \ \& \ \alpha = \frac{d\omega}{dt}\right]$$

In vector notation, $\vec{a} = \vec{\alpha} \times \vec{r}$

Simple Harmonic Motion

A motion of a body which is repeated identically after fixed interval of time is called periodic motion. It is also called harmonic motion.

In general, Simple harmonic motion (S.H.M.) is defined as the periodic motion of a body in which the force (or acceleration) is always directed towards the mean position and its magnitude is proportional to its displacement from the mean position. It is the simplest type of oscillatory motion in which the displacement of the particle varies sinusoidally with time. The force acting on the particle executing SHM is called the restoring force.

$$F = -Kx \quad \text{———} \quad (1)$$

where, K = Positive constant called force constant.

The eq.(1) is also called force law of SHM.

SI Unit – N/m

If m be the mass and a be the acceleration of the particle

$$F = ma = -Kx$$

$$a = -\left(\frac{K}{m}\right)x \quad \text{———} \quad (2)$$

$$a = -\omega^2 x$$

Characteristics of Linear SHM.

1. Motion is periodic along a straight line
2. Force (or acceleration) is directed towards the mean position.
3. Force (or acceleration) is directly proportional to its displacement from the mean position.
4. Velocity of a particle is maximum at the centre and minimum at the extreme position.

Equation of S.H.M.

We consider the S.H.M. as a projection of the movement of a particle performing uniform circular motion taken on diameter of a circle.

Consider a particle P performing uniform circular motion in anti-clockwise direction.

Let,

ω = Angular velocity of a particle in circular motion

r = Radius of circular path called radius vector

C = Centre of circular path.

The particle is initially at position P and C is its projection. Particle moves from P to P_1 in time t second, θ is the angular displacement of a particle in t second.

In $\triangle CP_1B_1$,

$$\sin\theta = \frac{CB_1}{CP_1} = \frac{y}{r}$$

$$\therefore y = r\sin\theta$$

$$y = r\sin\omega t \quad [\because \theta = \omega t]$$

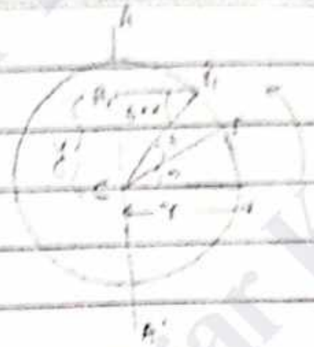
This is the equation of SHM.

r is the maximum possible displacement of a particle in SHM and it is called amplitude a

$$\therefore y = a\sin\omega t \quad [\because r = a]$$

General case :-

In $\triangle CP_1N_1$



$$\sin(\theta+\alpha) = \frac{CN_1}{CP_1}$$

$$\sin(\theta+\alpha) = \frac{y}{r}$$

$$y = r \cdot \sin(\theta+\alpha)$$

$$y = a \sin(\omega t+\alpha)$$

$$\left[\because r = a = \text{Amplitude } \& \ \theta = \omega t\right]$$

This is an equation of SHM in general.

where α = Initial phase angle, also called epoch

If particle starts from extreme position, then $\alpha = 90^\circ$

$$y = r \cdot \sin(\omega t + 90^\circ)$$

$$y = r \cdot \cos \omega t$$

$$y = a \cos \omega t$$

This is the equation for displacement of a particle in SHM, if the particle is initially at extreme position.

Time Period

A particle in SHM repeats its motion after a regular time interval. The time taken to complete one revolution is called the time period T.

$$T = \frac{2\pi}{\omega}$$

$$\because \omega^2 = \frac{k}{m}$$

$$\omega = \sqrt{\frac{k}{m}}$$

$$\therefore T = 2\pi \cdot \sqrt{\frac{m}{k}}$$

where, k = Force constant

m = Mass of the particle

Frequency & Angular Frequency

The reciprocal of time period is called the frequency. It is represents the number of oscillations per unit time.

$$\nu = \frac{1}{T} = \frac{\omega}{2\pi}$$

$$\nu = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

If θ is an angular displacement, I is the moment of Inertia, then angular acceleration is

$$\alpha = \frac{-k\theta}{I}$$

$$\alpha = -\omega^2\theta$$

$$\therefore \omega^2 = \frac{k}{I}$$

$$\omega = \sqrt{\frac{k}{I}}$$

The time period of oscillation is

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{k}}$$

And frequency of oscillation is

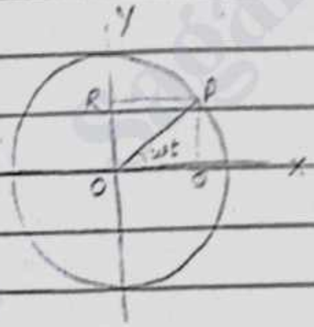
$$\nu = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{I}}$$

$\therefore$ The quantity $\omega = \sqrt{kI^{-1}}$ is the angular frequency.

SHM as a projection of uniform circular motion on any diameter.

The projection of uniform circular motion on a diameter of the circle follows SHM.

Consider a particle P moving on a circle of radius r with a constant angular speed ω. The radius OP will make an angle $\theta = \omega t$ with the x-axis at time t.



Consider the projection of the position vector OP on the x-axis moves on the circle is given as

$$x = r \cos \omega t \quad \text{—} \; ①$$

The y-coordinates of the particles at the time t is

$$y = r \sin \omega t \quad \text{—} \; ②$$

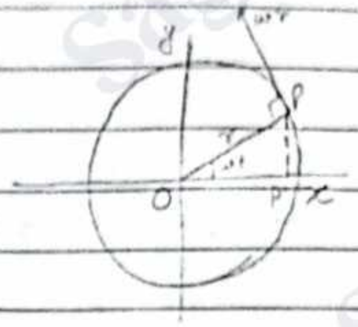
Equation ① Shows that the foot of perpendicular Q executes a SHM on the x-axis. The amplitude is a and angular frequency is ω and eq. ② Shows the foot of perpendicular executes on Y-axis.

Note :- Oscillatory motion is that motion in which a body moves to and fro or back and forth repeatedly about a fixed point, in a definite interval of time. Example - Motion of pendulum of clock.

Derivation of Displacement, Velocity and Acceleration of a body executing SHM.

Let a particle oscillating back and forth about the origin of an x-axis b/w the point P and Q.

Case 1:- When a particle starts from mean position.



If a particle starts from mean position, then displacement, velocity and acceleration are given by

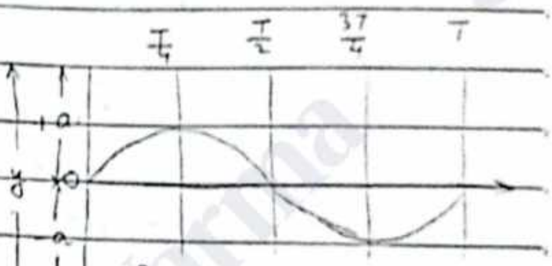
$$y = a \sin \omega t$$

$$V = \frac{dy}{dt} = a\omega \cos \omega t$$

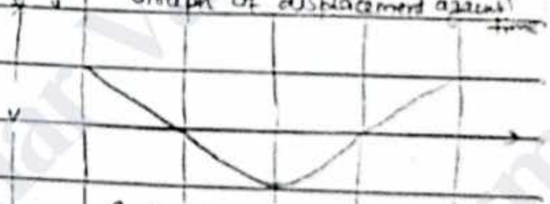
$$\text{Acceleration} = \frac{dv}{dt} = -a\omega^2 \sin \omega t.$$

Putting $\omega = \frac{2\pi}{T}$ in the above equation, we can determined the value for displacement, velocity & time.

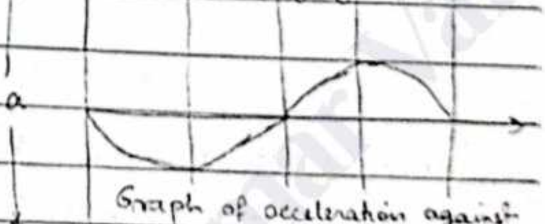
Time (t)	0	T/4	T/2	3T/4	T
$\theta = \omega t$	0	$\pi/2$	π	$3\pi/2$	2π
Displacement	0	a	0	-a	0
Velocity	$a\omega$	0	$-a\omega$	0	$a\omega$
Acceleration	0	$-a\omega^2$	0	$a\omega^2$	0



Graph of displacement against time

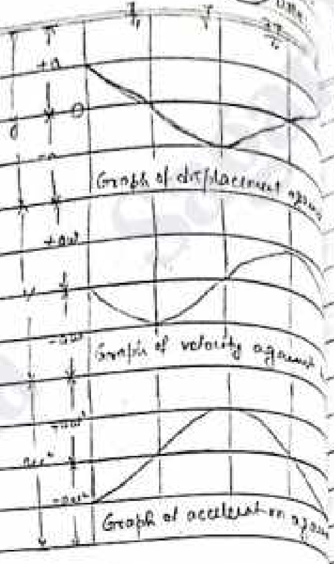


Graph of velocity against time



Graph of acceleration against time

Case 2:- When a particle start from extreme position.



If a particle start from extreme position, then the displacement, velocity and acceleration are given by

$y = a\cos\omega t$

$v = \frac{dy}{dt} = -a\omega\sin\omega t$

$acc^n = \frac{dv}{dt} = -a\omega^2\cos\omega t$

Putting $\omega = \frac{2\pi}{T}$ in the above equation, we can determine the values of displacement, velocity and acceleration for different values of time t.

	Time (t)	0	T/4	T/2	3T/4	T
	$\theta = \omega t$	0	$\pi/2$	π	$3\pi/2$	2π
(cos)	Displacement	a	0	-a	0	a
(sin)	Velocity	0	$-a\omega$	0	$a\omega$	0
(-cos)	Acceleration	$-a\omega^2$	0	$a\omega^2$	0	$-a\omega^2$

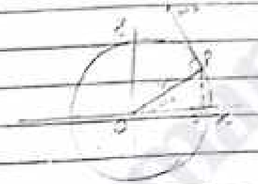
Note:- Centripetal force is the force which acts along the radius of circle at every point and is always directed towards the centre which the body moves. Its magnitude is $m\omega^2 r$.

Centrifugal force is the force which acts opposite to the centripetal force. It always act away from the centre of the path or to throw the body from the centre of circular path. Its magnitude is $m\omega^2 r$.

Derivation of Displacement, Velocity and Acceleration of a body executing SHM.

Let a particle oscillating back and forth about the origin of an x-axis b/w the point P and Q.

Case 1:- When a particle starts from mean position.

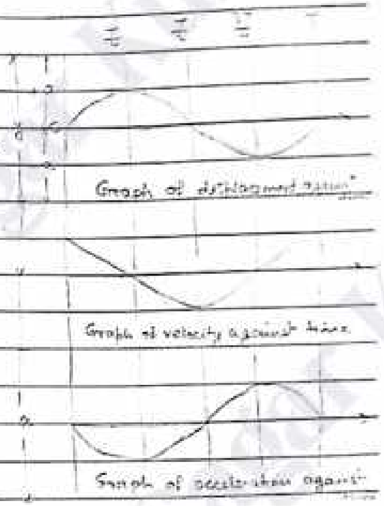


If a particle starts from mean position, then displacement, velocity and acceleration are given by

$y = a\sin\omega t$

$v = \frac{dy}{dt} = a\omega\cos\omega t$

Acceleration $= \frac{dv}{dt} = -a\omega^2\sin\omega t$.



Putting $\omega = \frac{2\pi}{T}$ in the above equation, we can determined the value for displacement, velocity & time.

	Time (t)	0	T/4	T/2	3T/4	T
	$\theta = \omega t$	0	$\pi/2$	π	$3\pi/2$	2π
sin	Displacement	0	a	0	-a	0
cos	Velocity	$a\omega$	0	$-a\omega$	0	$a\omega$
-sin	Acceleration	0	$-a\omega^2$	0	$a\omega^2$	0

Case 2:- When a particle starts from extreme position.


Graph of displacement against time
Graph of velocity against time
Graph of acceleration against time

If a particle starts from extreme position, then the displacement, velocity and acceleration are given by

$$y = a\cos\omega t$$

$$v = \frac{dy}{dt} = -a\omega\sin\omega t$$

$$acc^n = \frac{dv}{dt} = -a\omega^2\cos\omega t$$

Putting $\omega = \frac{2\pi}{T}$ in the above equation, we can determine the values of displacement, velocity and acceleration for different values of time t.

Time (t)	0	$T/4$	$T/2$	$3T/4$	T
$\theta = \omega t$	0	$\pi/2$	π	$3\pi/2$	2π
(cos) Displacement	a	0	$-a$	0	a
(sin) Velocity	0	$-a\omega$	0	$a\omega$	0
(-cos) Acceleration	$-a\omega^2$	0	$a\omega^2$	0	$-a\omega^2$

Note:- Centripetal force is the force which acts along the radius of the circle at every point and is always directed towards the centre along which the body moves. Its magnitude is $m\omega^2 r$.

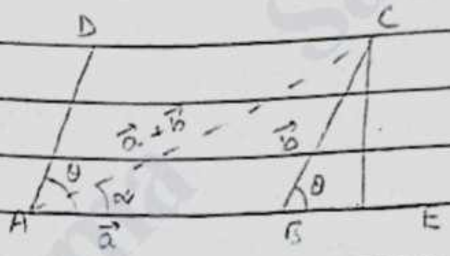
Centrifugal force is the force which acts opposite to the centripetal force. It always act away from the centre of the path or tends to throw the body from the centre of circular path. Its magnitude is $m\omega^2 r$.

Equality of Vectors

Two vectors (representing two values of the same physical quantity) are called equal if there magnitudes and directions are same.

Addition of vectors.

If $\vec{a}$ and $\vec{b}$ are the two vectors to be added, which shows triangle rule of vector addition.



Therefore,

The magnitude of $\vec{a}+\vec{b}$ is

$$R = \sqrt{a^2 + b^2 + 2ab\cos\theta}$$

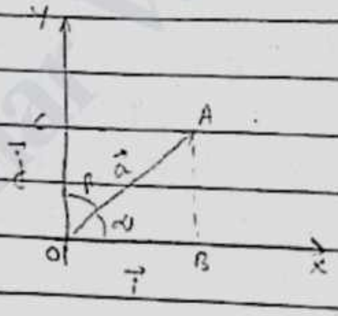
Its angle with $\vec{a}$ is α, then

$$\tan\alpha = \frac{b\sin\theta}{a + b\cos\theta}$$

Resolution of vectors.

Here, $\vec{a} = \vec{OA}$ in XY plane drawn from origin O.

If $\vec{i}$ and $\vec{j}$ denote vectors of unit magnitude along OX and OY respectively, we get



$$\vec{a} = a\cos\alpha\,\vec{i} + a\cos\beta\,\vec{j}$$

If we have two or more vectors, having rectangular cordinate axes, $\vec{a}$, $\vec{b}$ and $\vec{c}$ in XY plane along z component, then

$$\text{Resultant } R = \sqrt{(a_x + b_x + c_x)^2 + (a_y + b_y + c_y)^2}$$

Along an angle α,

$$\tan\alpha = \frac{a_y + b_y + c_y}{a_x + b_x + c_x}$$

Dot product OR Scalar Product of two vectors

It is defined as

$$\vec{a} \cdot \vec{b} = ab\cos\theta$$

where, a & b are the magnitude of $\vec{a}$ and $\vec{b}$ respectively.
θ is the angle between them.

Cross Product OR Vector Product of two vectors

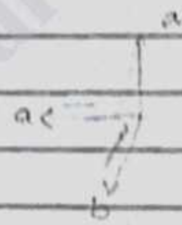
It is defined as

$$|\vec{a} \times \vec{b}| = ab\sin\theta$$

where, a and b are the magnitudes of $\vec{a}$ and $\vec{b}$ respectively.
θ is the smaller angle b/w the two.

Right Hand Screw Rule

The right hand screw rule can be used when a direction must be determined based upon a rotational direction or vice versa.
Direction of the resultant of the cross product of two vectors.
For example, If $\vec{a}$ is in east direction and $\vec{b}$ is in north direction, then resultant $\vec{a} \times \vec{b}$ is in the upward direction.



It is a common mnenomic rule for understanding orientation of axes in 3-D space. It is also convenient method for quickly finding the direction of a cross product of 2 vectors.

WORK, ENERGY & POWER

Work

When a force acts on an object and the object actually moves in the direction of force, then the work is said to be done by the force.

Work done by the force is equal to the product of the force and the displacement of the object in the direction of force.

The work done by a force on a particle during a displacement has been defined as

$$W = \int \vec{F} \times d\vec{s} = Fs\cos\theta$$

where θ is the smallest angle between F and s.

Work is a scalar quantity. Its SI unit is Joule and CGS unit is erg.

$\therefore$ 1 Joule $= 10^7$ erg

Work done by a force is zero, if

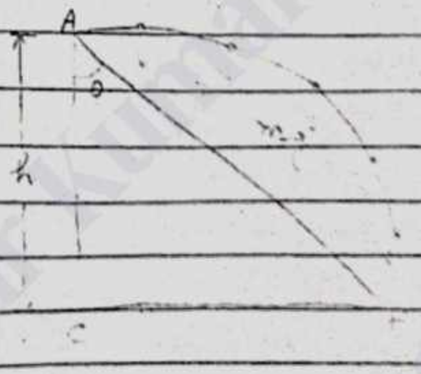
(A) Body is not displaced actually, i.e., $s = 0$.

(B) Body is displaced perpendicularly to the direction of force, i.e., $\theta = 90°$.

Work done by a force is positive, if angle between F and s is acute angle. If it is negative, then angle formed b/w them is obtuse angle.

Work done by a constant force depends on the initial and final position and not on the actual path followed between initial and final position.

The force of gravity ($m\vec{g}$) is constant in magnitude and direction if the particle moves near the surface of the earth. Suppose a particle moves from A to B along some curve and that $\vec{AB}$ makes an angle θ with the vertical.



The work done ~~to~~ by the force of gravity during the transit from A to B is

$$W = mg\,(AB)\cos\theta$$
$$W = mgh$$

where, h = Height descend by the particle

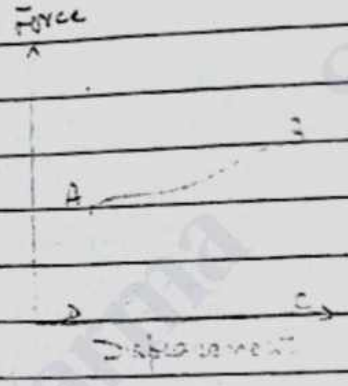
If a particle ascends a height h, then the work done by the force of gravity will be

$$W = -mgh$$

Work done in different conditions

1. Work done by a variable force is given by

$$W = \int F \times ds$$



It is equal to the area under the force - displacement graph along the with proper sign.

Work Done = Area ABCDA

2. Work done in displacing any body under the action of a number of forces is equal to the work done by the resultant force.

3. In equilibrium (Static or dynamic), the resultant force is zero, therefore, resultant work done is zero.

4. If work done by a force during a rough trip of a system is zero, then the force is conservative, otherwise it is called non-conservative force.

Conservative force - Gravitational force, electrostatic force, magnetic force

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5. Work done by the force of gravity on a particle of mass is given by

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where, g = acceleration due to gravity
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6. Work done in compressing or stretching a spring is given by

$$W = \frac{1}{2} kx^2$$

where, K = Spring constant
x = Displacement from mean position

7. When one end of a spring is attached to a fixed vertical support and a block attached to the free end moves on a horizontal table from $x = x_1$ to $x = x_2$, then,

$$W = \frac{1}{2} K(x^2x_2 - x^2x_1)$$

8. Work done by the couple for an angular displacement θ is given by

$$W = i * \theta$$

where, i = Torque of the couple.

Power

Power is defined as the rate at which work is done upon an object. Power is a time-based quantity which is related to how fast a job is done.

Power = Rate of doing work = Work Done / Time Taken

If under a constant force F of a body is displaced through a distance s in time t, then the power

$$P = \frac{W}{t} = \frac{F \times s}{t}$$

$\because \frac{s}{t} = v$; uniform velocity with which body is displaced,

$\therefore \quad P = F \times V$

$$P = Fv\cos\theta$$

where θ is the smaller angle between F and v.

Power is a scalar quantity and its SI unit is Watt (w)

Dimensional formula – $M^1L^2T^{-3}$

Other units are

1 Kilowatt (KW) = 1000 Watt

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Average Power is defined as the total energy consumed in the total time taken by an object.

In simple language, we can say that average power is the average amount of work done or energy converted per unit of time.

Energy.

Energy of a body is its capacity of doing work. It is also defined as the ability to do a task. It can neither be produced nor lost but can be converted from one type to the next.

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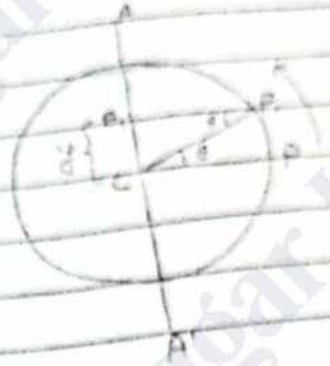
Equation of S.H.M.



We consider the S.H.M. as a projection of the movement of a particle performing uniform circular motion taken on diameter of a circle.

Consider a particle P performing uniform circular motion in anti-clockwise direction.

Let,

ω = Angular velocity of a particle in circular motion.

r = Radius of circular path called radius vector.

C = Centre of circular path.

The particle is initially at position P and C is its projection. Particle moves from P to P_1 in time t second. θ is the angular displacement of a particle in t second.

In ΔCP_1B_1,

$$\sin\theta = \frac{CB_1}{CP_1} = \frac{y}{r}$$

$$\therefore y = r \cdot \sin\theta$$

$$y = r \sin\omega t \quad [\because \theta = \omega t]$$

This is the equation of SHM.

r is the maximum possible displacement of a particle in SHM and it is called amplitude a.

$$\therefore y = a \sin\omega t \quad [\because r = a].$$

General case:-

In a $\triangle CP_1P_2$

$\sin(\theta+\alpha) = \frac{CP_1}{CP_2}$

$\sin(\theta+\alpha) = \frac{y}{r}$

$y = r.\sin(\theta+\alpha)$

$y = a\sin(\omega t+\alpha)$

$[\because r = a = \text{Amplitude} \ \& \ \theta = \omega t]$

This is an equation of SHM in general.

where α = Initial phase angle, also called epoch

If particle starts from extreme position, then $\alpha = 90°$

$y = r.\sin(\omega t + 90°)$

$y = r.\cos\omega t$

$y = a\cos\omega t$

This is the equation for displacement of a particle in SHM, if the particle is initially at extreme position.

Time Period

A particle in SHM repeats its motion after a regular time interval. The time taken to complete one revolution is called the time period T.

$$T = \frac{2\pi}{\omega}$$

$$\because \omega^2 = \frac{k}{m}$$

$$\omega = \sqrt{\frac{k}{m}}$$

$$\therefore T = 2\pi.\sqrt{\frac{m}{k}}$$

where, k = Force constant

m = Mass of the particle

Frequency & Angular Frequency

The reciprocal of time period is called the frequency. It is represented by the number of oscillations per unit time.

$$\nu = \frac{1}{T} = \frac{\omega}{2\pi}$$

$$\nu = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

If θ is an angular displacement, I is the moment of Inertia, then angular acceleration is

$$\alpha = \frac{-k\theta}{I}$$

$$\alpha = -\omega^2\theta$$

$$\therefore \omega^2 = \frac{k}{I}$$

$$\omega = \sqrt{\frac{k}{I}}$$

The time period of oscillation is

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{k}}$$

And frequency of oscillation is

$$\nu = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{I}}$$

$\therefore$ The quantity $\omega = \sqrt{kI^{-1}}$ is the angular frequency.

SHM as a projection of uniform circular motion on a diameter.

The projection of uniform circular motion on a diameter of circle follows SHM.

Consider a particle P moving on a circle of radius r with a constant angular speed ωt. The radius OP will make an angle θ = ωt with the x-axis at time t.

Consider the projection of the position vector OP on the x-axis moves on the circle is given as

$$x = a \cos \omega t \quad \text{—— (1)}$$

The y-coordinates of the particles at the time t is

$$y = r \sin \omega t \quad \text{—— (2)}$$

Equation (1) shows that the foot of perpendicular Q executes SHM on the x-axis. The amplitude is a and angular frequency is ω and eq. (2) shows the foot of perpendicular executes on y-axis.

Note :- Oscillatory motion is that motion in which a body moves to and fro or back and forth repeatedly about a fixed point, in definite interval of time. Example - Motion of pendulum of a clock.

Equality of Vectors

Two vectors (representing two values of the same physical quantity) are called equal if there magnitudes and directions are same.

Addition of vectors.

If $\vec{a}$ and $\vec{b}$ are the two vectors to be added, which shows triangle rule of vector addition.

Therefore,

The magnitude of $\vec{a}+\vec{b}$ is

$$R = \sqrt{a^2 + b^2 + 2ab\cos\theta}$$

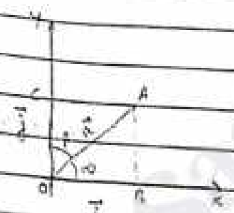
Its angle with $\vec{a}$ is α, then

$$\tan \alpha = \frac{b \sin\theta}{a + b\cos\theta}$$

Resolution of vectors.

Here, $\vec{a} = \vec{OA}$ in XY plane drawn from origin O.

If $\vec{i}$ and $\vec{j}$ denote vectors of unit magnitude along OX and OY respectively, we get



$$\vec{a} = a\cos\alpha\,\vec{i} + a\cos\beta\,\vec{j}$$

If we have two or more vectors, having rectangular coordinate axes, $\vec{a}$, $\vec{b}$ and $\vec{c}$ in XY plane along z component, then

$$\text{Resultant } R = \sqrt{(a_x + b_x + c_x)^2 + (a_y + b_y + c_y)^2}$$

Along an angle α,

$$\tan\alpha = \frac{a_y + b_y + c_y}{a_x + b_x + c_x}$$

Dot product or Scalar Product of two vectors

It is defined as

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Unit - 4

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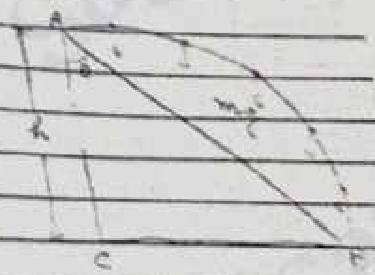

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Mechanical Energy

The sum of kinetic and potential energy at any point remains constant throughout the motion. It doesnot depend on time. This is known as law of conservation of mechanical energy.

It is of two types –

1. Kinetic energy – is the energy possessed by any object by virtue of its motion. It is acquired by a moving body is equivalent to average work performed by the body just before it comes to rest.

Kinetic energy of an object is given by

$$K.E. = \frac{1}{2}mv^2 = p^2/2m$$

where, m = Mass of an object

v = Velocity of the object

p = mv = momentum of the object

2. Potential energy – is the energy possessed by any object by virtue of its position or configuration. It is generated by the body as a consequence of its position or state.

Potential energy of an object is given by

$$P.E. = mgh$$

Potential energy depends upon frame of reference.

(a) Elastic PE – If a spring of spring constant K is stretched through a distance x, then elastic potential energy of the spring is determined by

$$P.E._{elastic} = \frac{1}{2}Kx^2$$

(b) Electric PE – when two point charges q_1 and q_2 is separated by a distance of r in vacuum is given by

$$U = \frac{1}{4\pi\varepsilon_0} \times \frac{q_1 q_2}{r}$$

~~Here, $\frac{1}{4\pi\varepsilon_0} = 9.0 \times 10^{10}$ Nm^2/C^2 (constant)~~

Work Energy Theorem

The work done on a body by applying force is equal to the change in kinetic energy of the body. This is defined as Work - Energy Theorem.

$$W = \int_{x_1}^{x_2} F \cdot dx = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$$

$$= K_f - K_i = \cancel{\Delta KE}\ \Delta KE$$

where, K_i = Initial KE

K_f = Final KE.

Regarding the work-energy theorem, it is worth ~~noting~~ noting that

(i) If W_{net} is positive, then $K_f - K_i$ = positive, ie, $K_f > K_i$ or KE is positive, will increase and vice-versa.

(ii) This theorem can be applied to non-inertial frames also. In a non-inertial frame it can be written as -

Work done by all the forces (including Pseudo force) = change in KE in non-inertial force.

Mass - Energy Equivalence

According to Einstein, the mass can be transformed into energy and vice-versa.

When Δm mass disappears, then produced energy is

$$E = \Delta m c^2$$

where, c = speed of light in vaccum.

$$c = 3 \times 10^8 \ m/s$$

About 8.18×10^{-14} J of energy may be converted to form an electron and obtain by destroying the electron in Einstein's special theory of relativity.

Representation of work by using graph

A graph is drawn by taking component of applied force along y-axis and displacement along x-axis. Area underneath this graph is equal to work done. If the applied force is constant, then the graph will be a straight line parallel to x-axis.

Suppose, due to the action of this force, the body moves from position x_1 to x_2, then work done for small displacement will be

$$W = F(x_2 - x_1) = F \Delta x$$

That is,

Work = Applied Force × Displacement of the point of action of the force along its direction

$$W = FS$$

If the displacement is along the direction making an angle θ with the applied force, then the work done is measured by applied force along the component of displacement along the direction of force.

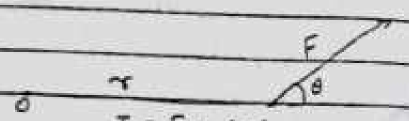
$$W = FS \cos\theta$$

Work Done by a Torque

In Physics, work is an important quantity which refers to the amount of displacement that a force produces when applied to any object. If some force F applied on any object and is displaced by some distance d, then the work done on that object by that force is the dot product of the force and the distance.

$$W = F \times \vec{d}$$

A/c to Newton's 2nd law of motion, we know that force causes a change in the linear acceleration of a body. Similarly, the torque causes a change in the angular acceleration of the body.



$$\tau = F \cdot r \sin\theta$$

Visualisation of Torque

Torque depends on the distance b/w the axis of rotation and the point of application. It is k/a moment arm or the lever arm.

The formula for torque produced by the force F acting at a point that is at a distance r from the axis of rotation is given by the vector cross product of F and r.

$$\tau = \vec{r} \times F$$

$$\tau = rF \sin\theta$$

It acts perpendicular to the plane of r and F a/c to the rules of the vector product.

SI unit - Nm.

Expression for work done by Torque

We know that, for the expression for work done by a force F that displaces an object by some distance r.

$$W = F \cdot r$$

$$W = Fr\cos\theta$$

This is the most basic expression for work.

Suppose, we have a body that is rotating with some angular velocity ω and the force applied on it at a distance r from the axis of rotation. Now we can formulate the relationship b/w the torque and the work done using this case.

We know that, a/c to the work-energy theorem, the amount of work done is equal to the change in kinetic energy. This theorem is also valid for rotational motion. So, work done by all the torques will be equal to change in KE

$$W_T = \Delta K.E$$

where, W_T = work done by the torque.

Work done by the torque is defined by the product of the angular force applied and the angular displacement produced by the applied torque.

Therefore,

$$W = \int \vec{\tau} \cdot d\vec{\theta}$$

Work done by constant torque can be written as

$$W = \tau\theta$$

This is the relation b/w torque and work done

From the above section,

$$\vec{\tau} = \vec{r} \times \vec{F}$$

Also from the rotational form of Newton's 2nd law

$$\vec{\tau} = I\alpha$$

where, I = Rotational mass or moment of inertia

α = Angular acceleration.

Thus, we can write,

$$W = \int I\alpha \, d\theta$$

$$W = \int I \cdot \frac{d\omega}{dt} \, d\theta \qquad \left[\because \alpha = \frac{d\omega}{dt}\right]$$

$$W = \int I \cdot \frac{d\theta}{dt} \, d\omega$$

$$W = \int I \, \omega \, d\omega$$

If we consider ω_0 as the initial angular velocity and ω as the final angular velocity, then the integration gives

$$W = \int_{\omega_0}^{\omega} I \cdot \omega \, d\omega$$

$$W = I\left[\frac{\omega^2}{2}\right]_{\omega_0}^{\omega}$$

$$W = \frac{I\omega^2}{2} - \frac{I\omega_0^2}{2}$$

$$W = \frac{I}{2}\left(\omega^2 - \omega_0^2\right)$$

Spring force

One end of a spring is attached to a fixed vertical support and the other end to a block which can move on a horizontal table. Work done on the block by the spring-force as the block moves from $x=0$ to $x=x_1$. The force on the block is k times the elongation of the spring. The force in this interval is kx and the displacement is ... The force and the displacement are opposite in direction.

So, $\vec{F} \cdot d\vec{r} = -F\,dx = -kx\,dx$ during this interval

The total work done as the block is displaced from $x=0$ to $x=x_1$ is

$$W = \int_0^{x_1} -kx\,dx$$

$$W = \left[-\frac{1}{2}kx^2\right]_0^{x_1}$$

$$W = \frac{-kx_1^2}{2}$$

If the block from $x=x_1$ to $x=x_2$, then the work done is

$$W = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2$$

GENERAL PROPERTIES OF MATTER.

5.1 Elasticity

Elasticity

The property of a deformable body due to which the body regains its original shape upon the removal of the forces causing the deformation is called elasticity.

Deforming Force

When a force applied on a body produces a change in the normal positions of the molecules of the body, which results in a change in the configuration of the body either in length, volume or shape, then force applied is called deforming force.

Restoring Force

Restoring force is a force which gives rise to an equilibrium in a physical system. If the system is perturbed away from the equilibrium, the restoring force will tend to bring the system back towards equilibrium.

Action of Spring — is an idealized spring exerts a force that is proportional to the amount of deformation of the spring from its equilibrium length, exerted to oppose the deformation.

Elastic and Plastic Body

Elastic Body — is the body which regains the original shape after removal of external forces.

A body which regains its original configuration immediately and completely after the removal of deforming force from it, is called perfectly elastic body.

For example, phospher bronze and quartz are nearly perfectly elastic bodies.

Plastic body - is the body which doesnot regain its original shape after removal of external forces.
A body which doesnot regain its original configuration at all on the removal of deforming force, howsoever small the deforming force may be called perfectly plastic body.
For example, mud, paraffin wax and putty are nearly perfectly plastic bodies.

~~Stress~~

Rigidity

The property on account of which a body does not change its size and shape even a large force is applied on it, is called rigidity.
No body is perfectly rigid, but stone can be considered as a rigid body.

Plasticity

The property on account of which a body does not regain its original size and shape on removal of applied force is called as plasticity.

Stress

If an internal or external restoring force F is applied uniformly per unit cross-sectional area is called stress.

$$\text{Stress} = \frac{\text{Applied force}}{\text{Cross-sectional area}}$$

$$\sigma = \frac{F}{A}$$

SI unit - N/m^2

CGS unit - $dyne/cm^2$

Dimension - $M^1L^{-1}T^{-2}$

Types of stress

There are two types of stress. They are

(i) Normal stress (ii) Tangential or shearing stress.

(i) Normal stress - when a deforming force acts normally over an area of a body, then the internal restoring force set up per unit area is called normal stress.

Normal stress is of 3 types.

(a) Tensile stress - The stress measured in connection with the changes in length is called tensile stress.

$$\text{Tensile stress} = \frac{\text{Applied force}}{\text{Cross sectional Area}} = \frac{Mg}{\pi r^2}$$

(b) Compressive stress — when decrease in length of the wire or compression of the body due to applied force, is called compressive stress, to the cross-sectional area.

(c) Hydraulic or volume stress — If the force is applied and there is change in volume, then the corresponding stress is called hydraulic stress.

$$\text{Volume stress} = \frac{\text{Applied force}}{\text{Area}}$$

$$= \text{Change in Pressure } (dP)$$

(ii) Tangential or Shearing Stress — when a tangential force F applied on the top face change in shape of body of area A, is called shearing stress.

$$\text{Shearing stress} = \frac{\text{Tangential force}}{\text{Area}}$$

Strain

The fractional change in the dimension of a body produced by the external force acting on it is called strain.
In other words, change in dimension per unit original dimension is called as strain.
Strain has no unit and dimensions.
Strain is of three types, these are :-

(i) Longitudinal Strain — When a deforming force produces a change in length alone, the strain produced in the body is k/a Longitudinal strain. It is given as

$$\text{Longitudinal Strain} = \frac{\text{Change in length } (\Delta l)}{\text{original length } (l)}$$

(ii) Volumetric strain — when a deforming force produces a change in volume alone, the strain produced in the body is k/a volumetric strain. It is given as

$$\text{Volumetric strain} = \frac{\text{change in volume } (\Delta V)}{\text{original volume } (V)}$$

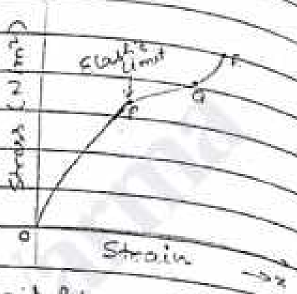
(iii) Shearing strain – When a deforming force produces change in the shape of body without changing its volume is called shearing strain. It can also be defined as the angular distortion produced in the body by which means that it is the ratio of the displacement of the surface of body to the height of the body.

$$\text{Shearing strain} = \frac{\text{Displacement}}{\text{Height}}$$

$$s = \frac{d}{l}.$$

Elastic Limit

A solid like steel wire stretched by the application of a load. The relationship between stress and strain as shown in figure, we observed that



(i) when stress is zero, strain is also zero.

(ii) when the stress is gradually increased, the strain increases proportionality till the point P is reached.

(iii) Point Q is called yield point due to extension of steel wire beyond Q

(iv) Further increase the stress beyond Q, Maximum stress corresponding to R is called breaking stress.

Defination:- The stress corresponding to the limiting value of the load, which when applied and subsequently released, doesn't produce permanent deformation is called as elastic limit.

Hooke's Law

Hooke's law is a fundamental law of elasticity. It states that the extension produced in the wire is directly proportional to the load applied within elastic limit, i.e.,

$$\text{Extension} \propto \text{Load Applied}$$

It is applicable to all types of deformation like bending, twisting, compression, etc. Therefore, modified form of Hooke's law is given as:-

Stress developed, within elastic limit, is directly proportional to the strain produced in a body

$$\text{Stress} \propto \text{Strain}$$

$$\text{Stress} = E \times \text{Strain}$$

where, E is a constant called Modulus of Elasticity or co-efficient of elasticity of material of a body.

$$\therefore E = \frac{\text{Stress}}{\text{Strain}}$$

Modulus of Elasticity

When a force is applied on a body, then its length changed or volume changed or shape of a body changes. Accordingly, there are three types of modulus of elasticity.

1. Young's Modulus of Elasticity (Y)

It is defined as the ratio of normal stress to the longitudinal strain within elastic limit. It is denoted by Y.

If l be the length of the wire and Δl be the increase in length produced by the force F. If A be the area of cross-section of the wire.

According to Statement,

$$Y = \frac{\text{Normal Stress}}{\text{Longitudinal Strain}}$$

$\because$ Normal Stress $= \frac{F}{A}$

Longitudinal Strain $= \frac{\Delta l}{l}$

$$\therefore \quad Y = \frac{F/A}{\Delta l/l}$$

$$Y = \frac{F \cdot l}{A \cdot \Delta l}$$

$$Y = \frac{mg\,l}{\pi r^2 \Delta l} \qquad \boxed{\because F = mg \ \& \ A = \pi r^2}$$

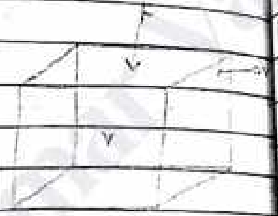
SI unit $= N/m^2$, CGS unit $=$ dyne/cm^2, Dimension $= M^1L^{-1}T^{-2}$

Material	Youngs Modulus (Y) $10^9\ N/m^2$	Ultimate Strength $10^6\ N/m^2$	Yield Strength
Steel	200	400	250
Iron	190	330	170
Copper	110	400	200
Aluminium	70	110	95
Glass	65	50	—
Concrete	30	40	—

2. Bulk Modulus of Elasticity (B) / (K)

It is defined as the ratio of normal stress to the volumetric strain, within the elastic limit.

$$B = \frac{\text{Normal Stress}}{\text{Volumetric Strain}}$$



Let V = original volume

ΔV = Decrease in volume

F = Applied force

A = Area of cross section on which force is applied.

$$\therefore \quad B = \frac{F/A}{\Delta V/V}$$

$$B = \frac{FV}{A\Delta V}$$

SI – N/m^2, CGS – Dyne/cm^2

Dimension – $M^1L^{-1}T^{-2}$

Modulus of Rigidity (Shear Modulus) of Elasticity

It is defined as the ratio of tangential stress to the shearing strain, within the elastic limit

Denoted by – G or η

$$G = \frac{\text{Tangential Stress}}{\text{Shearing Strain}}$$

$$G = \frac{F/A}{\Delta l/l}$$

$$G = \frac{Fl}{A\Delta l}$$

$$G = \frac{F/A}{\tan\theta} \qquad \left[\because \tan\theta = \frac{\Delta l}{l}\right] \left\{ \frac{\text{Lateral displacement of layer}}{\text{Its distance from fixed layer}} \right]$$

$$G = \frac{F}{A\tan\theta} \qquad \left[\because \theta \text{ is small angle} \ \therefore \tan\theta = \theta\right]$$

$$G = \frac{F}{A\theta}$$

Relation between Y, B & G

For a given material, there is a certain relation b/w them is given by :-

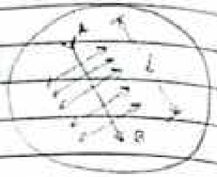
$$\frac{9}{Y} = \frac{1}{B} + \frac{3}{G}$$

$$\Rightarrow Y = \frac{9BG}{3B+G}$$

5.2 Surface Tension

Surface Tension

Surface Tension could be defined as the property of the surface of a liquid that allows it to resist an external force, due to the cohesive nature of the water molecules.



Let AB (l) be an imaginary line drawn tangentially anywhere on the surface of liquid, the force of surface tension (F) acts at right angles to the line AB on both sides along the tangent to the surface of liquid as shown in figure.

The force acting per unit length of such a line gives the quantitative measure of surface tension (T), is given by

$$T = \frac{F}{l}$$

SI unit - N/m^2

Dimension - $M^0L^1T^{-2}$

The surface tension is a scalar quantity b/c it has no specific direction for a given liquid.

The mutual attraction of the molecules of a liquid is called cohesion. When two liquids of different mass, density or when a liquid and a gas are in contact, then the surface of contact will be a curved surface k/s meniscus.

The value of surface tension depends on the radius of meniscus and varies not only with the nature of fluids in contact but also with the temperature.

Property of Surface Tension

Molecular force - Every molecule attracts another molecule. This attraction of force is called molecular force.

Cohesive force - is the force of attraction amongst the molecules of same substances.

Adhesive force - is the force of attraction acting between the molecules of different substances.

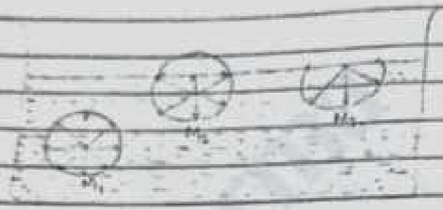
Molecular Range - is the maximum distance upto which the cohesive force and be can operate b/w two molecules. The order of molecular range in solids and liquids is 10^{-9} m.

Sphere of Influence - is the sphere drawn with the molecule at the center and radius equal to the molecular range.

Surface Energy - is defined as the amount of work done against the force of surface tension, informing the surface of liquid of given area of constant temperature.

Laplace's Molecular Theory

According to the theory of the phenomenon of surface tension explained in terms of Laplace's Molecular Theory, every liquid is made up of a large number of molecules.



Consider three molecules M_1, M_2 and M_3 of a liquid as shown in figure with their sphere of influence drawn around them.

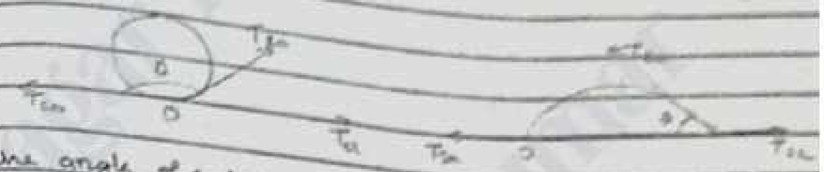
The sphere of influence of M_1 lies completely inside the liquid. It is equally attracted in all directions by molecules lying within its sphere of influence. The net force acting on such a molecule is zero.

The part sphere of influence of M_2 lies outside the liquid, which means it is not equally surrounded by neighbouring molecules. The molecules in lower part of sphere are more, get attracted downside. The net force acting on such a molecule is in downward direction.

Now consider a molecule M_3 for which the sphere of influence is exactly half outside the liquid. It is attracted by molecules from below. But, there is no attractive force from above the surface. As a result, it is acted upon by a force acting vertically downwards. Therefore, molecule M_3 experiences more downward resultant force than M_2 molecule.

Angle of contact and its significance

Consider water in one glass container and mercury in another container, as shown in figure, the surface of the liquid will be curved at the point of contact.



The angle of contact (θ) is the angle enclosed between the tangents to the liquid surface at the point of contact and the solid surface inside the liquid.

ST unit - radian.

Suppose surface tension corresponding to these interface are

T_{LA} = Surface tension of liquid-air

T_{SA} = Surface tension of solid-air

T_{LS} = Surface tension of liquid-solid.

When liquid on the solid surface is at rest, then the molecules in this region where these three interface meet are in equilibrium. So, the net force acting on them will be zero.

For molecule O, to be in equilibrium, we have,

$$T_{SL} + T_{LA}\cos\theta = T_{SA}$$

$$\cos\theta = \frac{T_{SA} - T_{SL}}{T_{LA}}$$

If a glass strip is dipped in water and mercury, the angle of contact (θ) for both, we determine

(i) It depends upon the nature of liquid & solid

(ii) It depends upon the medium that exists above the free surface of liquid.

(iii) It is fixed for a given pair of liquid and solid and surrounding medium.

The angle of contact (θ) for

(a) Pure water in a glass is zero, i.e, $\theta = 0°$

(b) Mercury and glass is about $135°$.

Capillarity or Capillary Action

Capillarity is the ability of a liquid to move through a secondary liquid due to attraction and is one of the fundamental physical properties of all fluids. It can be defined as the rate of which liquid move across a surface or between two surfaces. Capillarity action is the ~~force~~ or an effort made to push the liquid by fighting the gravitational force of attraction. This fall occurs when the liquid faces a surface tension.

(P.T.O)
(Continue)

Consider a capillary tube dipped in a liquid. It is observed that the liquid rise in it above the general level of liquid in the vessel if the angle of contact is acute, i.e. water rises up inside the capillary and if it is depressed down the general level of liquid, the angle of contact is obtuse, i.e. mercury depresses down inside the capillary. Thus rise or fall of a liquid inside the capillary is called as capillarity

Surface Tension of a Liquid in a Capillary Tube OR Relation b/w Surface Tension (T), Capillary Rise (h) and Radius of Capillary (r).

The height (h) through which a liquid will rise in a capillary tube of radius (r), which wets the walls of tube will be given by

$$h = \frac{2T\cos\theta}{r\rho g}$$

$$T = \frac{hr\rho g}{2\cos\theta}$$

Where, T = Surface tension force
h = Height of liquid or level difference
r = Radius of capillary tube
ρ = Density of liquid
θ = Angle of contact

5.3 Viscosity

Viscosity

Viscosity is the property of liquid on account of which liquid tries to oppose the relative motion between its different layer.

Consider the flow of liquid on the horizontal plane surface. It is observed that different liquid layers move with different velocities. Consider layer P and Q. Layer P which has more speed, tries to accelerate layer Q and vice versa. Thus, because of different tendencies of different layers, a frictional force of fluid is created between two layer called viscous force which tries to oppose the relative motion between different layer, i.e, tries to reduce the difference b/w their speeds called viscocity.

Velocity Gradient

Consider any two layer P and Q with velocities $(v+dv)$ and v respectively and dx is the change in vertical distance b/w the layers. Hence,

Velocity Gradient is defined as the change in velocity per unit change in vertical distance of liquid layer.

$$\therefore \text{Velocity gradient} = \frac{dv}{dx}$$

Unit of velocity gradient $\rightarrow$ s^{-1}.

Newton's Law of Viscosity

The law states that the viscous force F developed b/w two liquid layer is

(1) directly proportional to the surface area 'A' of liquid layer.

(2) directly proportional to velocity gradient

Thus,

$$F \propto A$$

$$F \propto \frac{dv}{dx}$$

On combining,

$$F \propto A \cdot \frac{dv}{dx}$$

$$F = \eta A \cdot \frac{dv}{dx}$$

where, η is called coefficient of viscosity, which is constant. It changes from liquid to liquid.

$$\therefore \eta = \frac{F}{A \cdot \frac{dv}{dx}}$$

SI unit – $N\,s/m^2$

CGS unit – $dyne\text{-}s/cm^2$

$$1\,\text{Poise} = 1\,dyne\text{-}s/cm^2 = \frac{1}{10}\,N\text{-}s/m^2$$

Coefficient of Viscosity (η) of a liquid is defined as viscous force developed between two liquid layers of unit surface area for unit velocity gradient.

Flow of Liquid through a Tube.

In case of flow of liquid flow through a tube, the liquid layer which flows along axis has max. velocity and layers which are in contact with wall of tube of tube has minimum velocity.

Streamline or Laminar Flow

It is the flow of the liquid in which every particle of liquid moves in the same direction (i.e., parallel or inline) of flow of liquid. Here, the velocity at every point within the liquid remains constant.

Critical Velocity

The value of velocity of flow of liquid upto which flow is streamline is called critical velocity (V_c).

If velocity of flow of liquid increases and crosses critical velocity then particle starts moving in random, i.e., flow becomes turbulent.

Thus, critical velocity is defined as the velocity of flow of liquid at which streamline flow changes into turbulent flow.

Turbulent Flow

The flow of liquid in which every particle is not moving in line and they move in random direction is called turbulent flow.

This happens when speed of flow of water is more than certain value.

velocity is given by the relation

$$V = \frac{\eta R}{\rho r}$$

where,

V - velocity of flow of liquid
η = Coefficient of viscosity of liquid
R = Reynold's number
ρ = Density of liquid
r = Radius of the tube

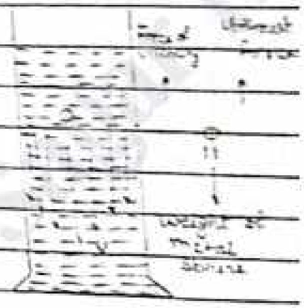
Significance of Reynold's Number -

when $R < 2000$, then liquid in flow is Streamline
when R in b/w 2000 and 3000, then liquid flow is unstable
when $R > 3000$, then liquid flow is turbulent.

Free Fall of Spherical Body Through Viscous Medium

⇒ Derivation of Viscous Force

Suppose a small metal sphere of radius r and density d is free to fall under gravity in a viscous liquid of density ρ as shown in figure. When this sphere covers a small distance, it attains a constant terminal velocity V. At this position, the viscous force F becomes equal to the weight of the sphere and the upward thrust on the account of bouyancy.



volume of sphere = $\frac{4}{3}\pi r^3$

weight of sphere = $\frac{4}{3}\pi r^3 dg$

Now,

Upthrust force = Loss of weight of body in liquid
= weight of displaced liquid
= weight of equal volume of the liquid
= $\frac{4}{3}\pi r^3 \rho g$

viscous force, F = Weight of sphere − weight of equal volume of liquid

$$F = \frac{4}{3}\pi r^3 dg - \frac{4}{3}\pi r^3 \rho g$$

$$F = \frac{4}{3}\pi r^3 g (d-\rho)$$

Terminal Velocity – It is the constant velocity with which a body falls through liquid column.

Stoke's law

Stoke's law states that the force of viscosity experienced by a small metal sphere falling freely through a viscous medium with terminal velocity is directly proportional to

(i) radius of metal sphere 'r'
(ii) terminal velocity 'v'
(iii) Co-efficient of viscosity of liquid 'η'

$$F \propto \eta r v$$

$$F = 6\pi\eta r v$$

This is our Stoke's law formula.

η by Stoke's Method

Consider a metal sphere placed on the surface of liquid taken in glass jar. It is observed that after covering certain distance, metal sphere attains a constant velocity called terminal velocity. Metal sphere falling freely through a liquid experiences three forces

(i) Weight of the metal sphere in the downward direction
(ii) Force of viscosity in the upward direction
(iii) Upthrust force (force of Bouyancy) in the upward direction.

Bouyant Force – is the force with which liquid lifts the body dipped into it. It is also called upthrust force.

By Archemedis' Principle,

Upthrust force = Loss of weight of body in liquid
= Weight of displaced liquid

Force of viscosity + Upthrust force = weight of metal surface

∴ Total upward force = Downward force

$$\therefore 6\pi\eta r v + \text{weight of the displaced liquid} = \text{weight of the metal sphere}$$

$$\eta = \frac{2}{9}\frac{r^2 g (d-\rho)}{v}$$

where, η = Co-efficient of viscosity of liquid
r = radius of metal sphere
d = density of metal (heavier)
ρ = density of liquid (lighter)
v = terminal velocity.

Since $T_1 > T_2$, then, amount of heat flowing (Q) from C_1 at steady state is directly proportional to

1. Cross-sectional area 'A' of rod.
2. Temperature difference b/w two planes i.e. $(T_1 - T_2)$
3. Time (t) for which heat flows.

and is inversely proportional to

1. Distance 'd' b/w two planes or distance b/w two thermometers

Thus,

$$Q \propto A$$
$$Q \propto (T_1 - T_2)$$
$$Q \propto t$$
$$Q \propto \frac{1}{d}$$

∴ Combining,

$$Q \propto \frac{A \times (T_1 - T_2) \times t}{d}$$

$$Q = K \cdot \frac{A \times (T_1 - T_2) \times t}{d} \quad \text{—— (1)}$$

where, K = constant of proportionality k/a co-efficient of thermal conductivity.

K = depends on material of a bar.

Thus, coefficient of thermal conductivity is defined as amount of heat conducted in one second, in steady state of temperature through unit cross-sectional area of an element of material of unit thickness with unit temperature difference between its opposite faces.

Unit of co-efficient of thermal conductivity

SI − $W m^{-1} K^{-1} = J m^{-1} K^{-1} s^{-1}$

CGS − $cal\ cm^{-1}\ °C^{-1}\ sec^{-1}$

MKS − $kcal\ m^{-1}\ °C^{-1}\ sec^{-1}$

Unit-6

HEAT

Transmission of Heat

Heat is the form of energy transferred b/w two systems or a system and its surroundings by virtue of temperature difference.

SI unit of heat energy − Joule

CGS unit of heat energy − Calorie

1 Calorie = 4.186 Joule

There are three modes of transmission of heat by which such a transmission can take place. They are :-

① Conduction
② Convection
③ Radiation

1. Conduction — is a process of transfer of heat from a body at higher temperature to a part of body at lower temperature without bodily (actual) movement of particles. It takes place through solid.

2. Convection — is a process of transfer of heat from of body at higher to lower temperature without actual movement of particles. It takes place through liquid. Convection can be natural or forced. In natural convection, gravity plays an important part. It involves bulk transport of different parts of the fluid.

In forced convection, material is forced to move by a pump and maintaining it at a uniform temperature.

3. Radiation — is the process of transfer of heat in which heat is transferred from one place to other directly without the necessity of intervening medium. Heat radiations can pass through vacuum.

Aerial Expansion or Co-efficient of Aerial Expansion or Surface Expansion or Superficial Expansion (β)

The coefficient of aerial expansion (β) of a material is defined as increase in area per unit original area at 0°c per unit increase in temperature.

Consider thin metal sheet PQRS of area A_0

Let A_0 = Original area (PQRS) at 0°c
A_t = New area (P'Q'R'S') at t°c
$(A_t - A_0)$ = Increase in area (shaded portion)
$(t - 0)$ = Increase in temperature

Then, co-efficient of aerial expansion 'β' is given by

$$\beta = \frac{A_t - A_0}{A_0 \times t}$$

Unit - per °c (/°c)

Cubical Expansion or Co-efficient of Cubical (volume) Expansion (γ)

The coefficient of cubical expansion 'γ' of a material is defined as increase in volume per unit original volume at 0°c, per unit increase in temperature. Its unit is /°c

Consider a metal cube of volume V_0

Let, V_0 = Original volume at 0°c
V_t = New volume at t°c
$V_t - V_0$ = Increase in volume
$(t - 0)$ = Increase in temperature.

Then, coefficient of cubical expansion is given by

$$\gamma = \frac{V_t - V_0}{V_0 \cdot t}$$

Thermal conductivities of some materials

Metals

Materials	Thermal conductivities	Non-metals Material	Thermal conductivities
Silver	406	Concrete	0.8
Copper	385	Body fat	0.2
Aluminium	205	Felt	0.04
Brass	109	Glass	0.8
Steel	50.2	Ice	1.6
Lead	34.7	Water	0.8
Mercury	8.3	Wood	0.12
		Glass wool	0.04
		Insulating bricks	0.15

Gases.

Air	0.024
Argon	0.016
Hydrogen	0.14

Temperature Gradient

The temperature gradient is defined as the change in temperature per unit length of rod. The unit of TG is °c/m or °K/m

The factor $\frac{T_1 - T_2}{d}$ in eq. ① is called temp gradient

$$TG = \frac{T_1 - T_2}{d}$$

where, T_1 and T_2 = Temperature at plane 1 and plane 2
and d = distance b/w plane 1 and plane 2

Now eqn ① becomes

$$Q = K \cdot A \cdot (T.G) \times t$$

Thus, co-efficient of thermal conductivity (K) of a material can also be defined as amount of heat flowing through a rod of unit area, in one second for unit temperature gradient at steady state.

Expansion of Solid

Whenever solid is heated, it expands. If solid is in thin rod form, then after heating, its length increases. If it is in thin sheet form, then its area increases and if it is in cube form, then after heating, its volume increases.

Accordingly, there are three types of coefficient of expansion

Linear Expansion OR Coefficient of Linear Expansion (α)

Consider a metal rod of length L_0

Let,

L_0 = Original length of rod at 0°c

L_t = New length at t°c

$l_t = (L_t - L_0)$ = Increase in length (Shaded portion)

$(t - 0)$ = Increase in temperature

Then, Co-efficient of linear expansion is given by

$$\alpha = \frac{L_t - L_0}{L_0 \times t}$$

Defination – The ~~expan~~ Co-efficient of linear expansion 'α' of a material is defined as increase in length per unit original length at 0°c, per unit increase in temperature.

Unit – per °c (/°c)

~~Aerial Expansion OR Co-efficient of Aerial Expansion OR Surface Expansion~~
~~Expansion OR Surf~~

Relation between α, β and γ

For a given material, α, β and γ are constant. Also, it is observed that for a given material, α, β & γ are interrelated with each other.

For a given material,

β is twice of α → $\beta = 2\alpha$

and γ is thrice of α → $\gamma = 3\alpha$

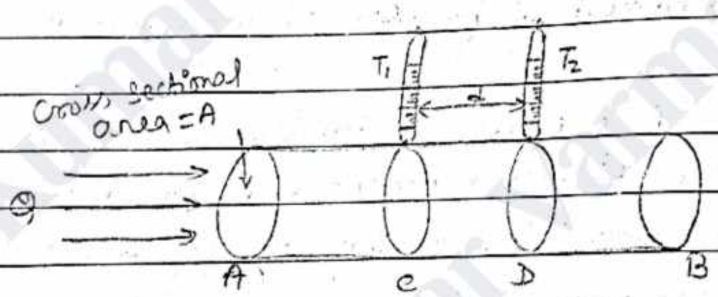
Thus, for a given material, relation between α, β and γ is as follows:

$\alpha : \beta : \gamma = 1 : 2 : 3$

Good & Bad Conductor of Heat

The material through which heat conducts easily and speedily is called good conductor of heat. For example, Copper, Iron, aluminium. Good conducting material is used as heat sink in electronic circuits such as heat sink in electronic circuits which absorbs heat and protects the components from overheating.
The material through which heat doesnot conduct is called bad conductor of heat. For example, wood, wool, plastic. Bad conducting material like thermocole is used in ice box due to which ice melts slowly.

Thermal Conductivity & Co-efficient of Thermal Conductivity (Law of Thermal Conductivity).



Suppose AB is a metal bar, of cross-sectional area A as shown in figure. Consider the bar to be in steady state. No amount of heat is lost to the surroundings by means of radiation. Consider two planes C and D in the bar.
Let,
Q = the amount of heat flowing from C to D.
d = distance b/w C and D or distance b/w two thermometer
T_1 = Temperature of plane C
T_2 = Temperature of plane D.