$$\sum_j \sum_i P(x_i/y_j)\, p(y_j) \log \frac{1}{p(y_j)}$$

$$\sum_i P(x_i/y_j)$$

$$P(x_0/y_1) + P(x_1/y_1)$$

$$0 \xrightarrow{1} 0$$

$$0 \xrightarrow{1} 1$$

$$H(X|Y) = p(y_1) \sum p(x_i|y_1) \log_2 \frac{1}{p(x_i|y_1)}$$

$$+ p(y_0) \sum p(x_i|y_0) \log_2 \frac{1}{p(x_i|y_0)}$$

$$= \sum_i \sum_j p(y_j) \cdot p(x_i|y_j) \log_2 \frac{1}{p(x_i|y_j)}$$

$$H(X|Y) = 0.645$$

$I(X,Y) \leftrightarrow I(X;Y) = H(X) - H(X|Y)$ Noisy.

$$= H(Y) - H(Y|X).$$

Types of channel-

1) Noise-free. $H(X|Y) = 0$
2) Noisy $I(X;Y) > 0 \quad \therefore H(X) > H(X|Y)$
3) Ambiguous $H(X) = H(X|Y) \quad I(X,Y) = 0$

$$H(X;Y) = \sum_i \sum_j P(x_i, y_j) \log_2 \frac{1}{P(x_i, y_j)}$$

$$= \sum_i \sum_j P(x_i, y_j) \log_2 \frac{1}{P(x_i|y_j)\, P(y_j)}$$

$$H(X;Y) = H(Y) + H(Y|X)$$

$$= H(X) + H(X|Y)$$

$|Y) = \sum_i \sum_j P(x, y) \log \frac{1}{P(x/y)}$

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$$H(X;Y) = H(Y) + H(Y/X).$$

$$H(X;Y) = \sum \sum P(x, Y) \log \frac{1}{P(X,Y)}$$

$$;Y) = \sum \sum P(x/y) P(\cancel{y}) \left[\log \frac{1}{P(X/Y)} + \log \frac{1}{P(Y)} \right]$$

$$= \sum \sum P(X/Y) P(\cancel{Y}) \cdot \log \frac{1}{P(X/Y)}$$

$$/Y) \quad + \sum_j \sum_i P(X/Y) \cdot P(Y) \log \frac{1}{P(Y)}.$$

$$\to H(X/Y) + \sum P(Y) \log \frac{1}{P(Y)}$$

$H($

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Property of Mutual Info^n [1 Problem] in exam

1) The mutual info of a channel is symmetric.
i.e $I(x;y) = I(y;x)$

Proof $I(x;Y) = H(X) - H(X|Y)$

$$= \sum P(x_i) \log \frac{1}{P(x_i)} - \sum_i \sum_j P(x_i, y_j) \log \frac{1}{P(x_i|y_j)}$$

$$= \sum_i \sum_j P(x_i) \cdot P(y_j|x_i) \cdot \log \frac{1}{P(x_i)}$$

$$- \sum_i \sum_j P(x_i; y_j) \log \frac{1}{P(x_i|y_j)}$$

$$I(x;Y) = \sum_i \sum_j P(x_i; y_j) \log \frac{P(x_i|y_j)}{P(x_i)}$$

$$I(Y;X) = \sum \sum P(x_i; y_j) \log \frac{P(y_j|x_i)}{P(y_j)}$$

Same

2) Mutual Inf^n is always non-negative.

i.e $I(x;y) \geq 0$

Proof $I(x;y) = \sum \sum P(x_i, y_j) \log_2 \frac{P(x_i|y_j)}{P(x_i)}$

$$= \sum \sum P(x_i, y_j) \log_2 \frac{P(x_i, y_j)}{P(x_i) \cdot P(y_j)}$$

$\frac{P(x_i, y)}{P(y)}$

$$= \log_2 e \sum \sum P(x_i, y_j) \ln \frac{P(x_i, y_j)}{P(x_i) \cdot P(y_j)}$$

$P(X/Y)\,P(Y) = P(Y/X)$

$$\boxed{\ln x \le x-1}$$

use this

$$-\ln x \geq 1-x$$

$$\ln\frac{1}{x} \geq 1-x$$

$$(x,y) \geq \log_2 e \sum_i \sum_j P(x_i, y_j)\left[1 - \frac{P(x_i)\cdot P(y_j)}{P(x_i, y_j)}\right]$$

$$\geq \log_2 e\left(\sum_i \left[\sum_j P(x_i, y_j) - \sum P(x_i)\cdot P(y_j)\right)\right)$$

$$\geq \log_2 e\left(\underbrace{\sum_i\sum_j P(x_i, y_j)}_{1} - \underbrace{\sum_i\sum_j P(x_i)\,P(y_j)}_{1}\right)$$

$$\boxed{I(x,y) \geq 0}$$

$$I(x;y) = H(X) + H(Y) - H(X;Y)$$

$$I(X;Y) = H(X) - H(X|Y)$$

$$H(X;Y) = H(\cancel{Y}) + H(\cancel{X}/\cancel{Y}).$$

$$I(X;Y) + H(X;Y) = H(X) + H(Y)$$

$$\boxed{I(X;Y) = H(X) + H(Y) - H(X;Y)}$$

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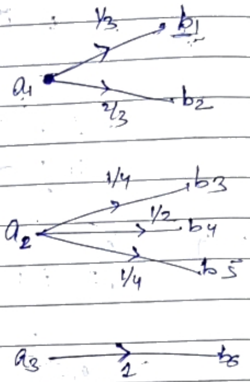
Q

$P(a_1 b_1)P(b_1) = P(b_1|a_1)\cdot P(a_1)$

$\boxed{P(a_1|b_1) = \frac{P(b_1|a_1)\cdot P(a_1)}{P(b_1)}}$

$\boxed{P(b_1) = \sum_i P(a_i)P(b_1|a_i)}$



$P(a_1|b_1) = \frac{P(b_1|a_1)\cdot P(a_1)}{\sum P(a_i)P(b_1|a_i)}$

$= \frac{\frac{1}{3}\times P(a_1)}{P(a_1)\times\frac{1}{3}}$

$\boxed{P(a_1|b_1) = 1}$

$H(A|B) = \sum_A\sum_B P(A,B)\cdot\log\frac{1}{P(A|B)}$

$P(a_1|b_2) = \frac{P(b_2|a_1)\times P(a_1)}{P(b_2)}$

$= \sum_A\sum_B P(B)\times P(A|B)\log\frac{1}{P(A|B)}$

$= \sum_A P(b_1)\cdot P(A_i/b_1)\log\frac{1}{P(A_i|b_1)}$ a_1/b_1, a_1/b_2

$+ P(b_2)P(A_i|b_2)\log$

$+ P(b_3)$

$+ P(b_4)$

$+ P(b_5)$

$+ P(b_6)$

$= \sum P(b_1)\cdot P(a_i|b_i)\log\frac{1}{P(a_1|b_1)}$ + rest are zero for b_1 $=1$

Similarly all

$\boxed{H(A|B) = 0}$ hence, noise free channel

here also,

a_1	1
a_2	1
a_3	1
a_4	0
a_5	0
a_6	0

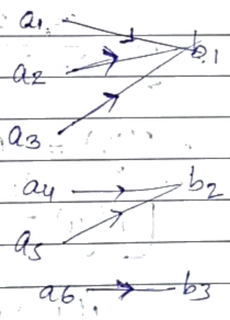
$$\begin{array}{c|cccccc} & b_1 & b_2 & b_3 & b_4 & b_5 & b_6 \\ \hline a_1 & 1/3 & 2/3 & 0 & 0 & 0 & 0 \\ a_2 & 0 & 0 & 1/4 & 1/2 & 1/4 & 0 \\ a_3 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}$$

<u>Channel Matrix</u>

→ if in channel matrix, for every column if there is only single entry then the channel is <u>Noise free</u> channel.

<u>Deterministic channel</u> :

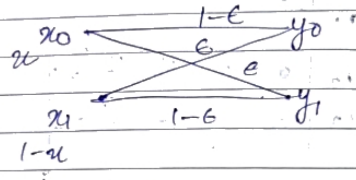
here also, $H(A/B) = 0$.



$$\begin{array}{c|ccc} & b_1 & b_2 & b_3 \\ \hline a_1 & 1 & 0 & 0 \\ a_2 & 1 & 0 & 0 \\ a_3 & 1 & 0 & 0 \\ a_4 & 0 & 1 & 0 \\ a_5 & 0 & 1 & 0 \\ a_6 & 0 & 0 & 1 \end{array}$$

→ if in channel matrix, for every row if there is only single entry then the channel is <u>Deterministic channel</u>.

Binary Entropy



$I(X;Y) = H(Y) - H(Y/X)$

$P(y_0) = P(x_0)\cdot P(y_0/x_0) + P(x_1)\cdot P(y_0/x_1)$

$= x(1-\epsilon) + (1-x)\epsilon$

$= x - x\epsilon + \epsilon - x\epsilon$

$\boxed{P(y_0) = x+\epsilon - 2x\epsilon} \to A$

$P(y_1) = P(x_0)P(y_1/x_0) + P(x_1)\cdot P(y_1/x_1)$

$\boxed{P(y_1) = x(\epsilon) + (1-x)(1-\epsilon)} \to B$

$\boxed{P(y_0) + P(y_1) = 1}$

$\Rightarrow \cancel{x+\epsilon-2x\epsilon + x\epsilon + 1 - \epsilon - x + x\epsilon} \Rightarrow 1$

$\boxed{H(Y) = A\log\frac{1}{A} + B\log\frac{1}{B}}$

$H(Y/X) =$

$P(x)H(y_0/x)$ $y=0:$

$P(x)H(y_1/x)$ $\cdot y=1:$

$H(Y/X) =$

$H_E = \boxed{H(Y/X)}$

Bi

[N

So, Mutual

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$$H(Y/x) = \sum_i P(x_i) \sum_j P(y_j/x_i) \log \frac{1}{P(y_j/x_i)}$$

$y=0$: $P(x_0) \cdot P(y_0/x_0) \cdot \log \frac{1}{P(y_0/x_0)}$

$+ P(x_1) \cdot P(y_0/x_1) \log \frac{1}{P(y_0/x_1)}$.

$y=1$: $P(x_0) \cdot P(y_1/x_0) \log \frac{1}{P(y_1/x_0)}$

$+ P(x_1) \cdot P(y_1/x_1) \log \frac{1}{P(y_1/x_1)}$

$$H(Y/X) = \epsilon(\mu + 1 - \mu) \log \frac{1}{\epsilon} + (1-\mu+\mu)(1-\epsilon) \log \frac{1}{1-\epsilon}$$

$$= \boxed{H(Y/X) = \epsilon \log \frac{1}{\epsilon} + (1-\epsilon) \log \frac{1}{1-\epsilon}}$$

$\longrightarrow$ Binary entropy function
[No use of / Involvement of Source Prob.].

80, Mutual Inforᵐ -

$$I(Y;X) = H(Y) - H(Y/X)$$

$$= A \log \frac{1}{A} + B \log \frac{1}{B} - H_\epsilon \quad \text{--- (4)}$$

(ϵ is constant) $\leftarrow A = u(1-\epsilon) + \epsilon(1-u)$

$B = u\epsilon + (1-u)(1-\epsilon)$

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first differentiate :-

$$I' = -\frac{A}{A}A' - A'\log A - \frac{B}{B}B' - B\log B.$$

$$A' = 1-\epsilon+(-\epsilon) = 1-2\epsilon$$

$$B' = \epsilon + (\epsilon-1) = 2\epsilon-1.$$

$$\boxed{A' = -B'}$$

$$I' = -\cancel{A'} - A'\log A - \cancel{B'} - B'\log B$$

$$= -(A'\log A + B'\log B)$$

$$= -(A'\log A - A'\log B)$$

$$= -A'\left(\log \frac{A}{B}\right)$$

$$I' = (2\epsilon-1)\log \frac{u(1-\epsilon)+\epsilon(1-u)}{u\epsilon+(1-u)(1-\epsilon)}$$

To get max. info $\rightarrow I' = 0$

$\therefore$ $(2\epsilon-1)\log\frac{A}{B} = 0.$

$\epsilon = 1/2$ or $A = B$

$u(1-\epsilon)+\epsilon(1-u) = 2u\epsilon+1-u-\epsilon$

$u+\epsilon-2u\epsilon = 2u\epsilon+1-u-\epsilon$

$2u+2\epsilon-4u\epsilon = 1.$

$$\boxed{u+\epsilon-2u\epsilon = 1/2}$$

$$u(1-2\epsilon) = \frac{1}{2}-\epsilon$$

$\Rightarrow u$

so, $\epsilon = 1/$

Channel capacity $\leftarrow C = H(Y$

$H(Y) =$

$A = \frac{1}{2}$

$B = \frac{\epsilon}{2}$

$\Rightarrow \boxed{H(Y)}$

$\boxed{C = 1}$

also, I''

$) + \epsilon(1-u)$
$(1-u)(1-\epsilon)$

...entiate :

$A' \log A - \frac{B}{B} B' - B \log B.$

$\epsilon) = 1 - 2\epsilon$

$-1) = 2\epsilon - 1.$

$\boxed{A' = -B'}$

$- \cancel{B'} - B \log B$

$+ B' \log B)$

$- A' \log B)$

$\frac{A}{B})$

$\frac{u(1-\epsilon) + \epsilon(1-u)}{u\epsilon + (1-u)(1-\epsilon)}$

nfo $\rightarrow I' \leq 0$

$= 0.$

$A = B$

$u(1-\epsilon) + \epsilon(1-u) = 2u\epsilon + 1 - u - \epsilon$

$u + \epsilon - 2u\epsilon = 2u\epsilon + 1 - u - \epsilon$

$2u + 2\epsilon - 4u\epsilon = 1.$

$\boxed{u + \epsilon - 2u\epsilon = 1/2}$

$= \frac{1}{2} - \epsilon$

$\Rightarrow u \times \cancel{\epsilon} = \cancel{220}\ \frac{1}{2}$

$\Rightarrow \boxed{u = \frac{1}{2}}$

so, $\epsilon = 1/2$ or $u = 1/2$

Channel capacity

$C = H(Y) - H(Y/X).$ at $u = 1/2.$

$H(Y) = A \log \frac{1}{A} + B \log \frac{1}{B}$

$A = \frac{1}{2}(1-\epsilon) + \frac{\epsilon}{2} = \frac{1}{2}$

$B = \cancel{\frac{\epsilon}{2}} + \frac{1}{2} - \cancel{\frac{\epsilon}{2}} = \frac{1}{2}$

$\boxed{H(Y) = \frac{1}{2} + \frac{1}{2} = 1}$

$\boxed{C = 1 - H_\epsilon} = 1 - \left[\epsilon \log \frac{1}{\epsilon} + (1-\epsilon) \log \frac{1}{1-\epsilon}\right]$

also, I'' to show

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Channel Coding :
- → Error Detection
- → Error correction

$b_5\, b_4 b_3 b_2 b_1 b_0$

$1\,0\,1\,1\,1\,0\,0$ (last digit marked P)

$P = b_0 \oplus b_1 \oplus b_2 \oplus b_3 \oplus b_4 \oplus b_5$

$= 0$

Channel Coding
- Block codes
  - → Popular Block codes
  - → Linear
  - → Cyclic – upto mid sem.
  - → Bch
- Convolution

Block Codes

(n, k)

- n → length of codeword
- k → No. of info^m bits
- $(n-k)$ → no. of bit redudens.

Q for a gi…
probabili…
error d…
probably

Soln
$P_c = \ldots$
$P_e = \ldots$
$P_f = \ldots$
$\boxed{P_c + P_e + \ldots}$

Product Co…

Error Detection

Error correction

$b_1 \oplus b_2 \oplus b_3 \oplus b_4 \oplus b_5$

convolution

no. of info^n bits

of bit redundancy

Q for a given (5,4) even parity codes find the probability of correct decoding, probability of error decoding, failure decoding when probably of error is 0.1.

Solu^n $P_c = {}^5C_0 \, p^0 (1-p)^4$ $\quad p = 0.1$

$P_e = {}^5C_2 \, p^2 (1-p)^3 + {}^5C_4 \, p^4 (1-p)$.

$P_f = {}^5C_1 \, p^1 (1-p)^4 + {}^5C_3 \, p^3 (1-p)^2 + {}^5C_5 \, p^5 (1-p)^0$

$\boxed{P_c + P_e + P_f = 1}$

Product Codes [can detect a single bit of error]

Minimum Error decoding Rules -

$D_{ME}(b) = a^*$

$P(a^*/b) \geq P(a_i/b) \quad \forall i$

$\boxed{\dfrac{P(b/a^*)\,P(a^*)}{P(b)} \geq \dfrac{P(b/a_i)\,P(a_i)}{P(b)}}$

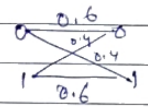
$P(b/a^*) \geq P(b/a_i)$

Maximum likelihood decoding rule.

#

$$P = \begin{bmatrix} 0.6 & 0.4 \\ 0.4 & 0.6 \end{bmatrix}$$

		$P(a_i)$
a_1	000	0.4
a_2	011	0.2
a_3	101	0.1
a_4	110	0.3



$\boxed{b = 111}$ $\boxed{D(b) \overset{?}{=} a}$

$P(a^*/b) \geq P(a_i/b) \quad \forall i$

~~Error maximum likelihood decoding rule~~

$P(b/a^*)\,P(a^*) \geq P(b/a_i)\,P(a_i)$

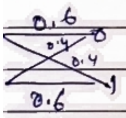
Rules -

b) $\forall i$

$\boxed{\frac{P(b/a_i)\,P(a_i)}{P(b)}}$

(a_i)

rule.



$\forall i$

~~decoding rule~~

$P(b/a_i)\,P(a_i)$

$P(b/a_1) = P(111/000) = P(1/0)\cdot P(1/0)\cdot P(1/0)$
$= (0.4)^3 = 0.064$

$P(b/a_2) = P(111/011) = P(1/0)\cdot P(1/1)\cdot P(1/1)$
$= 0.4\times 0.6\times 0.6$
$= 0.144$

$P(b/a_3) = P(111/101) = P(1/1)\cdot P(1/0)\cdot P(1/1)$
$= 0.144$

$P(b/a_4) = P(111/110) = P(1/1)\cdot P(1/1)\cdot P(1/0)$
$= 0.144$

$P(a_1)\cdot P(b/a_1) = 0.4\times 0.064 = 0.0256$
$P(a_2)\cdot P(b/a_2) = 0.2\times 0.144 = 0.0288$

~~Now, minimum error decoding rules~~

$P(a_3)\cdot P(b/a_3) = 0.1\times 0.144 = 0.0144$

$\boxed{P(a_4)\cdot P(b/a_4) = 0.3\times 0.144 = 0.0432}$

max. value

$\therefore$ b can be map to a_4

$\boxed{111 \longrightarrow 110}$

Hamming Distance Decoding rules :-

$a = 110110$
$b = 101111$

$d(a,b) = 3$ [Hamming distance of a & b]

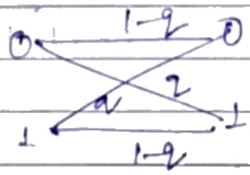
$d(a,b) = \underbrace{HW}_{\text{Hamming Weight}}(a \oplus b) = HW(011001) = 3$

HW → no. of non zero elements (bits) in $a \oplus b$

$P(b|a) = q^D (1-q)^{N-D}$

$q \rightarrow$ error probability (Cross probability)

$D \rightarrow$ no. of bits with error $HW(a \oplus b)$.



$P(b|a) = q^D (1-q)^{N-D}$

if $q < 0.5$ then only reliable transmission is possible.

D should also min.

by Minimum decoding rule.

$$\boxed{d(a^*, b) \leq d(a_i, b) \quad \forall i.}$$

if we have multiple choice then unable to correct the error.

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oding rule :-

mming distance of a & b]
eight
b) = HW(011001) = 3
ero elements (bits) in

-D

probability (Cross probability)
~~[illegible]~~ no. of bits with error
a ⊕ b).

) = $q^D (1-q)^{N-D}$
$q < 0.5$ then only
liable transmission
possible.
hould also min.

$\boxed{d(a_i, b) \ \forall i.}$

multiple choice then
correct the error.

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Q codewords.

a_1 000 $\qquad b = 0\,1\,0$

a_2 001 $\qquad d(a_1, b) = 1$ ⟷ this both are min & same

a_3 011 $\qquad d(a_2, b) = 2$

a_4 111 $\qquad d(a_3, b) = 1$

$d(a_4, b) = 2$

here we ~~just~~ can, not correct as we have multiple choice for min., so, we can just detect the error.

if $b = 110$

$d(a_1, b) = 2$
$d(a_2, b) = 3$
$d(a_3, b) = 2$
$d(a_4, b) = 1$ * ⟶ this is the correct code

Linear codes: one codeword will consist of zero vector

* If their exist 2 codewords say, C_i, C_j
then $C_k = C_i + C_j$

Minimum Hamming weight of non-zero codewords.

(3,2)

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\#

		HW
C_0	000	
C_1	011	(2)
C_2	101	(2)
C_3	110	(2)

$$\begin{array}{c|cccc} & C_0 & C_1 & C_2 & C_3 \\ \hline C_0 & C_0 & C_1 & C_2 & C_3 \\ C_1 & C_1 & C_0 & C_3 & C_2 \\ C_2 & C_2 & C_3 & C_0 & C_1 \\ C_3 & C_3 & C_2 & C_1 & C_0 \end{array}$$

$$d(C_n) = \min_{HW} \left(d(C_i, C_j) \right) \quad i \neq j$$

$$\begin{array}{c|cccc} & C_0 & C_1 & C_2 & C_3 \\ \hline C_0 & 0 & 2 & 2 & 2 \\ C_1 & 2 & 0 & 2 & 2 \\ C_2 & 2 & 2 & 0 & 2 \\ C_3 & 2 & 2 & 2 & 0 \end{array}$$

$$\boxed{d(C_n) = 2}$$

Assignment

Q. Construct (7,4) even parity hamming codes.

Q. Prove that (7,4) hamming code is a linear codes.

Q. Write the algorithmic steps to compute the basis vector of the given set.

Q. Find the basis vector of (7,4) hamming code.

(7,4)

Syndrome value

$Q \cdot H^T =$

\# Theorem 1

B is the

the corro

are repr

(3,2)

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	c_1	c_2	c_3
	c_1	c_2	c_3
	c_0	c_3	c_2
	c_3	c_0	c_1
	c_2	c_1	c_0

$) = \min_{all} (d(c_i, c_j)) \quad i \neq j$

c_0	c_1	c_2	c_3
	2	2	2
	0	2	2
	2	0	2
	2	2	0

$\boxed{2}$

evenparity
(7,4) hamming codes-

…at (7,4) hamming code is
codes.

…the algorithmic steps
…the basis vector of the

…sis vector of (7,4)
…e.

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(7,4)

7	6	5	4	3	2	1
D_4	D_3	D_2	P_4	D_1	P_2	P_1

\# $c \longrightarrow v$

$S = v \cdot H^T$ → Paritycheck matrix

Syndrome value; receive vector

if $S = (0 \cdots 0)$ then, v is valid vector.
else, Error is present in v.

$G \cdot H^T = 0$

$G = [I \mid P]$

$H = [P^T \mid I]$

\# **Theorem 1**: If Block code K_n detect upto t errors, if and only if its min. distance is greater than t.

B is the recived Codewords and a^* is the corrosponding valid Codeword of B. & rest are represented by a_i.

$d(b, a_i) > 0$

$d(a^*, b) + d(b, a_i) \geq d(a^*, a^i)$

$\Rightarrow d(b, a_i) \geq d(a^*, a^i) - d(a^*, b)$

as, $d(b, a_i) > 0$

$d(a^*, a_i) - d(a^*, b) > 0.$

$\Rightarrow \boxed{d(a^*, a_i) > d(a^*, b)}$

also, $\boxed{d(a^*, b) = t} \rightarrow$ [initial assumptions]

$\Rightarrow \underbrace{d(a^*, a_i)}_{K_n} > t$

$\therefore \boxed{d(K_n) > t}$

Theorem 2: A block code K_n correct upto t errors, if and only if its min. distance is greater than 2t.

$d(b, a^*) = t$

$d(b, a^*) < d(b, a_i) \quad \forall i$

$d(a^*, b) + d(b, a_i) \geq d(a^*, a_i)$

$d(a^*, b) \geq d(a^*, a_i) - d(b, a_i)$

$\cancel{d(a^*, a_i) - d(b, a_i)} < d(b, a_i)$

$\cancel{d(a^*, a_i) < 2d(b, a_i)}$

$\cancel{d(a^*, a_i) < 2t}$