Now using polynomial

$g^{11} = 1 \quad g^{12} = 1 \quad g^{13} = 1 \quad g^{14} = 1$

$g^{21} = 0 \quad g^{22} = 1 + D \quad g^{23} = D \quad g^{24} = 1$

$g^{31} = 0 \quad g^{32} = D \quad g^{33} = 1 + D^2 \quad g^{34} = 1 + D^2$

$u_0 = D + D^2$

$u_1 = 1 + D$

$u_2 = 10 + D^2$

$$v^j(D) = \sum_{i=1}^{M} u^i(D) \otimes g^{ij}(D)$$

$v^1(D) = (1+D^2)\cdot 1 + (1+D)\cdot 0 + (D+D^2)\cdot 0 = 1 + D^2$

$v^2(D) = D^2 + D^3$

$v^3(D) = 1 + D^2 + D^3 + D^4$

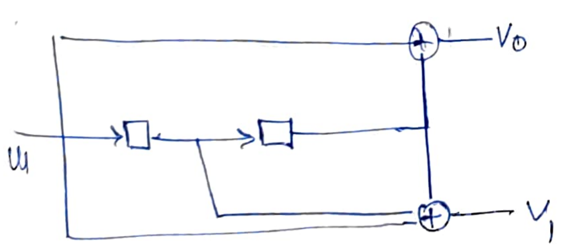
$v^4(D) = D^3 + D^4$

(n, k, m)

$\boxed{V(D) = v^1(D^n) + D\cdot v^2(D^n) \ldots D^{n-1} v^n(D^n)}$ (multiplexing operation)

$v(D) = v^1(D^4) + D v^2(D^4) + D^2 v^3(D^4) + D^3 v^4(D^4)$

$= 1 + D^8 + D(D^8 + D^{12}) + D^2(1 + D^8 + D^{12} + D^{16}) + D^3(D^{12} + D^{16})$



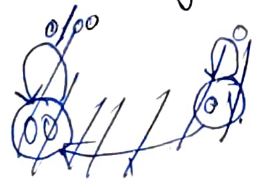
(Received word)

Rec = 01, 11, 10, 10, 00, 11, 10

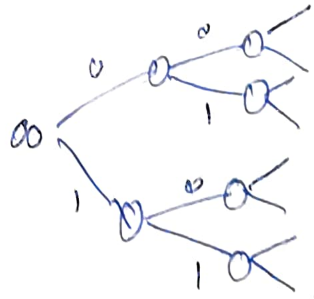
(2, 1, 2)

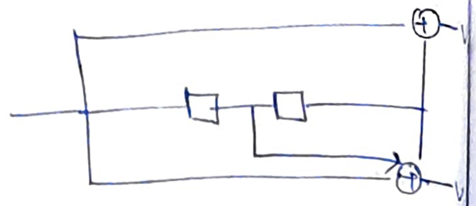
Memory content in shift register : 00, 01, 10, 11

Making automata for shift register.



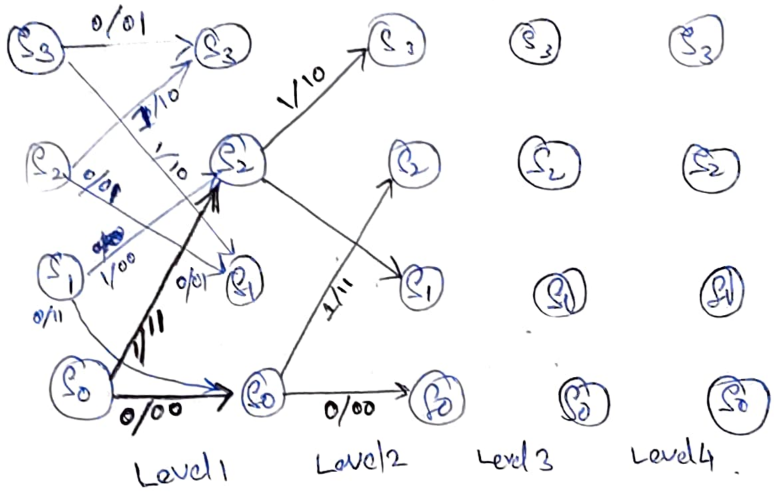
Tree diagram:



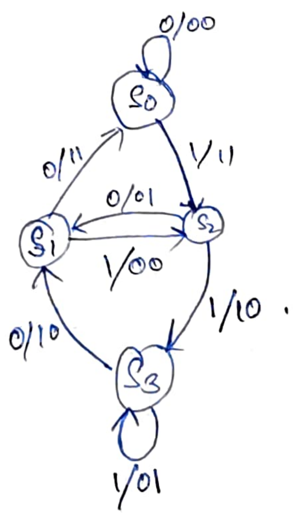


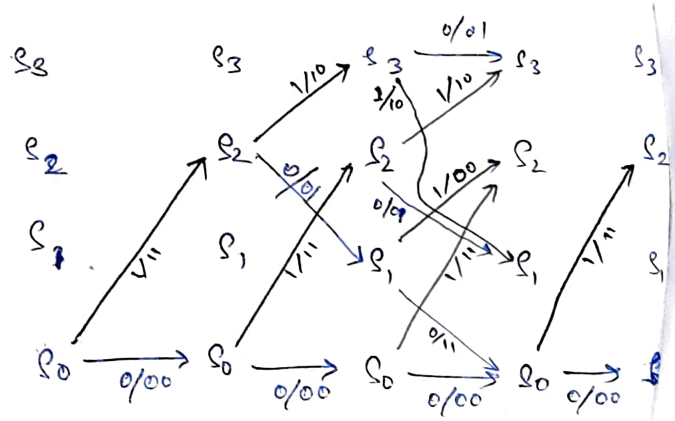
States
S_0 00
S_1 01
S_2 10
S_3 11

Trellis diagram:



input / output
i.e $u_1 / v_0 v_1$





(copy same level 3 in level 4)

Initial shift register has content 00 i.e S_0 state. So level I can start only from S_0. Draw lines only for state ~~states~~ that are reachable.

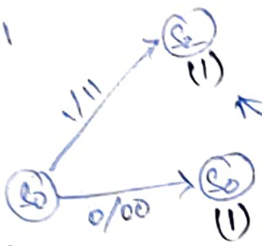
In decoding, we can use state diagram & Trellis's diagram.

Decoding using Trellis diagram:

Received: 01 11 10 10 00 11 10

If received is wrong use transition with minimum Hamming distance.
Write Hamming distance as weight.

Received 01



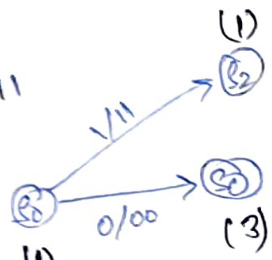
$H(01, 00) = 1$

$H(01, 11) = 1$

Hamming weights.

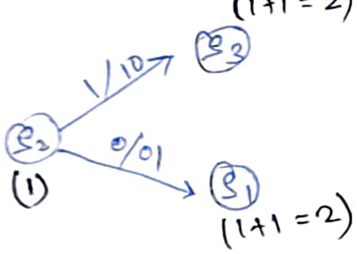
Here 2 possible path: Explore all possible path.

Received 11

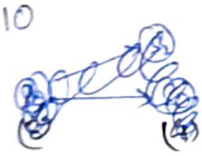


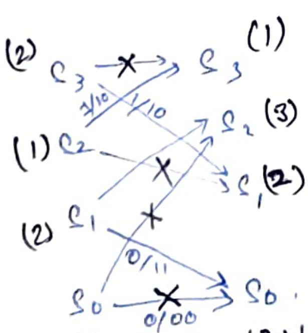
$H(11, 00) = 2$ ∴ add previous. $2+1 = 3$.

$H(11, 11) = 0$ ∴ $0+1 = 1$



Received 10





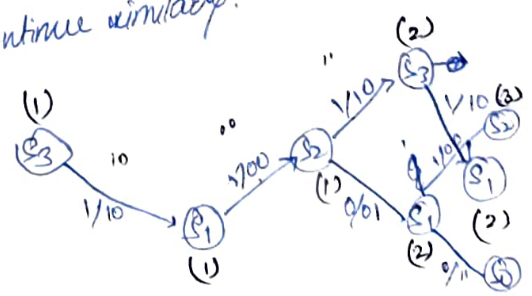
this path not considered.

$\left(\begin{array}{l}2+1=3\\ \text{or, } 3+1=4\end{array}\right)$ Take minimum.

→ In case of tie keep both paths.

$\boxed{\text{Take } S_0 \text{ to } S_0 \text{ Path}}$

Continue similarly:



Here we have 2 possible path