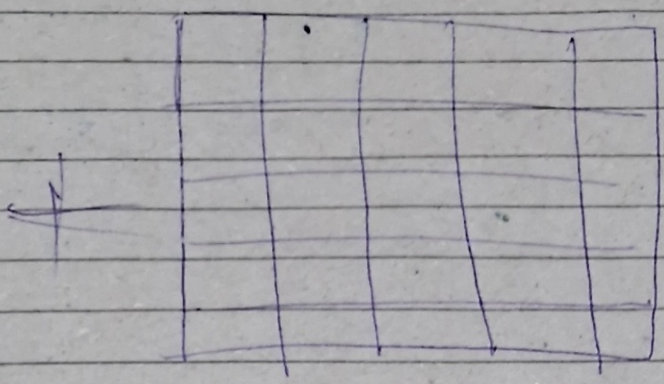
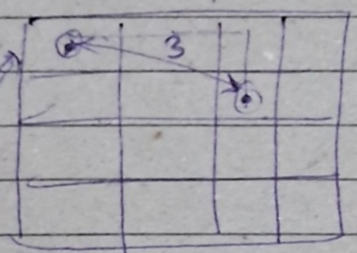


K

$^{nm}C_K$

Σ manhattan distance



K = ?

d = 3 X howmanytimes?

no. of arrangements for which the coins would have been at places were (0,1) & (1,2) = $^{mn-2}C_{K-2}$ (constant value)

this many times dist 3 would be added

so any pair of ~~vertices~~ cells would be added = ${}^{mn-2}C_{k-2}$ times

dist b/w all pairs of (i,j)

$$\sum\sum d_{ij} \cdot {}^{mn-2}C_{k-2}$$

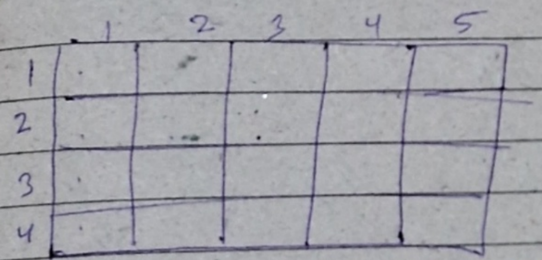
$${}^{mn-2}C_{k-2} \underbrace{\sum_{j=i+1}^{mn}\sum_{i=1}^{mn} d_{ij}}$$

problem breakdown

⇒ find sum of manhattan distance b/w all pairs of cells

⇒ Contribution technique

⇒ can be done in $(mn)^2 \longrightarrow$ TLE



$$\sum_{x_1=1}^{4}\sum_{x_2=1}^{4}\sum_{y_1=1}^{5}\sum_{y_2=5}^{5}\left[|x_1-x_2|+|y_1-y_2|\right]$$

it counts (a,b) and (b,a) so twice basically so it need to be divided by 2

$$\sum_{x_1=1}^{4}\sum_{x_2=1}^{4}\sum_{y_1=1}^{5}\sum_{y_2=1}^{5}\left[\boxed{|x_1-x_2|}+\boxed{|y_1-y_2|}\right]$$

$$2\sum_{x_1=1}^{n}\sum_{x_2=1}^{n}\sum_{y_1=1}^{m}\sum_{y_2=1}^{m}\left[|x_1-x_2|\right]$$

$$\sum_{x_1=1}^{n}\sum_{x_2=1}^{n}\left(|x_1-x_2|\right)\cdot\sum_{y_1=1}^{m}\sum_{y_2=1}^{m}(1) \to m^2$$

$$m^2\cdot\sum_{x_1=1}^{n}\sum_{x_2=1}^{n}|x_1-x_2|$$

~~basically~~ ~~what~~ it looks like this (x_1, x_2)

$(1,1)$ - - - $(1,n)$

$(2,1)$ - - - $(2,n)$

$\vdots$

$(n,1)$ - - - (n,n)

let $n=4$

$x_2 \backslash x_1$	1	2	3	4
1	0	1	2	2
2	1	0	1	2
3	2	1	0	1
4	3	2	1	0

$|x_1-x_2|$

⇒ both triangles are identical

⇒ so calculate the sum of 1 then double it to calculate the sum of all pairs of (x_1, x_2)

sum of 1 Δ $= \underset{3}{3}\cdot\underset{1\text{ times}}{1} + 2\cdot 2 + 1\cdot 3$

$$= \underset{\text{times}}{(n-1)}\cdot\underset{1}{1} + (n-2)\cdot 2 + \dots + (n-(n-1))\cdot(n-1)$$

$$= \sum_{i=1}^{n-1}(n-i)\cdot i$$

total sum $= 2\sum_{i=1}^{n-1}(n-i)\cdot i$

$$\frac{n\times(n+1)}{2} \qquad \frac{n(2n+1)(2n+1)}{6}$$

DATE: __/__/____
PAGE: ______

$$= \cancel{2}\left[n\sum_{i=1}^{n-1} i - \sum_{i=1}^{n-1} i^2\right] \qquad \frac{2n-2+1}{2n-1}$$

$$= \cancel{2}\left[n \cdot \frac{(n-1)n}{\cancel{2}} - \frac{\overset{(n-1)(n)(2n-1)}{\cancel{n(n+1)(2n+1)}}}{3\cancel{6}}\right]$$

$$= n(n-1)\left[n - \frac{(2n-1)}{3}\right]$$

$$= \frac{n(n-1)(n+1)}{3}$$

$$\text{so } m^2 \sum_{x_1=1}^{n} \sum_{x_2=1}^{n} |x_1 - x_2| = \frac{m^2 \cdot n(n-1)(n+1)}{3}$$

$$\text{total } x\text{-distance} = \frac{m^2 \cdot n(n-1)(n+1)}{3}$$

$$\text{total } y\text{-distance} = n^2 \cdot \frac{m(m-1)(m+1)}{3}$$

$$\text{final sum} = \frac{1}{2}\left[\frac{m^2 n(n-1)(n+1)}{3} + \frac{n^2 m(m-1)(m+1)}{3}\right]$$

$$= \frac{1}{6}\left[m^2 n(n-1)(n+1) + n^2 m(m-1)(m+1)\right]$$

$$\text{final sum} = \qquad \times \; {}^{mn-2}C_{k-2}$$

$(ab) \bmod m = (a \bmod m \cdot b \bmod m) \bmod m$

$\frac{a}{b} \bmod m = (a \times b^{-1}) \bmod m$

$b \cdot b^{-1} = 1 \bmod m$

$m = 7 \quad b = 2 \Rightarrow b^{-1} = 4$

$2 \times 4 = 8 \bmod 7 = 1$

Good Write

DATE: __/__/__
PAGE:

If b and p are coprime then

$(b^{p-1} \equiv 1) \bmod p$

$\Rightarrow \quad b \cdot b^{p-2} \equiv 1$

$\downarrow$

$b^{p-2} = b^{-1}$

$m = 7 \quad b = 2 \qquad b^{-1} = b^{7-2} = b^{5} = 2^{5} = 32 \% \bmod 7$
$= 4$

so here rather than dividing by 6 we will multiply with 6^{-1}

$^{n}C_{r} = \dfrac{n!}{(n-r)!\,r!}$