**Re-minor exam** (*Oct 29, Marks: 15, Duration: 1 hour*) **COL703, Aug-Nov 2022**

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**Name:** **Entry No.:**

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*Please note that you are not allowed to carry with you any notes, cheat-sheet, or any electronic devices including your phone and smart-watches. Also note that there will be zero tolerance for dishonest means like copying solutions from others, and even letting others copy your solution. If you are found indulging in such an activity, your exam paper will be seized immediately and you will be given a zero without any evaluation.*

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1. **[5 marks]** Show that $\alpha \vee \beta$ is consistent *iff* either $\alpha$ is consistent or $\beta$ is consistent. Use Hilbert's proof system. Recall that $\alpha$ is said to be *consistent* if $\nvdash \neg\alpha$.

2. **[5 marks]** Let S be a set of clauses – $\{C_1, \ldots, C_m\}$, where each $C_i$ is a disjunction of $n_i$ literals – $\{l_{i1}, \ldots, l_{in_i}\}$. Let us define $S^*$ to be the set of clauses

$$\bigcup_{i=1}^{m} \bigcup_{1 \leq j < k \leq n_i} \{\{l_{ij}, l_{ik}\}\}$$

   Prove that $S$ is renamable Horn if and only if $S^*$ is satisfiable.

   Recall that a *renamable Horn formula* is a CNF formula that can be turned into a Horn formula by negating (all occurrences of) some of its variables. For example,

$$(p_1 \vee \neg p_2 \vee \neg p_3) \wedge (p_2 \vee p_3) \wedge (\neg p_1)$$

   is renamable Horn because it can be turned into a Horn formula by negating $p_1$ and $p_2$.

3. **[2 marks]** Give a natural deduction proof of PBC using only basic rules.

4. **[3 marks]** Given the premises $(p \to q)$ and $(r \to s)$, use resolution to prove the conclusion $(p \vee r \to q \vee s)$.