A COL 202 Lecture 6 (31st Jan)

Normal Forms

Negation Normal Form (NNF)

A wff is in Negation Normal Form if it uses only $\wedge$, $\vee$, and literals.

(atoms/prop. variables and their negation)

$$\neg(\overset{\alpha}{p} \to \overset{\beta}{(p \wedge q)})$$

$$p \wedge (\neg p \vee \neg q)$$

$$a \to b \equiv \neg a \vee b$$

$$\neg(a \vee b) \equiv \neg a \wedge \neg b$$

$$\neg(a \wedge b) \equiv \neg a \vee \neg b$$

$$\neg(\neg p \vee (p \wedge q))$$

$$\boxed{\neg(\neg p)} \wedge \neg(p \wedge q)$$

$$p \wedge \neg(p \wedge q)$$

$$p \wedge (\neg p \vee \neg q)$$

Every wff is logically equivalent to a wff in NNF.

Disjunctive Normal Form (DNF)

A wff is in DNF if it is a disjunction of one or more terms where each term is a conjunction of literals.

$\underbrace{(p \wedge q \wedge \neg r) \boxed{\vee} (q \wedge \neg s)}_{\text{cubes}}$

Every wff is logically equivalent to a wff in DNF.

p

$p \vee q$

$(p \wedge q)$

$\neg(p \to (p \wedge q))$ into DNF.

$p \wedge \neg q$ $+1+1+1+1+1$

$\neg p \vee \neg q$

$p \vee \neg q$

$=$

1 $\boxed{\neg(p \to (p \wedge q))}$

2 $\neg(\neg p \vee (p \wedge q))$

3 $p \wedge \neg(p \wedge q)$

4 $p \wedge (\neg p \vee \neg q)$

$\cancel{(p \wedge \neg p)} \vee (p \wedge \neg q)$

$p \wedge \neg q$

Conjunctive Normal Form (CNF)

A wff is in CNF if it is a conjunction of one or more terms where each term is a disjunction of one or more literals.

$\neg(p \to (p \wedge q))$ into CNF.

$(p \vee \neg p \vee \neg q) \;||$

$(a \vee b \vee \neg c) \wedge (d \vee \neg e) \wedge (c \vee f)$

$(a \vee b)$

Every wff is equivalent to a wff in CNF.

Conversion from truth table

$(a \vee b \vee \neg c)$

$\wedge (a \vee \neg b \vee c)$

a	b	c	f
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

$\to (\neg a \wedge \neg b \wedge \neg c)$

$\vee (\neg a \wedge b \wedge c)$

$\vee (a \wedge \neg b \wedge \neg c)$

$\vee (a \wedge b \wedge c)$

$\cancel{\neg(\neg a \wedge \neg b \wedge c)}$

$$\neg(p \to (p \wedge q)) = f$$

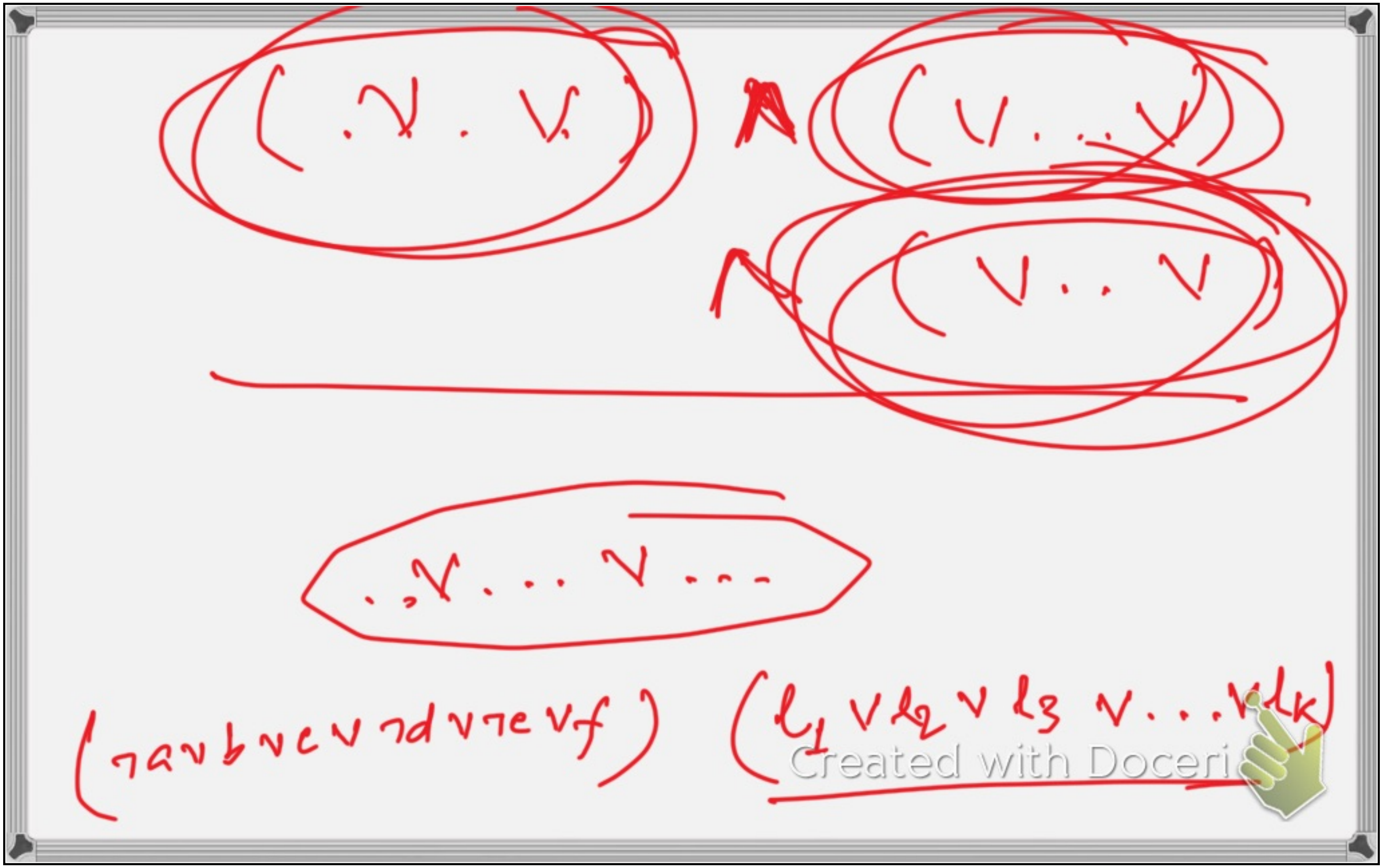
p	q	f
0	0	0
0	1	0
1	0	1
1	1	0

$$(p \wedge \neg q)$$

$$\boxed{(p \vee q) \wedge (p \vee \neg q) \wedge (\neg p \vee \neg q)}$$

$$(p \vee q) \wedge (p \vee \neg q) \wedge (\neg p \vee \neg q)$$

	p	q
$\times$	0	0
$\times$	0	1
	1	0
$\times$	1	1


$(. \vee . \vee .) \wedge (\vee \ldots \vee) \wedge (\vee \ldots \vee)$
$\ldots \vee \ldots \vee \ldots$
$(\neg a \vee b \vee c \vee \neg d \vee \neg e \vee f)$
$(l_1 \vee l_2 \vee l_3 \vee \ldots \vee l_k)$
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Validity check in CNF is easy.

Satisfiability checking in DNF is easy.