ACOL 202

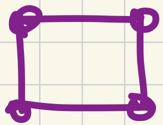
Lecture 26

(02 May 2025)

Regular graph

A d-regular graph is one where all nodes have degree d.

If a graph is d-regular for any d, it is called a regular graph.



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A 1-regular graph is called a perfect matching.

(If every node has degree ≤ 1 then the graph is called a matching.)

<u>Planar graph</u> G is planar if it is possible to draw G on a plane (a piece of paper) such that no two edges cross.

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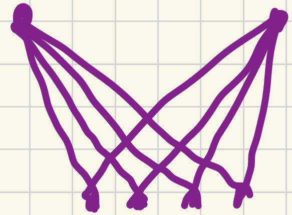
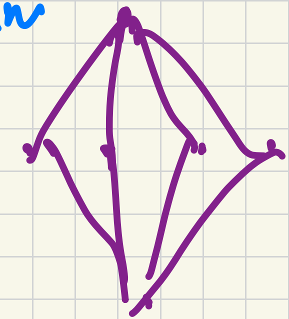
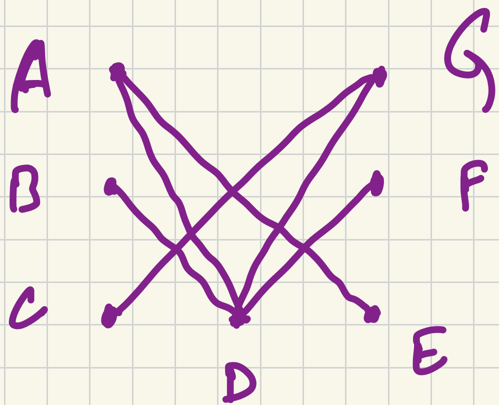
Is this graph planar?



Yes. We can draw it like this.

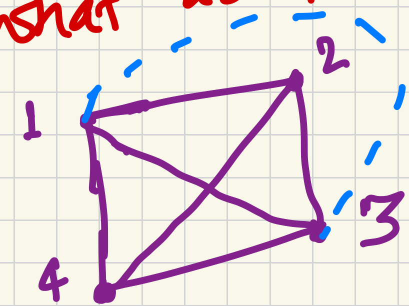


Is this graph planar?

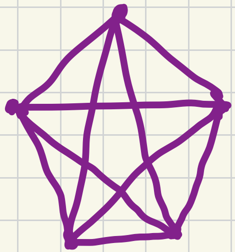


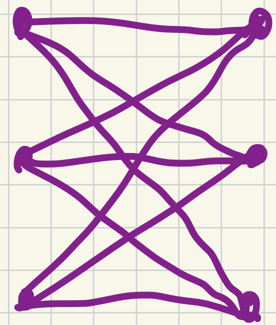
Yes. It can also be redrawn such that no edges cross.

What about this?



Yes. We can move one of the diagonals outside the square.





K_5, $K_{3,3}$

Examples of non-planar graphs

Those who are interested may read further about this (Kuratowski's Theorem).

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Paths sequence of vertices

$v_1, v_2, \ldots, v_k$

such that each $(v_i, v_{i+1}) \in E$.

simple path is one in which no vertex repeats.

Connected if there is a path from u to v for every $u, v \in V$.

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Connected components

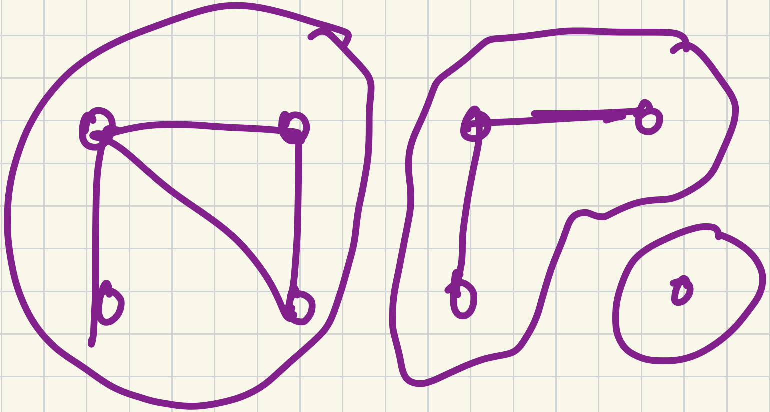
A connected component of $G = (V, E)$

is a set $C \subseteq V$

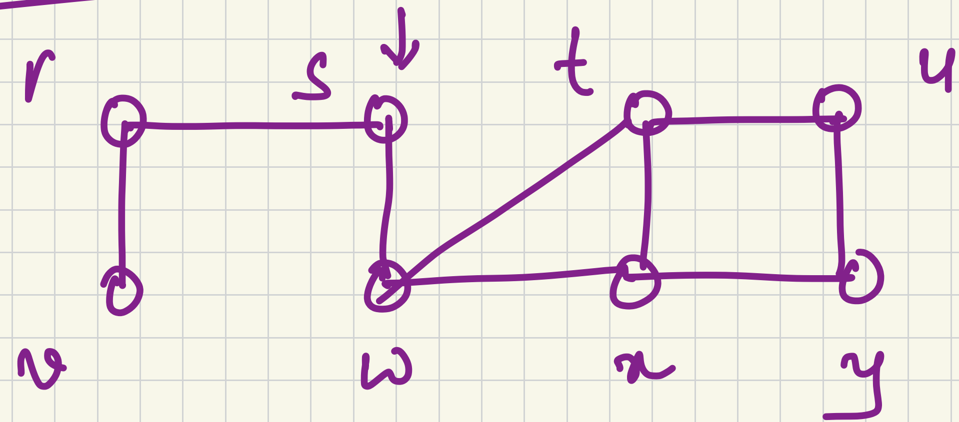
such that i) $\forall\, s, t \in C$ $\boxed{s \text{ and } t \text{ are connected}}$

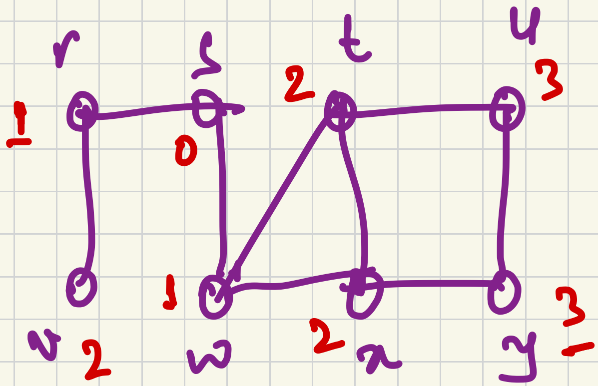
ii) $\forall\, x \in V \setminus C$

x is not connected to any vertex in C.



Breadth-first search (BFS)





color all the vertices white in the beginning

color a vertex gray when it is enqueued

color it black when it is dequeued

black	
	s(0)
s(0)	r(1) | w(1)
r(1)	w(1) | v(2)
w(1)	v(2) | t(2) | x(2)
v(2)	t(2) | x(2)
t(2)	x(2) | u(3)
x(2)	u(3) | y(3)
u(3)	y(3)
y(3)	empty

BFS(G, s) ← G is the graph ; s is the starting vertex

for each $u \in V(G)$ // $G = (V, E)$
  color[u] = white
  label[u] = ∞ // label stores the distance at which the vertices are discovered ; ∞ means not discovered yet

color[s] = gray ; label[s] = 0

$Q \leftarrow \{s\}$ // put s in the queue Q

while Q is non-empty
  $u \leftarrow$ head[Q] // pick the vertex at the beginning of the queue
  for each v that is adjacent to u
    if v is white
      color[v] = gray ; label[v] = label[u] + 1
      add v in the queue Q
  remove u from Q ; color[u] = black

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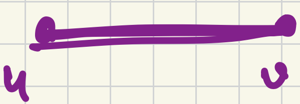
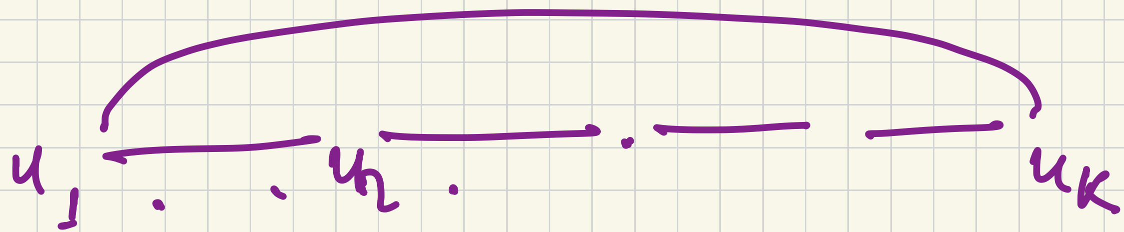
Cycles A cycle $\langle u_1, u_2, \ldots u_k, u_1 \rangle$ is a path of length ≥ 2 such that no edge is traversed twice.





A cycle is simple if each u_i, $i \in [1..k]$ is distinct.

<u>Acyclic</u> if there are no cycles.

<u>Trees</u> Connected, acyclic graphs.

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Claim Let $T = \langle V, E \rangle$ be a tree. Then $|E| = |V| - 1$.

Proof: (By induction)

Base case ($n=1$) • [Single vertex; no edges are possible with a single vertex; therefore $|E| = 0 = |V| - 1$]

Inductive step:

(We will first prove a lemma.)

Idea of the proof

Every connected acyclic graph with >1 vertices must have a $\boxed{\text{vertex of degree 1}}$.

Upon removal of $\boxed{\text{this}}$ vertex the graph is still connected and acyclic.

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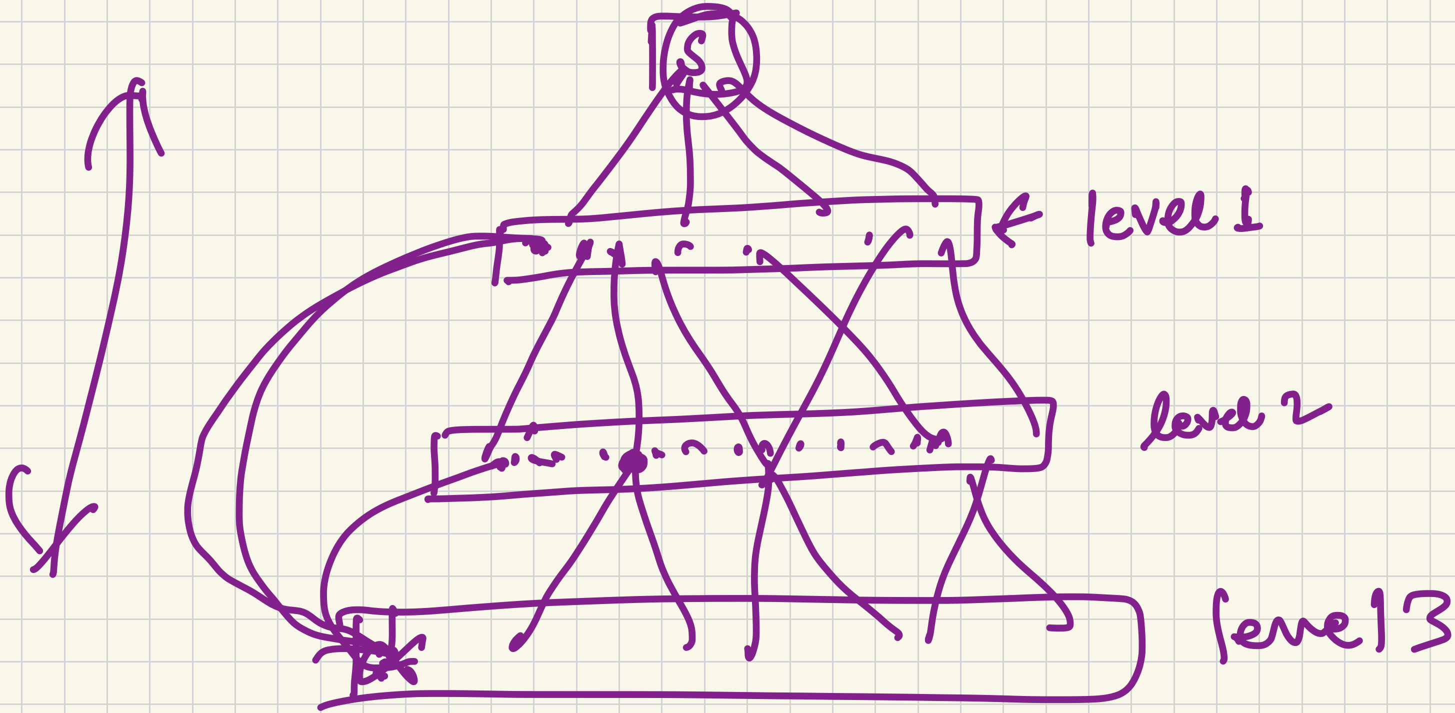
BFS

BFS Tree



Next lecture : DFS , Dijkstra's algorithm

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