Quiz 7 (*Nov 7, Marks: 3, Duration: 30 mins*) **COL703, Aug-Nov 2022**

Name: **Entry No.:**

1. **[1 marks]** Let F be a closed formula in Skolem form, $F = \forall x_1 \forall x_2 \dots \forall x_n\ G$, where G, in CNF, is given below:

$$\{\{T(a)\}, \{\neg T(x), \neg Q(f(x))\}, \{Q(f(y)), \neg P(x), Q(x)\}, \{P(x), \neg T(y)\}\}$$

   Use resolution to prove that F is unsatisfiable.

2. **[1 marks]** Recall the statement of Herbrand's theorem: Let $F = \forall x_1 \forall x_2 \dots \forall x_n\ F^*$ be a closed formula in Skolem form. Then F is satisfiable if and only if it has a Herbrand model.

   We claim that this is not true when F is an arbitrary formula. Consider, $S = P(a) \wedge \exists x\ \neg P(x)$ over a signature that consists of a constant symbol a and a predicate symbol P.

   Argue that S has a model, but it has no Herbrand models.

3. **[1 marks]** Consider the modal logic formula $(p \rightarrow \Box\Diamond q)$. Find a model in which it is true, and one in which it is false.

   A model $\mathcal{M} = ((W, R), V)$ is said to satisfy a formula ϕ if it is true in every world of the model $(\mathcal{M}, w \vDash \phi)$.