

ACOL 202

Extra lecture
(24th April)

$W_1 \quad W_2 \quad W_3 \quad \dots \quad W_k$

$1_2 \quad 1_2 \quad 1_2 \quad \dots \quad 1_2$

$0 \quad 1 \quad 2 \quad \dots \quad \dots$

$\frac{k}{2}$

$$0 + 1 + 2 + 3 + \dots + (k-1)$$

$$= \frac{(k-1)(k-1+1)}{2} = \frac{(k-1)k}{2}$$

Sum
-rule

$$|A \cup B| = |A| + |B|$$

$|A \cap B|$
is empty

Inclusion
- exclusion
rule

$$|A \cup B| = |A| + |B| - |A \cap B|$$

Exercise (from last lecture)
How many integers between
1 and 1000 (both inclusive)
that are evenly divisible by any of
2, 3 or 5.

A set of numbers in $[1, 1000]$ evenly
divisible by 2

B 3

C 5

$$|A| = 500, |B| = 333, |C| = 200$$

$$|A \cap B| = 166, |B \cap C| = 66,$$

$$|A \cap C| = 100$$

$$|A \cap B \cap C| = 33$$

$$|A \cup B \cup C| =$$

Product-rule

The cardinality of $A \times B$
(Cartesian product of A and B)
is the product of
the cardinalities of
A and B.

Example

How many 32-length bitstrings
are possible?

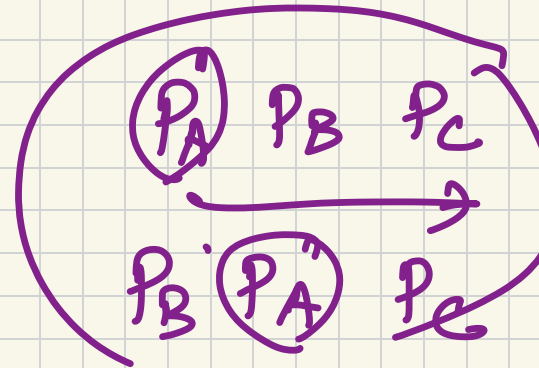
32
2

() () . . . () () . . . ()
1 2 22

Example

Suppose there are 8 people who are participating in the finals of 100-meter sprint at Olympics. Then how many different trios of medalists are possible.

$$8 \times 7 \times 6$$



Example

How many different opening moves are possible in a chess game?

20

Ans.

Example

Count the number of 4-digit PINs that start with three repeated digits.

WWWW
10 1 1 10

0001

1112

100 Ans

Permutation

A permutation of a set S is an ordering of the elements of S .

How many different permutations are possible for a set of size n ?

$$n!$$

$$\begin{array}{c} \overbrace{\quad} \\ \underbrace{\quad}_n \end{array} \quad \begin{array}{c} \overbrace{\quad} \\ \underbrace{\quad}_{n-1} \end{array} \quad \begin{array}{c} \overbrace{\quad} \\ \underbrace{\quad}_{n-2} \end{array} \quad \dots \quad \begin{array}{c} \overbrace{\quad} \\ \underbrace{\quad}_1 \end{array} \\ = n! \end{array}$$

Exercise

What fraction of integers
between 0 and 9,999,999,999
10 times

(all written as 10-digit numbers,
including any leading zeros)
have no repeated digits ?

$$\frac{10!}{10^{10}}$$

Ans

Exercise

A prefix-free code is a set C of bitstrings with the property that no $x \in C$ is a prefix of another $y \in C$.

What is the number of prefix-free codes that we can have with bitstrings of length 1 or 2.

0, 1, 00, 01, 10, 11

Exercise
Write a Python
program and check this!

$\{0, 1\}$

$\{0, 1\}$

$\{\emptyset, 1\}$

$\{\emptyset, 1, \dots\}$

1

$\{11, 10\}$ 4

$\{00, 01\}$ 4

$\{00, 01, 10, 11\}$
16

25

Example

Suppose two teams A and B are playing a best-of-five series of games. The teams play until one team has won three games, at which point the match is over.

How many different sequences of outcomes are possible?

B A B A A

B B B

Suppose A wins the match.

AAA || A never loses

13AAA
ABAA
AABA || A loses once

(20) ~~18~~

W W W W W
A A A A
0 1 2 3

(6)

Mapping-rule

Let A and B be arbitrary sets.

Then there exists

- i) an onto function $f: A \rightarrow B$ iff $|A| \geq |B|$
- ii) a one-to-one function \dots iff $|A| \leq |B|$
- iii) a bijection \dots iff $|A| = |B|$

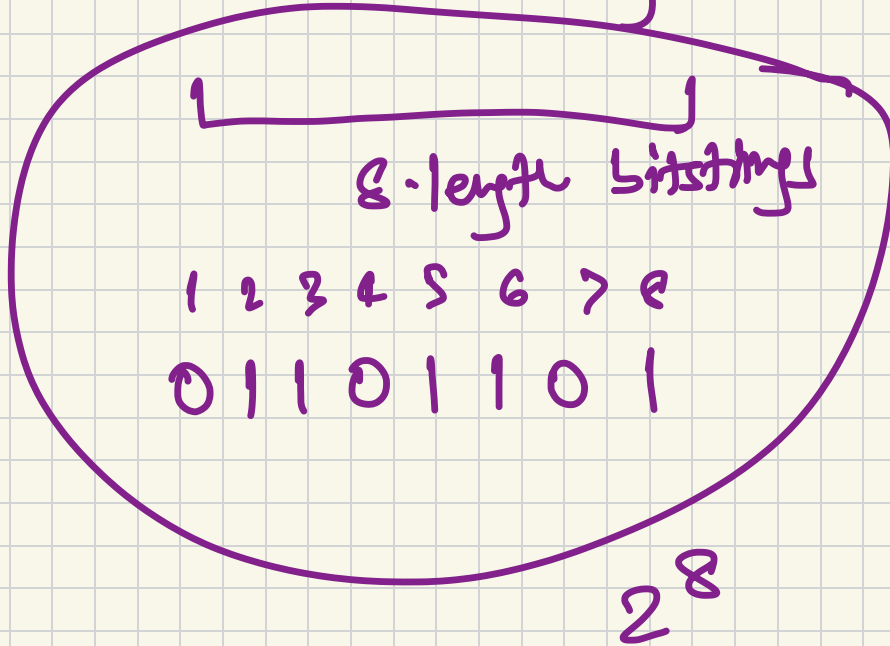
Example

What is

$$|\mathcal{P}\{1, \dots, 8\}|$$

bijection

between



and this powerset

$$\{2, 3, 5, 6, 8\}$$

Claims

for any finite set X

$$|P(X)| = 2^{|X|}$$

① Permutation

(seen earlier in
inclusion-exclusion
rule.)

How many different ways can you arrange
the letters in the following
words?

i)

PERL

PE₁ER

ii)

PE₁ER₂

PE₁ER₂

iii)

SMALLTALK