ACOL 202

Lecture 10

(14th Feb)

In case of prop. logic, it was straightforward to give a meaning to the logical formulas.

Propositions had a truth value and then there were truth tables that could give interpretation to the logical symbols $\wedge, \vee, \neg, \rightarrow$

$LT(x, y)$

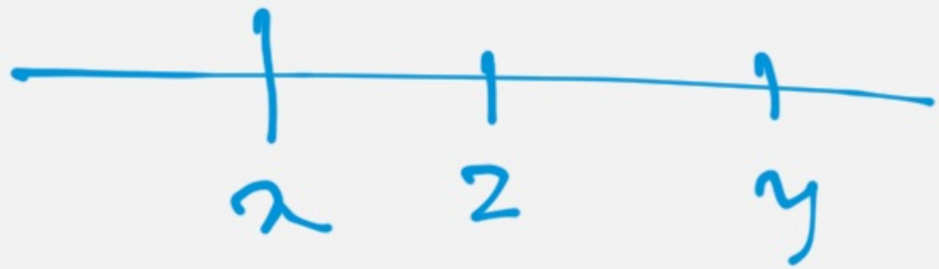
$\boxed{x < y}$

Closed formulas / sentences

Formulas with no free variables

$$\forall x \forall y \forall z \left(R(x,y) \wedge R(y,z) \rightarrow R(x,z) \right)$$

$$\forall x \forall y \left(R(x,y) \to \exists z \left(\neg(z=x) \land \neg(z=y) \land R(x,z) \land R(z,y) \right) \right)$$



over real numbers.

$R(x,y)$ $\quad x \le y$

$$\boxed{\forall x \; P(x)}$$

$$\neg \left(\exists x \; \neg P(x) \right)$$

$$\neg \exists x \; P(x) \equiv \forall x \; \neg P(x)$$

$$\neg \left(\forall x \; P(x) \right) \equiv \exists x \; \neg P(x)$$

$$\exists x \; P(x) \equiv \neg \forall x \; \neg P(x)$$

Order of quantification matters.

$W(x, y)$ x has watched the movie y.

$W(\text{Ayachi}, \text{Indian 2})$

$\forall x \, \exists y \; W(x, y)$

$\exists y \, \forall x \; W(x, y)$

Proofs

Claim Let n be a positive integer and let k be an integer satisfying

$$2 \leq k \leq n.$$

Then, $(n!+1)$ is not evenly divisible by k.

Proof. k evenly divides $n!$

The next integer that k evenly divides is $(n!+k)$.

But $n! < n!+1 < n!+k$

(Why? $k \geq 2$)

Therefore, $(n!+1)$ is not evenly divisible by k.

Proved.

→ Direct proof

A positive integer is divisible by 4 iff the last two digits is divisible by 4.

$d_0 d_1 d_2 \ldots$ (reverse order)

$$n = d_0 \times 1 + d_1 \times 10 + d_2 \times 100 + d_3 \times 1000$$

$$\frac{n}{4} = \frac{d_0}{4} + \frac{10 d_1}{4} + \ldots$$

Claim The product of two rational numbers is a rational number.

Proof by contradiction

$\sqrt{2}$ is not rational.

Suppose not. $\sqrt{2} = p/q$ (in lowest terms)

$p^2 = 2q^2$

$\therefore$ p^2 is divisible by

$\therefore$ p is even. $\therefore$ $p^2/2$ is even. $\therefore$ q is even.

Prove the contrapositive

$$p \to q$$

$$\neg q \to \neg p$$

Claim If $|x|+|y| \neq |x+y|$ then $xy < 0$.

(Proof by case-splitting)

Claim

Let $x \in \mathbb{R}$.

Then $-|x| \leq x \leq |x|$

Case I: $x \geq 0$

Case II: $x < 0$

Case I : $x \geq 0$

$$-x \leq 0 \leq x$$

$$-|x| = -x \leq 0 \leq x = |x|$$

Case II : $x \leq 0 \leq -x$

$$-|x| = x \leq 0 \leq -x = |x|$$