Quiz 4 (*Nov 11, Marks: 6, Duration: 30 mins*) **COL750, Jul-Dec 2024**

Name: **Entry No.:**

1. **[3 marks]** Consider the following clauses:

   `A1.` ($y = 0 \vee x = 7$)
   `A2.` ($y \neq 0 \vee x \geq 7$)
   `A3.` ($x \neq 7$)
   `B1.` ($x = 7 \vee x < 7$)
   `B2.` ($x \neq 7 \vee x > 10$)
   `B3.` ($x \leq 10$)

   Let A be the formula given by the set of clauses {`A1, A2, A3`} and let B be the formula given by the set {`B1, B2, B3`}. If A and B are inconsistent, use the resolution proof to compute an interpolant for them. If not, argue that they are not inconsistent.

   Note that the literals in the formula above are not propositional variables. But we can treat them as propositional variables. For instance, you could treat ($x \neq 7$) as p, and thus ($x = 7$) becomes $\neg p$.

2. **[3 marks]** Consider how the IC3 algorithm works. Suppose we have found frames $F_0, F_1, \ldots, F_k$ such that they satisfy the following properties:

   (a) $I \to F_0$ (F_0 contains the initial set of states I)
   (b) $F_i \to F_{i+1}$ $(0 \leq i < k)$ (frames are monotonic)
   (c) $F_i \to P$ $(0 \leq i \leq k)$ (none of the frames contain a bad, i.e. $\neg P$, state)
   (d) $F_i \wedge T \to F'_{i+1}$ $(0 \leq i < k)$ (F_i over-approximates i-step reachability)

   At this point, it is checked whether $F_k \wedge T \to P'$? Suppose this does not turn out to be the case (i.e., suppose that this implication fails). Then there must exist an F_k state s that is one transition away from violating P. Now, if P is an invariant, then $\neg s$ must be inductive relative to some F_i. We may wonder what is the maximum i, $(0 \leq i \leq k)$, such that $\neg s$ is guaranteed to be inductive relative to F_i?

   Claim: $\neg s$ is inductive relative to F_{k-2}, if not a later frame.

   Write `Yes/No` to indicate whether this claim is correct or not. And give an argument to explain your answer.