ACOL 202 Lecture 4 (24th Jan)

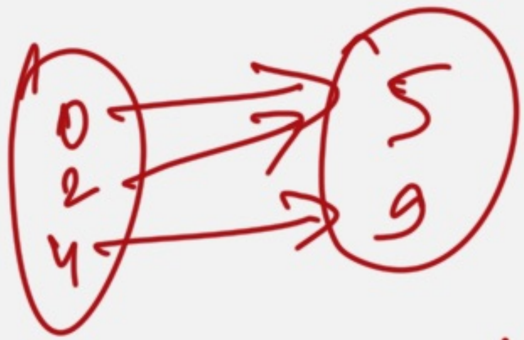
Onto and one-to-one functions

A function $f: A \to B$ is called onto (surjective) if f's codomain equals f's range.

i.e., for every $b \in B$ there exists at least one $a \in A$ such that $f(a) = b$

$A = \{0, 2, 4\}$

$B = \{5, 9\}$



Onto function $h : B \to A$

One to one functions

injective function

A function $f: A \to B$ is called one-to-one if

$$f(a_1) = f(a_2) \text{ implies that } a_1 = a_2.$$

A function that is both onto (surjective) and one-to-one (injective) is called a bijective function.

not $\{True, False\} \rightarrow \{True, False\}$

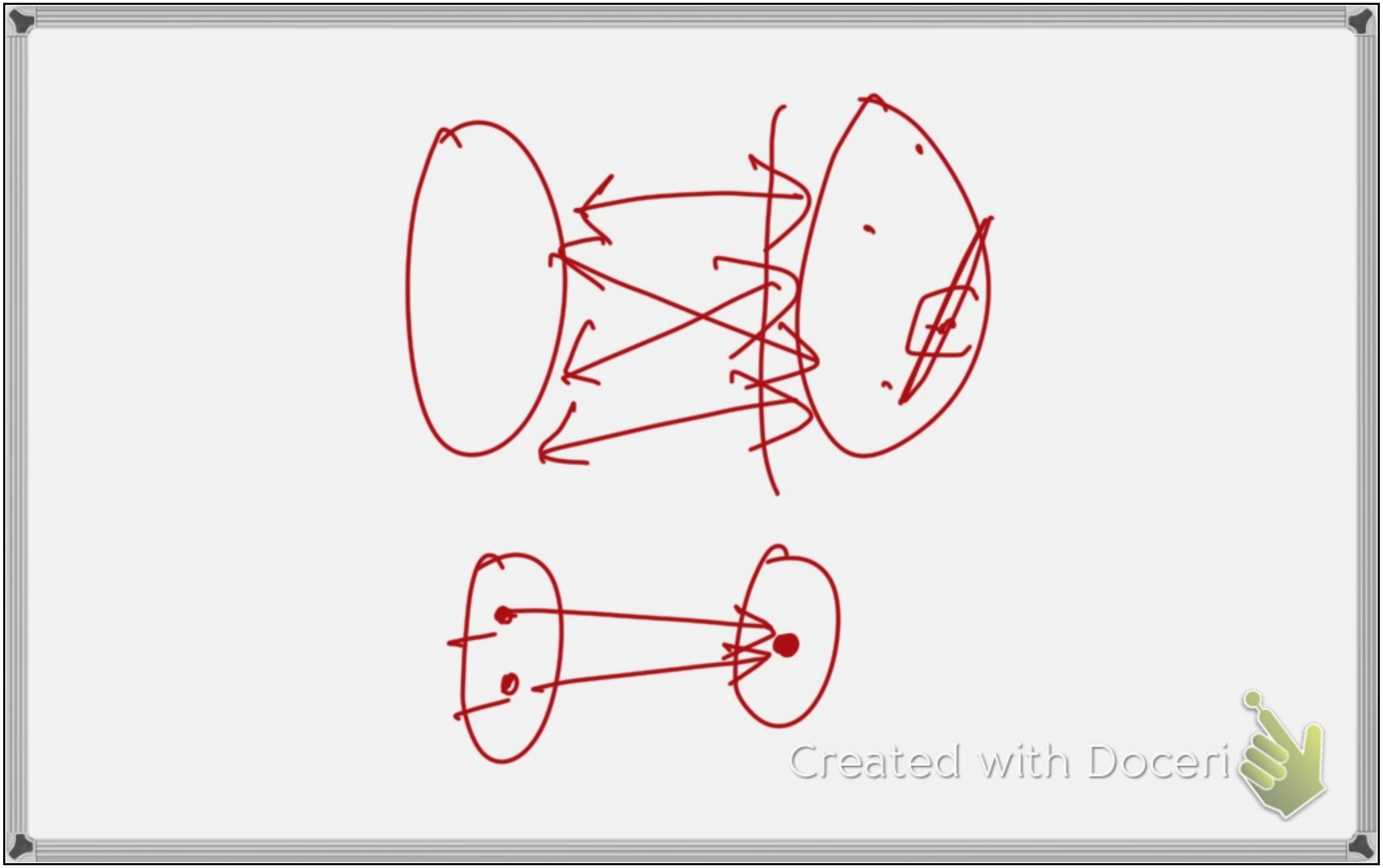
negation

$$\left.\begin{aligned} not(True) &= False \\ not(False) &= True \end{aligned}\right\}$$

Inverse of a function

Let $f: A \to B$ be a bijective function.

$f^{-1}: B \to A$ is called the inverse of f where whenever $f^{-1}(b) = a$ $f(a) = b$.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$f : \mathbb{R} \to \mathbb{R}$

$f(x) = \frac{x}{2}$ $f^{-1}(y) = 2y$

not^{-1} is same as the not function.

$\underline{f^{-1} \circ f}$ $\underline{\text{identity}}$

Polynomials

$f : \mathbb{R} \to \mathbb{R}$

$f(x) = a_0 + a_1 x + a_2 x^2 + \dots + a_k x^k$

where each $a_i \in \mathbb{R}$

$a_k \neq 0$

$k \in \mathbb{Z}^{\geq 0}$

Degree of a polynomial is the largest power of x with a non-zero coefficient.

Roots $\{ x \in \mathbb{R} \mid f(x) = 0 \}$

Logic

valid reasoning

↓

precise, formal language to express our statements/arguments to be able to reason formally.

Making valid deductions (conclusions) using a formal language

Example

If the train arrives late and there are no taxis at the station then John will be late for his meeting.

John was not late for his meeting.

The train did arrive late.

From these facts, one can conclude that there were taxis at the station.

Another example

p not q

If it is raining and Mark is not carrying his umbrella then he will get wet.

r

Mark is not wet. Not r

It is raining. p

Therefore, Mark is carrying his umbrella with him.

q

Both these sentences seem to have a common structure

If p and not q then r.

$\wedge$ $\neg$

Not r.	p	Therefore, q.

Propositional logic

→ proposition (declarative sentence)

→ The sum of 3 and 5 numbers is 8.

T/F → All student registered for ACOL 202 are present in the class today.

We won't consider non-declarative sentences, e.g "Could you please pass me the pen?"

In particular, we consider atomic propositions/sentences

↓ assigned some symbols

↓ be connected with logical connectives to form more complex statements.

Example

p : The number 5 is even.

q : The number 5 is prime

r : The number 5 is bigger than the number 2.

$\neg p$ The number 5 is not even.

$q \wedge r$ The number 5 is prime and the number 5 is bigger than the number 2.

$(q \wedge r) \rightarrow \neg p$

If the number 5 is prime and it is ~~greater~~ bigger than 2 then it is not even.

$\neg$	not
$\wedge$	and
$\vee$	or
$\rightarrow$	implies

Binding priorities

$\neg$ binds more tightly than $\wedge$ and $\vee$

$\wedge$ and $\vee$ bind more tightly than $\to$

$\to$ is right associative

$p \to q \to r$ denotes $p \to (q \to r)$

$\boxed{(p \wedge q) \to (\neg r \vee q)}$

$\boxed{((p \wedge q) \to (\neg(r) \vee q))}$

$\boxed{p \wedge q \to \neg r \vee q}$

Propositional formulas

strings over propositional atoms, logical symbols, and left- and right- brackets.

but not everything is allowed, of course.

$(\wedge \vee)( ))(\neg\, pqr$ does not make any sense.

We would like our formulas to be "well-formed".

Well-formed formulas (wff)

- → propositional atoms are wffs
- → If α is a wff $(\neg\alpha)$ is a wff.
- → If α and β are wffs $(\alpha \wedge \beta)$, $(\alpha \vee \beta)$, $(\alpha \to \beta)$ are wffs.
- → nothing else is a wff.

$$\underbrace{(p \wedge q)}_{\alpha} \rightarrow \underbrace{((\neg r) \vee q)}_{\beta}$$

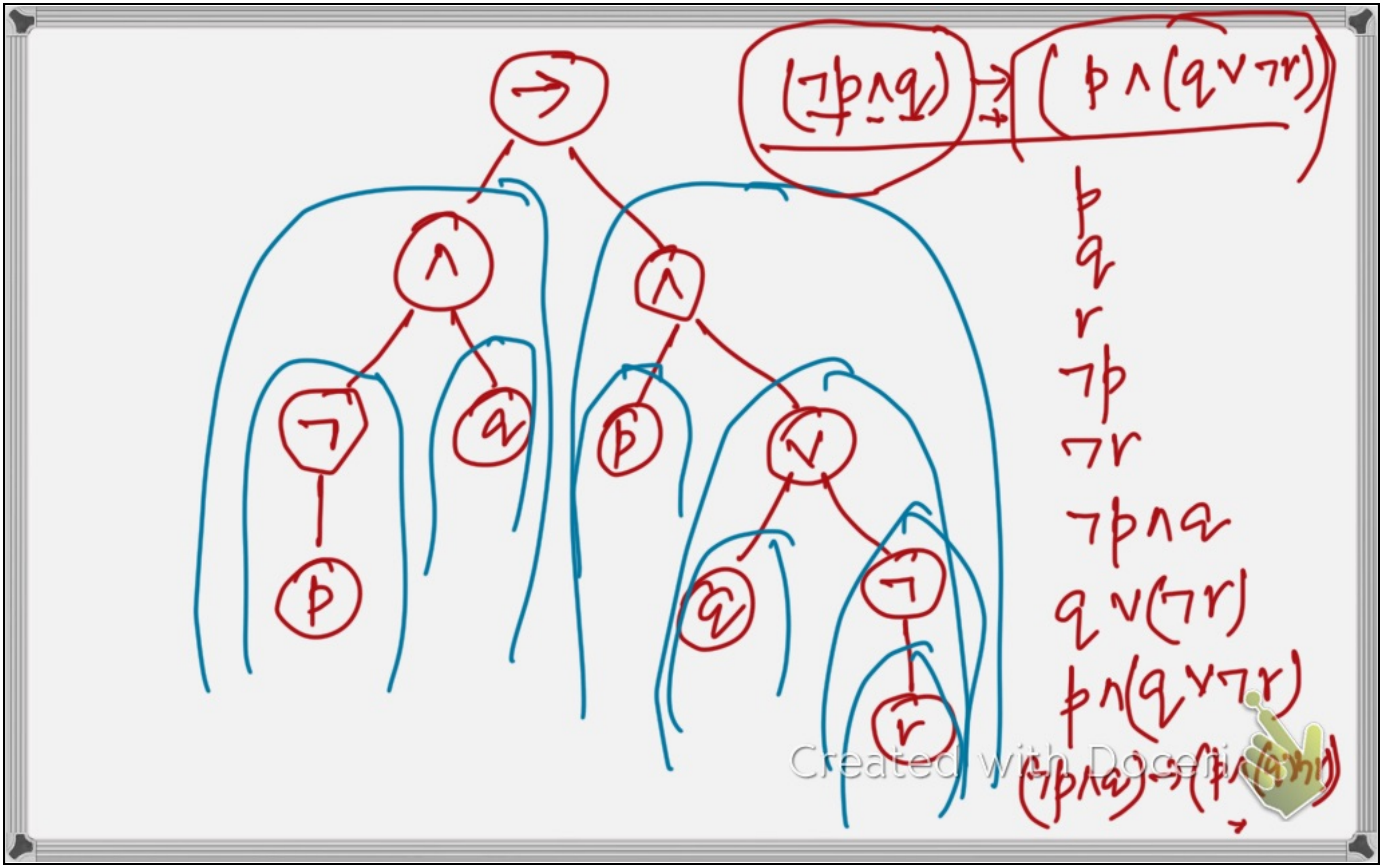
$\alpha_1 \wedge \alpha_2$

p ✓ $\qquad q$ ✓

$\beta_1 \qquad \beta_2$

$\neg r \qquad q$ ✓

$\neg (r)$


(¬p∧q) → (p∧(q∨¬r))
p
q
r
¬p
¬r
¬p∧q
q∨(¬r)
p∧(q∨¬r)
(¬p∧q)→(p∧(q∨¬r))
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$\boxed{p \wedge q}$

Look at the meaning of the logical symbol

$\boxed{\text{Truth tables}}$ ←

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