1. Use the predicates

   $R(x, y)$: *x respects y*
   $A(x, y)$: *x attended y*
   $P(x)$: *x is a professor*
   $S(x)$: *x is a student*
   $L(x)$: *x is a lecture*

   and the nullary function symbol (i.e. a constant)

   m: Mary

   to translate the following into predicate logic.

   (a) Mary respects every professor.
   (b) Some professor respects Mary.
   (c) Mary respects herself.
   (d) No students attended every lecture.
   (e) No lecture was attended by every student.
   (f) No lecture was attended by any student.

2. Come up with an appropriate set of predicates, and write down their meanings, and use the predicates to translate the following into predicate logic.

   (a) All red things are in the box.
   (b) Only red things are in the box.
   (c) No animal is both a cat and a dog.
   (d) Every prize was won by a girl.
   (e) A girl won every prize.

3. **[2 marks]** Use the truth table given below to express **F** in conjunctive normal form.

p	q	r	**F**
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

4. Use the truth table given in the previous question to express **F** in disjunctive normal form.

5. Consider the predicate logic formula $\exists x\ (P(y, z) \wedge (\forall y\ (\neg Q(y, x) \vee P(y, z))))$ where P and Q are predicate symbols with two arguments.

(a) Draw the parse tree of the formula.

(b) Identify all bound and free variable leaves in the formula.

(c) Is there a variable in the formula which has both free and bound occurrences?

6. Let P be a predicate symbol with arity 3. Draw the parse tree of $\neg(\forall x\ ((\exists y\ P(x,y,z))\wedge(\forall z\ P(x,y,z))))$. Also indicate the free and bound variables in the parse tree.

7. Express the following by formulas of first-order logic, using predicate $H(x)$ for x *is happy*, $R(x)$ for x *is rich*, $G(x)$ for x *is a graduate*, and $C(x,y)$ for y *is a child of* x.

   (a) Any person is happy if all their children are rich.

   (b) All graduates are rich.

   (c) Someone is a graduate if they are a child of a graduate.

   (d) All graduates are happy.

8. **[2 marks]** Write down the following as sentences of predicate logic. Use $B(x)$ for x *is a barber* and $S(x,y)$ for x *shaves* y.

   (a) Every barber shaves all persons who do not shave themselves.

   (b) No barber shaves any person who shaves himself.