Quiz 6 (*Oct 20, Marks: 3, Duration: 40 mins*) **COL703, Aug-Nov 2022**

Name: **Entry No.:**

1. **[0.5 marks]** Prove the validity of $S \to \forall x\, Q(x) \vdash \forall x\, (S \to Q(x))$, using natural deduction, where S is a nullary predicate (essentially, a propositional atom).

2. **[0.5 marks]** Prove the validity of $P(b) \vdash \forall x\, (x = b \to P(x))$, using natural deduction.

3. **[0.5 marks]** Consider the following predicate-logic sentences.

   ϕ_1: $\forall x\, P(x, x)$
   ϕ_2: $\forall x \forall y\, (P(x, y) \to P(y, x))$
   ϕ_3: $\forall x \forall y \forall z\, (P(x, y) \wedge P(y, z) \to P(x, z))$

   These sentences express that P is reflexive, symmetric, and transitive.

   Show that transitivity is not semantically entailed by the other two properties. In other words, give a model (an assignment) that satisfies ϕ_1 and ϕ_2, but does not satisfy ϕ_3.

4. Consider a predicate logic formula $\phi := \psi_1 \wedge \psi_2 \wedge \psi_3$, where

   ψ_1: $\forall x \exists y\, R(x, y)$
   ψ_2: $\forall x\, \neg R(x, x)$
   ψ_3: $\forall x \forall y \forall z\, (R(x, y) \wedge R(y, z) \to R(x, z))$

   - **[0.5 marks]** Is ϕ satisfiable? Justify your answer.
   - **[1 marks]** Can ϕ have a finite model (i.e., an assignment where the universe has only finitely many elements)? Give such a finite model, or argue otherwise.