1. Let $\mathcal{P}(S)$ denote the power set of the set S. Write down the set $\mathcal{P}(\mathcal{P}(\{1\}))$.
2. Define $S = \{1, 2, ..., 100\}$. Let $W = \{x \in S \mid x \bmod 2 = 0\}$, $H = \{x \in S \mid x \bmod 3 = 0\}$, and $O = S - H - W$. Is $\{W, H, O\}$ a partition of S? Justify your answer.
3. **[2 marks]** Let us define the *similarity* score of two sets A and B to be the value of $|A \cap B|$. Now suppose we have a collection of sets $A_1, A_2, \ldots, A_n$. Consider the following claim.

   Claim: Suppose that the set A_v is the most similar set to the set A_u in this collection (apart from A_u itself). Then A_u is necessarily the set that is most similar to A_v (apart from A_v itself).

   If the claim is true, argue why it must be true. Otherwise, give an example to demonstrate that the claim does not hold.
4. Why doesn't the function $f : \{0, \ldots, 23\} \to \{0, \ldots, 11\}$ where $f(n) = n \bmod 12$ have an inverse?
5. Consider the function $f : \{0, 1, \ldots, 7\} \to \{0, 1, \ldots, 7\}$ defined by $f(x) = (x^2 + 3) \bmod 8$. What is the value of $f(3)$ and $f(7)$?
6. Let us define $f : \{0, 1, 2, 3\} \to \{0, 1, 2, 3\}$ as $f(x) = x^2 \bmod 4$. Is f onto?
7. **[2 marks]** Let $f(x) = 3x + 1$ and let $g(x) = 2x$. Identify a function h such that $g \circ h$ and f are identical.
8. Consider the function $f(x) = (x^3 + 2x) \bmod 8$. If we think of this as a function $f : \{0, 1, 2, 3\} \to \{0, 1, \ldots, 7\}$, would f be one-to-one?
9. Consider $h : \mathbb{R}^{\geq 0} \to \mathbb{R}^{\geq 1}$, where $h(x) = 3^x$. What is the inverse of h?
10. If $p(x)$ and $q(x)$ are both polynomials with degree 7, what is the smallest and largest possible degree of $f(x) = p(x) + q(x)$?
11. If $p(x)$ and $q(x)$ are both polynomials with degree 7, what is the smallest and largest possible degree of $f(x) = p(x) \cdot q(x)$?