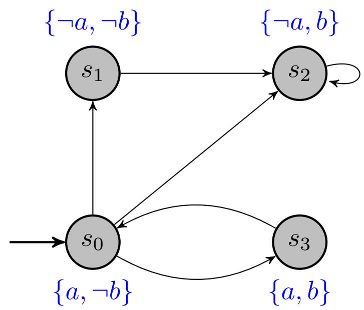
1. **[1.25 marks]** Consider the model shown in the figure below.



For each of the formulas ϕ shown below, use NuSMV to (i) determine whether the formula ϕ is valid, and (ii) persuade NuSMV to exhibit some path, wherever possible, some path which satisfies ϕ.

   (a) `G a`
   (b) `a U b`
   (c) `a U X (a ∧ ¬b)`
   (d) `X ¬b ∧ G (¬a ∨ ¬b)`
   (e) `X (a ∧ b) ∧ F (¬a ∧ ¬b)`

2. **[2 marks]** Which of the following pairs of CTL formulas are equivalent? For those which are, argue briefly why they are equivalent. For those which are not, create a NuSMV file with a model and the two formulas, each as a property to check, such that one property is true of the model and the other is false.

   (a) EF ϕ and EG ϕ
   (b) EF ϕ $\vee$ EF ψ and EF $(\phi \vee \psi)$
   (c) AF ϕ $\vee$ AF ψ and AF $(\phi \vee \psi)$
   (d) AF $\neg\phi$ and $\neg$EG ϕ
   (e) EF $\neg\phi$ and $\neg$AF ϕ
   (f) A[ϕ_1 U A[ϕ_2 U ϕ_3]] and A[A[ϕ_1 U ϕ_2] U ϕ_3]
   (g) $\top$ and AG ϕ $\rightarrow$ EG ϕ
   (h) $\top$ and EG ϕ $\rightarrow$ AG ϕ

3. **[0.5 marks]** Assume $\Sigma = \{0, 1\}$. Let L be the language of ω-words over Σ that **do not** contain 01.

   - Give an ω-regular expression for L.
   - Give an NBA (Non-deterministic Büchi Automata) for L.

4. **[0.25 marks]** Suppose U is the regular language $a(a + b)^*a$. What is the NBA for U^ω?

5. **[2 marks]** The **F** operator in LTL is used to say that a property is true sometime in the *future*. Let us now introduce the **O** operator (short form for *Once*) to say that property was true sometime in the *past*.

The formal semantics of **O** can be defined as follows. For an ω-word α, let α^i denote the suffix of α starting from the i^{th} position. Then:

$\alpha^i \models \mathbf{O}\phi$ if $\exists\, j \leq i$ such that $\alpha^i \models \phi$, and $\alpha \models \mathbf{O}\phi$ if $\alpha^0 \models \mathbf{O}\phi$

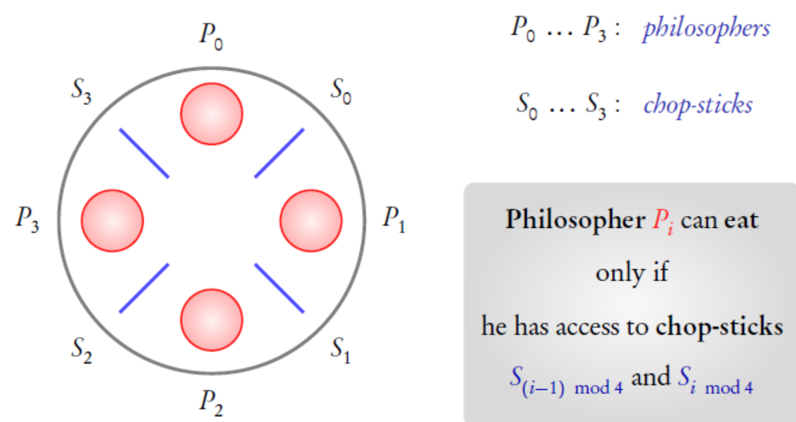
Let p_1 and p_2 be atomic propositions. Take the alphabet $\mathbb{B}^2 = \{\binom{0}{0}, \binom{0}{1}, \binom{1}{0}, \binom{1}{1}\}$ where the top element indicates the value for p_1 and the bottom one indicates the value of p_2. Let $\psi := \mathbf{G}(p_1 \rightarrow \mathbf{O}p_2)$.

(a) Give two examples of ω-words over $\mathbb{B}^2$: one which satisfies ψ and one which does not satisfy ψ.

(b) Show that ψ can be rewritten into an equivalent LTL formula which uses only the standard Until operator **U** and the boolean connectives $(\neg, \wedge, \vee, \rightarrow)$.

(c) Construct a Non-deterministic Büchi Automata recognizing the language of ψ.

6. **[2 marks]** Consider the Dining Philosopher's problem. A table holds four chop-sticks and four bowls of noodles (arranged as pictured).



Four philosophers sit around the table. A philosopher shares his right chop-stick with his right neighbor and his left chop-stick with his left neighbor. Each philosopher cycles through three states: thinking, hungry and eating (in that order). After thinking for a while, a philosopher gets hungry. In order to eat, he needs to hold his left and right chop-sticks. A philosopher can only pick one chop-stick at a time. When he is done eating, he releases the chop-sticks and goes back to thinking. Since the philosophers are sharing chop-sticks, they need to find a method for them not to starve to death.

Your task is to implement a protocol in NuSMV which satisfies the property that every philosopher gets to eat infinitely often.

7. **[1 marks]** Let $\mathcal{A} = (Q, \{q_0\}, \Sigma, \delta, F)$ be an NFA for a language U. Consider the following method of constructing a Non-deterministic Büchi Automata $\mathcal{B}$ for the language U^ω.

- $\mathcal{B}$ has the same set of states Q, and the initial state is q_0.
- All transitions in $\mathcal{A}$ are present in $\mathcal{B}$.
- In addition, for each transition $q \xrightarrow{a} q_F$ in $\mathcal{A}$ with $q_F \in F$, add a transition $q \xrightarrow{a} q_0$ in $\mathcal{B}$.
- Make q_0 as the only final (good) state in $\mathcal{B}$.

Note that the above procedure does not add a new state.

Will the above procedure result in an NBA for U^ω. If yes, give a proof. If not, give a counterexample.

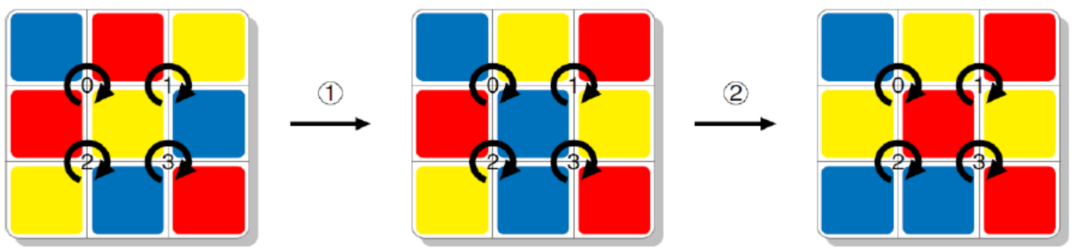
8. **[0.75 marks]** Let $\Sigma = \{a, b, c\}$. Construct a Büchi automata (deterministic or non-deterministic) for the following languages.

   (a) set of all ω-words where abc occurs at least once

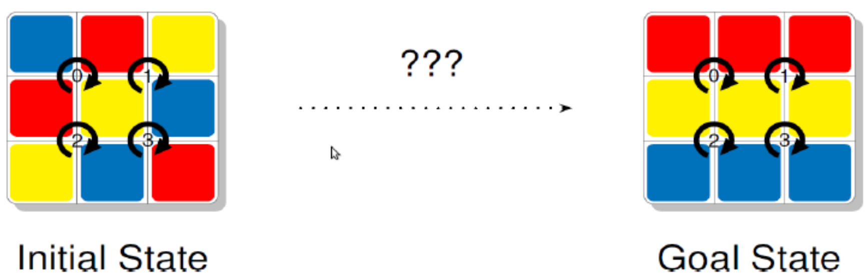
   (b) set of all ω-words where abc occurs infinitely often

   (c) set of all ω-words where abc occurs finitely often

9. **[1.25 marks]** Consider the rotation of tiles as shown in the picture below.



Model this in NuSMV to obtain the sequence of rotations that may take us from the initial state to the goal state as given below.



10. **[1 marks]** Let $\Sigma = \{a, b\}$. Define $L_{b \geq a} := \{\alpha \in \Sigma^\omega |$ in every finite prefix of α, the number of b's is $\geq$ the number of a's$\}$.

   (a) Give an example of an ω-word present in $L_{b \geq a}$.

   (b) Give an example of an ω-word which is not in $L_{b \geq a}$.

   (c) Is $L_{b \geq a}$ ω-regular? Justify.