
Name:

Entry No.:

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1. [1 marks] Prove, using natural deduction, that the sequent $(\exists x \phi) \vee (\exists x \psi) \vdash \exists x (\phi \vee \psi)$ is valid.
 2. [0.5 marks] Write down a formula in first order logic with equality that captures the requirement that a unary predicate P holds for exactly three different elements.
 3. [0.5 marks] Write down the formula that you will get from the following substitution.

$$(\forall x P(x, y) \wedge Q(x))[t/x]$$

4. Consider the following structures over a signature with a single binary relation symbol R .

$$\begin{aligned} U_{\mathcal{A}} &= \mathbb{N} \text{ and } R_{\mathcal{A}} = \{(n, m) \in \mathbb{N} \times \mathbb{N} \mid \text{such that } n < m\} \\ U_{\mathcal{B}} &= \mathbb{Z} \text{ and } R_{\mathcal{B}} = \{(n, m) \in \mathbb{Z} \times \mathbb{Z} \mid \text{such that } n < m\} \\ U_{\mathcal{C}} &= \mathbb{Q} \text{ and } R_{\mathcal{C}} = \{(n, m) \in \mathbb{Q} \times \mathbb{Q} \mid \text{such that } n < m\} \end{aligned}$$

- [0.5 marks] Give a predicate logic sentence (i.e., formula with no free variables) that is satisfied by \mathcal{B} but not by \mathcal{A} .
- [0.5 marks] Give a predicate logic sentence (i.e., formula with no free variables) that is satisfied by \mathcal{C} but not by \mathcal{B} .