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Name:

Entry No.:

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Please note that you are not allowed to carry with you any notes, cheat-sheet, or any electronic devices including your phone and smart-watches. Also note that there will be zero tolerance for dishonest means like copying solutions from others, and even letting others copy your solution. If you are found indulging in such an activity, your exam paper will be seized immediately and you will be given a zero without any evaluation.

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1. Recall that if we have clauses  $(p \vee q)$  and  $(r \vee \neg q)$ , then we can apply resolution to get the clause  $(p \vee r)$ . Does this mean that the formula  $(p \vee q) \wedge (r \vee \neg q)$  is equivalent to  $(p \vee r)$ ? In particular:
  - (a) **[2 marks]** Is  $(p \vee q) \wedge (r \vee \neg q) \vdash (p \vee r)$  valid? If so, give a natural deduction proof for this. If not, justify your answer using a suitable valuation.
  - (b) **[2 marks]** Is  $(p \vee r) \vdash (p \vee q) \wedge (r \vee \neg q)$  valid? If so, give a natural deduction proof for this. If not, justify your answer using a suitable valuation.
  - (c) **[2 marks]** Are the two formulas  $((p \vee q) \wedge (r \vee \neg q))$ , and  $(p \vee r)$  equisatisfiable, i.e. one is satisfiable *iff* the other one is satisfiable? Why or why not?
2. **[4 marks]** Let us define a new notion of consistency: call  $X$  consistent if there is no formula  $\alpha$  such that  $X \vdash \alpha$  and  $X \vdash \neg\alpha$ . Prove that  $X$  is consistent *iff* every finite subset of  $X$  is consistent.
3. Recall, from our discussion in the class, that Horn-SAT is in P. Consider the following variants of the Horn satisfiability problem.
  - (a) **[1.5 marks]** *Dual-Horn SAT*: a dual variant of Horn SAT in which each clause has at most one negative literal. Argue that Dual-Horn SAT is in P.
  - (b) **[3.5 marks]** *Double-Horn-SAT*: the problem of deciding whether a given Horn formula has at least two models. Argue that Double-Horn-SAT is also in P.