1. **[1 mark]** Is the following a tautology? Justify your answer.
   $p \rightarrow (q \rightarrow p)$

2. **[1 mark]** Are $\neg(p \leftrightarrow q)$ and $\neg p \leftrightarrow q$ logically equivalent? Justify your answer.

3. **[1 mark]** Prove that implication is not associative by giving a truth assignment in which $(p \rightarrow (q \rightarrow r))$ and $((p \rightarrow q) \rightarrow r)$ have different truth values.

4. **[1.5 mark]** Consider the following popular puzzle. When asked for the ages of her three children, Mrs. Baker says that Alice is her youngest child if Bill is not her youngest child, and that Alice is not her youngest child if Carl is not her youngest child. Encode these facts, and the necessary background knowledge that only one of the three children can be her youngest child, into propositional logic formulas. Use propositions $a$, $b$ and $c$ to denote that Mrs. Baker's youngest child is Alice, Bill and Carl, respectively. Find out, with the help of truth tables, who the youngest child is.

5. **[1.5 mark]** Let $p$ and $q$ be atomic propositions, and $\alpha$ and $\beta$ be propositional logic formulas on $p$ and $q$. Consider the following definitions for $\alpha$ and $\beta$:

   - $\alpha = (p \rightarrow \neg\beta)$
   - $\beta = (q \rightarrow \neg\alpha)$

   Show that there are exactly two pairs of propositional logic formulas $(\alpha, \beta)$ which satisfy the above definitions. Note that logically equivalent formulas should not be considered as different formulas while solving this problem.