ACOL 215 (15/09/2025)

Logic Gates → binary logic

↳ circuits

understand or find out when one circuit is equivalent to another one

~~simplifying them~~

↳ Boolean Algebra

→ a set of element –

→ a set of operators –

→ a set of axioms / postulates –

$\underbrace{\text{axioms}}$ → theorem

Example of some common postulates

1. Closure A sets S is closed wrt a binary operator "op" if for every pair of element of S,

"op" specifies how we can obtain a unique element of S.

Example

$\mathbb{N}$ is closed under $+$

$\mathbb{N}$ is not closed under $-$

Associative law

op is associative whenever

$$x \text{ op } (y \text{ op } z) = (x \text{ op } y) \text{ op } z$$

$$x * (y * z) = (x * y) * z$$

$\forall x, y, z \in S$

Commutative law

$$x * y = y * x$$

$\forall x, y \in S$

Identity element

A set S is said to have an identity element wrt $*$ (a binary operator) if there exists $e \in S$ such that

$$e * x = x * e = x$$

for every $x \in S$.

Example

Set of integers
binary operator $+$
identity element is 0
$x + 0 = 0 + x = x$.

Inverse A set S having an identity element e wrt a binary operator $*$ is said to have an inverse whenever for every $a \in S$ $\exists\, y \in S$ such that

$$a * y = e.$$

Example Integers $+$ 0

a $-a$

Distributive law

If $*$ and $+$ are two binary operator on S, $*$ is said to distribute over $+$ whenever

$$x * (y+z) = (x*y) + (x*z)$$

Example

Field is an algebraic structure.

$\underbrace{\mathbb{R}, +, \cdot}$ → forms the field of real numbers

$+$ addition

$\cdot$ multiplication

0 additive identity

1 multiplicative "

$-a$ additive inverse

$1/a$ if $a \neq 0$ mult. "

distributive law

$\cdot$ distributes over $+$

$a \cdot (b+c) = a \cdot b + a \cdot c$

$a + b \cdot c \neq (a+b) \cdot (a+c)$

Boolean algebra is an algebraic structure defined by a set of elements B, together with binary operators $\cdot$ and $+$ provided that the following postulates hold:

1. Closure wrt $\cdot$ — $+$

2. 0 is identity wrt $+$

$$0+x = x+0 = x$$

1 is identity wrt $\cdot$

$$1\cdot x = x\cdot 1 = x$$

3. Commutative wrt $+$ and $\cdot$

$$x + y = y + x$$

$$x \cdot y = y \cdot x$$ ✓

4. $+$ distributes over $\cdot$

$$x + (y \cdot z) = (x + y) \cdot (x + z)$$

$\cdot$ distributes over $+$

$$x \cdot (y + 2) = x \cdot y + x \cdot 2$$

5. For every $x \in B$, there is an $x' \in B$ such that (complement of x)

$$x + x' = 1$$

$$x \cdot x' = 0$$

6. There exist at least two elements x and $y \in B$ such that $x \neq y$.

Note that the associative law is not a part of Boolean algebra postulates.

But it holds.

We are going to be looking at two-valued Boolean algebra

$B = \{0, 1\}$

operator $\cdot$, $+$

x	y	$x+y$	$x \cdot y$
0	0	0	0
0	1	1	0
1	0	1	0
1	1	1	1
		OR	AND

complement

x	x'
0	1
1	0

NOT

x	y	z	$y+2$	$x\cdot(y+2)$	$x\cdot y$	$x\cdot 2$	$(x\cdot y)+(x\cdot 2)$

Two-valued Boolean Algebra

Boolean Algebra

Switching Algebra

Theorems and properties of Boolean algebra

identity	a)	$\boxed{x+0=x}$	b)	$x \cdot 1 = x$
Complement	a)	$\boxed{x+x'=1}$	b)	$x \cdot x' = 0$
Commutative	a)	$x+y=y+x$	b)	$x \cdot y = y \cdot x$
associativity	a)	$x+(y+z)=(x+y)+z$	b)	$x(yz)=(xy)z$
distributive	a)	$x(y+z)=xy+yz$	b)	$\boxed{x+yz=(x+y)(x+z)}$

Dual

Theorem

1. a) $\boxed{x + x = x}$ b) $x \cdot x = x$

2. a) $x + 1 = 1$ b) $x \cdot 0 = 0$

3. (Involution) $(x')' = x$

4. (DeMorgan) $(x+y)' = x'y'$ $(xy)' = x' + y'$

5. (Absorption) $x + xy = x$ $x(x+y) = x$

Proof 1 a) $x + x = x$

$$x + x = (x+x)\cdot 1 \quad \text{identity b}$$
$$= (x+x)\cdot(x+x') \quad \text{complement a}$$
$$= x + xx' \quad \text{distributive}$$
$$= x + 0$$
$$= x$$

$\vdots$

Proved

Exercise

Complete the other proofs