ACOL 215

(03 Sept. 2025)

Number systems

→ Binary, Octal, Decimal, Hexadecimal

$$(B65F)_{16} = (1011011001011111)_2$$

$$(110101)_2 = 32 + 16 + 4 + 1$$

$2^5 \quad 2^4 \quad 2^2 \quad 2^0$

$$= (53)_{10}$$

Electrical signals → bits → 0, 1

Groups of bits → represent information

1 byte = 8 bits

can encode one keyboard character

$(FF)_{16}$

0 . . . 255 in decimal

00 to FF in hexadecimal

$= f \times 16^1 + f \times 16^0 = 15 \times 16 + 15$

$= 255$

Computer memory capacity is usually given in bytes.

(Kilo byte)	KB	2^{10} bytes
(Mega byte)	MB	2^{20} bytes
(Giga byte)	GB	2^{30} bytes
(Tera byte)	TB	2^{40} bytes

Arithmetic operations

Same rules ; but only the r-allowable digits should be used while operating in base r.

$$
\begin{array}{r}
1111 \\
1011101 \\
+\ 100111 \\
\hline
1010100
\end{array}
\qquad\qquad
\begin{array}{r}
101101 \\
-\ 100111 \\
\hline
000110
\end{array}
$$

Number-base conversions

(base-r to decimal)

equivalent

$(0011)_8 = 1\times 8^1 + 1\times 8^0$

$= 9$

$(1001)_2 = 1\times 2^3 + \underbrace{\quad}_{0} + 1\times 2^0$

$= 8 + 1 \quad 9$

Decimal to binary

$(41)_{10}$ to binary $\quad (101001)_2$

	Quotient	Remainder
$41/2$	20	1
$20/2$	10	0
$10/2$	5	0
$5/2$	2	1
$2/2$	1	0
$1/2$	0	1

Convert $(153)_{10}$ to Octal

	Quotient	Remainder
$153/8$	19	1
$19/8$	2	3
$2/8$	0	2

$(231)_8$

$(0.6875)_{10}$ to binary

$0.6875 \times 2 = 1.3750$ 1

$0.3750 \times 2 = 0.7500$ 0

$0.7500 \times 2 = 1.5000$ 1

$0.5000 \times 2 = 1.0000$ 1

$(0.1011)_2$

$$(41.6875)_{10}$$

$$(101001.1011)_2$$

Idea: convert the integer and the fraction separately

$(0.513)_{10}$ to octal

$(153.513)_{10}$ to octal

$(231.4065)_8$

$$0.513 \times 8 = 4.104 \quad 4$$
$$0.104 \times 8 = 0.832 \quad 0$$
$$0.832 \times 8 = 6.656 \quad 6$$
$$0.656 \times 8 = 5.248 \quad 5$$
$$0.248 \times 8 = 1.984 \quad 1$$
$$0.984 \times 8 :$$

$(0.4065)_8$

up to 4 significant digits

Complement of numbers

→ radix complement — r's complement

→ diminished radix complement — (r-1)'s complement

where r is the radix / base.

binary

2's complement
1's complement

decimal

10's complement
9's complement

Def^n Given a number N in base r having n digits, the $(r-1)$'s complement of N is defined as

$$\boxed{(r^n - 1) - N}$$

Example

9's complement of 546700

$n = 6$

$$(10^6 - 1) - 546700$$

$$= 999999 - 546700$$

$$
\begin{array}{r}
999999 \\
- \; 546700 \\
\hline
453299
\end{array}
$$

Another example

1's complement of 1011000

7 digits

$n = 7$

$(2^7 - 1) - 1011000$

127

1111111

$$
\begin{array}{r}
1111111 \\
- \; 1011000 \\
\hline
0100111
\end{array}
$$

Radix complement

Arithmetic operations

In-person class on Monday, Sept. 8th.

Lab

8th and 9th