ACOL 202

Lecture 2 (17^{th} Jan)

Basic Data Types

Most basic unit of data:

bit

1 / 0

$\boxed{\text{true / false}}$

on / off

yes / no

every piece of data on a (digital) computer is stored as a sequence of bits

$\boxed{\text{Boolean}}$

George Boole (British Mathematician)

Numbers

Integers, Reals, Rationals

$\mathbb{Z}$ $\mathbb{R}$ $\mathbb{Q}$

A real number that is not rational is called an irrational number.

$[a, b]$ (a, b) $[a, b)$

$(a, b]$

Absolute value, floor, ceiling

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

The floor of a real number x is the largest integer which is less than or equal to x.

The ceiling . . . . real no. x is . . . smallest integer which is greater than or equal to x.

Exponentiation

$b^0 = 1$ (by def^n, even when $b = 0$)

$b^n = b \cdot b \cdot b \ldots b$ (n times)

$b^{m/n} = \left(b^{\frac{1}{n}}\right)^m$

where $b^{1/n}$ denotes the number y such that $y^n = b$.

$b^{1/n}$ is called the n^{th} root of b $\sqrt[n]{b}$

$$b^{1/2} = \sqrt{b}$$

$$4^{3/2} = 8$$

When the exponent is negative,

b^{x} is defined as $\dfrac{1}{b^{-x}}$

$$25^{-3/2} = \frac{1}{125}$$

What if the exponent is irrational?

→ approximate by finding the closest rational

Properties of exponentiation

$$b^{x+y} = b^x \cdot b^y$$

$$(b^x)^y = b^{xy}$$

$$(ab)^x = a^x \cdot b^x$$

a, b - real
x, y - rationals
irrationals

Logarithms (Inverse operation to exponentiation)

$\log_b x$ is the number of times that we must multiply b by itself so that we get x.

$$\log_3 81 = 4$$

For a real number $x \leq 0$
and any base b

$\log_b x$ is undefined.

Also, logarithms base 1 are undefined.

Properties

$$\log_b 1 = 0$$

$$\log_b b = 1$$

$$\log_b xy = \log_b x + \log_b y$$

$$\log_b x^y = y \log_b x$$

"Change of base" formula

$$r \qquad \log_b x = \frac{\log_c x}{\log_c b} \qquad \log_c x = p, \quad \log_c b = q$$

$\log_b x = r$

$\log_c x = p$

$\log_c b = q$

$b^r = x$

$c^p = x$

$c^q = b$

$c^{qr} = b^r = x$

$= c^p$

$qr = p$

$\log_2(x)$ $\lg x$

$\lg x$

Created with Doceri

Modulus & Division

$\frac{9}{2} = 4.5$ regular

$\lfloor 9/2 \rfloor = 4$ integer division

↳ remainder 1

For any integer $k > 0$ and n, the integer $n \bmod k$ is the remainder when we divide n by k.

$$-3 \bmod 5 = 2$$

$$\boxed{n \bmod k = n - k \cdot \left\lfloor \frac{n}{k} \right\rfloor}$$

$n \bmod k$ always lies between 0 and $(k-1)$.

Even, Odd, Primes and Composite

for non-negative integers

defined for integers > 1.

Summation

$$\sum_{i=1}^{n} i \qquad 1+2+3+\ldots+n$$

$$\sum_{i=0}^{n} (n-i) = \sum_{j=0}^{n} j$$

$$\sum_{i=1}^{3}\sum_{j=1}^{4} v_{ij}$$

v_{ij}

$$\begin{array}{c|cccc} \text{row} \backslash \text{column} & 1 & 2 & 3 & 4 \\ \hline 1 & 5 & 7 & 9 & 2 \\ 2 & 3 & 4 & 6 & 1 \\ 3 & 12 & 8 & 9 & 5 \end{array}$$

$$\boxed{v_{11} + v_{12} + v_{13} + v_{14}}$$
$$+ \; v_{21} + v_{22} + v_{23} + v_{24}$$
$$+ \; v_{31} + v_{32} + v_{33} + v_{34}$$

$$\sum_{j=1}^{4} v_{1j} + \sum_{j=1}^{4} v_{2j} + \sum_{j=1} v_{3j}$$

$$\sum \qquad \prod$$

$$\prod_{i=1}^{n} i = n!$$

$$\sum_{i=1}^{0} x_i = 0 \qquad \prod_{i=1}^{0} x_i = 1$$

Sets

Unordered collection of objects

(typically denoted by a capital letter)

S

Membership

$x \in S$

$x \notin S$

Cardinality $|S|$ the number of elements in S

Explicit and Implicit Representation

The set of all perfect 2-digit squares

$\{16, 25, 36, 49, 64, 81\}$

is the same as

$\{n \in \mathbb{Z} : \sqrt{n} \in \mathbb{Z} \text{ and } n \geq 10 \text{ and } n \leq 99\}$

Empty set $\{\ \}$ ϕ

Set complement

The complement of a set A, wrt the universe U, written as $\overline{A}$, is the set of all elements of U that are not contained in A.

Set Union and Intersection

$$A = \{1, 3, 5\} \qquad B = \{2, 4, 6\}$$

$$A \cup B = \{1, 2, 3, 4, 5, 6\}$$

$$A \cup B = \{x : x \in A \text{ or } x \in B\}$$

$$A \cap B = \{x : x \in A \text{ and } x \in B\}$$

Set difference

$A \setminus B$

the set of all elements of A that are not in B.

Set equality

when they contain exactly the same elements.

Subsets, Supersets

$A \subseteq B$

$A \subset B$

Disjoint

empty intersection

Sets of sets

$\{\{\}, \{1, 2, 3\}\}$

Power set

$$P(\{1,2,3\})$$

$$= \{ \{\}, \{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}, \{1,3\}, \{1,2,3\} \}$$