Mid-term Exam (*Sept. 12, Marks: 20, Duration: 90 mins*) **COL750, Jul-Dec 2024**

Name: **Entry No.:**

Please note that you are not allowed to carry with you any notes, cheat-sheet, or any electronic devices including your phone and smart-watches. Also note that there will be zero tolerance for dishonest means like copying solutions from others, and even letting others copy your solution. If you are found indulging in such an activity, your exam paper will be seized immediately and you will be given a zero without any evaluation.

1. **[5 marks]** Consider the set of LTL/CTL formulas
$\mathcal{F} = \{F\ q \rightarrow F\ p, AF\ q \rightarrow AF\ p, AG\ (q \rightarrow AF\ p)\}$.

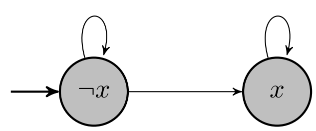
   (a) Is there a model in which all the above formulas hold true? **(1 marks)**

   (b) For each $\phi \in \mathcal{F}$, is there a model such that ϕ is the only formula in $\mathcal{F}$ that is satisfied in that model? **(3 marks)**

   (c) Find a model in which no formula of $\mathcal{F}$ holds true. **(1 marks)**

   Justify your answer in each case.

2. **[3 marks]** Represent the following transition system as an ROBDD.



3. **[4 marks]** *Claim:* It is possible to find two (finite) transition systems T_1 and T_2 (without terminal states, and over the same set of atomic propositions) and a CTL (state) formula ϕ such that *traces*$(T_1) =$ *traces*(T_2) and T_1 satisfies ϕ but T_2 does not satisfy ϕ.

   If the claim above is false, prove that it is so. If it is not, provide a T_1, T_2, and ϕ in support of the (truth of the) claim.

   A *trace* of a transition system T is the sequence of labels (i.e., sets of atomic propositions) observed along a path of T. The set of all traces of T is denoted by *traces*(T).

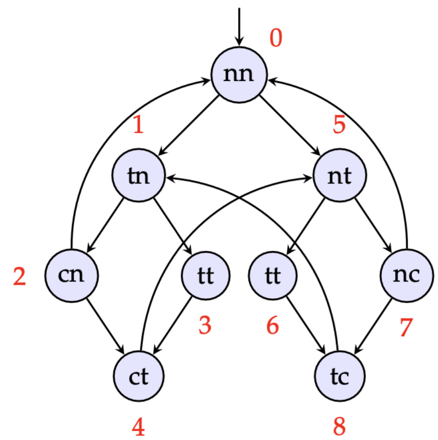
   Recall the syntax of CTL state and path formulas:

$$\text{State formula } \phi := \mathit{true} \mid p_i \mid \phi_1 \wedge \phi_2 \mid \neg\phi \mid \mathbf{E}\alpha \mid \mathbf{A}\alpha$$
$$\text{Path formula } \alpha := \mathbf{X}\phi \mid \phi_1\mathbf{U}\phi_2 \mid \mathbf{F}\phi \mid \mathbf{G}\phi$$

4. **[3 marks]** Recall the CTL Model Checking algorithm that we had studied in the class. We started by converting a given CTL formula in Existential Normal Form, and then gave procedures to tackle EX, EU, and EG formulas. If we wish to argue that the CTL Model Checking algorithm terminates, we must argue that the procedures that we had seen for EX, EU, and EG formulas indeed terminate for

a given finite model $\mathcal{M} = (S, \to, \mathcal{L})$. The procedure for EX did not involve any loop. But the ones for EU and EG did have *repeat-until* loops. Argue why the procedures for EU and EG must terminate for any finite model. Can you put an upper bound on the number of loop-iterations for these procedures, in terms of the cardinality of S? Justify your answer.

5. **[5 marks]** Recall the mutual exclusion example that we discussed in the class. We had two processes, each of which could be one of the three states – *not-trying* (n), *trying (to get into critical section)* (t), and *critical section* (c). The state transition diagram for the example has been reproduced below.



The nine states have been numbered, 0 to 8, and the label in each state denotes the atomic propositions that are true in that state. For example, the state numbered 5 is labelled 'nt' to denote that the first process is in the *non-trying* state, whereas the second process is in the *trying* state. In particular, it means that n_1 and t_2 are the (only) atomic positions that are true in the state numbered 5.

Consider the following CTL formulas, capturing *safety* and *liveness* properties.

Safety: $\neg$ **EF** $(c_1 \land c_2)$
Liveness: $\mathbf{AG}(t_1 \to \ \mathbf{AF}\ c_1)\ \land\ \mathbf{AG}(t_2 \to \ \mathbf{AF}\ c_2)$

- **[1 marks]** Does safety hold trivially? Why?
- **[4 marks]** Use the CTL model checking algorithm to check if $\mathbf{AG}(t_1 \to \ \mathbf{AF}\ c_1)$ holds in the initial state. (Note that we are checking only the first part of the liveness property. The second conjunct can be checked similarly, and is therefore not a part of this question.) Please write all the steps clearly, with justification (*do not take the shortcut of drawing and repeatedly annotating the same diagram – that may make it difficult for me to understand the annotation/justification*).