1. Prove by mathematical induction that $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.
2. Let $r \in \mathbb{R}$ such that $r \neq 1$. Use mathematical induction to prove that

$$\forall n \in \mathbb{Z}^{\geq 0}, \quad \sum_{i=0}^{n} r^i = \frac{r^{n+1}-1}{r-1}.$$

3. **[2 marks]** Use mathematical induction to prove that $\forall n \geq 2$, $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \ldots + \frac{1}{\sqrt{n}} > \sqrt{n}$.
4. Prove the following using mathematical induction.
   (a) $\forall n \geq 3$, $2n+1 < 2^n$.
   (b) $\forall n \geq 1$, $2^{2n} - 1$ is divisible by 3.
   (c) $\forall n \geq 0, k \geq 2$, $k^n - 1$ is evenly divisible by $(k-1)$.
5. Prove, by strong induction, that you can make exact change for any amount greater than 7 cents using only 3 and 5 cent coins.
6. Prove by induction that any convex polygon P with $k \geq 3$ vertices can be decomposed into a set of $k-2$ traingles whose interiors do not overlap.
7. Consider the set S defined recursively as follows:
   - *(base case)* $3 \in S$, and
   - *(recursive step)* if $x \in S$ and $y \in S$ then $x + y \in S$.

   Prove that S equals the set of all positive integers divisible by 3.
8. Prove by induction on n that $8^n - 3^n$ is divisible by 5 for any non-negative integer n.
9. **[2 marks]** Let F_n denote the n^{th} Fibonacci number. Prove that $F_n \bmod 2 = 0$ if and only if $n \bmod 3 = 0$.
10. Let us define a *power of two* as either (i) 1, or (ii) $2 \cdot k$, where k is a power of two. Prove by structural induction that the product of any two powers of two is itself a power of two.
11. Call a logical proposition truth-preserving if the proposition is true under the all-true truth assignment. Prove the following claim by structural induction on the form of the proposition:

    Any logical proposition that uses only the logical connectives $\vee$ and $\wedge$ is truth-preserving.