Mid-term exam (*20 Marks, 13th Oct., 2 hours*) **ACOL215, IITD-AD, Jul-Dec 2025**

Name: **Entry No.:**

Important points to note:

- Clearly show all the steps of your solutions with brief explanations wherever needed. Writing unexplained sequences of bits and characters would not fetch any marks.
- Stay concise and answer to-the-point. Needlessly long explanations will be penalised.
- There will be zero tolerance for dishonest means, including copying or sharing of solutions, and use of calculators, mobile phones, smart watches, or any other electronic gadgets during the exam (irrespective of whether inside or outside the exam hall). Offenders will get an F grade straight away. This will also attract strict disciplinary action.

1. **[7 marks]** Recall the Binary-Coded Decimal (BCD) Code that we had seen in the class. Here is $(185)_{10}$ in BCD representation: (0001 1000 0101). Note that BCD is an example of what are called *weighted codes*. In a weighted code, each bit position is assigned a weighting factor in such a way that each digit can be evaluated by adding the weights of all the 1's in the coded combination. In that sense, BCD codes can also be thought of as 8421 weighted codes. For example, the code 0110 represents 6, which is obtained by adding 4 and 2, the sum of weights corresponding to the 1's that appear in the code.

   Similar to the 8421 code discussed above, let us assume that we are working with 2421 (weighted) codes which work exactly like 8421 codes, except that the weighting factors are 2, 4, 2, and 1, instead of 8, 4, 2, and 1.

   (a) **[1 marks]** Write the decimal numbers 4, 6, 7, and 9 in 2421 representation.

   (b) **[1 marks]** 2421 code is an example of a *self-complementing code*. A self-complementing code has the property that 9's complement of a decimal number is obtained directly by changing 1's to 0's and 0's to 1's, i.e. by complementing each bit in the coded representation.

   Demonstrate this self-complementing property of 2421 code using the decimal number 387 as an example.

   (c) **[1.5 marks]** Give a brief argument explaining why the 2421 code are self-complementing.

   (d) **[1.5 marks]** Are BCD codes also self-complementing? Prove or disprove.

   (e) **[2 marks]** Are excess-3 codes also self-complementing? Prove or disprove. Recall that excess-3 code for a decimal digit d is the same as the 4-digit binary code for $(d + 3)$.

2. **[4 marks]** Find all the prime implicants of the following Boolean function, and determine which ones are essential. Also, create a sum of products expression for the function, and use that to design a circuit for it only using NAND gates. (Do not use inverter gates; you may assume that complemented inputs are available to you if needed.)

$$F(w, x, y, z) = \Sigma(0, 1, 3, 6, 7, 8, 9, 12, 13, 14, 15)$$

3. **[2 marks]** Recall that a gate can be extended to multiple inputs if the binary operation it represents is commutative and associative. Show that XNOR gates can be extended to multiple inputs.

4. **[5 marks]**

   (a) **[2 marks]** Design a half-subtractor circuit with inputs x and y and outputs D and B_{out}. The circuit should subtract the bits $x - y$ and place the difference in D, and the borrow in B_{out}. Also draw the circuit diagram for it.

   (b) **[3 marks]** Design a full-subtractor circuit with three inputs x, y, B_{in} and two outputs D and B_{out}. The circuit subtracts $x - y - B_{in}$ to get the difference D, where B_{in} is the input borrow and B_{out} is the output borrow. Draw a circuit diagram that shows how a full-subtractor may be implemented using two half-subtractors.

5. **[2 marks]** Find the radix complement of $(1750)_8$, and verify that your answer is correct by taking the radix complement again.