COL703: Logic for Computer Science (Aug-Nov 2022)

Lectures 9 & 10 (Hilbert's Proof System, Compactness, Strong Completeness)

Kumar Madhukar

madhukar@cse.iitd.ac.in

September 8th and 12th, 2022

Exercises from the last class

- Show that $\alpha \vee \beta$ is consistent iff either α is consistent or β is consistent.

Exercises from the last class

- Show that $\alpha \vee \beta$ is consistent iff either α is consistent or β is consistent.
- Show that if $\alpha \wedge \beta$ is consistent then both α and β are consistent.

Exercises from the last class

- Show that $\alpha \vee \beta$ is consistent iff either α is consistent or β is consistent.
- Show that if $\alpha \wedge \beta$ is consistent then both α and β are consistent.
- What about the converse?

Completeness Proof

β consistent $\quad\rightarrow\quad$ β satisfiable

- every consistent set can be extended to a maximal consistent set (MCS)
- let X be an MCS; for all formulas α, $\quad v_X \models \alpha$ *iff* $\alpha \in X$
  (where v_X is the valuation that every atomic proposition in X to *true*)

Questions from the last class

- $\alpha \to \neg\neg\alpha$
- uniqueness of MCS

Derivability and Logical Consequence

Strong Completeness

Let $X \subseteq \Phi$ and $\alpha \in \Phi$. Then $X \vDash \alpha$ *iff* $X \vdash \alpha$.

Compactness Theorem

Let $X \subseteq \Phi$ and $\alpha \in \Phi$. Then $X \vDash \alpha$ *iff* there exists $Y \subseteq_{\mathit{fin}} X, Y \vDash \alpha$.

Finite Satisfiability

Let $X \subseteq \Phi$. Then, X is satisfiable *iff* every $Y \subseteq_{fin} X$ is satisfiable.

Proof of Compactness Theorem

Proof of Strong Completeness

(left as an exercise)

Next week

- SAT Solving
- Binary Decision Diagrams

Thank you!