ACOL 215

(16th Sept.)

Prove $x + 1 = 1$

$$
\begin{aligned}
x + 1 &= (x+1) \cdot 1 && \text{identity (b)} \\
&= (x+1)(x+x') && \text{complement (a)} \\
&= x + 1 \cdot x' && \text{distributive (b)} \\
&= x + x' && \text{identity (b)} \\
&= 1 && \text{complement (a)}
\end{aligned}
$$

Prove

$$x \cdot 0 = 0$$

$$\begin{aligned}
x \cdot 0 &= x \cdot 0 + 0 && \text{identity (a)}\\
&= x \cdot 0 + x \cdot x' && \text{complement (b)}\\
&= x(0 + x') && \text{distributive (a)}\\
&= x \cdot x' && \text{identity (a)}\\
&= 0 && \text{complement (b)}
\end{aligned}$$

Note that there was a typo in the lecture slides of 15th Sept. The rule distributive (a) is $x(y+z) = xy + xz$.

Exercise Prove that

$$x + xy = x$$

$$
\begin{aligned}
x + xy &= x \cdot 1 + xy && \text{identity (b)} \\
&= x(1+y) && \text{distributive (a)} \\
&= x(\underline{y+1}) && \text{commutative} \\
&= x \cdot \underline{1} && \text{Proved (Earlier)} \\
&= x && \text{identity (b)}
\end{aligned}
$$

Exercise

$$x(x+y) = x$$

De Morgan's Theorem

$$(x+y)' = x'y'$$

(Prove using truth tables)

x	y	$x+y$	$(x+y)'$	x'	y'	$x'y'$
0	0	0	1	1	1	1
0	1	1	0	1	0	0
1	0	1	0	0	1	0
1	1	1	0	0	0	0

← same →

Operator precedence

( ), NOT, AND, OR $\longrightarrow$

Boolean Functions

binary variables, constants : 0 and 1, logical operation symbols

$F_1 = x + y'z$

$F_2 = xyz + x'yz + xy'z + x'y'z$

$f_1 = x + y'z$

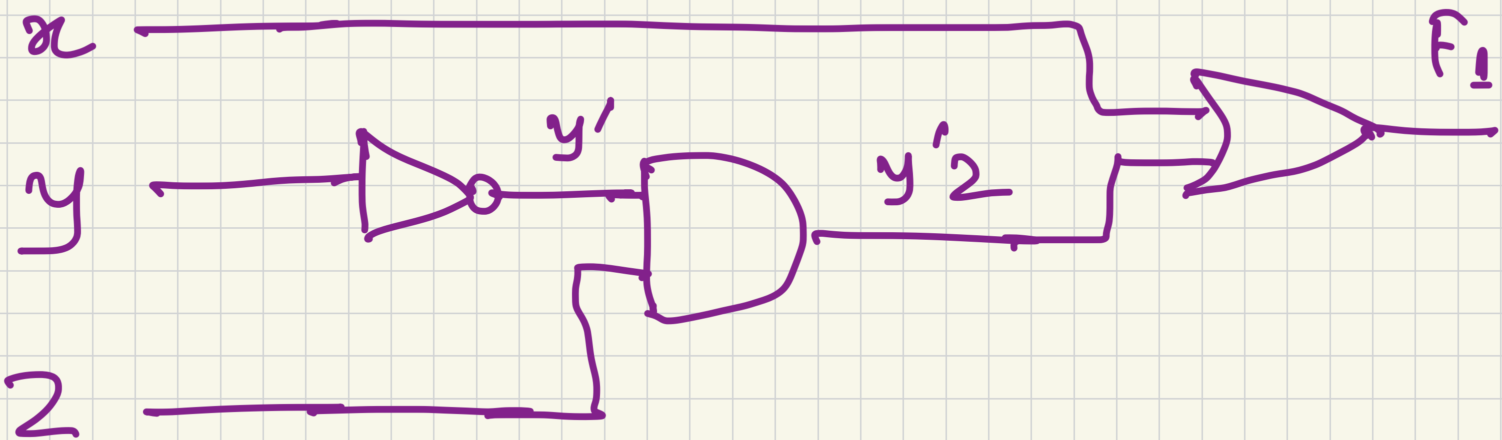
f_1 equals when $x = 1$

or $y = 0$ and $z = 1$

x	y	z	$x+y'z$	f_2
0	0	0	0	0
0	0	1	1	1
0	1	0	0	0
0	1	1	0	1
1	0	0	1	0
1	0	1	1	1
1	1	0	1	0
1	1	1	1	1

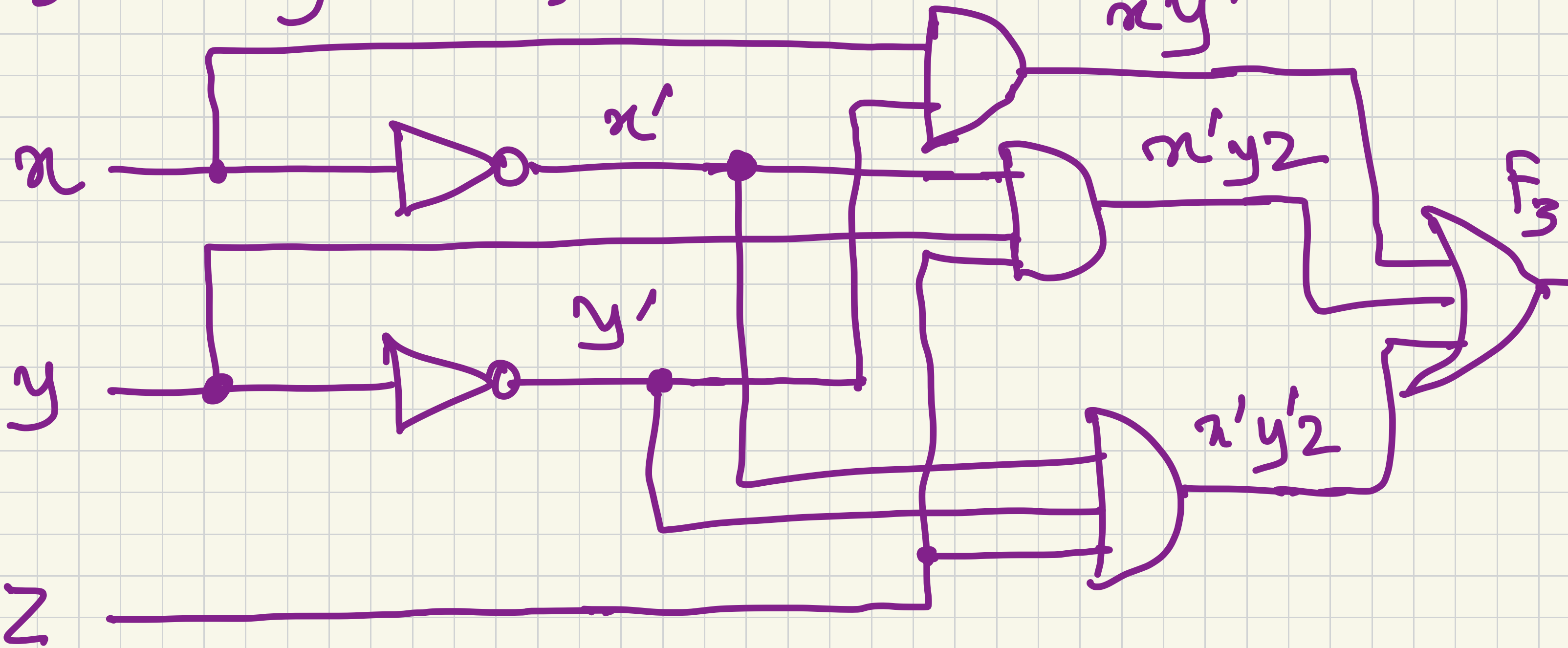
Logic diagram of F_1

$x + y'z$



$$F_3 = \underline{x'y'z} + \underline{x'yz} + \underline{xy'}$$

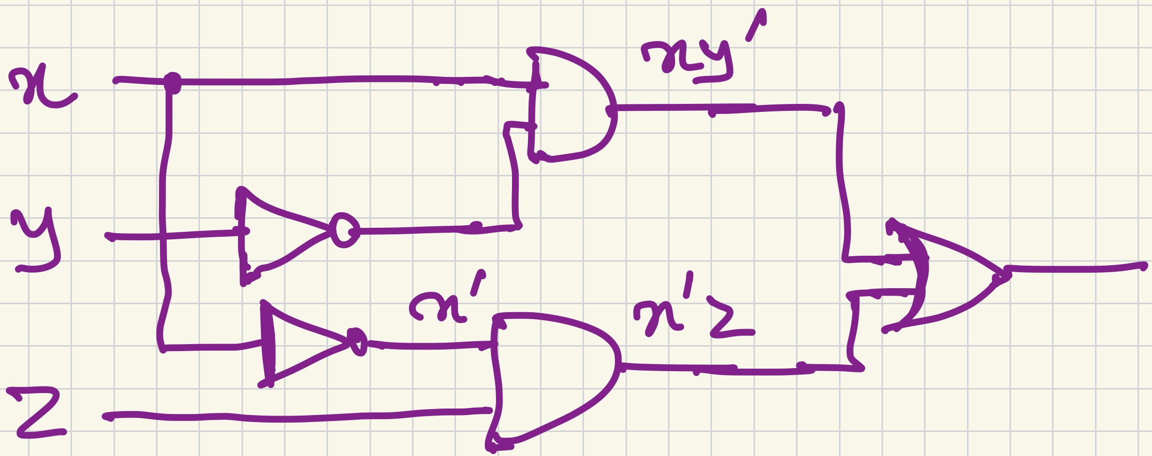
Logic diagram of F_3



$$F_3 = x'y'z + x'yz + xy'$$

$$= x'z(y'+y) + xy'$$

$$= \underline{x'z} + \underline{xy'}$$



Simplify :

i) $x \cdot (x' + y)$

$= x \cdot x' + xy = 0 + xy$

$= xy$

ii) $(x+y) \cdot (x+y')$

x

iii) $xy + x'z + yz$

whenever this is 0

this can't be 1

$$xy + x'z + yz$$

$$= xy + x'z + yz(x + x')$$

$$= \underline{xy} + x'z + \underline{yzx} + yzx'$$

$$= xy(\underbrace{1+z}_{1}) + x'z(\underbrace{1+y}_{1})$$

$$= xy + x'z$$

iv) $\boxed{(x+y)\,(x'+z)}\,(y+z)$

iii) and ix) consensus theorem

Complement of a function

The complement of a function F (denoted as F') is obtained by interchanging 1's and 0's in the value of F.

Algebraically, we can use DeMorgen's laws to get the complement,

$$(A+B+C)' = (A+X)' \quad \text{when } X = (B+C)$$

$$= A'X'$$

$$= A'(B+C)'$$

$$= A'(B'C')$$

$$= A'B'C'$$

Generally, for n terms/variables

$$(A_1 + A_2 + \dots + A_n)' = A_1' \cdot A_2' \cdot \dots \cdot A_n'$$

$$(A_1 A_2 A_3 \dots A_n)' = A_1' + A_2' + A_3' + \dots + A_n'$$

Example $F = x' \cdot y \cdot z' + x' \cdot y' \cdot z$

What is F'?

$(x + y' + z) \cdot (x + y + z')$

Exercise $F = x(y'z' + yz)$

Find F'.

$x' + yz' + zy'$