Logical equivalence proof from 13th Oct. lecture

A mistake in the proof done in the class

Kumar Madhukar

There was a mistake in the proof done in the class on October 13th, in showing logical equivalence of $(\forall x\ F \wedge G)$, and $\forall x\ (F \wedge G)$ (of course, under the assumption that x does not occur free in G).

We started with an assignment $\mathcal{A}$ that models $\forall x\ F \wedge G$, and argued that $\mathcal{A} \models \forall x\ F$, and therefore $\mathcal{A}_{[x \mapsto a]} \models F[a/x]$ for every a in the universe of $\mathcal{A}$.

The last (red) bit above is not exactly true. It is true only when a is a term, and $\mathcal{A}$ maps x not to a, but to whatever it maps the term a to. So, it would be correct to write $\mathcal{A}_{[x \mapsto \mathcal{A}[\![a]\!]]} \models F[a/x]$, and that too only when a is a term. The correctness of this follows from the translation lemma. Anyway, this is not needed here.

For the proof, you can simply say $\mathcal{A}_{[x \mapsto a]} \models F$ for every a in the universe of $\mathcal{A}$, because you know that $\mathcal{A} \models \forall x\ F$.

The rest of the proof goes through as we had discussed. From $\mathcal{A} \models G$, we know that $\mathcal{A}_{[x \mapsto a]} \models G$. Why? Relevance lemma (because x is not free in G, so $\mathcal{A}$ and $\mathcal{A}_{[x \mapsto a]}$ do not differ in the assignment of free variables).

I apologise for this mistake and the confusion.