Quiz 4 (*Sep 19, Marks: 4, Duration: 50 mins*) **COL703, Aug-Nov 2022**

Name: **Entry No.:**

1. **[1 marks]** Prove the following lemma, which we had used for proving the Compactness theorem (recall that Φ was used to denote the set of propositional logic formulas):

   $$\textit{For all } Z \subseteq \Phi \textit{ and all } \beta \in \Phi, \quad Z \vDash \beta \quad \textit{iff} \quad Z \cup \{\neg\beta\} \textit{ is not satisfiable.}$$

2. **[1 marks]** Show that if $\alpha \wedge \beta$ is consistent, then both α and β are consistent. Recall that α is said to be consistent if $\nvdash \neg\alpha$.

3. **[1 marks]** Consider the CNF formula ϕ shown below. Write a 3-CNF formula ψ such that ψ is satisfiable *iff* ϕ is satisfiable.

   $\phi = (\neg x_1 \vee \neg x_4) \wedge (x_1 \vee \neg x_2 \vee \neg x_3) \wedge (\neg x_2 \vee \neg x_3 \vee x_4 \vee x_1) \wedge (x_1)$

4. **[1 marks]** Let $X \subseteq \Phi$ and $\alpha \in \Phi$, where Φ denotes the set of propositional logic formulas. The use of strong completeness ($X \vDash \alpha \quad \textit{iff} \quad X \vdash \alpha$) can lead to an alternative, shorter, proof of compactness ($X \vDash \alpha \quad \textit{iff there exists} \quad Y \subseteq_{fin} X,\ Y \vDash \alpha$). Give that proof.