

From these facts, it is possible to conclude r .

$p, q \Rightarrow r, p \Rightarrow s, \neg s \vee q$

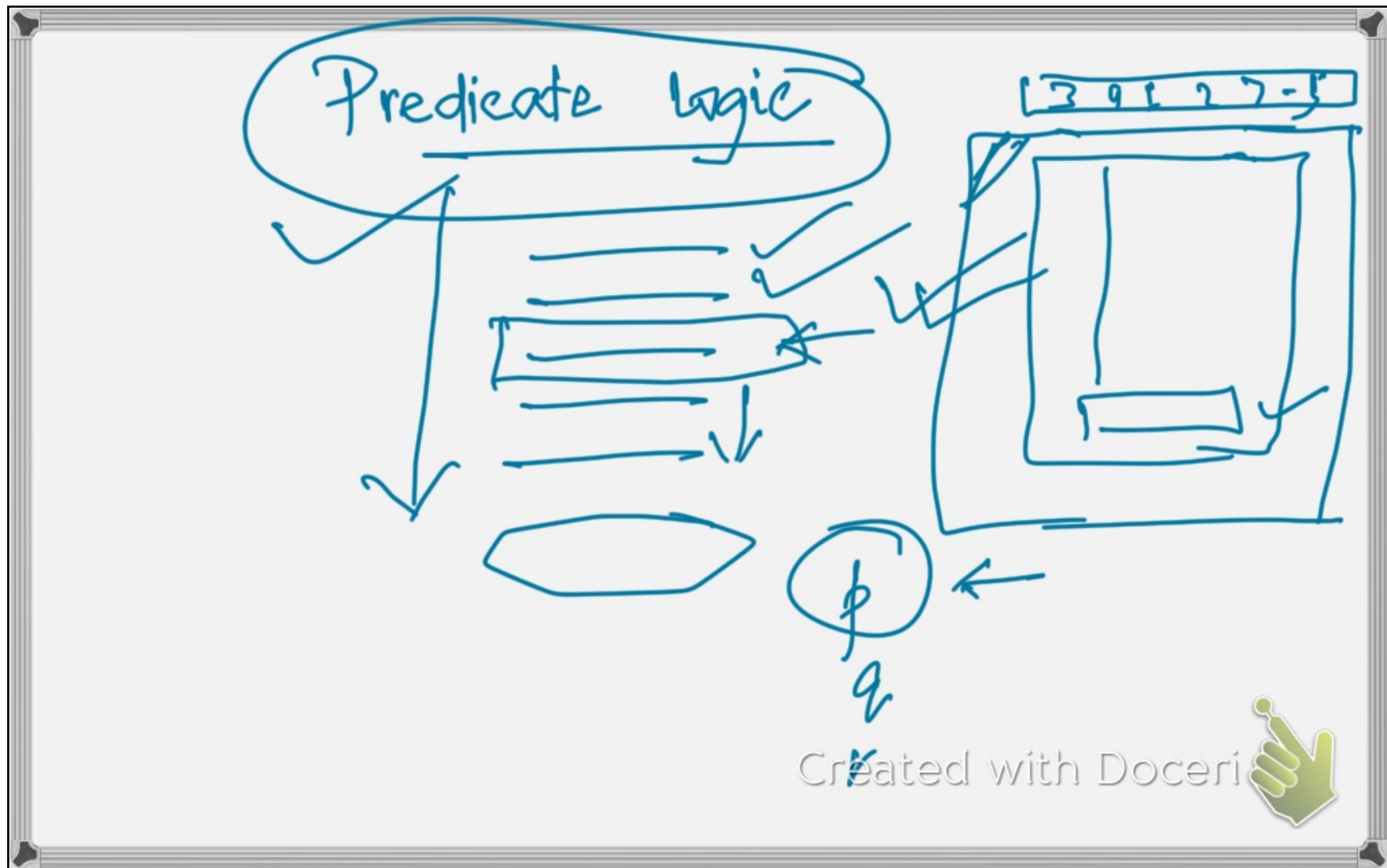
p
 $p \Rightarrow s$

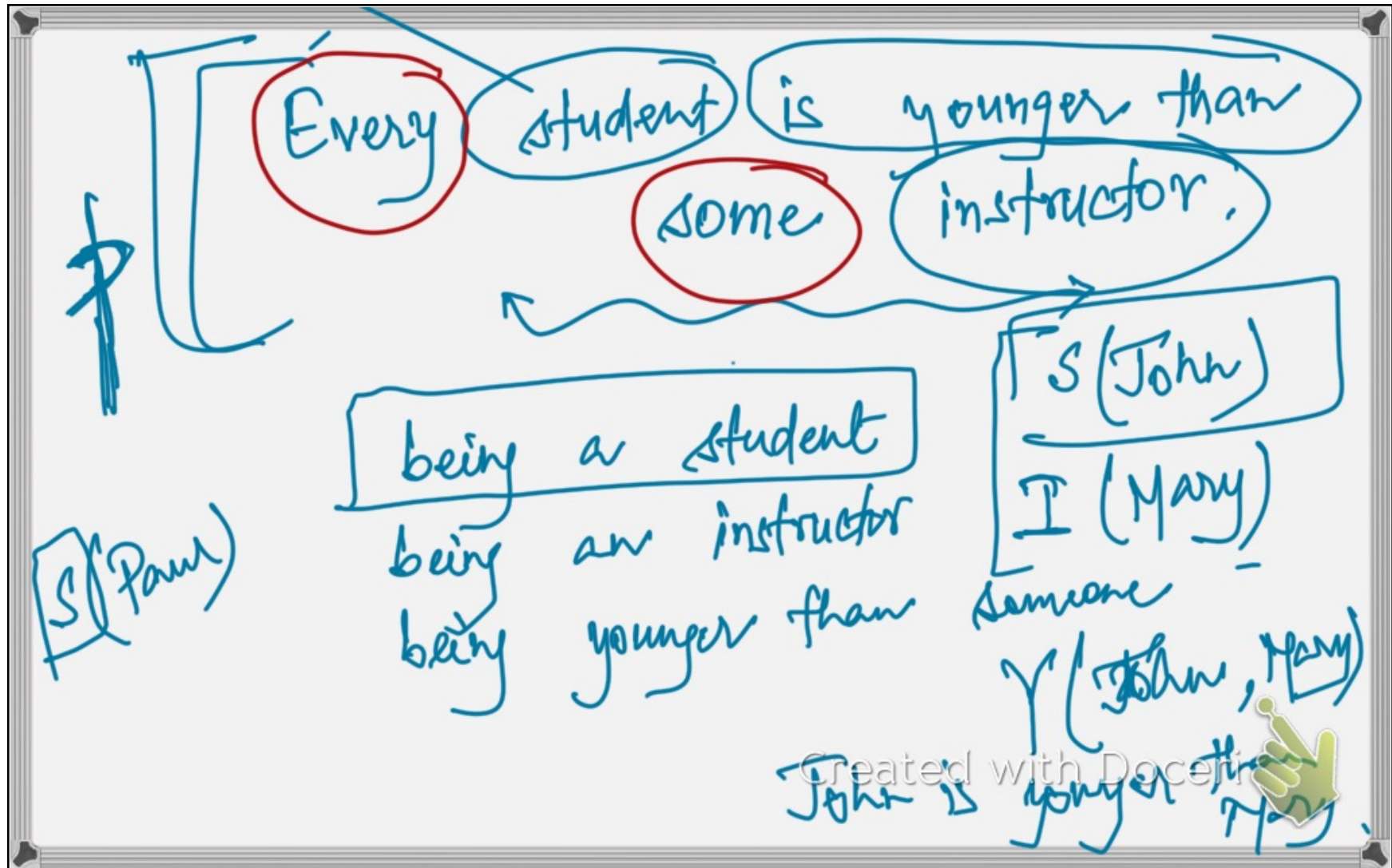
s

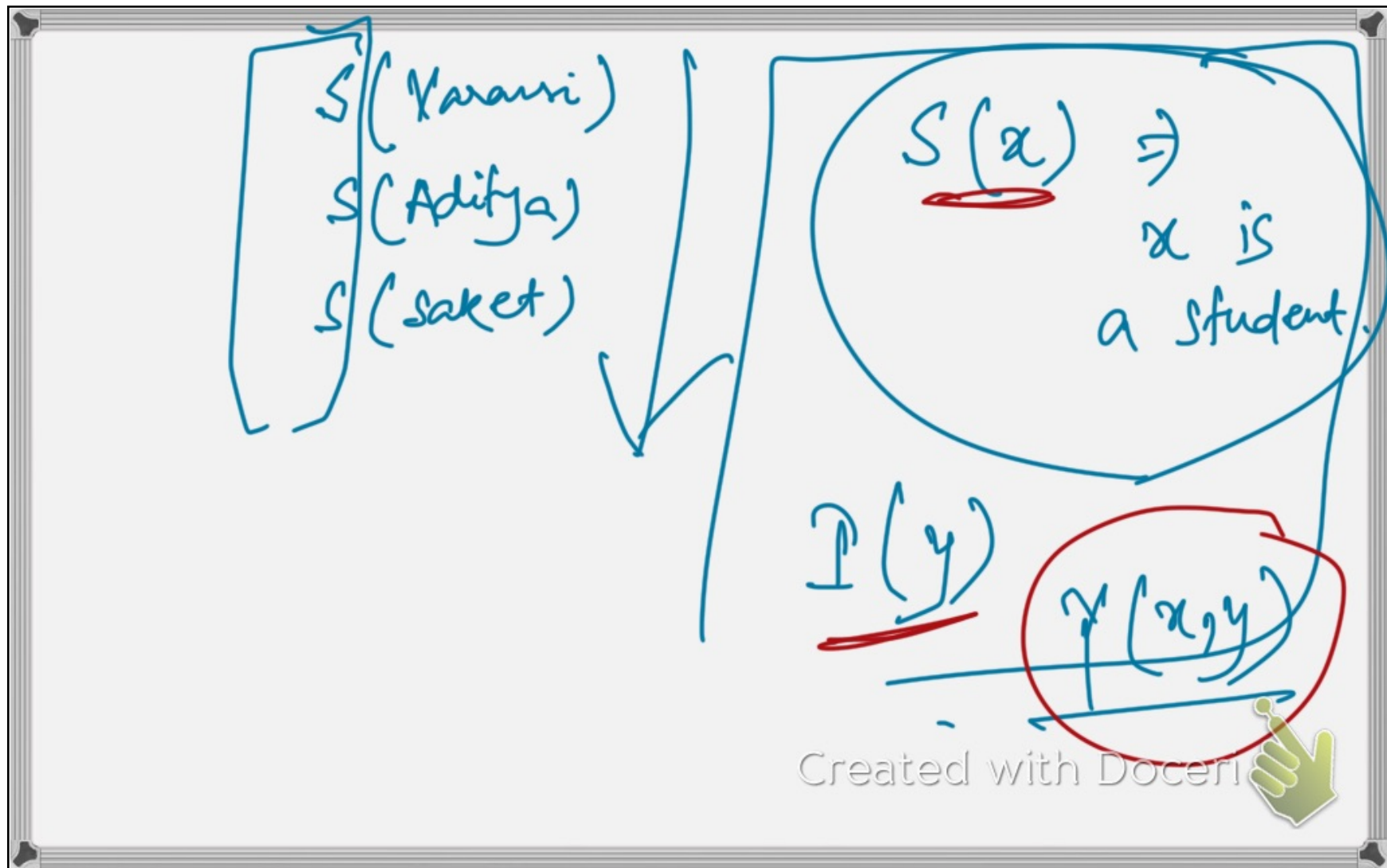
$p \quad q$
 $p \wedge q$

logical derivations

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Quantifiers

always be attached to some variable

\forall for all

\exists there exists

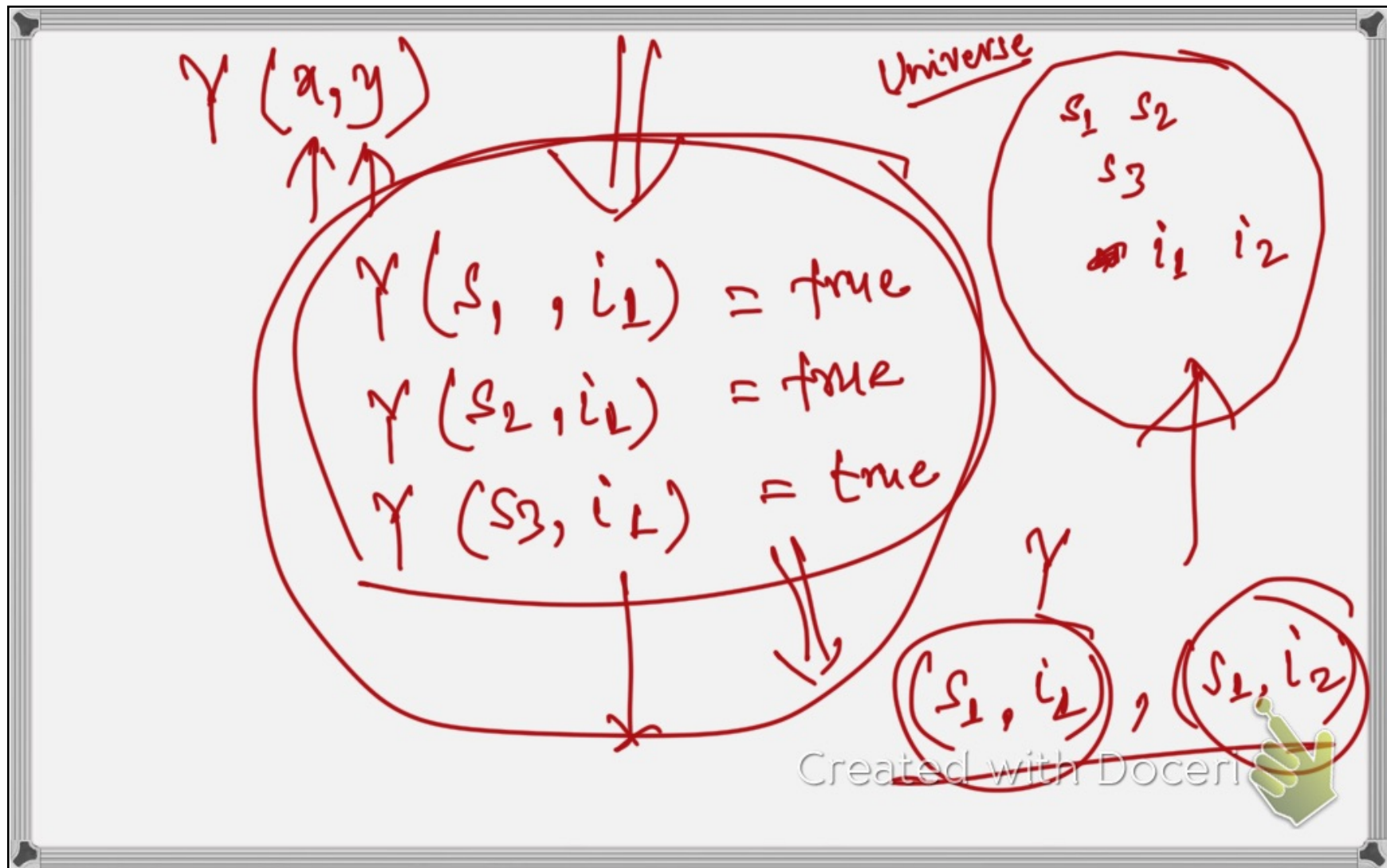
$\forall x (S(x) \rightarrow \exists y (I(y) \wedge Y(x, y)))$

if-then

For every x ,
there is some y

if x is a student, then
such that y is an instructor
and x is younger than y .

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Example

Not all birds can fly.

$B(x)$

$F(y)$

x is a bird
 y can fly

$\exists x$ such that $B(x)$ and $\neg F(x)$

$\neg (B(x) \wedge \neg F(x)) \quad \neg \forall x (B(x) \rightarrow F(x))$

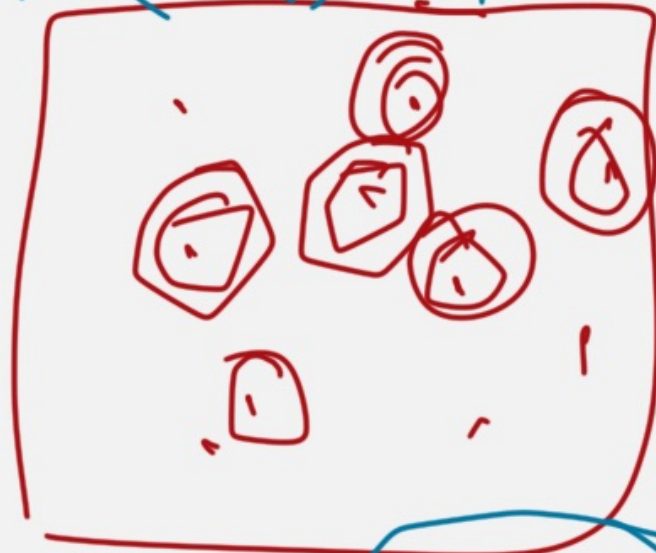
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$$\forall c (Y(c, m(c)))$$

Every child is
younger than its
mother.

$$\forall c (C(c) \rightarrow Y(c, m(c)))$$



$$\forall c, m \quad Y(c, m) \wedge M(c, m)$$

$$\forall x \forall y (C(x) \wedge M(x, y)) \rightarrow Y(x, y)$$

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Ann and Mary have the
same maternal grandmother.

$$M(\underline{M(x, y)}, z)$$

$$M(T, z)$$

$$M(\underline{MLn})$$

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$$\exists u \exists v \exists w$$

$$(M(\text{Ann}, u) \wedge M(\text{Mary}, v))$$

$$\Rightarrow (M(u, w) \wedge M(v, w))$$

$$p \Rightarrow q$$

F

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↓

p	q	$p \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

↓

$\forall x \forall y \forall u \forall v \quad = (x, y, u, v)$

$M(Ann, x) \wedge M(x, y)$
 $\wedge M(Mary, u) \wedge M(u, v)$

$\Rightarrow (y = v)$

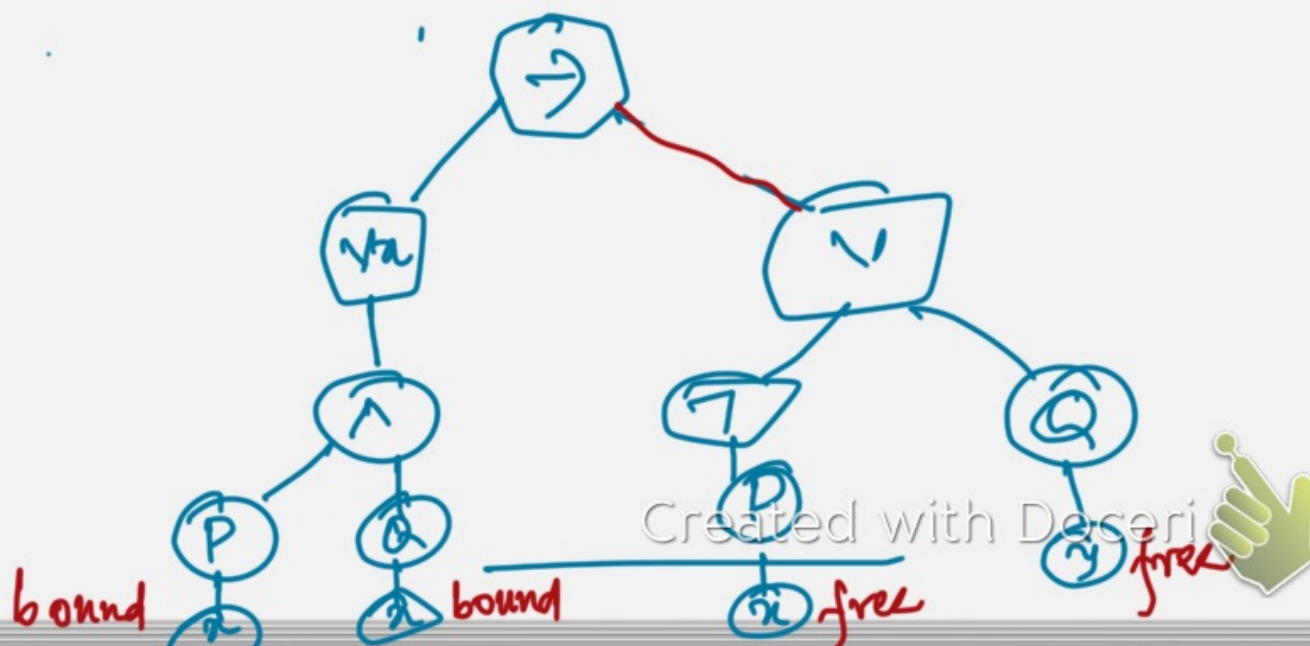
\downarrow

$m(m(Ann)) \stackrel{m(x)}{=} m(m(Mary))$

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Formal syntax

$$(\forall x (P(x) \wedge Q(x))) \rightarrow (\neg P(x) \vee Q(y))$$



Closed formula / sentence

Formulas with no free variables.

$$\forall x \forall y \forall z \left(R(x, y) \wedge R(y, z) \rightarrow R(x, z) \right)$$

closed

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