Quiz 2 (6 marks) (*March 25*) **ACOL202, IITD-AD, Jan-May 2025**

Name: **Entry No.:**

You may assume the following facts in this quiz without proving them. But you must explicitly indicate this by writing something like "*(using fact i)*" wherever you are using the i^{th} fact from here.

1. If p is a prime number, and $a, b \in \mathbb{Z}$, then $p \mid ab$ if and only if $p \mid a$ or $p \mid b$.
2. $k^n - 1$ is evenly divisible by $k - 1$, for any $n \geq 0$ and $k \geq 2$.

1. **[1 mark]** Show that if a and b are both positive integers, then $(2^a - 1) \bmod (2^b - 1) = 2^{(a \bmod b)} - 1$.
2. **[2 marks]** Show that if a and b are positive integers, then $gcd(2^a - 1, 2^b - 1) = 2^{gcd(a,b)} - 1$. Use mathematical induction.
3. **[1.5 marks]** Let a and b be relatively prime. Let c be relatively prime to both a and b. Prove that c and ab are also relatively prime.
4. **[1.5 marks]** A *palindromic bitstring* is a string of `0`'s and `1`'s that reads the same front-to-back as it does from back-to-front. For example, `0010100` is a palindromic bitstring, where `011` is not. Here is a recursive definition of palindromic bitstrings.

   - The empty string ϵ is a palindromic bitstring.
   - The string `0` (consisting of a single `0`) is a palindromic bitstring.
   - The string `1` (consisting of a single `1`) is a palindromic bitstring.
   - If `s` is a palindromic bitstring, so is `0s0`.
   - If `s` is a palindromic bitstring, so is `1s1`.

   Let $n_0(s)$ and $n_1(s)$ denote, respectively, the number of `0`'s and `1`'s in a palindromic bitstring s. Use induction to prove that $n_0(s) \cdot n_1(s)$ is even for any palindromic bitstring s.