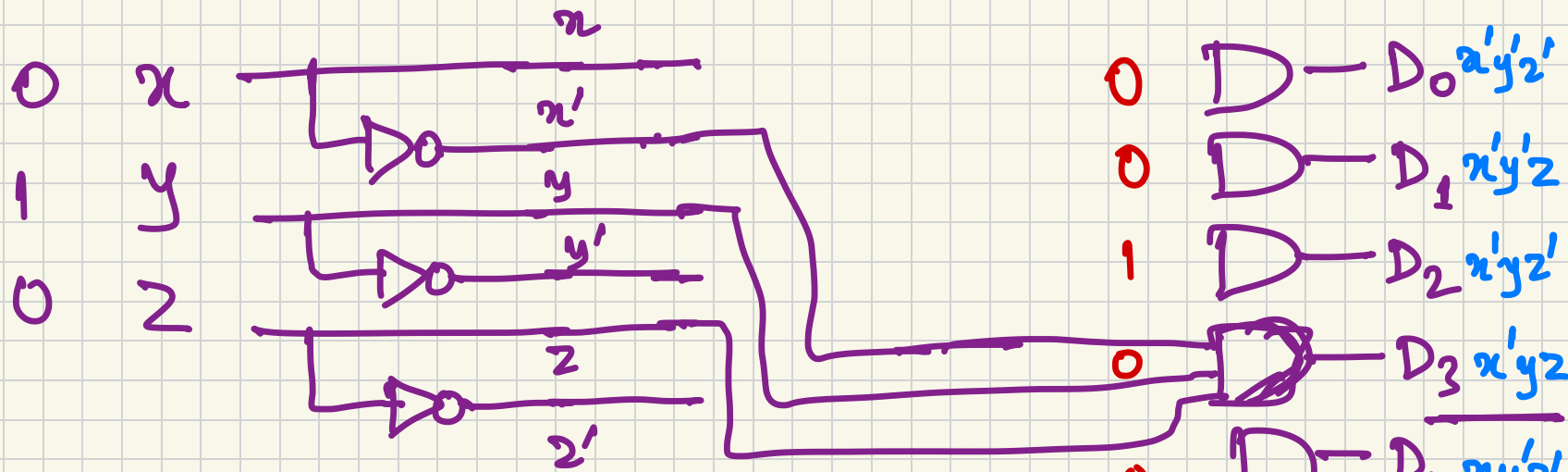## Decoders

A decoder is a combinational circuit that converts binary information from n input lines to a maximum of 2^n output lines.

In particular, we look at n-to-m-line decoders, where $m \leq 2^n$.

A binary code of n bits can represent 2^n distinct elements of coded information.

x

0 x

1 y

0 z

x'

y

y'

z

z'

0 D_0 $x'y'z'$

0 D_1 $x'y'z$

1 D_2 $x'yz'$

0 D_3 $x'yz$

0 D_4 $xy'z'$

0 D_5 $xy'z$

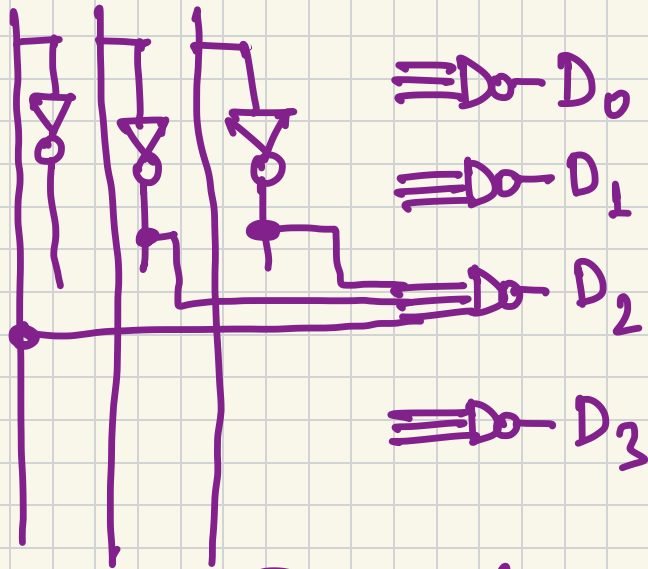0 D_6 xyz'

0 D_7 xyz

Some decoder are
constructed with } for
NAND gates } efficiency

→ complemented output

Decoders can have one or more enable inputs to control the circuit operation.

E	A	B		D_0	D_1	D_2	D_3
1	X	X		1	1	1	1
0	0	0		0	1	1	1
0	0	1		1	0	1	1
0	1	0		1	1	0	1
0	1	1		1	1	1	0

Two-to-four line decoder with an enable input constructed with NAND gates

complemented input & complement
enable input

NAND
gates

Outputs are
enabled when
$E = 0$
(active - low
enabled)

A decoder with enable
input can function as a demultiplexer

→ a circuit that receives information
from a single line and directs it to
one of 2^n possible output lines,

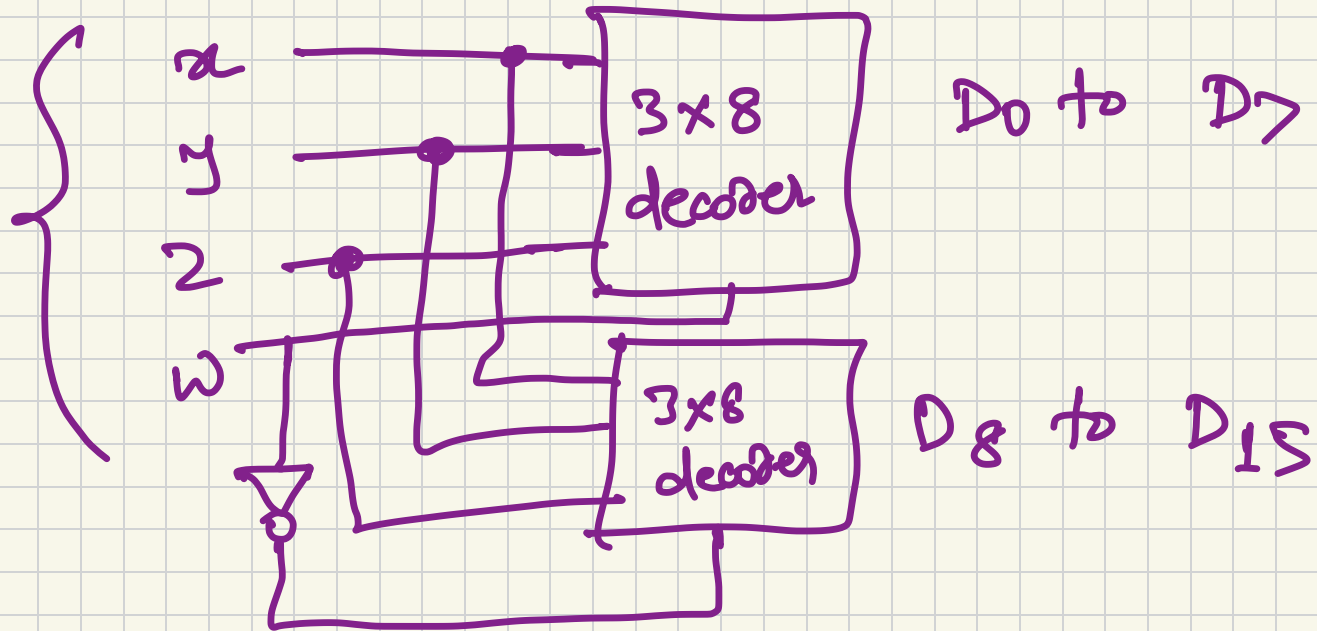E can be thought of as data input line and A, B as selection input lines.

→ for different selection inputs, E is directed to different outputs.

when AB is 10, E is directed to D_2.

AB is 11, E is directed to D_3.

A decoder with an enable input is also called a decoder demultiplexer.

Decoders with enable inputs can
be connected to form a larger
decoder circuit.

x

y

2

w

3×8
decoder

D_0 to D_7

3×8
decoder

D_8 to D_{15}

Implement a full-adder with a decoder.

x	y	z		C	S
0	0	0		0	0
0	0	1		0	1
0	1	0		0	1
0	1	1		1	0
1	0	0		0	1
1	0	1		1	0
1	1	0		1	0
1	1	1		1	1

$$S(x,y,z) = \sum(1,2,4,7)$$

$$C(x,y,z) = \sum(3,5,6,7)$$

x

y

z

3×8
decoder

0
1
2
3
4
5
6
7

S

C

Encoder

inverse operation of a
decoder

2^n (or fewer) input lines

$\rightarrow$ n output lines.

Example: octal to binary encoder

$$x = D_4 + D_5 + D_6 + D_7$$
$$y = D_2 + D_3 + D_6 + D_7$$
$$z = D_1 + D_3 + D_5 + D_7$$

D_0	D_1	D_2	D_3	D_4	D_5	D_6	D_7	x	y	z
1	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	1
0	0	1	0	0	0	0	0	0	1	0
0	0	0	1	0	0	0	0	0	1	1
0	0	0	0	1	0	0	0	1	0	0
0	0	0	0	0	1	0	0	1	0	1
0	0	0	0	0	0	1	0	1	1	0
0	0	0	0	0	0	0	1	1	1	1

If D_5 and D_6 are 1 at the same time the output is 1 1 1 (which is absurd).

⤷ Fixed by assigning higher priority to higher index input.

If all inputs are 0, then output is 000 which is the code when $D_0 = 1$.

⤷ Fixed with a valid bit indicator.

D_0	D_1	D_2	D_3
0	0	0	0
1	0	0	0
X	1	0	0
X	X	1	0
X	X	X	1

$\rightarrow$

x	y	V
X	X	0
0	0	1
0	1	1
1	0	1
(1)	1	1

small columns under left table:
- D_0: 0 0 0
- D_1: 0 0 1
- D_2: 0 1 0
- D_3: 1 1 1

bottom left: x_1 X_0 X_0 (1)

	00	01	11	10
00	x	1	1	0
	0	0	1	0
01	1	1	1	0
11	1	1	1	0

$D_2 D_3$

$D_0 D_1$

	00	01	11	10
00	X	1	1	1
01		1	1	1
11		1	1	1
10		1	1	1

$\overbrace{}^{D_2}$

$\underbrace{}_{D_3}$

$x = D_2 + D_3$

$y?$

$D_1 \cdot D_2' + D_3$

$V = D_0 + D_1 + D_2 + D_3$

Karnaugh map with axes labeled $D_2 D_3$ (columns) and $D_0 D_1$ (rows).

Column headers: 00, 01, 11, 10

Row headers: 00, 01, 11, 10

$D_0 D_1$ \ $D_2 D_3$	00	01	11	10
00	X	1	1	0
01	1	1	1	0
11	1	1	1	0
10	0	1	1	0

D_3

$D_1 D_2$

D_3

Multiplexers

I_0

I_1

S

I_0

I_0

1

I_2

0

I_0

0

I_0

output

| $S = 0$ | I_0 |
| $S = 1$ | I_1 |