ACOL 215

(23rd Sept.)

Exercise For the boolean function

$$F = A'C + \underline{A'B} + AB'C + BC$$

i) express F as a sum of minterms

$$F = \Sigma(1, 2, 3, 5, 7)$$

ii) find the minimal sum of products expression for F

$$F = C + A'B$$

Exercise Simplify $F(x,y,z) = \sum(0,1,2,5)$

$$f(x,y,z) = y'z + x'z'$$

Exercise Simplify $f(x,y,z) = \sum(0,2,3,4,6)$

$$f(x,y,z) = z' + x'y$$

Four - variable K-map

Same as three variable K-map

→ one square represent one minterm (giving a term with 4 literals)

→ two adjacent squares represent a term with three literals

→ four adjacent squares . . . . two literals

→ eight . . . . . . one literals

→ sixteen - . - constant function 1.

y

wx \ yz	00	01	11	10
00	m_0 $w'x'y'z'$	m_1 $w'x'y'z$	m_3 $w'x'yz$	m_2 $w'x'yz'$
01	m_4	m_5	m_7	m_6
11	m_{12}	m_{13} $wxy'z$	m_{15}	m_{14} $wxyz'$
10	m_8	m_9	m_{11}	m_{10}

z

x

w

Exercise Simplify

$$F(w,x,y,z) = \sum(0,1,2,4,5,6,8,9,12,13,14)$$

$$\underline{z'x} + \underline{z'w'} + \underline{y'}$$

$wx \backslash yz$	00	01	11	10
00	m_0 1	m_1 1	m_3	m_2 1
01	m_4 1	m_5 1	m_7	m_6 1
11	m_{12} 1	m_{13} 1	m_{15}	m_{14} 1
10	m_8 1	m_9 1	m_{11}	m_{10}

y (over columns 11 and 10), z (under columns 01 and 11), x (beside rows 01 and 11), w (beside rows 11 and 10)

y' $\quad z'w' \quad z'x$

Implicants

The product terms of a sum-of-products representation (of a Boolean function) are called implicants of the function.

When the implicant has a value 1, the function has a value 1.

A prime implicant is a product term obtained by combining the maximum possible number of adjacent squares.

Consider $F(A,B,C,D) = \sum(0,2,3,5,7,8,9,10,11,13,15)$

AB \ CD	00	01	11	10
00	m_0 1	m_1	m_3 1	m_2 1
01	m_4	m_5 1	m_7 1	m_6
11	m_{12}	m_{13} 1	m_{15} 1	m_{14}
10	m_8 1	m_9 1	m_{11} 1	m_{10} 1

C, D, A, B

$BD + B'D' + \underline{AD} + CD$

$BD + B'D', + + AD + B'C$

AB'	8, 9, 10, 11
AD	9, 11, 13, 15
$\boxed{BD}$	$\boxed{5, 7, 13, 15}$
CD	3, 7, 15, 11
$\boxed{B'D'}$	$\boxed{0, 2, 8, 10}$
$B'C$	3, 2, 11, 10

essential

$BD + B'D' + AB' + CD$

$BD + B'D' + AB' + B'C$

Five-variable K-map

Works similarly but cumbersome.

Product of sums simplification

→ You can compute a simplified sum of products expression for f'
and then take complement

Write 0's where there are no 1's
and merge them.

Example

$F(A,B,C,D) = \sum(0,1,2,5,8,9,10)$

AB \ CD	00	01	11	10
00	m_0 1	m_1 1	m_3 0	m_2 1
01	m_4 0	m_5 1	m_7 0	m_6 0
11	m_{12} 0	m_{13} 0	m_{15} 0	m_{14} 0
10	m_8 1	m_9 1	m_{11} 0	m_{10} 1

C

B

A

D

$F' = CD + AB + BD'$

$F = (C'+D') \cdot (A'+B') \cdot (B'+D)$