ACOL 202

29th April

$$f(n) = O(g(n))$$

$$f(n) = \Omega(g(n))$$

$$f(n) = \Theta(g(n))$$

Made with Goodnotes

Selection Sort

64 25 12 22 11

find smallest, swap with the first element
find smallest among the remaining, swap with the second element

and so on . . .

Made with Goodnotes

101 203 3 64 25 12 22 11 5 8

11 25 12 22 64 4 7

11 12 25 22 64 3

11 12 22 25 64

How many primitive steps

$$n + n-1 + n-2 + \dots + 1$$

$$\frac{n(n+1)}{2} \qquad O(n^2)$$

Made with Goodnotes

Find the smallest in the array/list is $O(n)$ operation.

Linear search

$\boxed{2 \quad 7 \quad 3 \quad 5 \quad 13 \quad 23 \quad 15 \quad 11}$, 4

$O(n)$

Made with Goodnotes

Merge sort

11 7 91 43 5 68 32 1

11 7 | 91 43 | 5 68 | 32 1

11 | 7 | 91 | 43 | 5 | 68 | 32 | 1

7, 11 | 43, 91 | 5, 68 | 1, 32

7, 11, 43 91

1, 5, 32, 68

1, 5, 7

Made with Goodnotes

$$\underline{T(n)} = \underline{2T\left(\frac{n}{2}\right)} + c \cdot n$$

$$T(n) = T(n-1) + c \cdot n$$

$$T(n) = T(n/2) + c$$

Made with Goodnotes

$$T(n) = \cancel{T(n-1)} + c \cdot n$$

$$\cancel{T(n-1)} = \cancel{T(n-2)} + c \cdot (n-1)$$

$$\cancel{T(n-2)} = \cancel{T(n-3)} + c \cdot (n-2)$$

$$\cancel{T(n-3)} = \cancel{T(n-4)} + c \cdot (n-3)$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$\cancel{T(2)} = T(1) + c \cdot 2$$

$$T(n) = \underline{T(1)} + c \cdot 2 + c \cdot 3 + \dots + c \cdot n$$

$$= c + c \cdot 2 + c \cdot 3 + \dots + cn$$

$$= c \, n(n+1)/2$$

$$T(n) = O(n^2)$$

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$$T(n) = T(n/2) + c$$

$$= T(n/4) + c + c$$

$$= T(n/8) + c + c + c$$

$$= T(n/16) + c + c + c + c$$

$$= T(n/32) + 5c$$

$$= T\left(\frac{n}{2^6}\right) + \underline{6c}$$

$$T(1) + c \cdot \log(n)$$

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$$T(n) = 2T(n/2) + c \cdot n$$

Exercise Solve this recurrence.

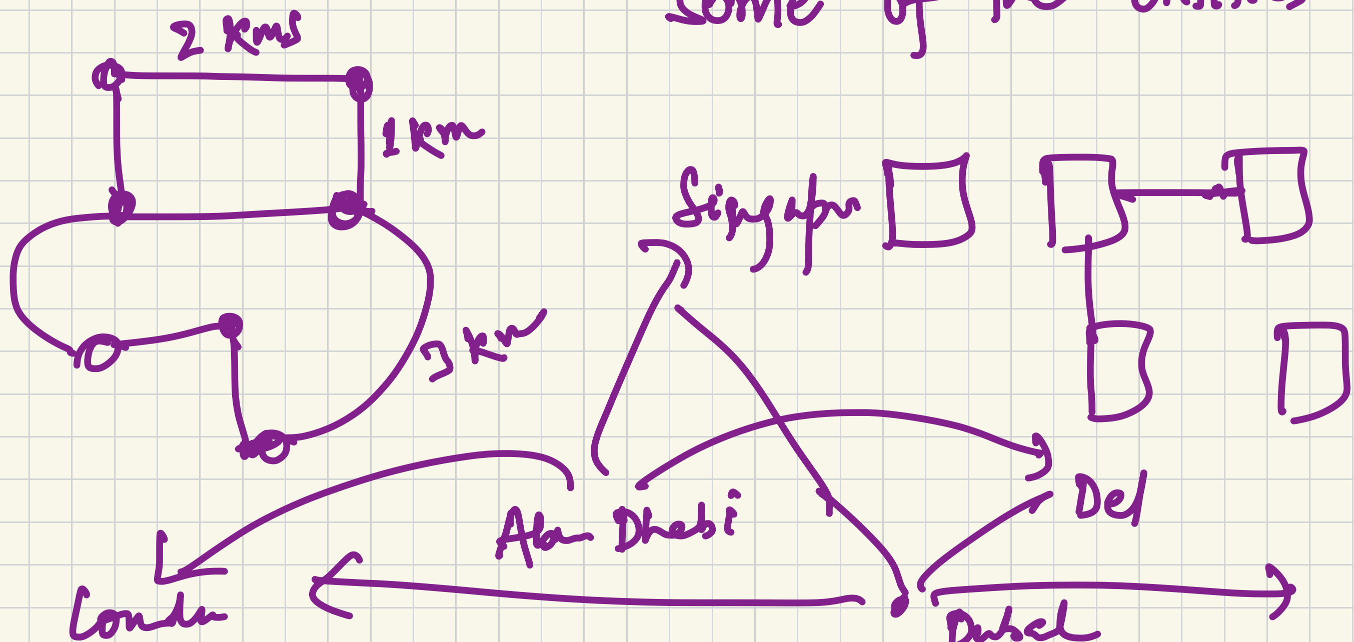
Made with Goodnotes

Graph Theory

Graphs are networks

→ a collection of entities

→ pairwise relationship between some of the entities.



Made with Goodnotes

An **undirected graph** $G = (V, E)$
is a pair (V, E)
where V is the set of vertices
E is the set of edges.

$(\underline{\underline{u, v}})$

$\downarrow$

This is an ordered pair if G is a directed graph.

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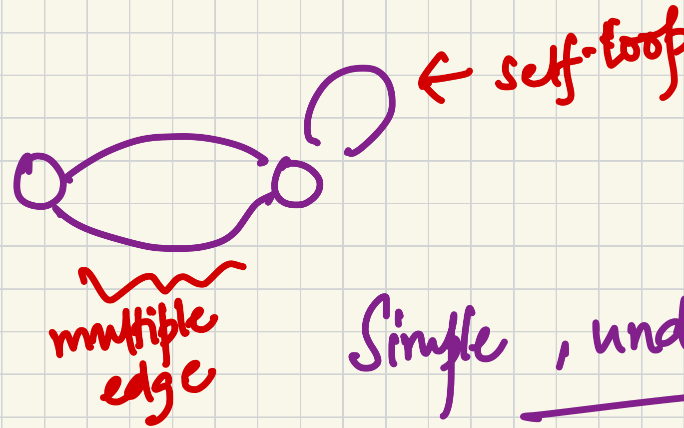
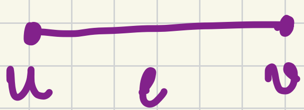
A graph is simple if it has no multiple edges or self-loops.



Simple, undirected graphs



u and v are adjacent

u is a neighbor of v

u and v are endpoints of e

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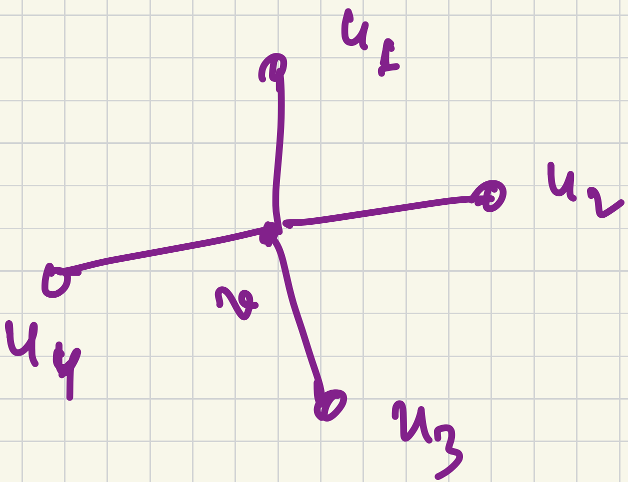
Neighborhood

$$v = \{ u \mid (u,v) \in E \}$$

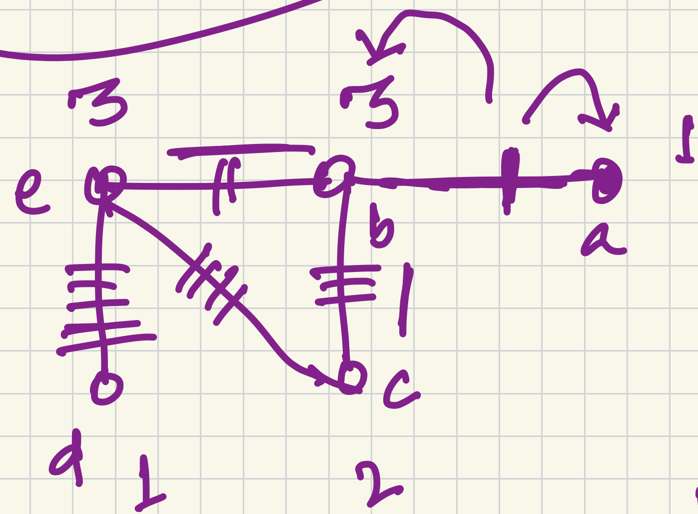


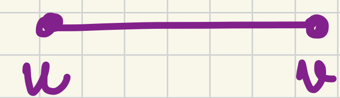
$$\text{degree}(v)$$

$=$ size of the neighborhood of v.

claim

$$\sum_{v \in V} \text{degree}(v) = 2|E|$$





$$\sum \text{degrees} = 3+3+1+1+2$$

$$= 10 \qquad = 2*5$$

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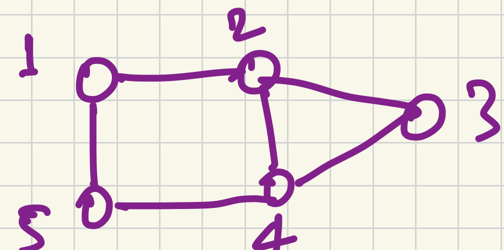
If n_{odd} is the number of nodes with an odd degree, then n_{odd} must be even.

Representing Graphs



Adjacency Matrix || Adjacency List

	1	2	3	4	5
1	0	1	0	0	1
2	1	0	1	1	0
3	0	1	0	1	0
4	0	1	1	0	1
5	1	0	0	1	0

1	2	5	
2	1	4	3
3	2	4	
4	5	3	2 - ..
5	1	4	

Made with Goodnotes

Graph Isomorphism

$G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$

are isomorphic if there exists a bijection $f : V_1 \to V_2$

such that

$\forall \; u_1, v_1 \in V_1$

edge $(u_1, v_1) \in E_1$ iff

edge $(f(u_1), f(v_1)) \in E_2$.

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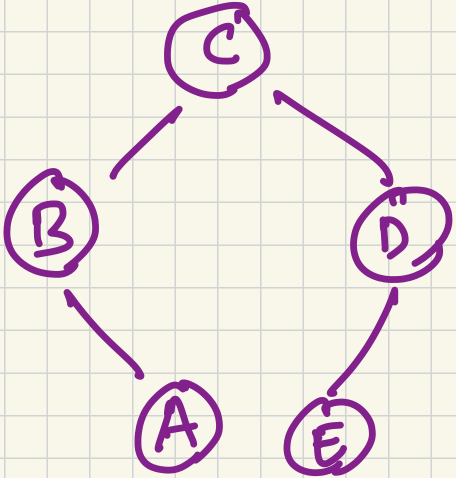
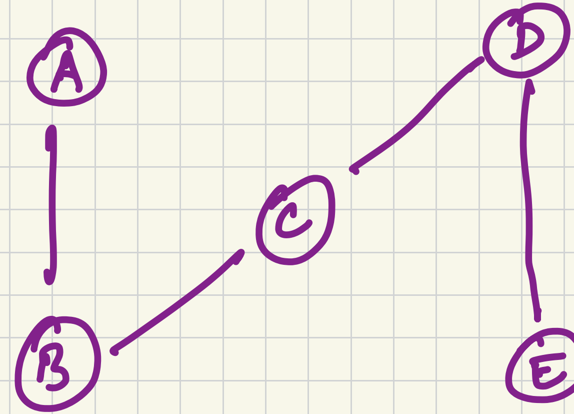
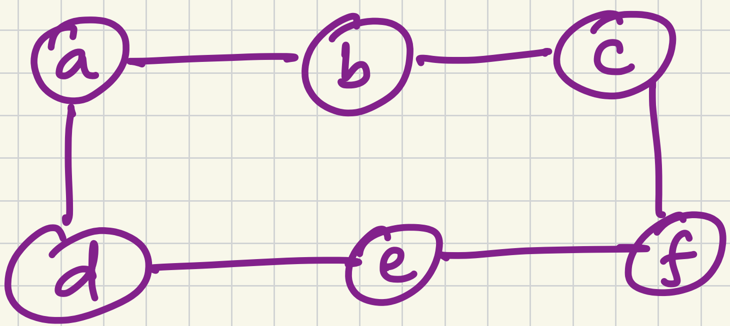
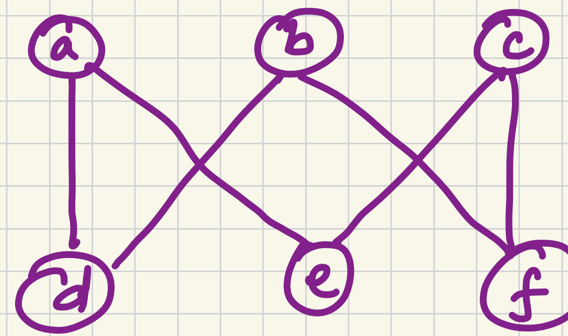




isomorphic





isomorphic

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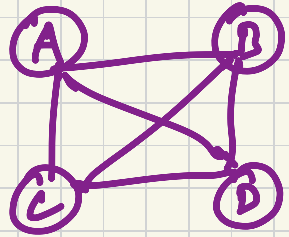

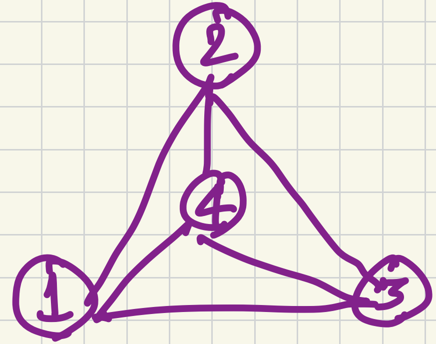

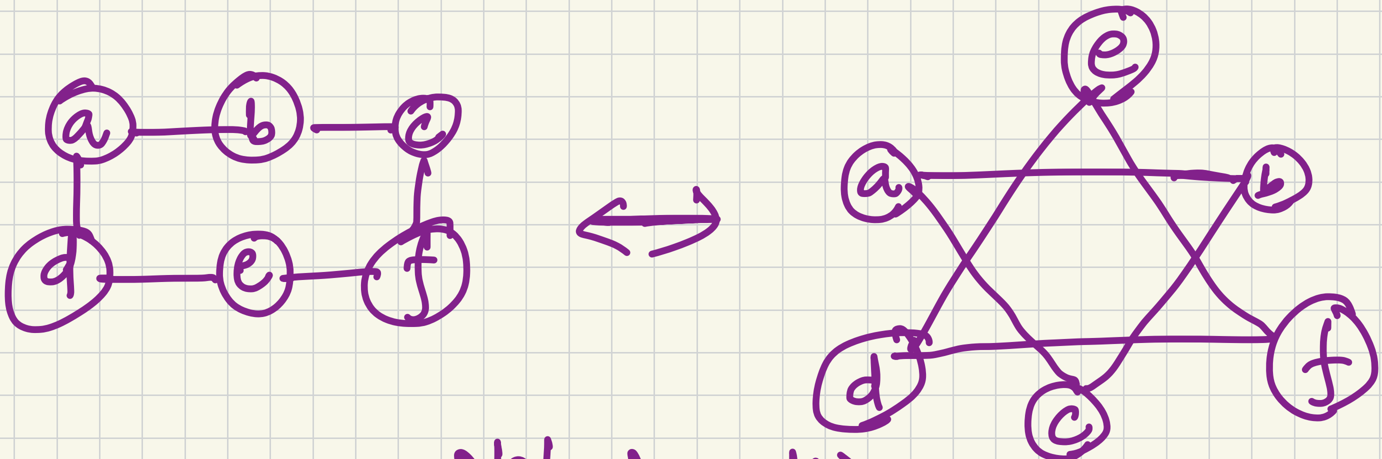

Not isomorphic

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Subgraph of a graph $G = (V, E)$

is another graph $G' = (V', E')$

where $V' \subseteq V$

and $E' \subseteq E$

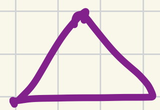
and for every $e \in E'$,

the endpoints of e are in V'.

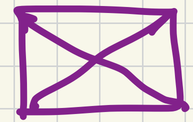
Made with Goodnotes

Special types of graphs

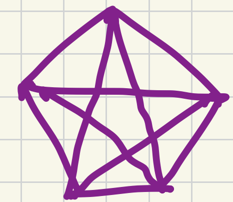
Complete graph (clique)



K_3



K_4



K_5

How many edges does a complete graph of n vertices have: $\binom{n}{2}$.

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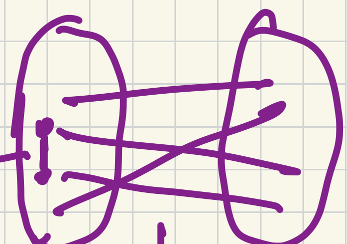
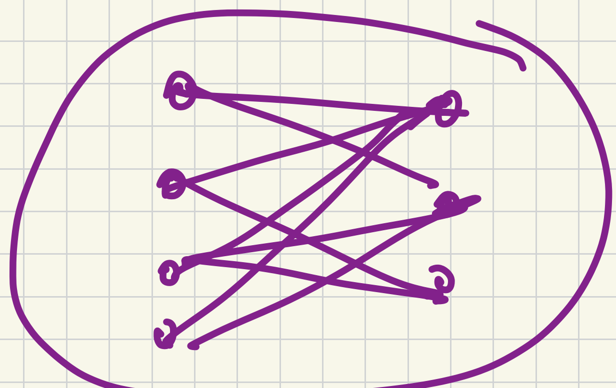
Bipartite graph $G = (V, E)$

where V can be partitioned into L and R such that for every edge $e \in E$ one endpoint of e is in L and the other endpoint is in R.



No edges between two vertices on the same side.

edges going across



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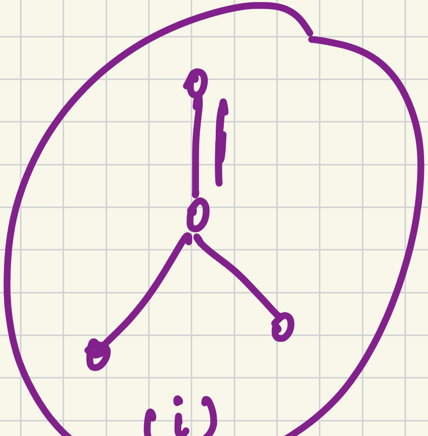
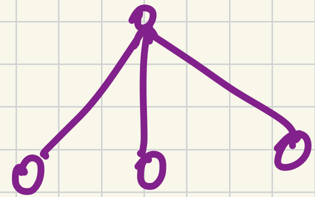
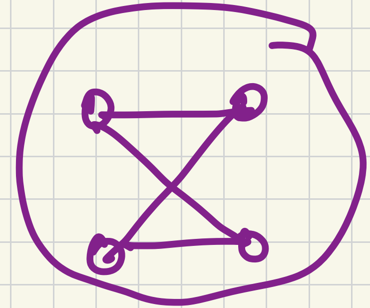


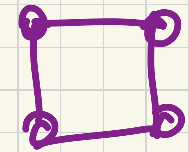
(i)



(ii)



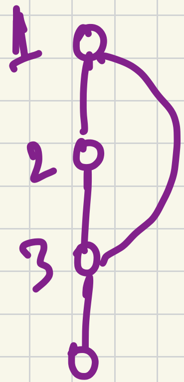
(iii)



(iv)

i) and ii) are bipartite. They are isomorphic.

So are (ii) and (iv).



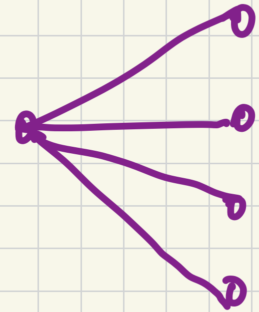
Not bipartite

Suppose, $1 \in L$.

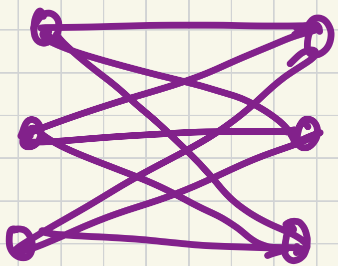
Then both 2 and 3 must be in R.

But that is not possible because (2,3) is an edge!

Complete bipartite graphs



$K_{1,4}$



$K_{3,3}$

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