1. Are $\lfloor x \rfloor + \lfloor y \rfloor$ and $\lfloor x + y \rfloor$ always equal? Why or why not?

2. Assume n is an integer and $n \geq 1$. Is $\lfloor \frac{n+1}{2} \rfloor = \lceil \frac{n}{2} \rceil$ true? Explain.

3. A *perfect number* is a positive integer n that has the following property: n is equal to the sum of all positive integers $k < n$ that evenly divide n. For example, 6 is a perfect number, because 1, 2, and 3 are the positive integers less than 6 that evenly divide 6, and $6 = 1 + 2 + 3$. Find the next perfect number after 6. You may find it easy to write a program that finds the next perfect number after 6.

4. Compute the value of the summation: $\sum_{i=1}^{6} i \cdot 2^i$

5. Rewrite the following set by exhaustively listing its elements.

$$\{n \in \mathbb{Z} \mid 0 \leq n \leq 20 \quad \text{and} \quad n \bmod 5 = n \bmod 7\}$$

6. [**4 marks**]   Prove that set intersection distributes over set union, i.e.

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$