Major exam (*Nov 26, Marks: 25, Duration: 2 hours*) **COL703, Aug-Nov 2022**

Name: **Entry No.:**

1. **[2.5 marks]** Consider the following first-order logic formula:

$$\exists x \ (P(x) \rightarrow \forall y \ P(y))$$

If we consider the universe as the students in COL703, and interpret $P(x)$ as '*x likes pizza*', then the statement above means that *there is someone in the class who if (s)he likes pizza, then everyone in the class likes pizza.*

You are required to either give an interpretation that makes this formula false (you are free to use or adapt the interpretation given above), or prove that the formula is valid (and explain why the interpretation above is not contradictory).

2. **[3.5 marks]** We claim that ROBDDs can be used to count all solutions to a propositional satisfiability problem. Write down an algorithm that computes the number of satisfying truth assignments of a propositional formula, given its ROBDD.

3. **[2 marks]** Use the axioms (A1-A3) given below, and Modus Ponens, to derive $\alpha \rightarrow \alpha$.

- $\alpha \rightarrow (\beta \rightarrow \alpha)$ (A1)
- $(\neg\beta \rightarrow \neg\alpha) \rightarrow ((\neg\beta \rightarrow \alpha) \rightarrow \beta)$ (A2)
- $(\alpha \rightarrow (\beta \rightarrow \gamma)) \rightarrow ((\alpha \rightarrow \beta) \rightarrow (\alpha \rightarrow \gamma))$ (A3)

4. Let F and G be propositional logic formulas such that $F \vDash G$.

   (a) **[1.5 marks]** Show that if F and G have no variables in common, then either F is unsatisfiable, or G is valid.

   (b) **[2.5 marks]** Now let F and G be arbitrary formulas. Show how you will construct a formula H, mentioning only propositional variables common to F and G, such that $F \vDash H$ and $H \vDash G$. [*Note that such a formula H always exists. In order to convince yourself, you may use induction on the number of propositional atoms in F that are not in G. This is not a part of the question, though.*]

5. **[3 marks]** Prove that a set of Horn clauses is unsatisfiable if and only if it has a positive unit resolution refutation. A *positive unit resolution refutation* is a resolution refutation containing only positive unit inferences. A *positive unit inference* is a resolution inference in which one of the hypotheses is a unit clause containing a positive literal only.

6. **[2 marks]** Show that $\Box(\phi \wedge \psi) \leftrightarrow (\Box\phi \wedge \Box\psi)$ is a valid modal logic formula.

7. **[3.5 marks]** Let us consider the sentences that follow. Everyone who loves all animals is loved by someone. Anyone who kills an animal is loved by no one. Ramesh loves all animals. Either Ramesh or Curiosity killed the cat, who is named Molly. Did Curiosity kill Molly? Use resolution to answer this.

8. **[4.5 marks]** Prove that if Γ is an unsatisfiable set of clauses, then Γ has a refutation containing only positive resolution inferences. A *positive resolution inference* is one in which one of the hypothesis is a positive clause. A clause is said to be *positive* if it contains only positive literals.