ACOL 215

(17th sept.)

Suppose we have two binary variables x and y.

Literals : x , x' , y , y'

Consider the products :

$$\underbrace{xy \,,\ xy' \,,\ x'y \,,\ x'y'}_{\text{standard products (minterms)}}$$

Consider the sums :

$$\underbrace{(x+y) \,,\ (x+y') \,,\ (x'+y) \,,\ (x'+y')}_{\text{standard sums (maxterms)}}$$

f_2	f_1	x	y	z	Minterms		Maxterms	
					Terms	Symbols	Terms	Symbols
0	0	0	0	0	$x'y'z'$	m_0	$x+y+z$	M_0
0	1	0	0	1	$x'y'z$	m_1	$x+y+z'$	M_1
0	0	0	1	0	$x'yz'$	m_2	$x+y'+z$	M_2
1	0	0	1	1	$x'yz$	m_3	$x+y'+z'$	M_3
0	1	1	0	0	$xy'z'$	m_4	$x'+y+z$	M_4
1	0	1	0	1	$xy'z$	m_5	$x'+y+z'$	M_5
1	0	1	1	0	xyz'	m_6	$x'+y'+z$	M_6
1	1	1	1	1	xyz	m_7	$x'+y'+z'$	M_7

$$f_1 = m_1 + m_4 + m_7$$

$$f_2 = m_3 + m_5 + m_6 + m_7$$

$$f_1' = m_0 + m_2 + m_3 + m_5 + m_6$$

$$\boxed{f_2' = \underline{m_0 + m_1 + m_2 + m_4}}$$

$$f_1' = x'y'z' + x'yz' + x'yz + xy'z + xyz'$$

$$(f_1')' = (x+y+z) \cdot (x+y'+z) \cdot (x+y'+z') \cdot (x'+y+z') \cdot (x'+y'+z)$$

$$f_1 = \underline{M_0 \cdot M_2 \cdot M_3 \cdot M_5 \cdot M_6}$$

$$(f_2')' = f_2$$

$$= \underline{M_0 \cdot M_1 \cdot M_2 \cdot M_4}$$

$$f_2' = \underline{\sum(0,1,2,4)}$$

$$f_2 = \underline{\prod(0,1,2,4)}$$

Boolean functions expressed as sum of minterms or product of maxterms are said to be in canonical form.

Express $F = A + B'C$ as a sum of minterms.

$$A = A(B+B') = AB + AB'$$

$$= (AB+AB')(C+C')$$

$$= ABC + ABC' + AB'C + AB'C'$$

$$B'C = B'C(A+A') = AB'C + A'B'C$$

$$F = ABC + ABC' + \underline{AB'C} + AB'C' + \underline{AB'C} + A'B'C$$

$$= \underbrace{ABC}_{m_7} + \underbrace{ABC'}_{m_6} + \underbrace{AB'C}_{m_5} + \underbrace{AB'C'}_{m_4} + \underbrace{A'B'C}_{m_1}$$

$$\underline{F(A,B,C) = \Sigma(1,4,5,6,7)}$$

Product of maxterms

$x + yz$
$= (x+y)(x+z)$

$$F = xy + x'z$$

$$= (xy + x')(xy + z)$$

$$= (x' + x)(x' + y)(x + z)(y + z)$$

$$x' + y = x' + y + zz'$$

$$= (x' + y + z)(x' + y + z')$$

$$x+z = x+z+yy'$$

$$= (x+y+z)(x+y'+z)$$

$$\boxed{F(x,y,z) = \Pi(0,2,4,5)}$$

$$y+z = y+z+xx'$$

$$= (x+y+z)(x'+y+z)$$

$$F = (x'+y+z)(x'+y+z')(x+y+z)(x+y'+z)$$
$$(x+y+z)(x'+y+z)$$

$$= \underset{M_4}{(x'+y+z)}\,\underset{M_5}{(x'+y+z')}\,\underset{M_0}{(x+y+z)}\,\underset{M_2}{(x+y'+z)}$$

Conversion between canonical forms

The complement of a function expressed as the sum of minterms equals the sum of minterms missing from the original function.

$$F(A,B,C) = \Sigma(1,4,5,6,7)$$

$$F'(A,B,C) = \Sigma(0,2,3)$$

$$= m_0 + m_2 + m_3$$

$$(F')'(A,B,C) = m_0' \cdot m_2' \cdot m_3'$$

$$= M_0 \cdot M_2 \cdot M_3$$

$$F = \underline{\prod} \underline{(0, 2, 3)}$$

Note: m_i and M_i are complements of each other.

This gives us a conversion procedure: Interchange Σ and $\prod$, and list the numbers missing from the original form.

Exercise

Write $F = xy + x'z$ in the sum of minterms form.

$$F = \underline{\sum(1,3,6,7)}$$

in the product of maxterms form

$$= \prod(0,2,4,5)$$

Standard forms

Sum of products

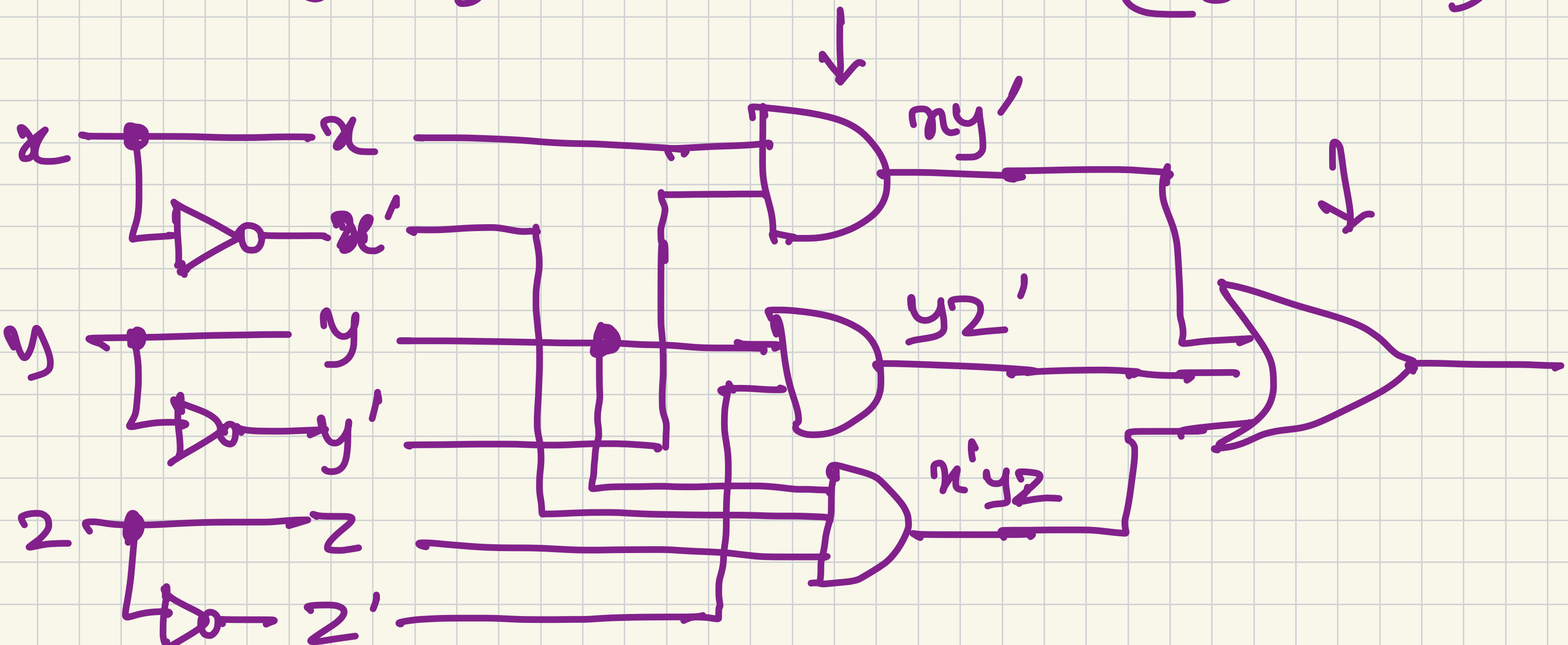
$F_1 = xy' + yz' + x'yz$

product of sums

$F_2 = (x+z) \cdot (y+z') \cdot (x'+y'+z)$

The standard form expressions give a two-level logical gate structure (logic-diagram)

$f = xy' + yz' + x'yz$



Exercise. Draw a two-level logic diagram to implement

$$F = BC' + AB + ACD$$

Exercise Express $F = A + B'C + AD$ as a sum of minterms.

$$F = \sum(2, 3, 8, 9, 10, 11, 12, 13, 14, 15)$$