ACOL 202

Lecture 5
(28th Jan)

Syntax of propositional logic
(last lecture)

Meaning of these formulas

$p \wedge q$

truth value

interpretation of this symbol

truth value

Truth tables

$\boxed{p} \wedge \boxed{q}$

1 True

0 False

$(\underline{\alpha} \wedge \underline{\beta})$

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

p	$\neg p$
T	F
F	T

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

$\top$ (top) True

$\perp$ (bottom) False

(if and only if)

(iff)

$p \Leftrightarrow q$

$(p \Rightarrow q) \wedge (q \Rightarrow p)$

Derived connectives

p	q	$(p \Rightarrow q)$	$(q \Rightarrow p)$	$(p \Rightarrow q) \wedge (q \Rightarrow p)$ / $p \Leftrightarrow q$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

Exclusive or, $\oplus$, xor

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

$(p \vee q) \wedge \neg(p \wedge q)$

Valuation / Assignment

→ Assigns T/F to every atom (atomic proposition) in a formula.

$\boxed{\text{Satisfiability}}$

p	q	r	s	α
T	T	⋮	⋮	T
T	T	⋮	⋮	T
F	F	⋮	⋮	F
T	F	⋮	⋮	F

$$\overset{\text{antecedent}}{(p} \Rightarrow \overset{\text{consequent}}{q)} \wedge (q \Rightarrow r) \wedge (\neg p) \wedge (\neg r)$$

$$F \Rightarrow T \quad \wedge \quad (T \Rightarrow T) \quad \wedge \quad T$$

✓ ✓ ✓

p	F
q	T
r	T

Validity

The formula is always true (for every valuation).

Unsatisfiability

$p \wedge \neg p$ is unsatisfiable.

$p \vee \neg p$ is valid.

A formula is valid iff its negation is unsatisfiable.

$p_1 \quad p_2 \quad p_3 \quad \ldots\ldots \quad \alpha \quad \neg\alpha$

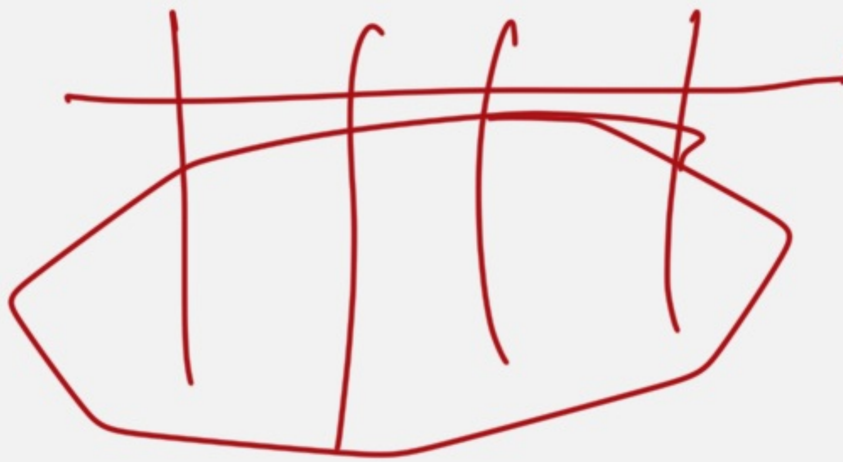
α	$\neg\alpha$
T	F
T	F
T	F
T	F

A valid formula is also called a tautology.

$$\frac{(p \wedge (p \Rightarrow q)) \Rightarrow q}{\text{is a tautology.}}$$

(Modus ponens) MP

$$\boxed{(\alpha \wedge (\alpha \Rightarrow \beta)) \Rightarrow \beta}$$



α	β
T	T
T	F
F	T
F	F

Law of excluded middle (LEM) $(p \vee \neg p)$ tautology

Prove that α is satisfiable iff $\neg\alpha$ is not valid.

forward ($\Rightarrow$) $\quad$ $\underline{\alpha \text{ is satisfiable}} \Rightarrow \neg\alpha$ is not valid

backward ($\Leftarrow$) $\quad$ $\neg\alpha$ is not valid $\Rightarrow$ α is satisfiable.

There must be a valuation

$v : \begin{Bmatrix}\text{atomic}\\\text{props}\end{Bmatrix} \to \{T, F\}$

which makes α true.

The same valuation v will make $\neg\alpha$ false

$\therefore$ $\neg\alpha$ is not true for every valuation.

$\therefore$ $\neg\alpha$ is not valid.

$\Leftarrow$ $\neg\alpha$ is not valid.

Let AP denote the set of atomic proposition.

Then a valuation is a function

$v : AP \to \{T, F\}$.

Logical equivalence

$$\alpha \equiv \beta$$

α and β are logically equivalent if they agree on all valuations.

Example

$\neg(p \wedge q)$ and $(p \wedge q) \Rightarrow \neg q$ are logically equivalent.

$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$

$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$$

$$\neg (p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg (p \vee q) \equiv \neg p \wedge \neg q$$

$$p \Rightarrow q \equiv \neg p \vee q$$

p	q	$\neg p$	$p \Rightarrow q$	$\neg p \vee q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

Implication

$p \Rightarrow q$

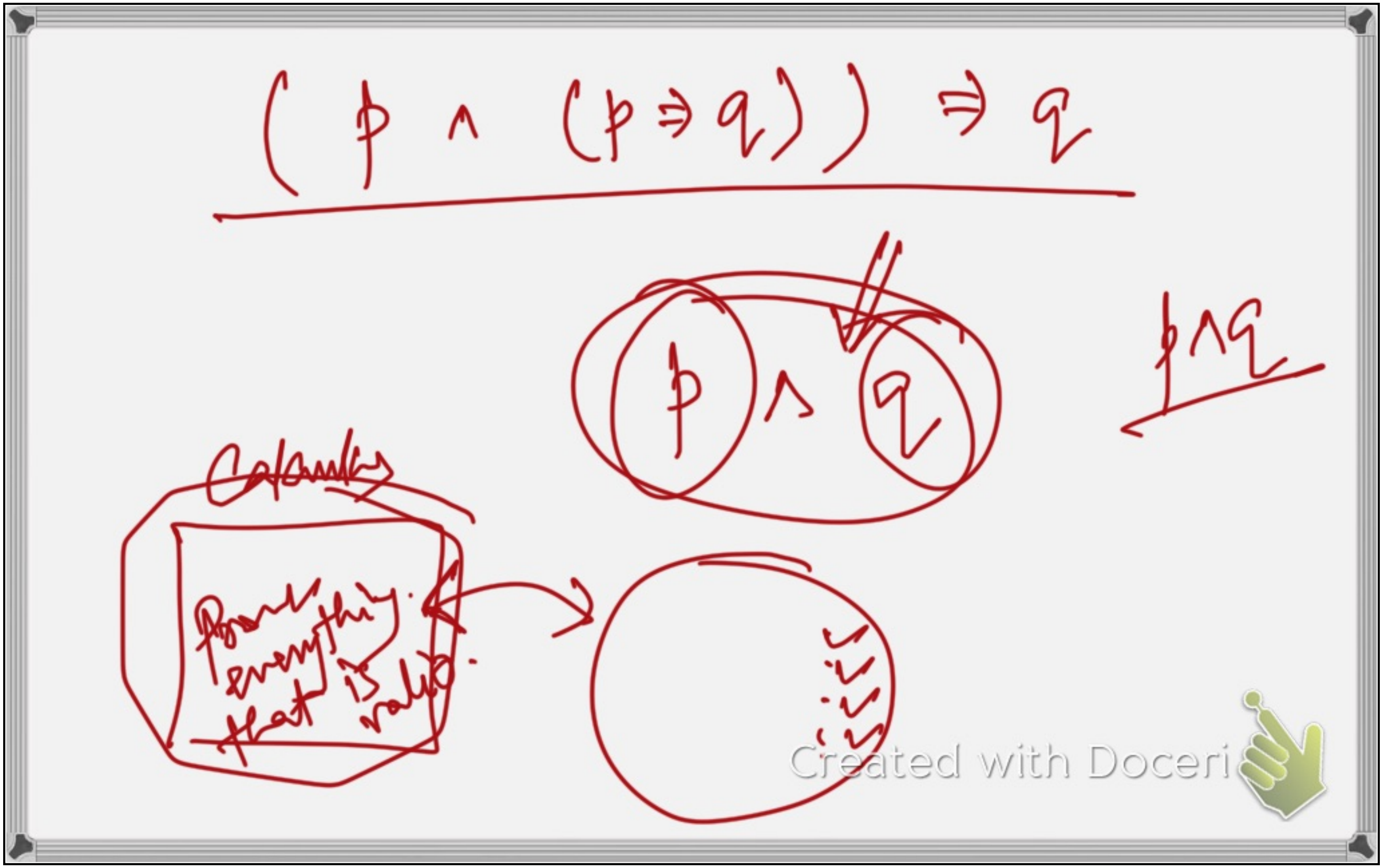
converse $q \Rightarrow p$

contrapositive $\neg q \Rightarrow \neg p$

inverse $\neg p \Rightarrow \neg q$

Claim The contrapositive is logically equivalent to the implication.

$$p \Rightarrow q \equiv \neg q \Rightarrow \neg p$$


$(p \wedge (p \Rightarrow q)) \Rightarrow q$
$p \wedge q$
$p \wedge q$
Calculus
Prove everything that is valid.
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Normal Forms

Predicate Logic

Next lecture