Assignment - 04

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Solution ① → given N inputs neurons,
H hidden neurons
C output neurons.

ⓐ Connection between input-hidden and hidden outputs layers.

So total weight $= N \times H + H \times C$

$= (N + C) \times H$

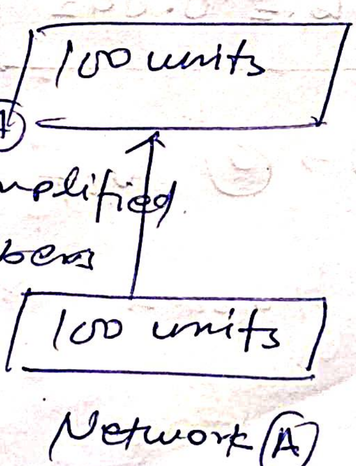
ⓑ If there are also direct connections from input to output.

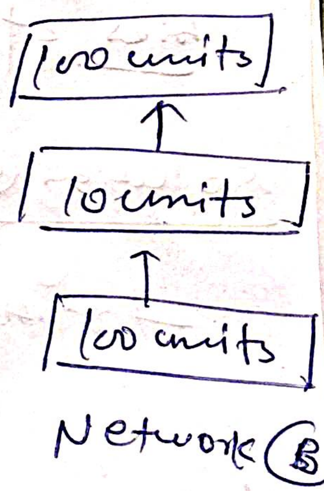
Total weights $= N \cdot H + H \cdot C + N \cdot C$

Solution ② → Two multiplayer perceptron with linear activation

ⓐ one advantage of Network Ⓐ

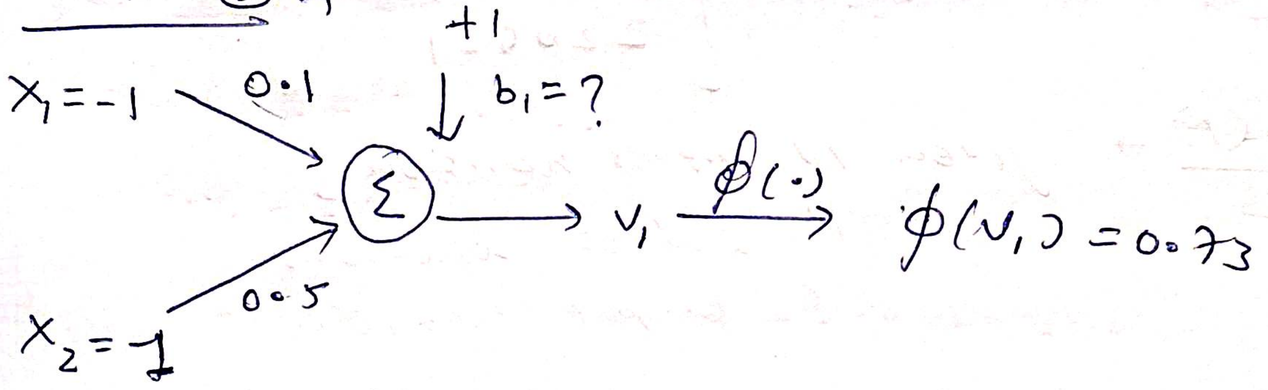
we can see A is more simplified model because of less numbers of hidden layer and weight. so we can say that it is less prone to overfitting than B





ⓑ B is deeper network than A so non-linear relationship can be better predicted and deeper network allows for hierarichial feature extraction

Solution (3) →



given $a = 2$, Now, $z = a(w_1 x_1 + w_2 x_2 + b_1)$

$z = 2(0.1 \times (-1) + 0.5 \times 1 + b_1)$

$\frac{1}{1+e^{-z}} = 0.73 \Rightarrow e^{-z} = \frac{27}{73}$

$z = \ln \frac{73}{27}$

$\therefore 2b_1 = 0.995 - 0.8$

$z = 0.995$

bias of $\boxed{b_1 = 0.097}$

Solution (4) → (a) NOT perceptron.

truth table →

x	NOT X
0	1
1	0

Let perceptron output be. $\begin{cases} 1 & \text{if } w \cdot x + b > 0 \\ 0 & o/w \end{cases}$ (wt, input)

Now → (i) when $x = 0$, $w \times 0 + b = b$
if $b > 0$ then output will 1 (input as 0)

(ii) when $x = 1$, $w \cdot 1 + b = w + b$
$w + b \leq 0$ then output well 0

so let $b = 2$, so $w + 2 \leq 0 \Rightarrow \boxed{w \leq -2}$

let $w = -2$, then output $= 1$ if $(-2x + 2 > 0)$

else 0

(b) NAND Perceptron

let A & B be input

A	B	AND	NAND
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	0

perceptron output is $= \begin{cases} 1 & \text{if } w_1 x_1 + w_2 x_2 + b > 0 \\ 0 & \text{o/w} \end{cases}$

Now → (1) for $A = 0$, $B = 0$

$$w_1 \cdot 0 + w_2 \cdot 0 + b = b > 0$$

(2) for $A = 0$, $B = 1$

$$w_1 \cdot 0 + w_2 \cdot 1 + b = w_2 + b > 0$$

(3) for $A = 1$, $B = 0$

$$w_1 \cdot 1 + w_2 \cdot 0 + b = w_1 + b > 0$$

(4) for $A = 1$; $B = 1$

$$w_1 \cdot 1 + w_2 \cdot 1 + b = w_1 + w_2 + b \leq 0$$

let $b = 2$ then $w_2 > -2$, $w_1 > -2$ and

$w_1 + w_2 \leq -2$

Now

$$w_1 = w_2 = -1.5$$

so for the (4) case: $-1.5 + (-1.5) + 2 = -3 + 2$

Hence final values are $= -1 \leq 0$

$w_1 = -1.5$, $w_2 = -1.5$, $b = 2$

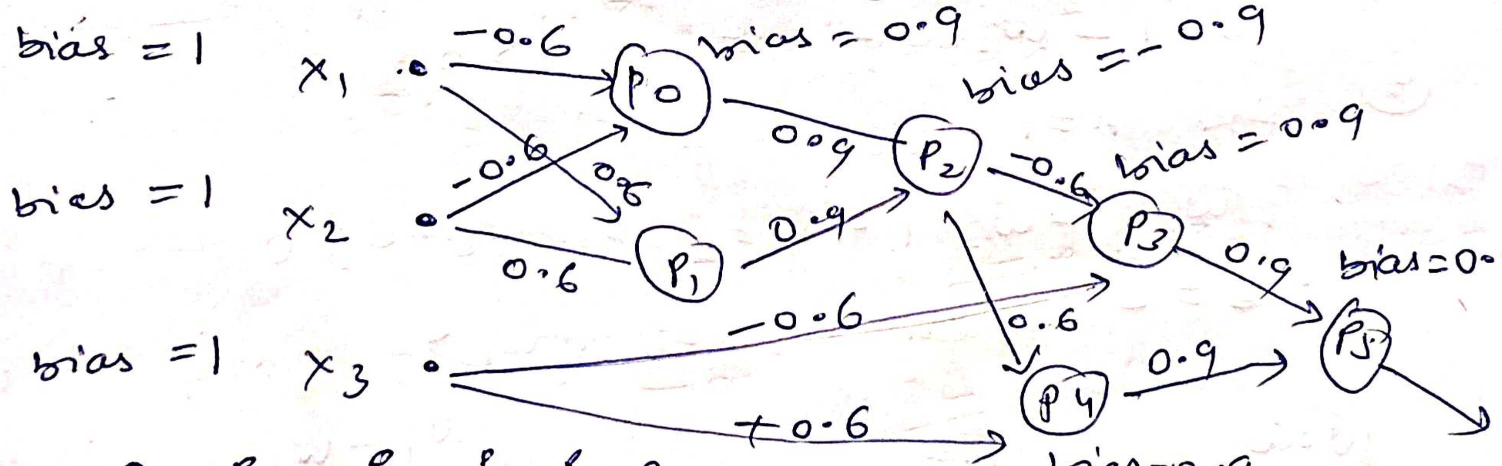
$$\text{output} = \begin{cases} 1 & \text{if } -1.5x_1 - 1.5x_2 + 2 > 0 \\ 0 & \text{o/w} \end{cases}$$

Solution - 5 ->

Let us suppose 1 = odd & 2 = even, therefore our input-output table will be

x_1	x_2	x_3	y
1	1	1	1
1	1	0	0
1	0	1	0
0	1	1	0
1	0	0	1
0	1	0	1
0	0	1	1
0	0	0	0

$y = ((x_1 \text{ XOR } x_2) \text{ XOR } x_3)$



P0, P1, P2, P3, P4 & P5 all have sign function

where $\text{sgn}(x) = \begin{cases} 1 & x > 0 \\ -1 & x < 0 \\ 0 & x = 0 \end{cases}$

Solution (6) :-

To derive update equation for an MLP that uses ReLU in its hidden units,

let d be the no of inputs

$\therefore x \in \mathbb{R}^d$ is input layer

$W^{(1)} \in \mathbb{R}^{H \times d}$ be ~~hidden layer~~ wt matrix for hidden layer

$b^{(1)} \in \mathbb{R}^H$ be the bias vector

hidden layer $z^{(1)} = W^{(1)}x + b^{(1)}$

ReLU activation function,

$$h = \text{ReLU}(z^{(1)})$$

$$= \max(0, z^{(1)})$$

for hidden layer to ouput layer

$W^{(2)} \in \mathbb{R}^{1 \times H}$, $b^{(2)} \in \mathbb{R}$

output : $\hat{y} = W^{2}(h) + b^{(2)}$

loss (MSE) $\boxed{L = \frac{1}{2}(y - \hat{y})^2}$

Backpropagation :-

$$\frac{dL}{d\hat{y}} = -(y - \hat{y})$$

$$\frac{dL}{dW^{(2)}} = \frac{dL}{d\hat{y}} \cdot \frac{d\hat{y}}{dW^{(2)}} = -(y - \hat{y}) \cdot h^T$$

$$\frac{\partial L}{\partial b_j^{(1)}} = -(y - \hat{y})$$

$$\frac{\partial L}{\partial h} = \frac{\partial L}{\partial \hat{y}} \cdot w^{(2)} \qquad \frac{\partial}{\partial z}(\mathrm{ReLU}(z)) = \begin{cases} 1 & z > 0 \\ 0 & z \leq 0 \end{cases}$$

$D = 1 \cdot (z^{(1)} > 0)$ is a diagonal matrix where
1 indicates active ReLU units.

$$\frac{\partial L}{\partial z^{(1)}} = \frac{\partial L}{\partial h} \cdot D$$

$$\frac{\partial L}{\partial w^{(1)}} = \frac{\partial L}{\partial z^{(1)}} \cdot X^T$$

$$\frac{\partial L}{\partial b^{(1)}} = \frac{\partial L}{\partial z^{(1)}}$$

Now →

$$w^{(2)} = w^{(2)} - \eta \frac{\partial L}{\partial w^{(2)}}$$

$$b^{(2)} = b^{2} - \eta \frac{\partial L}{\partial b^{(2)}}$$

$$w^{(1)} = w^{(1)} - \eta \frac{\partial L}{\partial w^{(1)}}$$

$$w^{(1)} = b^{(1)} - \eta \frac{\partial L}{\partial b^{(1)}}$$

Now →

$$w^{(2)} = w^{(2)} + \eta (y - \hat{y}) h^T$$

$$b^{(2)} = b^{(2)} + \eta (y - \hat{y})$$

$$w^{(1)} = w^{(1)} - \eta \left(\left(\frac{\partial L}{\partial \hat{y}} \cdot w^{(2)} \right) \cdot D \right) X^T$$

$$b^{(1)} = b^{(1)} - \eta \left(\left(\frac{\partial L}{\partial \hat{y}} \cdot w^{(2)} \right) \cdot D \right.$$