220621-ra-27062025

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given

We have a dataset of 200 samples:

- 120 positive (P)
- 80 negative (N)

1(a) Gini Index before splitting For a binary classification, the Gini index is:

$$\text{Gini} = 1 - p_1^2 - p_2^2$$

Where:

- p_1 = fraction of positive examples
- p_2 = fraction of negative examples

since

- Total samples = 200
- $p_1 = \frac{120}{200} = 0.6$
- $p_2 = \frac{80}{200} = 0.4$

$$\text{Gini} = 1 - (0.6)^2 - (0.4)^2 = 1 - 0.36 - 0.16 = \boxed{0.48}$$

1(b) Gini Index after a specific split The split results in:

- **Left subset**: 50 positive, 10 negative → 60 samples
- **Right subset**: 70 positive, 70 negative → 140 samples

Let's compute the **Gini impurity for each subset**, and then compute the **weighted Gini**.

Left Subset (60 samples):

- Positive = 50 $\quad p_1 = \frac{50}{60} = 0.8333$
- Negative = 10 $\quad p_2 = \frac{10}{60} = 0.1667$

$$\text{Gini}_{\text{left}} = 1 - (0.8333)^2 - (0.1667)^2 = 1 - 0.6944 - 0.0278 = \boxed{0.2778}$$

Right Subset (140 samples):

- Positive = 70 $\quad p_1 = \frac{70}{140} = 0.5$
- Negative = 70 $\quad p_2 = \frac{70}{140} = 0.5$

$$\text{Gini}_{\text{right}} = 1 - (0.5)^2 - (0.5)^2 = 1 - 0.25 - 0.25 = \boxed{0.5}$$

Compute Weighted Gini Index The weighted Gini is:

$$\text{Gini}_{\text{weighted}} = \frac{60}{200} \cdot \text{Gini}_{\text{left}} + \frac{140}{200} \cdot \text{Gini}_{\text{right}}$$

$$= 0.3 \cdot 0.2778 + 0.7 \cdot 0.5 = 0.0833 + 0.35 = \boxed{0.4333}$$

- **Before splitting**: Gini = 0.48
- **After splitting**: Gini = 0.4333
- **Decrease in impurity** = 0.48 − 0.4333 = **0.0467**

Since the Gini impurity **decreased**, this split is considered to have **improved the purity** of the data.

The **Sum of Squared Errors (SSE)** is calculated as:

$$\text{SSE} = \sum_i (y_i - \bar{y})^2$$

Where:

- $\bar{y}$ = mean of y values in the group
- SSE measures how much variation there is in y from the mean

We want to **split the data on x** and find the point where **SSE is minimized**

Possible Split Points for x :

We can only split between the values of x . Possible split points:

```
(1+2)/2 = 1.5
(2+3)/2 = 2.5
(3+4)/2 = 3.5
(4+5)/2 = 4.5
(5+6)/2 = 5.5
(6+7)/2 = 6.5
(7+8)/2 = 7.5
```

For each split point:

- Divide data into two groups:
  - Left: x split
  - Right: x > split
- Compute mean of y in each group
- Compute SSE for both groups
- Add them → total SSE

Summary

Split at x	Total SSE
1.5	331.0
2.5	205.0
3.5	125.0
4.5	**82.75**
5.5	92.0
6.5	125.5
7.5	166.0

The best split occurs at **x = 4.5**, with **minimum SSE = 82.75**

Part (b): Build the First Split of a Regression Tree Using SSE Now that we know the best split is at **x = 4.5**, we split the data into:

- **Left node (x 4.5)**: x = 1, 2, 3, 4 → y = [10, 12, 15, 18] → mean = 13.75
- **Right node (x > 4.5)**: x = 5, 6, 7, 8 → y = [21, 25, 28, 30] → mean = 26.0

Regression Tree (First Split Only):

```
         x  <= 4.5
        /        \
   y = 13.75      y = 26.0
```

Final Answer

(a) Best split point on x = 4.5

- SSE is minimized at this point: **Total SSE = 82.75**

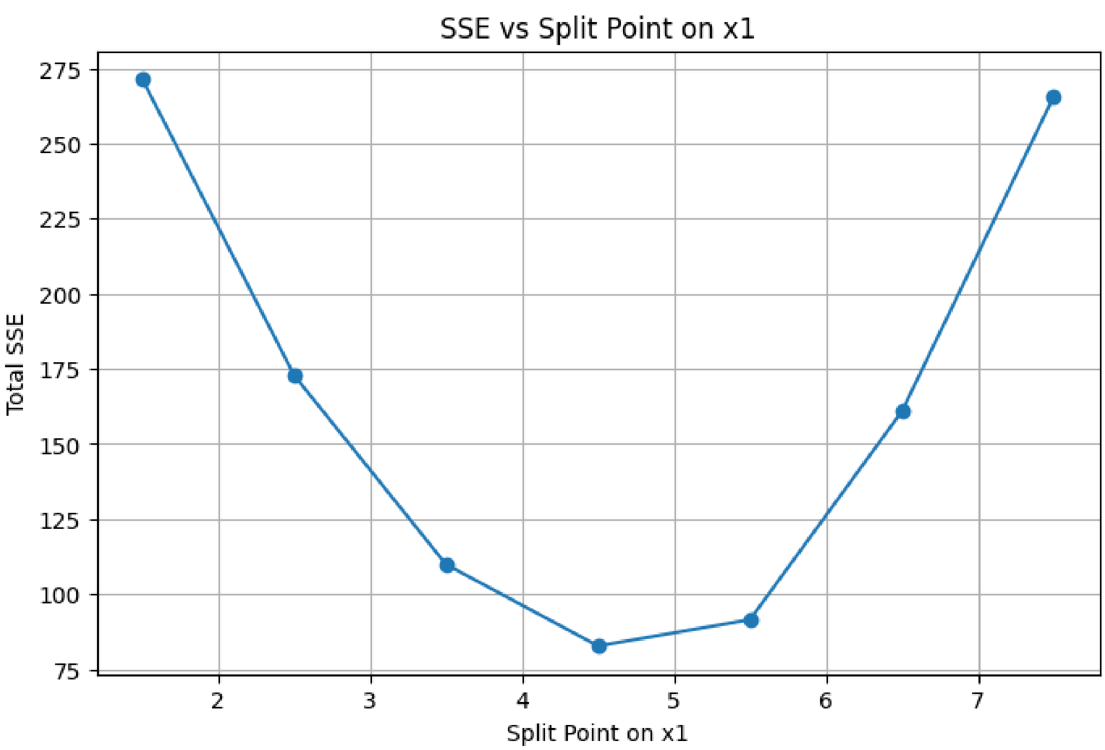
(b) First split of the regression tree:

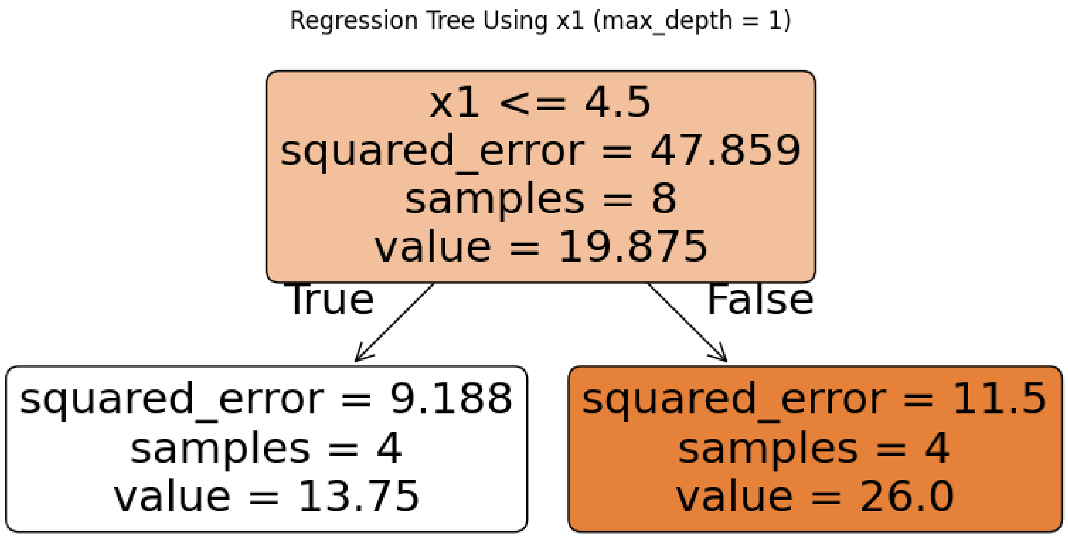
- Split on **x 4.5**
- Predictions:
  - Left: y = 13.75
  - Right: y = 26.0

[34]:
```python
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from sklearn.tree import DecisionTreeRegressor, plot_tree

# dataset
data = {
    'x1': [1, 2, 3, 4, 5, 6, 7, 8],
    'x2': [5, 6, 8, 10, 12, 15, 18, 20],
    'y':  [10, 12, 15, 18, 21, 25, 28, 30]
}
df = pd.DataFrame(data)

# Compute SSE for all possible split points on x1
splits = [(df['x1'][i] + df['x1'][i+1]) / 2 for i in range(len(df['x1']) - 1)]
sse_results = []

for split in splits:
    left = df[df['x1'] <= split]
    right = df[df['x1'] > split]

    mean_left = left['y'].mean()
    mean_right = right['y'].mean()

    sse_left = ((left['y'] - mean_left) ** 2).sum()
    sse_right = ((right['y'] - mean_right) ** 2).sum()

    total_sse = sse_left + sse_right
    sse_results.append((split, total_sse))

# Store SSE results and find best split
sse_df = pd.DataFrame(sse_results, columns=['Split_x1', 'Total_SSE'])
best_split = sse_df.loc[sse_df['Total_SSE'].idxmin()]
print("Best split point on x1:", best_split['Split_x1'])
print("Minimum Total SSE:", best_split['Total_SSE'])
```

```python
# Plot SSE vs split point
plt.figure(figsize=(8, 5))
plt.plot(sse_df['Split_x1'], sse_df['Total_SSE'], marker='o')
plt.xlabel("Split Point on x1")
plt.ylabel("Total SSE")
plt.title("SSE vs Split Point on x1")
plt.grid(True)
plt.show()

# Train a regression tree using x1 only (max_depth=1)
reg_tree = DecisionTreeRegressor(max_depth=1)
reg_tree.fit(df[['x1']], df['y'])

# Plot the regression tree
plt.figure(figsize=(10, 5))
plot_tree(reg_tree, feature_names=['x1'], filled=True, rounded=True)
plt.title("Regression Tree Using x1 (max_depth = 1)")
plt.show()
```

```
Best split point on x1: 4.5
Minimum Total SSE: 82.75
```





Since building the **best decision tree** using `Budget (B)` and `Reviews (R)` to predict whether a **movie is a Hit** (+) or **Flop** (−).

So need to Predict **Hit (H = +)** or **Flop (H = −)** using:

- Budget (B): 0 = Low, 1 = High
- Reviews (R): 0 = Negative, 1 = Positive

B (Budget)	R (Review)	H (Hit/Flop)	Count
0	0	−	5
0	0	+	1
0	1	−	0
0	1	+	4
1	0	−	3
1	0	+	1
1	1	−	2
1	1	+	0

Given Frequency Table: Let's convert this into a **data table**:

B	R	Label	Count
0	0	−	5
0	0	+	1
0	1	−	0

B	R	Label	Count
0	1	+	4
1	0	−	3
1	0	+	1
1	1	−	2
1	1	+	0

Total Entropy (before any split) calculating entropy to measure impurity.

$$\text{Entropy} = -p_+ \log_2 p_+ - p_- \log_2 p_-$$

Total Counts:

- Total $+ = 1 + 4 + 1 + 0 = \mathbf{6}$
- Total $- = 5 + 0 + 3 + 2 = \mathbf{10}$
- Total = 16

$$p_+ = \frac{6}{16} = 0.375, \quad p_- = \frac{10}{16} = 0.625$$

$$\text{Entropy}_{\text{total}} = -0.375 \log_2(0.375) - 0.625 \log_2(0.625) \approx 0.954$$

Try splitting on Budget (B)

Group B = 0:

- (0,0): 5 −, 1 +
- (0,1): 0 −, 4 +
- Total: 10
- Pos = 5, Neg = 5 $\rightarrow p_+ = 0.5,\ p_- = 0.5$

$$\text{Entropy}_B = 0 = -0.5 \log_2(0.5) - 0.5 \log_2(0.5) = 1.0$$

Group B = 1:

- (1,0): 3 −, 1 +
- (1,1): 2 −, 0 +
- Total: 6
- Pos = 1, Neg = 5 $p_+ = \frac{1}{6},\ p_- = \frac{5}{6}$

$$\text{Entropy}_B = 1 = -\left(\frac{1}{6} \log_2 \frac{1}{6} + \frac{5}{6} \log_2 \frac{5}{6}\right) \approx -[0.431 + 0.219] = 0.650$$

Weighted Entropy for split on B:

$$E_B = \frac{10}{16} \cdot 1.0 + \frac{6}{16} \cdot 0.650 = 0.906$$

Information Gain (IG) from split on B:

$$IG_B = 0.954 - 0.906 = \boxed{0.048}$$

Try splitting on Review (R)

0.0.1 Group R = 0:

- (0,0): 5 −, 1 +
- (1,0): 3 −, 1 +
- Total = 10
- Pos = 2, Neg = 8

$$p_+ = 0.2, \quad p_- = 0.8$$

$$\text{Entropy}_R = 0 = -0.2 \log_2 0.2 - 0.8 \log_2 0.8 \approx 0.722$$

0.0.2 Group R = 1:

- (0,1): 0 −, 4 +
- (1,1): 2 −, 0 +
- Total = 6
- Pos = 4, Neg = 2 $p_+ = \frac{4}{6}, \ p_- = \frac{2}{6}$

$$\text{Entropy}_R = 1 = -\left(\frac{4}{6} \log_2 \frac{4}{6} + \frac{2}{6} \log_2 \frac{2}{6}\right) \approx 0.918$$

0.0.3 Weighted Entropy for split on R:

$$E_R = \frac{10}{16} \cdot 0.722 + \frac{6}{16} \cdot 0.918 = 0.806$$

Information Gain (IG) from split on R:

$$IG_R = 0.954 - 0.806 = \boxed{0.148}$$

Attribute	Info Gain
Budget (B)	0.048
Review (R)	**0.148**

Selecting best attribute So, **Review (R)** should be the **root node** of the tree.

```
Using majority labels in leaf nodes:

### Breakdown:

* R = 0:

  * B = 0 → (5 -, 1 +) → Predict: **Flop (-)**
  * B = 1 → (3 -, 1 +) → Predict: **Flop (-)**
* R = 1:

  * B = 0 → (0 -, 4 +) → Predict: **Hit (+)**
  * B = 1 → (2 -, 0 +) → Predict: **Flop (-)**

---
```

if R == 0: Predict = Flop else: if B == 0: Predict = Hit else: Predict = Flop "'

Decision Tree using Cross-Entropy Impurity** **(a) Selecting the Root Attribute**

To determine the best root, we compute the **weighted cross-entropy** for each attribute:

Attribute	Weighted Entropy
capacity	**0.441**(best)
maintenance	0.737
price	0.932
airbag	0.942

- **capacity** gives the **lowest entropy**, meaning it best separates the data.

Answer (a): The best attribute for the root node is `capacity`.

(b) Next Best Split After Root

- Splitting on `capacity` gives 3 branches:
  - capacity = 2: All "no" → pure node → no split needed

- capacity = 5: All "yes" → pure node → no split needed
- capacity = 4: Mixed → further splitting required

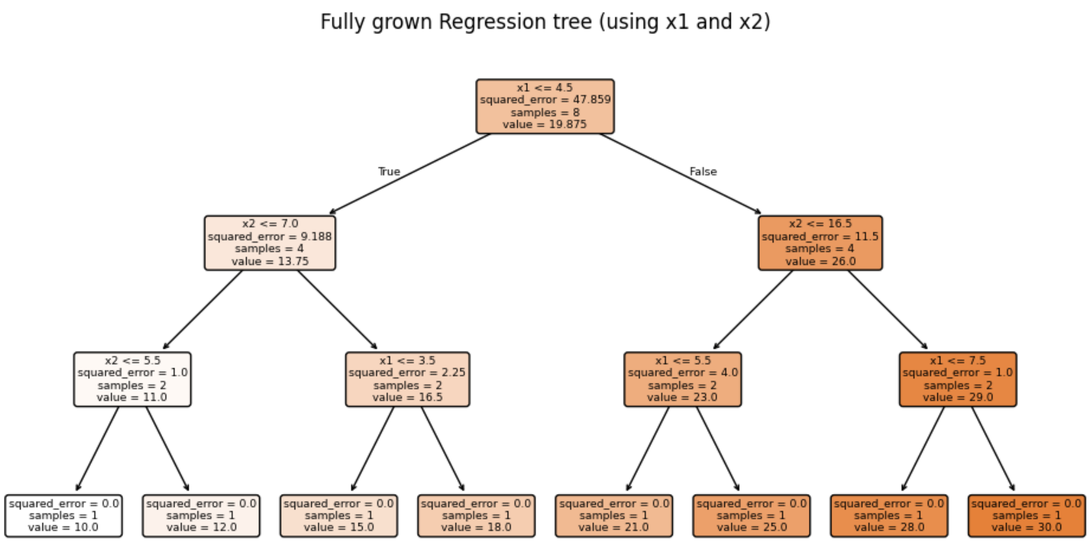
We then evaluate attributes (price, maintenance, airbag) on the **subset where capacity = 4** to find the next best split.

Answer (b): For the node where `capacity = 4`, perform a cross-entropy calculation again on the remaining attributes. The attribute with the lowest entropy in this subset becomes the **next split**.

0.0.4 Final answer:

- **Root node:** `capacity`
- **Further split needed only on:** `capacity = 4` group
- **Next best attribute:** Determined by applying entropy on that subgroup

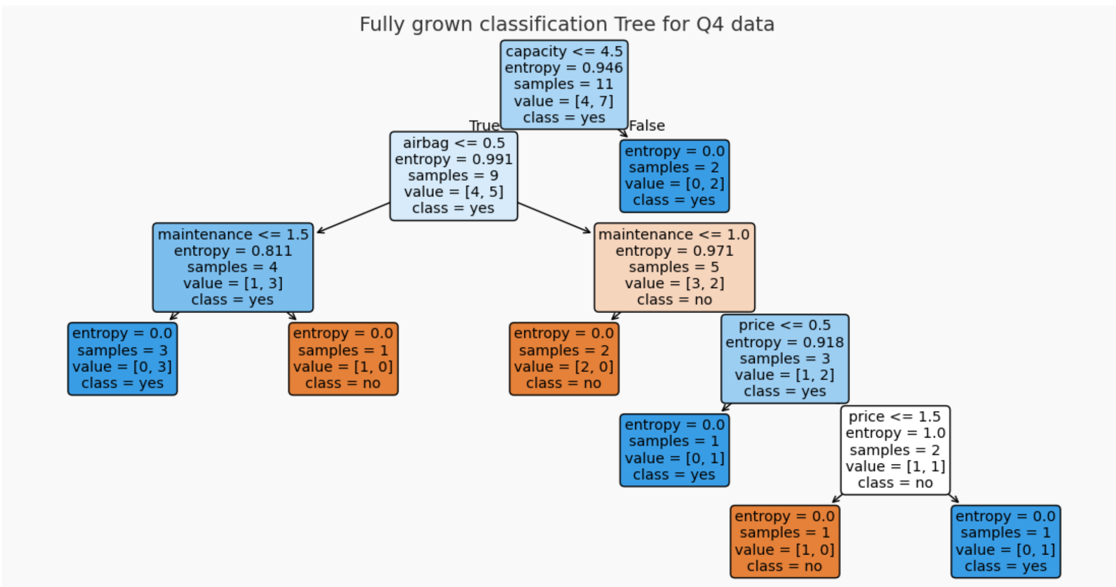
[35]:
```python
import pandas as pd
import matplotlib.pyplot as plt
from sklearn.tree import DecisionTreeRegressor, plot_tree

#  the dataset
data = {
    'x1': [1, 2, 3, 4, 5, 6, 7, 8],
    'x2': [5, 6, 8, 10, 12, 15, 18, 20],
    'y':  [10, 12, 15, 18, 21, 25, 28, 30]
}
df = pd.DataFrame(data)

#  Fiting a fully grown regression tree (no max depth limit)
reg_tree = DecisionTreeRegressor()
reg_tree.fit(df[['x1', 'x2']], df['y'])

#  Visualize the regression tree
plt.figure(figsize=(12, 6))
plot_tree(reg_tree, feature_names=['x1', 'x2'], filled=True, rounded=True)
plt.title("Fully grown Regression tree (using x1 and x2)")
plt.show()
```



[36]:

```python
import pandas as pd
from sklearn.preprocessing import LabelEncoder
from sklearn.tree import DecisionTreeClassifier, plot_tree
import matplotlib.pyplot as plt

# Define the dataset
data = {
    'price':       ['low', 'low', 'low', 'low', 'med', 'med', 'med', 'med', 'high', 'high', 'high'],
    'maintenance': ['low', 'med', 'low', 'high', 'med', 'med', 'high', 'high', 'med', 'high', 'high'],
    'capacity':    [2, 4, 4, 4, 4, 4, 2, 5, 4, 2, 5],
    'airbag':      ['no', 'yes', 'no', 'no', 'no', 'yes', 'yes', 'no', 'yes', 'yes', 'yes'],
    'profitable':  ['yes', 'no', 'yes', 'yes', 'no', 'yes', 'no', 'yes', 'yes', 'no', 'yes']
}

df = pd.DataFrame(data)

# Encode categorical features
le = LabelEncoder()
for col in ['price', 'maintenance', 'airbag', 'profitable']:
    df[col] = le.fit_transform(df[col])
```

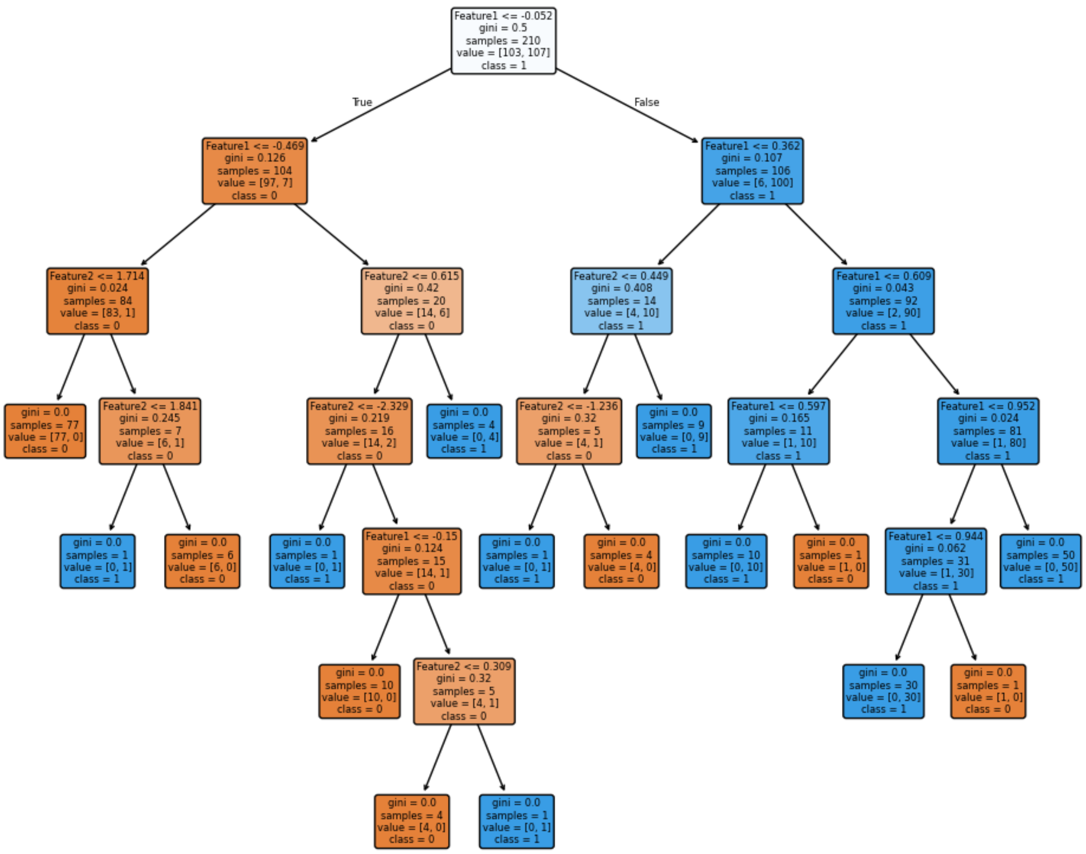
```python
#  Define features and label
X = df.drop(columns='profitable')
y = df['profitable']

# Train a fully grown classification tree
clf = DecisionTreeClassifier(criterion='entropy')  # Fully grown by default
clf.fit(X, y)

# Visualize the classification tree
plt.figure(figsize=(14, 7), facecolor="#f9f9f9")
plot_tree(clf, feature_names=X.columns, class_names=['no', 'yes'], filled=True,
 ↪rounded=True)
plt.title(" Fully grown classification Tree for Q4 data", fontsize=14,
 ↪color="#333333")
plt.show()
```



[37]:
```python
# part a:

import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from sklearn.tree import DecisionTreeClassifier, plot_tree
from sklearn.metrics import mean_squared_error

# Define file paths for training and testing datasets
```

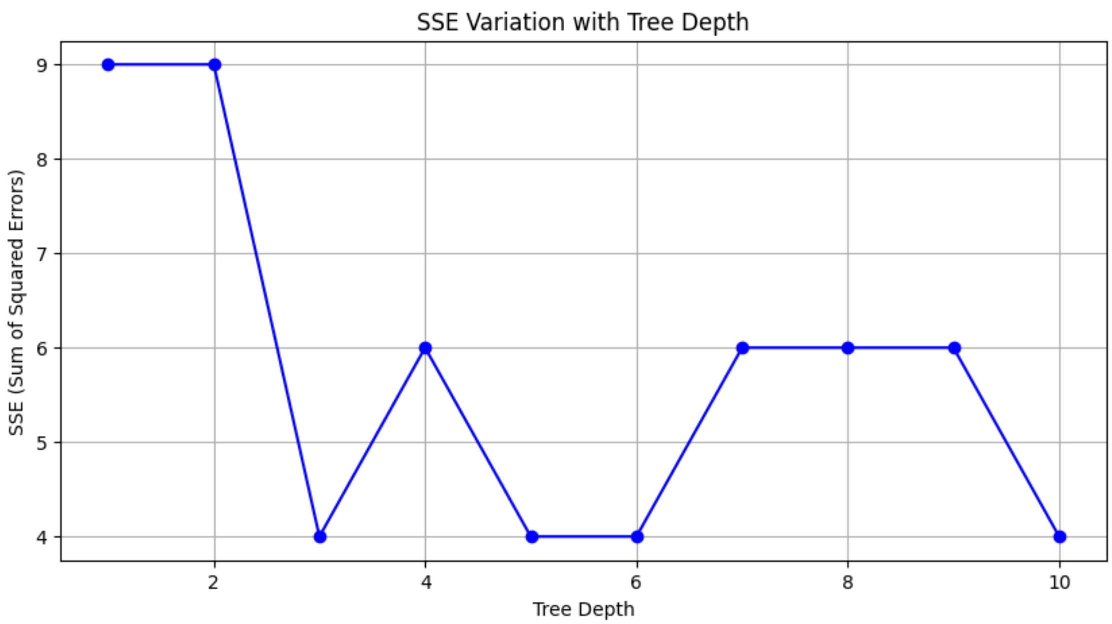
```python
train_file = "/content/A4_train.csv"
test_file = "/content/A4_test.csv"

#  datasets
train_data = pd.read_csv(train_file)
test_data = pd.read_csv(test_file)

# Extract features and labels
X_train_data, y_train_data = train_data[['Feature1', 'Feature2']],␣
↪train_data['Label']
X_test_data, y_test_data = test_data[['Feature1', 'Feature2']],␣
↪test_data['Label']

# Build a Decision Tree Classifier using the Gini criterion
dt_classifier = DecisionTreeClassifier(criterion="gini")
dt_classifier.fit(X_train_data, y_train_data)

# Visualize the fully grown decision tree
plt.figure(figsize=(12, 10))
plot_tree(dt_classifier, feature_names=['Feature1', 'Feature2'],␣
↪class_names=['0', '1'], filled=True, rounded=True)
plt.title("Fully Expanded Classification Tree")
plt.show()
```

Fully Expanded Classification Tree



[39]:
```python
# Part (b): Compute SSE at each depth
max_depths = list(range(1, 11))  # Exploring depths from 1 to 10
sse_results = []  # Storing SSE values for each depth

for depth in max_depths:
    tree_pruned = DecisionTreeClassifier(criterion="gini", max_depth=depth)
    tree_pruned.fit(X_train_data, y_train_data)

    predicted_labels = tree_pruned.predict(X_test_data)  # Generate predictions
    sse_value = np.sum((y_test_data - predicted_labels) ** 2)  # Compute SSE
    sse_results.append(sse_value)  # Store SSE value

# Plot SSE values against tree depth
plt.figure(figsize=(10, 5))
plt.plot(max_depths, sse_results, marker='o', linestyle='-', color='blue')
plt.xlabel("Tree Depth")
plt.ylabel("SSE (Sum of Squared Errors)")
```

```
plt.title("SSE Variation with Tree Depth")
plt.grid()
plt.show()

# part C: Identifying Optimal Pruning Depth
sse_changes = np.diff(sse_results)  # Compute differences in SSE between depths
best_depth = max_depths[np.argmin(np.abs(sse_changes))]  # Depth with least SSE␣
↪variation
print(f"Optimal Pruning Depth: {best_depth}")
```




```
Optimal Pruning Depth: 1
```

[42]:

```
#part d;

import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from sklearn.tree import DecisionTreeClassifier, plot_tree

# Train the decision tree model with optimal pruning depth
classifier_final = DecisionTreeClassifier(criterion="gini",␣
↪max_depth=best_depth)
classifier_final.fit(X_train_data, y_train_data)

# Visualize the pruned decision tree
plt.figure(figsize=(12, 5))
```

```
plot_tree(classifier_final, feature_names=['Feature1', 'Feature2'],
 ↪class_names=['0', '1'], filled=True, rounded=True)
plt.title(f"Optimally Pruned Classification Tree (Depth={best_depth})")
plt.show()
```