Assignment - 2 [EE708] 06/06/25

(1) $S_{xx} = \sum x_n^2 - N\bar{x}^2$ and $\hat{w}_1 = \frac{S_{xy}}{S_{xx}}$

$S_{xy} = \sum x_n y_n - N\bar{x}\bar{y}$ $\hat{w}_0 = \bar{y} - \hat{w}_1 \bar{x}$

$$S_{xx} = 543503.00 - \frac{250 \times (11211.00)^2}{250 \times 250}$$

$$= 40756.916$$

$$S_{xy} = 1996904.15 - \frac{11211 \times 44520.80}{250}$$

$$= 413.395$$

$$\hat{w}_1 = 413.395 / 40756.916$$

$$\boxed{\hat{w}_1 = 0.010143}$$

$$\hat{w}_0 = \frac{44520.80}{250} - 0.010143 \times \frac{11211}{250}$$

$$\boxed{\hat{w}_0 = 177.628347}$$

(b) for $x = 25$, $y = 177.628347 + 0.010143 \times 25$

$$\boxed{y = 177.881922}$$

(c) Residual = observed - predicted

$$= 170 - 177.881922$$

$$\boxed{\text{Residual} = -7.881922}$$

(d) Since residual -ve. the model overestimates by 7.881922

(2) $S_{xx} = \sum x_n^2 - n\bar{x}^2$.

$S_{xy} = \sum x_n y_n - \sum \bar{x}\bar{y}$

$$S_{xx} = 187.42 - \frac{14 \times 43 \times 43}{14 \times 14}$$

$$\boxed{S_{xx} = 25.3484485}$$

$S_{xy} = 1697.8 - 14(3.08) \times (40.8571433)$

$$\boxed{S_{xy} = -59.057966}$$

$$\hat{w}_1 = \frac{-59.057966}{25.3484480} = -2.329845$$

$\hat{w}_0 = \hat{y} - \hat{w}_1\bar{x} = 48.013099$

(b) for $x = 4.3$, $y = 48.013099 - 2.329845 \times 4.3$

$$\boxed{y = 37.9947635}$$

(c) point estimate of mean permeability for $X = 3.7$

$E[\hat{y}|x = 3.7] = 48.01296 + 3.7(2.329845)$

$$\boxed{y = 39.39770}$$

(d) Residual at $x = 3.7 \Rightarrow$ observed $y = 46.1$

$$\boxed{\text{Residual} = 6.70730}$$

3 ⓐ Regression Model

$\because \; y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \epsilon$

To → minimize $S = \sum_{i=1}^{n=10} (y_i - y_j)^2$

$$\frac{\partial S}{\partial \beta} = -2 \sum_{i=1}^{n=10} (y_i - \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i}) = 0$$

$$\sum_{i=1}^{n=10} y_i = 10\beta_0 + \left(\sum_{i=1}^{n=10} x_{1i}\right)\beta_1 + \beta_2 \sum_{i=1}^{n=10} x_{2i}$$

$$\frac{\partial S}{\partial \beta_1} = -2 \sum_{i=1}^{n=10} (y_i - (\beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i}))x_{1i} = 0$$

$$\sum x_i y_i = \beta_0 \sum x_i + \beta_1 \sum x_{1i}^2 + \beta_2 \sum x_{1i} x_{2i}$$

$$43550.8 = 223\beta_0 + \beta_1 5200.9 + 12352\beta_2$$

similarly for β_2

we get $104736.8 = 553\beta_0 + 12352\beta_1 + 31729\beta_2$

$$\underbrace{\begin{bmatrix} 10 & 223 & 553 \\ 223 & 5200.9 & 12352 \\ 553 & 12352 & 31729 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{bmatrix}}_{\beta} = \underbrace{\begin{bmatrix} \sum y \\ \sum x_1 y \\ \sum x_2 y \end{bmatrix}}_{b}$$

(b) estimating the parameter

$$\begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{bmatrix} = A^{-1}b \rightarrow \text{by solving this eqn we got}$$

$$\boxed{\begin{aligned} \beta_0 &= 171.055 \\ \beta_1 &= 3.7133 \\ \beta_2 &= -1.1259 \end{aligned}}$$

Regresn eqn:

$$\boxed{y = 171.055 + 3.7133x_1 - 1.1259x_2}$$

(c) pridicting strength for $x_1 = 18$ & $x_2 = 43\%$.

by eqn

$$y = 171.055 + 3.7133 \times 18 - 1.1259 \times 43$$

$$\boxed{y = 189.4807}$$

Solution (4)

250 male subjects · 13 physical characteristics

Regression coff. ~~$\beta = (x'x)^{-1}(x^T y)$~~

$$\boxed{\beta = (X^T X)^{-1} X^T y}$$

$$X^T X^{-1} = \begin{bmatrix} 2.19705 & -4.0042 \times 10^{-2} & -4.16 \times 10^{-2} \\ -0.4004 & 6.0077 \times 10^{-4} & -7.587 \times 10^{-} \\ -0.0041 4 & -7.3875 \times 10^{-5} & 2.5760 \times 10^{-} \end{bmatrix}$$

$$X^T y = \begin{bmatrix} 4757.9 \\ 334505.8 \\ 179706.7 \end{bmatrix}$$

by multiplication $\beta = \begin{bmatrix} -6744.13 \\ -171.15 \\ 1.76 \end{bmatrix}$

$$\boxed{y = -6744.13 - 171.15 x_1 + 1.76 x_2}$$

$\Rightarrow$ y = pridiction of % body fat

x_1 = weight ~~of~~ in meter

x_2 = ~~weight~~ size in inhs

Que 5 $\Rightarrow$ $f(x_1, x_2) = w_0 + w_1 x_1 + w_2 x_2 +$

$w_3 x_1 x_2 + w_5 x_2^2 + w_4 x_1^2$

To estimate least square for coff. w_i ($i = 0$ to 5)
in quadratic $\rightarrow$

we use the OLS method.

model can be written as

$$y = Xw + \epsilon$$

y: output vector ($N \times 1$) $\rightarrow$ size

X: design matrix ($N \times 6$)

w: vector of coff. to estimate.

$$w = [w_0 \rightarrow w_5]$$

ϵ = vector of residual

least square sol^n $\rightarrow$

$$S(w) = ||y - Xw||^2$$

$$= (y - Xw)^T (y - Xw)$$

$$= y^T y - 2w^T X^T y + w^T X^T X w$$

To min $S(w) \rightarrow$

$$\nabla_w S(w) = -2X^T y + 2X^T X w$$

so set $\nabla_w S(w) = 0 \rightarrow X^T X w = X^T y$

$$\boxed{w = (X^T X)^{-1} X^T y}$$