Excitation of terahertz plasmons eigenmode of a parallel plane guiding system by an electron beam

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Two parallel semiconductor plates, separated by a short distance, support surface plasmon eigenmode with amplitude maxima at the inner surfaces of the plates and minimum at the center. A relativistic sheet electron beam propagating through the space between the planes resonantly excites the surface plasma wave (SPW). The frequency of the driven SPW decreases with the energy of the beam while the growth rate increases. At the beam current ≈ 168 A the growth rate of 5.93×10^8 rad/s is achieved at the frequency ≈ 0.51 THz of SPW for the 5 mm $\hat{y}$ width and spacing between the two plates of ≈ 2.83 mm. The growth rate scales as 1/3 root of the electron beam current. © *2010 American Institute of Physics*. [doi:10.1063/1.3524368]

I. INTRODUCTION

Surface plasmons (SPs) have drawn vigorous attention in recent years due to their wide ranging applications.[1–8] SPs are ideally suited as sensors.[9,10] They propagate along the surface between a conductor and a dielectric or conductor and air with fields peaking at the interface and falling off exponentially away from it in either medium.[11–13] A tiny trace of a gas or DNA on the interface produces a measurable change in the propagation constant when one uses the attenuated total reflection configuration.[14–16] In this configuration a thin film of silver (or gold) is deposited on a glass prism. Laser light is impinged on the other face of the prism so as to fall on the prism-metal interface at an angle of incidence θ (called SP resonance angle θ_{SPW}) the laser gets mode converted into a surface plasma wave (SPW) at the metal-free space interface, and laser reflectivity sharply falls. θ_{SPW} is very sensitive to presence of gas or DNA on the free interface, hence aids their detection. SPs can increase the rate of laser ablation of materials by orders of magnitude, hence are suitable for rapid thin film deposition.

SPs are being explored for their potential in optics, magneto-optic data storage, microscopy,[17] and solar cells.[18–20] Their strong localization and resonant properties find application in biosensing, optoelectronics, metamaterials,[21] enhanced optical transmission through nanoapertures,[22] super-resolution imaging,[23] and negative refraction,[24–26] and enhanced nonlinear effects.[27,28] Welsh *et al.*[29] have described the influence of SP excitation on terahertz-pulse generation on a gold surface.

Significant efforts have been made on the resonant interaction of SPWs with electron beams. Liu and Tripathi[30] have studied the excitation of SPW over a planar metal surface by a sheet electron beam. SPW driven electron acceleration have been observed.[31,32] Kumar and Tripathi[33] have developed a formalism and explained some of these results. Kumar *et al.*[34] has examined the possibility of stimulated emission of a SPW on a metal–vacuum interface by electron-hole recombination in a forward biased *p-n* junction located near the interface.

In this paper, we study the excitation of SPW by a relativistic sheet electron beam propagating through the space between the semiconductors (n-InSb). The parallel plane structure is chosen to have a surface wave field minimum in the middle of the planes, so that an electron beam placed over there experiences a converging force and does not diverge. In Sec. II, we derive the dispersion relation for SPW. In Sec. III we study the beam response and obtain the growth rate. The results are discussed in Sec. IV.

II. SPW

Consider two semiconducting half spaces $x \leq -a$ and $x \geq a$, separated by free space. In each semiconductor the free electron density is n_o^o (cf. Fig. 1). A SPW propagates along $\hat{z}$ in this system. Taking the t, z variations in wave fields as $e^{-i(\omega t - k_z z)}$ and using $\nabla \cdot \vec{E} = 0$ in each region, one may write the electric field of the SPW (Ref. 35) as

$$\vec{E} = A_I\left(\hat{z} - \hat{x}\frac{ik_z}{\alpha_I}\right)e^{\alpha_I x}e^{-i(\omega t - k_z z)}, \quad x < -a, \tag{1}$$

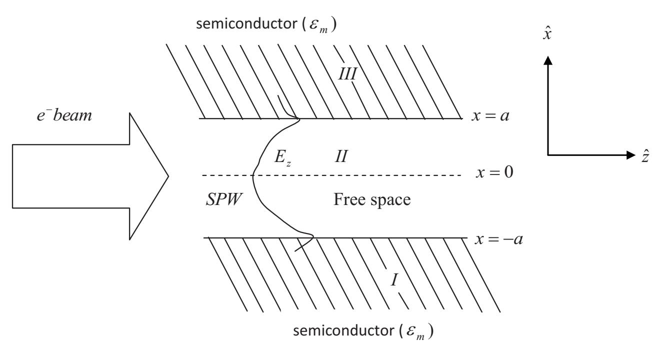

FIG. 1. Schematic of two semiconducting parallel plane guiding system with free space in between them. A relativistic sheet electron beam, injected in the free space region, excites the SPW.

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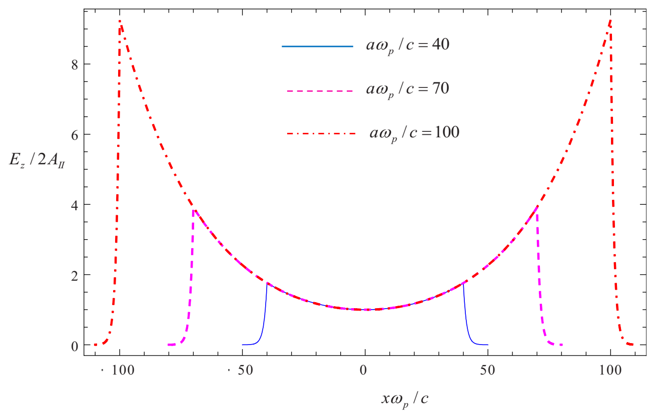

FIG. 2. (Color online) Plot of mode structure of SPW for $\omega/\omega_p=0.15$.

$$\vec{E}=\left[A_{II}\left(\hat{z}-\hat{x}\frac{ik_z}{\alpha_{II}}\right)e^{\alpha_{II}x}+A'_{II}\left(\hat{z}+\hat{x}\frac{ik_z}{\alpha_{II}}\right)e^{-\alpha_{II}x}\right]e^{-i(\omega t-k_z z)},$$
$$-a<x<a, \tag{2}$$

$$\vec{E}=A_{III}\left(\hat{z}+\hat{x}\frac{ik_z}{\alpha_I}\right)e^{-\alpha_I x}e^{-i(\omega t-k_z z)}, \quad x>a, \tag{3}$$

where $\alpha_I^2=k_z^2-\omega^2\varepsilon_m/c^2$, $\alpha_{II}^2=k_z^2-\omega^2/c^2$, $\varepsilon_m=\varepsilon_L-(\omega_p^2/\omega^2)(1-i\nu/\omega)$, ε_L is the lattice permittivity $\omega_p=(4\pi n_o^o e^2/m)^{1/2}$ is the plasma frequency, $-e$ and m are the electron charge and effective mass, and ν is the electron collision frequency. The mode structure of SP is plotted in Fig. 2. As seen from the plot the electric field is maximum at the inner semiconductor surfaces and reduces to zero inside the semiconductor. The continuity of E_z and εE_x (where ε is the permittivity of the medium) at $x=-a$ yields

$$A_I e^{-\alpha_I a}=A_{II}e^{-\alpha_{II}a}+A'_{II}e^{\alpha_{II}a},$$

$$\frac{\varepsilon_m\alpha_{II}}{\alpha_I}A_I=A_{II}e^{-\alpha_{II}a}-A'_{II}e^{\alpha_{II}a}. \tag{4}$$

Similar conditions at $x=a$ are

$$A_{II}e^{\alpha_{II}a}+A'_{II}e^{-\alpha_{II}a}=A_{III}e^{-\alpha_I a},$$

$$A_{II}e^{\alpha_{II}a}-A'_{II}e^{-\alpha_{II}a}=-\frac{\alpha_{II}\varepsilon_m}{\alpha_I}A_{III}e^{-\alpha_I a}. \tag{5}$$

For symmetric mode (E_z symmetric about $x=0$),

$$A'_{II}=A_{II},\ A_{III}=A_I,\ A_I=A_{II}2\cosh\alpha_{II}ae^{\alpha_I a},$$

$$\tanh\alpha_{II}a=-\frac{\alpha_{II}\varepsilon_m}{\alpha_I}, \tag{6}$$

$$k_z^2=\frac{\omega^2}{c^2}\varepsilon_m\frac{\varepsilon_m-\tanh^2\alpha_{II}a}{\varepsilon_m^2-\tanh^2\alpha_{II}a}. \tag{7}$$

For $\alpha_{II}a\gg 1$, Eq. (7) gives the conventional SPW dispersion relation corresponding to a single semiconductor-free space interface,

$$k_z^2=\frac{\omega^2}{c^2}\frac{\varepsilon_m}{1+\varepsilon_m}, \tag{8}$$

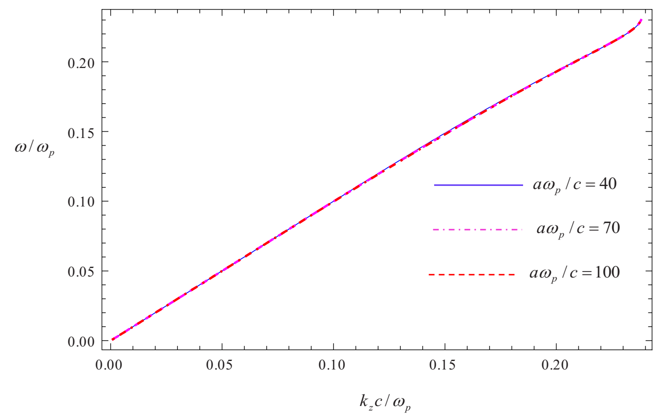

FIG. 3. (Color online) Plot of dispersion relation for SPW in double semiconductor structure for $\varepsilon_L=17$ and $\nu/\omega_p=0.0472$.

$$\alpha_{II}=\frac{\omega}{c}\frac{i}{\sqrt{1+\varepsilon_m}}. \tag{9}$$

For $\alpha_{II}a>1$, the dispersion relation can be solved iteratively, by writing $\tanh^2\alpha_{II}a\approx 1-4e^{-2\alpha_{II}a}$,

$$k_z^2=\frac{\omega^2}{c^2}\frac{\varepsilon_m}{1+\varepsilon_m}\left[1+\frac{\varepsilon_m}{\varepsilon_m^2-1}4e^{-2\alpha_{II}a}\right]. \tag{10}$$

In Fig. 3 we have plotted ω/ω_p versus $k_z c/\omega_p$ for $\varepsilon_L=17$, $a\omega_p/c=40,70,100$, $n_0=2.4\times10^{16}$ cm^{-3}, m is 0.015 times the free space electron mass. The frequency rises linearly with the wavenumber upto upper limit of frequency of SP, i.e., for higher value of wavenumber the behavior of the curve is the same as that of single semiconductor surface structure.

III. ELECTRON BEAM EXCITATION OF SPW

We launch a sheet electron beam of size r_{ob} (comparable to the spacing between planes), density n_{ob}, and velocity $v_{ob}\hat{z}$ in between the semiconductor with

$$n_{ob}=N_{ob}e^{-x^2/r_{ob}^2}. \tag{11}$$

The y width of the beam is b and the beam current is

$$I_b=\sqrt{\pi}N_{ob}r_{ob}bev_{ob}. \tag{12}$$

In the presence of the SPW, beam response is governed by the equation of motion

$$\frac{\partial}{\partial t}(\gamma\vec{v})+\vec{v}\cdot(\nabla\gamma\vec{v})=-\frac{e}{m}\left(\vec{E}+\frac{1}{c}\vec{v}\times\vec{B}\right), \tag{13}$$

where $\vec{B}=(c/i\omega)\nabla\times\vec{E}$ and $\gamma=(1-v^2/c^2)^{-1/2}$. We express $\vec{v}=v_{ob}\hat{z}+\vec{v}_1$, $\gamma=\gamma_o+\gamma_o^3 v_{ob}v_{1z}/c^2$, $\gamma\vec{v}=\gamma_o v_{1x}\hat{x}+\gamma_o^3 v_{1z}\hat{z}$, linearize Eq. (13) and solve it to get

$$v_{1x}=\frac{eE_x}{mi\omega\gamma_o}-\frac{ev_{ob}}{\gamma_o m\omega(\omega-k_z v_{ob})}\frac{\partial E_z}{\partial x}, \tag{14}$$

$$v_{1z}=\frac{eE_z}{mi\gamma_o^3(\omega-k_z v_{ob})}, \tag{15}$$

where $\gamma_o=(1-v_{ob}^2/c^2)^{-1/2}$. From the equation of continuity

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\vec{v}) = 0, \tag{16}$$

on expressing n as $n=n_{ob}+n_1$, one obtains the perturbed beam density

$$n_1 = \frac{v_{1x}}{i(\omega - k_z v_{ob})}\frac{\partial n_{ob}}{\partial x} + \frac{n_{ob}\nabla \cdot \vec{v}_1}{i(\omega - k_z v_{ob})}. \tag{17}$$

The perturbed current density is

$$\vec{J}_1 = -n_{ob}e\vec{v}_1 - n_1 e v_{ob}\hat{z}. \tag{18}$$

For the instability, we look for terms with $(\omega-k_z v_{ob})^2$ in the denominator. The first term on RHS of Eq. (18) does not have such a term, hence we discard it. Thus

$$\vec{J}_1 = -\hat{z}\frac{ev_{ob}}{i(\omega - k_z v_{ob})^2}\left[-\frac{ev_{ob}}{m\gamma_o\omega}\frac{\partial n_{ob}}{\partial x}\frac{\partial E_z}{\partial x} - \frac{ev_{ob}n_{ob}}{\gamma_o m\omega}\frac{\partial^2 E_z}{\partial x^2} + \frac{n_{ob}k_z eE_z}{m\gamma_o^3}\right]. \tag{19}$$

As $\partial^2/\partial x^2=\alpha_{II}^2$ and $k_z\cong\omega/v_{ob}$ for the Cerenkov resonance, the last two terms in Eq. (19) cancel each other. Thus

$$\vec{J}_1 = \hat{z}\frac{e^2v_{ob}^2}{im\omega\gamma_o(\omega - k_z v_{ob})^2}\frac{\partial n_{ob}}{\partial x}\frac{\partial E_z}{\partial x}. \tag{20}$$

The relevant Maxwell's equations are $\nabla\times\vec{E}=-(1/c)\partial\vec{H}/\partial t$, $\nabla\times\vec{H}=4\pi\vec{J}/c+(1/c)\partial\vec{D}/\partial t$. In the absence of the beam let the electric and magnetic fields of the SPW be $\vec{E}_s$ and $\vec{H}_s$. These fields satisfy

$$\nabla \times \vec{E}_s = i\frac{\omega}{c}\vec{H}_s, \tag{21}$$

$$\nabla \times \vec{H}_s = -\frac{i\omega}{c}\varepsilon'\vec{E}_s, \tag{22}$$

with appropriate boundary conditions at $x=-a$ and $x=a$ interfaces. Here $\varepsilon'=\varepsilon_m$ in medium I and III and $\varepsilon'=1$ for $-a<x<a$. In the presence of beam current, let

$$\vec{E} = A(t)\vec{E}_s, \quad \vec{H} = B(t)\vec{H}_s. \tag{23}$$

$\vec{E}$ and $\vec{H}$ satisfy the Maxwell's equations

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial\vec{H}}{\partial t},$$

$$\nabla \times \vec{H} = \frac{4\pi}{c}(\vec{J}_{1b} + \vec{J}_{1p}) + \frac{\varepsilon_L}{c}\frac{\partial\vec{E}}{\partial t}, \tag{24}$$

where $\vec{J}_{1p}=\sigma_m A\vec{E}_s+i(\partial\sigma_m/\partial\omega)(\partial A/\partial t)\vec{E}_s$, $\varepsilon_m=\varepsilon_L+i(4\pi\sigma_m/\omega)$, inside the semiconductor.

Outside the semiconductor $\varepsilon_L=1$, $\sigma=0$. Using Eq. (23) in Eq. (24)

$$\frac{\partial B}{\partial t} = -i\omega(A - B), \tag{25}$$

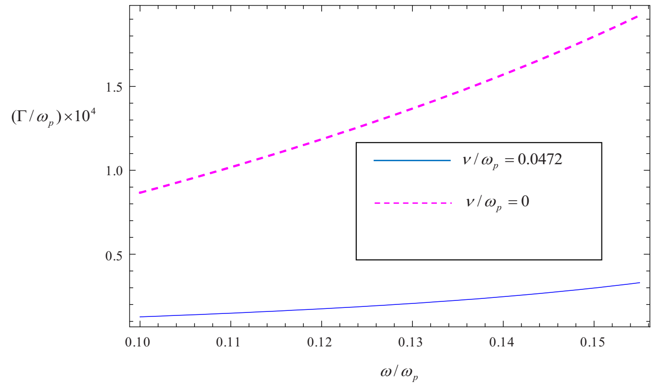

FIG. 4. (Color online) Plot of normalized growth rate vs normalized frequency for double semiconductor structure for $\omega_{pb}/\omega_p=10^{-3}$, $\varepsilon_L=17$, $\nu/\omega_p=0$, 0.0472, $a\omega_p/c=40$.

$$\left[\frac{\partial A}{\partial t}\frac{\partial}{\partial\omega}(\varepsilon'\omega) - i\omega\varepsilon'(A - B)\right]\vec{E}_s = -4\pi\vec{J}_{1b}. \tag{26}$$

Using Eq. (25) in Eq. (26), assuming $\partial B/\partial t\cong\partial A/\partial t$, multiplying the resulting equation by $\vec{E}_s^*$ and integrating over x from $-\infty$ to $+\infty$, one obtains

$$\frac{\partial A}{\partial t} + \Gamma_c A = \frac{-2\pi\int_{-\infty}^{+\infty}J_{1z}E_{sz}^*dx}{\int_{-\infty}^{+\infty}\vec{E}_s\cdot\vec{E}_s^*dx} = \frac{4\omega_{pb}^2v_{ob}^2\alpha_{II}G}{\gamma_o\omega(\omega - k_z v_{ob})^2 r_{ob}^2 D}A, \tag{27}$$

where $G=\int_{-a}^{a}\cosh(\alpha_{II}x)\sinh(\alpha_{II}x)e^{-x^2/r_{ob}^2}x\,dx$, $\Gamma_c=k_{zi}v_g$

$$D = [1+\cosh(2\alpha_{II}a)](\alpha_I^2 + k_z^2)/\alpha_I^3 + 2[2\alpha_{II}a(\alpha_{II}^2 - k_z^2) + (\alpha_{II}^2 + k_z^2)\sinh(2\alpha_{II}a)]/\alpha_{II}^3,$$

$\omega_{pb}=\sqrt{4\pi N_{ob}e^2/m_e}$ is electron beam plasma frequency, m_e is the free space electron mass, and v_g is the group velocity of the SPW. Take $\partial/\partial t=-i\delta$, $\omega=k_z v_{ob}+\delta$, one obtains

$$\delta^3 = \frac{4\omega_{pb}^2v_{ob}^2\alpha_{II}G}{\gamma_o\omega r_{ob}^2 D}, \tag{28}$$

The growth rate is

$$\Gamma = \mathrm{Im}\,\delta = \mathrm{Im}\left[\left(\frac{4\omega_{pb}^2v_{ob}^2\alpha_{II}G}{\gamma_o\omega r_{ob}^2 D}\right)^{1/3}\right]. \tag{29}$$

We have plotted normalized growth rate versus normalized frequency in Fig. 4 for the following parameters: $\omega_{pb}/\omega_p=10^{-3}$, $\varepsilon_L=17$, $\nu/\omega_p=0$, 0.0472, $a\omega_p/c=40$. As seen from the Fig. 4 that the growth rate decreases with collision frequency. The growth rate increases with increasing SPW frequency. As shown in Fig. 5, we have plotted normalized growth rate versus normalized spacing between the conducting plates. The growth rate while considering the collision frequency, initially with $a\omega_p/c$ attains a peak value and then decreases to $\approx 5.93\times 10^8$ rad/s.

IV. DISCUSSION

A parallel plane guiding system, with small separation between planes, supports a SPW. At low frequencies phase velocity is close to the velocity of light in vacuum. One may excite a terahertz wave using a relativistic electron beam.

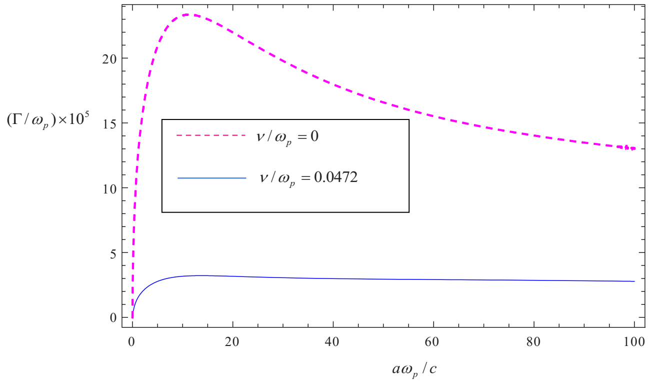

FIG. 5. (Color online) Plot of normalized growth rate vs normalized distance between the semiconductor plates for double semiconductor structure for $\omega_{pb}/\omega_p=10^{-3}$, $\varepsilon_L=17$ $\omega/\omega_p=0.15$, $\nu/\omega_p=0$, 0.0472, $\gamma_o=5.94$.

The double semiconductor structure helps in guiding the electron beam without diverging it as the field of SPW is minimum at the center as compared to single semiconductor structure. We have used InSb as the material as metals put up an impractically high requirement on beam energy. The energy of the beam is 3 MeV at $\omega/2\pi=0.51$ THz. The normalized growth rate increases with the normalized frequency. The growth rate decreases with the collision frequency. The growth rate of SPW attains a maximum value of 6.8×10^8 rad/s for an optimum value of 0.283 mm separation between the two semiconductor plates. At the beam current $\approx$168 A the growth rate of 5.93×10^8 rad/s is achieved at the frequency $\approx$0.51 THz of SPW for the 5 mm $\hat{y}$ width and spacing between the two plates of $\approx$2.83 mm. The growth rate scales as 1/3 root of the electron beam current.

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