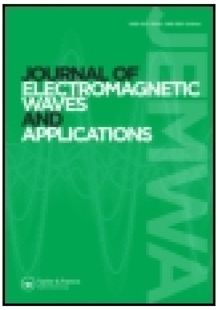

Journal of Electromagnetic Waves and Applications
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/tewa20

Modal study of plasma cladded cylindrical optical fiber

A.K. Mishra [a] , M. Kumar [b] , D. Kumar [c] & O.N. Singh II [a]

[a] Department of Applied Physics, Indian Institute of Technology (Banaras Hindu University), Varanasi, India, 221005

[b] Department of Electrical Engineering, Indian Institute of Technology (Banaras Hindu University), Varanasi, India, 221005

[c] Faculty of Engineering, Multimedia University, Cyberjaya, Malaysia

Version of record first published: 13 Apr 2013.

To cite this article: A.K. Mishra , M. Kumar , D. Kumar & O.N. Singh II (2013): Modal study of plasma cladded cylindrical optical fiber, Journal of Electromagnetic Waves and Applications, DOI:10.1080/09205071.2013.789411

To link to this article: http://dx.doi.org/10.1080/09205071.2013.789411

Journal of Electromagnetic Waves and Applications, 2013
http://dx.doi.org/10.1080/09205071.2013.789411

Taylor & Francis
Taylor & Francis Group

Modal study of plasma cladded cylindrical optical fiber

A.K. Mishra[a]*, M. Kumar[b], D. Kumar[c] and O.N. Singh II[a]

[a]*Department of Applied Physics, Indian Institute of Technology (Banaras Hindu University), Varanasi, India 221005;* [b]*Department of Electrical Engineering, Indian Institute of Technology (Banaras Hindu University), Varanasi, India 221005;* [c]*Faculty of Engineering, Multimedia University, Cyberjaya, Malaysia*

(*Received 21 January 2013; accepted 21 March 2013*)

A theoretical study of cylindrical dielectric waveguide having plasma in the cladding region is investigated. The eigen-value equations for the proposed waveguide and its power calculation have been evaluated using the usual boundary conditions. By using these equations modal dispersion, normalized group velocity, power confinement factor, and fractional modal power curves are drawn for some lower-order TE and TM modes. The nature and shape of the dispersion curve is usual but the group velocity of TM_{01} mode is largely affected in the presence of plasma. It may also be noted that for a fixed value of plasma density, the fractional modal power sustained by a particular fiber mode can be tuned at the optimum level by adjusting the operating frequency properly. Also, the maximum power is concentrated at the interface instead, at the waveguide axis.

Keywords: plasma frequency; operating frequency; modal dispersion; fractional power

1. Introduction

The dielectric waveguides filled with plasma either in core region or in cladding region has attracted much attention because of its spectacular propagation properties of millimeter, sub-millimeter, and optical waves. Plasma as a guiding media has many distinct characteristics over the other conventional guiding media. A considerable research was pursued considering plasma as a guiding media in bounded structures or waveguides with more or less emphasis on the metal plasma waveguides.[1–6] The optical energy propagating down the optical wire depends exclusively on the fiber geometry and materials used to fabricate it. The optical fibers loaded with unconventional materials, such as chiral materials, anisotropic materials, liquid crystals, and metamaterials have proved their far reaching importance as active or passive components in integrated optics and optoelectronics. Conceiving plasma as a guiding media in optical fibers can modify the characteristics of the propagating electromagnetic waves. Recently, a novel type of doubly clad step-index fiber, the inner cladding of which is filled with isotropic plasma, was investigated by Singh et al.[7] Their studies showed that the modes can be controlled by adjusting the width of plasma layer and plasma frequency. The wave theory gives an accurate physical picture about the structure of modes. Hairong et al. [8] discussed the dispersion characteristics and cut-off conditions of a plasma-cladded dielectric waveguide and reported some interesting results based on the modal theory. The effect of plasma loading in the travel-

*Corresponding author. Email: onkar_akm@yahoo.co.in, akm.rs.app@itbhu.ac.in

ling wave-tubes was also studied by many researchers [9–11] under the cold as well as hot plasma conditions and investigated the dispersion characteristics in more detail.

Considering the aforesaid investigations, we propose the cylindrical dielectric fiber having plasma in the cladding region. The refractive index (RI) profile is step-index and we consider isotropic and low-density plasma to avoid the damping loss. Our aim in this work is to investigate dispersion characteristics and fractional modal power based on the wave theory. As compared to dielectric media, plasma has a unique property of tunable RI that is manifested by either a change in plasma frequency or a change in operating frequency. So, the widening of signals (dispersion) appearing at the receiver end can be minimized by adjusting the RI. For the sake of simplicity, we present our theoretical analysis for the cylindrically-symmetric modes, but it can be applied for higher order modes also. The wave propagation in plasma-cladded optical fiber takes place through total internal reflection (TIR) phenomenon as it happens in dielectric waveguides. For the existence of TIR, the guiding region must be surrounded by a media with lower RI. To ensure this condition, plasma cannot be taken into the core of optical fiber, since no naturally occurring materials have RI lower than 1. Although our study is analytical in its approach and concerns mainly on theoretical calculations, the author has tried for the applicability of results in practical situations also. To develop the mathematical background of the structure, we have used the modal theory [16] for the discussion of dispersion characteristics and power calculations.

2. Theoretical analysis

The analysis is made by considering cold plasma (the interactions among the electrons and ions is negligible) under the assumption that the density of the plasma is very low and spatially isotropic. We consider the same structure (Figure 1) as taken by Hairong et al. [8], where the guiding region (core) is a dielectric surrounded by cylindrical plasma channels. Equivalently, one may consider as a dielectric cylinder placed coaxially with infinite, cold and isotropic plasma. The RI of plasma media is sufficiently accurately defined [13] by

$$n_2^2 = \left(1 - \frac{\omega_p^2}{\omega^2}\right) = (1 - \delta^2) \tag{1}$$

where $\delta = \frac{\omega_p}{\omega}$ is the plasma parameter, ω is the operating frequency, and ω_p is the plasma frequency and is defined by $\omega_p = \sqrt{\frac{e^2 n}{m_e \varepsilon_0}}$. The constants e, n, ε_0, and m_e are the charge, electron density, permittivity of free space, and mass of the electrons, respectively. The EM radiations launched into the core region of optical fiber is $E = Ae^{j(\omega t - \beta z)}$ where ω and β are the angular frequency and longitudinal propagation constant, respectively. We use cylindrical coordinate system (r, θ, z) in which z-axis is the direction of wave propagation. The derivation of eigen-

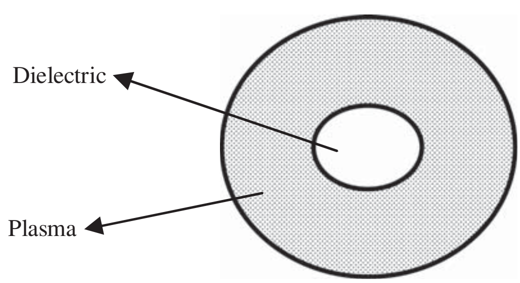

Figure 1. Geometry of optical fiber with cladding loaded with plasma.

value equation involves the solution of wave equation for electric and magnetic fields in core and cladding regions separately. These field solutions are in terms of well-known Bessel functions. The longitudinal components of the electric and magnetic fields [12] in the core and cladding regions of circular waveguide can be given as

$$E_{z_1} = AJ_v(ur)e^{jv\theta} \tag{2}$$

$$H_{z_1} = BJ_v(ur)e^{jv\theta} \tag{3}$$

$$E_{z_2} = CK_v(wr)e^{jv\theta} \tag{4}$$

$$H_{z_2} = DK_v(wr)e^{jv\theta} \tag{5}$$

where, $u^2 = k^2n_1^2 - \beta^2$, $w^2 = \beta^2 - k^2(1-\delta^2)$ and $k = \frac{2\pi}{\lambda}$ is the free space wave number. The subscripts 1 and 2 refer to the core and cladding regions, respectively and the constants A, B, C, and D are the field coefficients whose magnitude represents the strength of field. Other field components can be obtained by the use of appropriate Maxwell's equations in components form (Table 1). To calculate the field constants, we use the usual boundary conditions applied to dielectric–dielectric interface. These boundary conditions relate the behavior of fields from one media with the other. Matching the field components at the core-cladding boundary and manipulating a little bit of algebra, we get the eigen-value equation which follows:

$$\left\{\frac{J_v'(u\,a)}{u\,J_v(u\,a)} + \frac{K_v'(w\,a)}{w\,K_v(w\,a)}\right\} \times \left\{\frac{k^2n_1^2}{u}\frac{J_v'(u\,a)}{J_v(u\,a)} + \frac{k^2n_2^2}{w}\frac{K_v'(w\,a)}{K_v(w\,a)}\right\} - \frac{\beta^2\,v^2}{a^2}\left\{\frac{1}{u^2}+\frac{1}{w^2}\right\}^2 = 0 \tag{6}$$

This is the same eigen-value equation of step-index optical fiber as was derived by Snitzer [16] with the exception of having plasma in the cladding region. This is the standard equation in the sense that for the present structure, it is valid for any mode order. For the cylindrically symmetric distribution of fields $v=0$), this equation reduces to the eigen-value equation of TE and TM modes [8] which follows:

$$\frac{J_1(u\,a)}{J_0(u\,a)} + \frac{u}{w}\frac{K_1(w\,a)}{K_0(u\,a)} = 0 \quad (\mathrm{TE_{0m}\ mode}) \tag{7}$$

$$\frac{J_1(u\,a)}{J_0(u\,a)} + \frac{u}{w}\frac{(1-\delta^2)\,K_1(w\,a)}{n_1^2\,K_0(u\,a)} = 0 \quad (\mathrm{TM_{0m}\ mode}) \tag{8}$$

Table 1. Field components in core and cladding regions of optical fiber.

Core region ($r \leqslant a$)	Cladding region ($r > a$)
$E_{r_1} = -\frac{j\beta}{u^2}\{A\,u\,J_v'(u\,r) + B\frac{j\omega\,\mu\,v}{\beta\,r}J_v(u\,r)\}\,e^{jv\,\theta}$	$E_{r_2} = \frac{j\beta}{w^2}\{C\,w\,K_v'(w\,r) + D\frac{j\omega\,\mu\,v}{\beta\,r}K_v(w\,r)\}\,e^{jv\,\theta}$
$E_{\theta_1} = -\frac{j\beta}{u^2}\{A\frac{jv}{r}J_v(u\,r) - B\frac{\omega\mu\,u}{\beta}J_v'(u\,r)\}\,e^{jv\,\theta}$	$E_{\theta_2} = \frac{j\beta}{w^2}\{C\frac{jv}{r}K_v(w\,r) - D\frac{\omega\mu\,w}{\beta}K_v'(w\,r)\}\,e^{jv\,\theta}$
$H_{r_1} = -\frac{j\beta}{u^2}\{B\,u\,J_v'(u\,r) - A\frac{j\omega\,v\,n_1^2}{\beta\,r}J_v(u\,r)\}\,e^{jv\,\theta}$	$H_{r_2} = \frac{j\beta}{w^2}\{D\,w\,K_v'(w\,r) - C\frac{j\omega\,v\,n_2^2}{\beta\,r}K_v(w\,r)\}\,e^{jv\,\theta}$
$H_{\theta_1} = -\frac{j\beta}{u^2}\{B\frac{jv}{r}J_v(u\,r) + A\frac{\omega\,u\,n_1^2}{\beta}J_v'(u\,r)\}\,e^{jv\,\theta}$	$H_{\theta_2} = \frac{j\beta}{w^2}\{D\frac{jv}{r}K_v(w\,r) + C\frac{\omega\,w\,n_2^2}{\beta}K_v'(w\,r)\}\,e^{jv\,\theta}$

Two other parameters related to optical fibers are their normalized propagation constant (b) and normalized frequency (V), which are defined below

$$b = \left[\frac{\beta^2 - k^2 n_2^2}{k^2\,(n_1^2 - n_2^2)}\right]^{\frac{1}{2}}$$

$$V = k\,a\sqrt{n_1^2 - n_2^2}$$

For the cylindrical waveguides with vacuum core and plasma cladding, the V parameter is dependent on plasma frequency (ω_p) only and is independent of operating frequency (ω).[15] For such a waveguide, the dielectric constant of vacuum is 1 and hence, the V number becomes $V = \frac{\omega_p\, a}{c}$. Furthermore, the numerical aperture (NA), which is also a key parameter for the designing of an optical waveguide, is dependent on ω and ω_p both. At very high frequency compared to plasma frequency ($\omega \gg \omega_p$), the plasma parameter $\delta \approx 0$ and hence, NA ≈ 0. Under such circumstances, the waveguide accepts the minimum light launched from the source. But the optical waveguides having core other than vacuum ($\varepsilon_r > 1$), V-number and NA, both vary with the operating frequency. Thus, although there are no high-frequency limitations for the plasma-cladded single mode wave guides with vacuum core,[8] high-frequency limitations do occur for the single-mode operation of dielectric optical fibers with plasma cladding as $\omega_c \leqslant \frac{cV_c}{aNA}$.

2.1. Power calculations

The total power ($P_{\rm t}$) carried by a particular waveguide mode along the z-direction can be calculated by Poynting vector and is given by [12]

$$P_{\rm t} = \frac{1}{2}\mathrm{Re}\int_0^{\infty}\int_0^{2\pi}(E \times H^*).\,\hat{I}_z\,rdrd\theta \tag{9}$$

where, Re indicates the real part and asterisk represents a complex conjugate. The total normalized power is the sum of normalized powers flowing in the core and cladding regions and is equal to unity i.e.

$$P_{\rm t} = P_{\rm co} + P_{\rm cl} = 1 \tag{10}$$

Using Equations (2–5) along with Table 1 and performing algebraic manipulations, the fractional core power flow (the ratio of power flow in core to the total power) for cylindrically-symmetric modes reduces to

$$\eta_{\rm co} = \frac{P_{\rm co}}{P_{\rm t}} = \frac{\frac{n_1^2}{u^2}\int_0^1 J_1^2(u\,r)r\,dr}{\frac{n_1^2}{u^2}\int_0^1 J_1^2(u\,r)r\,dr + \frac{J_0^2(u\,a)\,n_2^2}{K_0^2(w\,a)w^2}\int_1^{\infty} K_1^2(w\,r)\,r\,dr} \tag{11}$$

In deducing above equation, the use of the following identity of Bessel function has been made:

$$\frac{J'_v(u\,a)}{u\,J_v(u\,a)} = \pm\frac{J_{v\mp 1}(u\,a)}{u\,a\,J_v(u\,a)} \mp \frac{v}{a\,u^2}$$

$$\frac{K'_v(w\,a)}{w\,K_v(w\,a)} = \frac{K_{v\pm 1}(w\,a)}{w\,a\,K_v(w\,a)} \mp \frac{v}{a\,w^2}$$

3. Results and discussions

To study the dispersion characteristics of the plasma-cladded optical fiber, we take the following parameters for a fair analysis.

Plasma frequency $(\omega_p) = 60$ GHz
RI of dielectric core $(n_1) = 1.2$
Operating frequency $(\omega) = 0.2$–1.2 PHz
Core radius $(a) = 5.0$ μm

In our present investigations, we have chosen the operating frequency in the optical range ($\omega \approx 10^{15}$ Hz) and fixed the plasma density at some desired level so that the assumption of cold plasma is restored. Moreover, if one wishes for the plasma frequency to be in the optical range of broad electromagnetic spectrum, the electron density becomes too high ($n \approx 10^{26}\,\mathrm{m}^{-3}$) so that the electron–ion collisions cannot be ignored and hence, the assumption of cold plasma remains no longer valid.

3.1. Dispersion analysis

The dispersion curve (Figure 2) is drawn between normalized propagation constant (b) and normalized frequency (V) keeping plasma frequency fixed. By varying the operating frequency, the RI changes according to relation (1). It is fairly evident from the dispersion curve that the guided modes occur in pairs with approximately same cut-off frequency (degenerate modes). As was expected, the nature and shape of dispersion curve is same as that for dielectric waveguides, since at higher frequencies ($\omega > \omega_p$), the wave reflects and refracts usually at core–cladding interface. Throughout our analysis, we have focused our study towards the cases when $\omega > \omega_p$ due to the following reasons: The wave propagation through any media is characterized by its permittivity, permeability, and the operating frequency. When $\omega > \omega_{p,}$ the electrical response to the media ensures that both $\varepsilon > 0$ and $\mu > 0$, thus leading to the RI $n > 0$. This reflects the pure wave propagation condition. But when $\omega < \omega_{p,}$ the situation is quite cumbersome. In this case, permittivity and permeability have opposite signs (i.e. $\varepsilon < 0$ and $\mu > 0$), leading to no wave propagation condition. So, this region of frequency is called as opaque region, quite similar to the behavior of metals at higher frequency.[14] In our analyses, we have assumed only TE and TM modes instead of fundamental HE_{11} mode due to the following reasons: The HE_{11} mode is basically a superposition of TE_{01} and TM_{01} modes and is symmetric about the transverse component of fields.[12] This mode exists at all frequencies and can be excited in any kind of dielectric waveguides but in the case of plasma-cladded optical fiber, the situation is quite different. From Equation (1), it is clear that when a mode is excited at a frequency lower than the plasma frequency, the RI of the cladding region becomes complex and it tends to an undefined value as the frequency approaches zero. In such cases, the definitions of V and b seem not defined. In Figure 3, the variation of normalized group velocity with the normalized frequency is shown for the lowest TE and TM modes. The group velocity of TE_{01} mode is comparatively less fluctuating than TM_{01} mode (Figure 3), but TM_{01} mode propagates with higher group velocity.

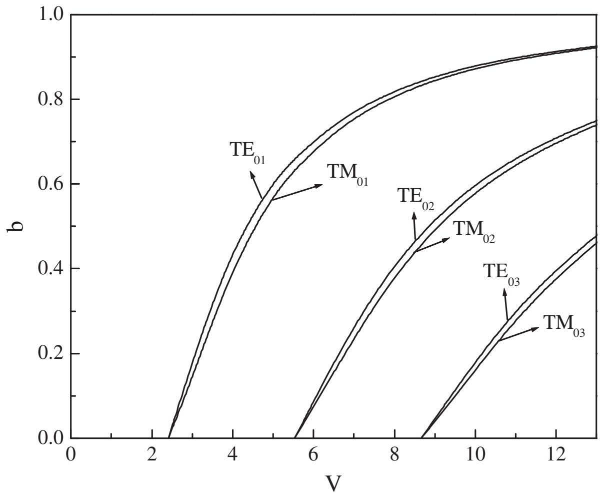

Figure 2. Dispersion curve of some lower order modes for $v=0$ and $a=5.0\,\mu m$.

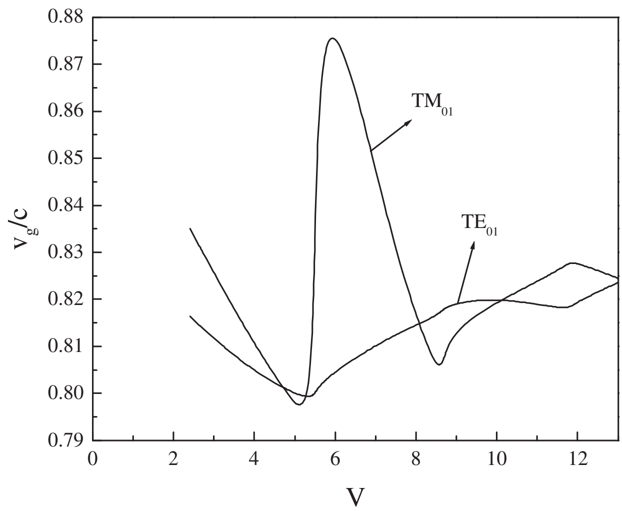

Figure 3. Normalized group velocity vs. V for some lower-order modes.

3.2. *Power distribution*

In Figure 4, the power confinement factor Γ (the ratio of the power confined in the core to the total launched power) is displayed against the normalized transverse distance ($R=r/a$). The power confinement factor basically displays the distribution of fractional power as a function of R in core and cladding regions separately at particular normalized frequency ω_n. For calculating Γ, we have assumed that whole power is launched into the core of the fiber. At frequencies near to the cut-off frequency ($\omega_n=3.63$), the total power confined in the core is only approx. 44% and 42% for TE_{01} and TM_{01} modes, respectively (Figure 4), and the remaining amount of power is flowing in the cladding of the fiber. Thus, at this frequency, majority of power flows in the cladding and is eventually lost. Moreover, it is fairly apparent

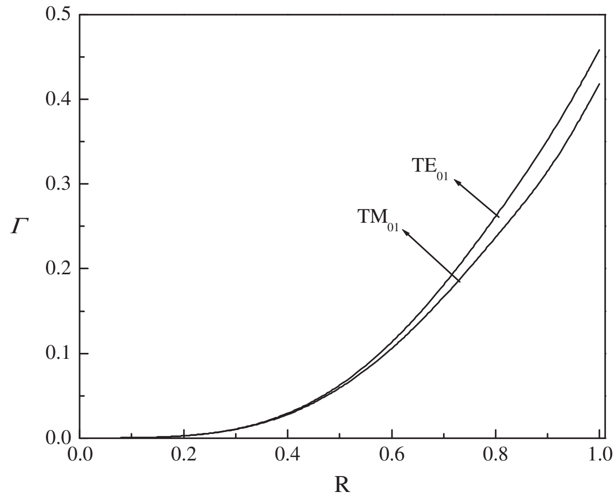

Figure 4. Variation of modal power confinement against R for $\omega_n = \dfrac{\omega a}{c} = 3.7$

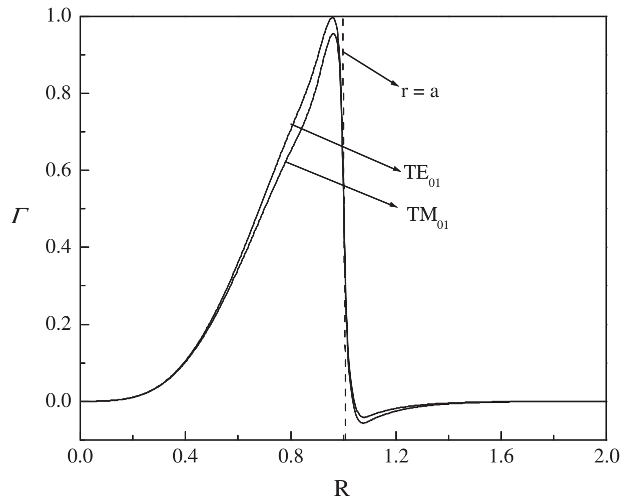

Figure 5. Variation of modal power confinement against R for $\omega_n = \dfrac{\omega a}{c} = 7.0$

from Figure 5 that when the waveguide is excited at a frequency ($\omega_n = 7.0$), well above the normalized cut-off frequency of a particular mode, Γ increases as we move from the waveguide axis ($R = 0$) and becomes maximum ($\Gamma \approx 1.0$) at the core – cladding interface ($R = 1$). This is the power confined between $R = 0$ and $R = 1$. Beyond $R = 1$, we have the cladding region in which a vanishing amount of power flows and it approaches zero when R is increased beyond 1. Moreover, it is clear from these figures. that the maximum power is concentrated at the core – cladding interface instead, near to the waveguide axis. In Figure 6, the fractional modal power (η_{co}) is plotted against the normalized frequency (V) for the TE_{01} and TM_{01} modes. Below the cut-off frequency of a particular mode, the whole power is flowing in the cladding of fiber. Exciting the waveguide at frequency above the cut-off frequency of

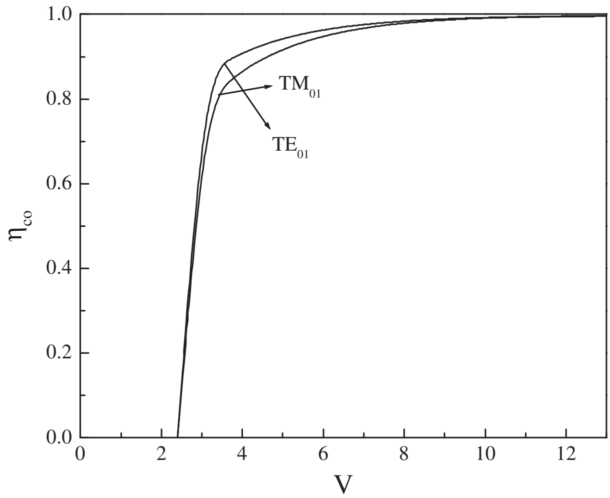

Figure 6. Fractional modal power for lowest circularly-symmetric mode.

the particular mode, the power sustained by the mode increases very sharply with frequency and saturates. This saturation of power indicates that by further increase in frequency, the power carrying capacity of the mode does not change.

4. Conclusions

The light gathering capacity and hence V number depends exclusively on the plasma parameter δ. The optical fibers loaded with plasma in the cladding region displays the usual modal dispersion characteristics for both TE and TM modes. The modal dispersion can be controlled by varying the plasma density and/or operating frequency. The modal group velocity of the plasma-cladded fiber is higher compared to that of the conventional dielectric optical fiber. The amount of power carried by a particular fiber mode depends upon the operating frequency and it is greater at higher frequency. Also, the maximum power is concentrated near the core – cladding interface instead, near to waveguide axis.

Acknowledgment

One of the authors (AKM) is grateful to Dr A.K. Tripathi, Department of Applied Physics, Indian Institute of Technology (BHU), Varanasi, India for his valuable discussions and suggestions.

References

[1] Singh V, Kumar D. Modal dispersion characteristics of a Bragg fiber having plasma in the inner cladding regions. Prog. Electromagn. Res. 2009;89:167–181.
[2] Hu BJ, Wei G. Numerical analysis of a plasma waveguide with finite thickness of cladding in external magnetic field. IEE Proc. Microwave Antennas Propag. 2001;148:115–118.
[3] Chatterton JD, Shohet JL. Guided modes and loss in a plasma filled Bragg waveguide. J. Appl. Phys. 2007;102:1–7.
[4] Kanjevac K, Babovic V, Kovacevic M. Improved numerical solutions to plasma loaded helical waveguide. Kragujevac J. Sci. 2003;25:31–40.
[5] Li C, Liu S, Zhang X, Dai Z. Theoretical studies on propagation of electromagnetic wave in preformed narrowed plasma channel. Proc. Int. Symp. Signals, Syst Electron. 2010;2:4244–4247.
[6] Shen HM. Plasma waveguide: a concept to transfer electromagnetic energy in space. J. Appl. Phys. 1991;69:6827–6835.

[7] Singh SP, Janam R, Jatan R, Singh V, Singh BD. Modal analysis and cutoff condition of a circular doubly clad optical waveguide with plasma in inner cladding region. J. Infrared Millimeter Terahertz Waves 2010;31:1381–1389.
[8] Hairong L, Changjian T, Pukun L. Mode theory of plasma cladding waveguide. J. Phys. D: Appl. Phys. 2007;40:2002–2009.
[9] Mishra G, Tripathi VK, Jain VK. Effect of plasma and dielectric loading on the slow-wave properties of a travelling wave tube. IEEE Trans. Electron Devices 1990;37:1561–1565.
[10] Mishra G, Yadav US. The effects of plasma loading on the dispersion characteristics of a travelling wave tube. J. Phys. D: Appl. Phys. 1992;25:173–177.
[11] Kumar M, Bhasin L, Tripathi VK. Plasma effects in a travelling wave tube. Phys. Scr. 2010;81:1–6.
[12] Pollock CR. Fundamentals of optoelectronics. Chicago (IL): Tom Casson; 1995.
[13] Krall NA, Trivelpiece AW. Principles of plasma physics. New Delhi: McGraw-Hill; 1973.
[14] Novotny L, Hafner C. Light propagation in a cylindrical waveguide with a complex, metallic, dielectric function. Phys. Rev. E. 1994;50:4094–4106.
[15] Shen HM, Pao HY. The plasma waveguide with a finite thickness of cladding. J. Appl. Phys. 1991;70:6653–6662.
[16] Snitzer E. Cylindrical dielectric waveguide modes. J. Opt. Soc. Am. 1961;51:491–498.