In order to calculate the local absorbed energy density $a(z)$ with a transfer matrix formalism, we are going to follow this 5- step process:

- Calculate r_s/t_s or r_p/t_p for every layer under consideration
- Calculate all the matrices $M_n = Tr \cdot Pr$
- Calculate $\tilde{M} \Rightarrow$ total reflectivity at layer 0 r; total transmission at last layer t
- Using r solve a system of equations for every layer to obtain v_n, w_n, i.e. amplitude of forward- backward traveling wave for each layer.
- Use all v_n, w_n to obtain the local absorption $a(z)$ at each point z

Each of those steps will correspond to one function in the code.

1 Fresnel formula

In order to represent the electric field, we want to use a superposition of an incoming and outgoing wave.

$$E(r) = E_f \cdot e^{ik_z r} + E_b \cdot e^{-ik_z r} \tag{1}$$

where,

$$k_z = \frac{2\pi n \cos(\theta)}{\lambda_{vac}} \tag{2}$$

If we are considering multiple layers, at each transition there will be a certain part that is going to be reflected and a part that is going to be transmitted $\rightarrow r_{n,n+1}, t_{n,n+1}$
To obtain the parameters $r_{n,n+1}$ and $t_{n,n+1}$ one can use the Fresnel formulas. Those vary slightly from s to p- polarized light. For s- polarized light one can find [1],

$$\begin{aligned} r_s &= \frac{n_1 \cos(\theta_1) - n_2 \cos(\theta_2)}{n_1 \cos(\theta_1) + n_2 \cos(\theta_2)} \\ t_s &= \frac{2 n_1 \cos(\theta_1)}{n_1 \cos(\theta_1) + n_2 \cos(\theta_2)} \end{aligned} \tag{3}$$

and for p-polarized light

$$
\begin{aligned}
r_p &= \frac{n_2 \cos(\theta_1) - n_1 \cos(\theta_2)}{n_2 \cos(\theta_1) + n_1 \cos(\theta_2)} \\
t_s &= \frac{2 n_1 \cos(\theta_1)}{n_2 \cos(\theta_1) + n_1 \cos(\theta_2)}
\end{aligned} \tag{4}
$$

2 Transfer Matrices

We can use this formulation to express a system of equations, where we formulate the intensity of the forward- and backward traveling wave at each layer n.

$$
\begin{bmatrix} v_n \\ w_n \end{bmatrix} = M_n \cdot \begin{bmatrix} v_{n+1} \\ w_{n+1} \end{bmatrix} \tag{5}
$$

where,

$$
M_n = Tr \cdot Pr = \begin{bmatrix} e^{-i\phi} & 0 \\ 0 & e^{i\phi} \end{bmatrix} \cdot \begin{bmatrix} 1 & r_{n,n+1} \\ r_{n,n+1} & 1 \end{bmatrix} \frac{1}{t_{n,n+1}} \tag{6}
$$

with,

$$
\phi = \frac{\delta_n 2\pi n \cos(\theta)}{\lambda_{vac}} \tag{7}
$$

where δ_n = thickness of layer n.

3 Total reflectivity/ transitivity

Following the expression of eq. (5) we can now multiply the matrices M_n for all layers under consideration, and formulate an expression which relates the amount of intensity reflected at the first layer r and the amount of intensity transmitted through the last layer t.

$$
\begin{bmatrix} 1 \\ r \end{bmatrix} = \tilde{M} \cdot \begin{bmatrix} t \\ 0 \end{bmatrix} = \begin{bmatrix} \tilde{M}_{00} & \tilde{M}_{01} \\ \tilde{M}_{10} & \tilde{M}_{11} \end{bmatrix} \begin{bmatrix} t \\ 0 \end{bmatrix} \tag{8}
$$

where $\tilde{M} = M_1 M_2 M_3 \dots$.
Equation (8), shows that, if we are multiplying the matrices M for every layer under consideration, we can obtain information about the total reflected and the total transmitted intensity. I.e. $t = \frac{1}{\tilde{M}_{00}}$ and $r = \frac{\tilde{M}_{10}}{\tilde{M}_{00}}$.

4 Forward- backward amplitudes for each layer

In order to calculate the amplitude of the forward- and backward traveling wave, a system of equations has to be solved for each layer. E.g.

$$\begin{bmatrix} 1 \\ r \end{bmatrix} = \begin{bmatrix} 1 & r_{0,1} \\ r_{0,1} & 1 \end{bmatrix} \frac{1}{t_{0,1}} \begin{bmatrix} v_1 \\ w_1 \end{bmatrix} \Rightarrow \begin{bmatrix} v_1 \\ w_1 \end{bmatrix} = M_1 \begin{bmatrix} v_2 \\ w_2 \end{bmatrix} \dots \tag{9}$$

This is, once the reflectivity at the 0´th layer is obtained, we can go from there and solve a system of equations for every layer, in order to obtain the amplitude of the forward-backward traveling wave v_n and w_n.

Absorbed density

Once the amplitudes v_n, w_n are obtained for ever layer, the absorbed energy density $a(z)$ can be calculated as a function of space.

$$a(z) = A_1 e^{2zIm(k_z)} + A_2 e^{-2zIm(k_z)} + A_3 e^{2izRe(k_z)} + A_3^* e^{-2izRe(k_z)} \tag{10}$$

where for s- polarized light [1]

$$\begin{aligned} A_1 &= |w|^2 \cdot \frac{Im[n\cos(\theta)k_z]}{Re[n_0\cos(\theta_0)]} \\ A_2 &= |v|^2 \cdot \frac{Im[n\cos(\theta)k_z]}{Re[n_0\cos(\theta_0)]} \\ A_3 &= vw^* \cdot \frac{Im[n\cos(\theta)k_z]}{Re[n_0\cos(\theta_0)]} \end{aligned} \tag{11}$$

and for p- polarized light

$$\begin{aligned} A_1 &= |w|^2 \frac{2Im[kz]Re[n\cos{(\theta^*)}]}{Re[n_0\cos{(\theta_0^*)}]} \\ A_2 &= |v|^2 \frac{2Im[kz]Re[n\cos{(\theta^*)}]}{Re[n_0\cos{(\theta_0^*)}]} \\ A_3 &= vw^* \frac{2Re[kz]Im[n\cos{(\theta^*)}]}{Re[n_0\cos{(\theta_0^*)}]} \end{aligned} \tag{12}$$

Calculating the respective amplitudes for the forward- and backward- traveling wave in every layer is therefor the key to obtain the local energy absorption function $a(t)$.

References

[1] Steven J. Byrnes. Multilayer optical calculations. *arXiv:1603.02720v3*, 2018.