Optimizing Bus Routes in Lucknow Region using VRPTW

Introduction

- Public transportation systems play a crucial role in urban mobility, providing essential services for millions of people worldwide. In cities like Lucknow, India, efficient management of bus routes is essential for ensuring timely and cost-effective transportation services. The Vehicle Routing Problem with Time Windows (VRPTW) is a mathematical optimization problem well-suited for optimizing bus routes by determining the most efficient routes and schedules while considering time constraints at bus stops.

Objective

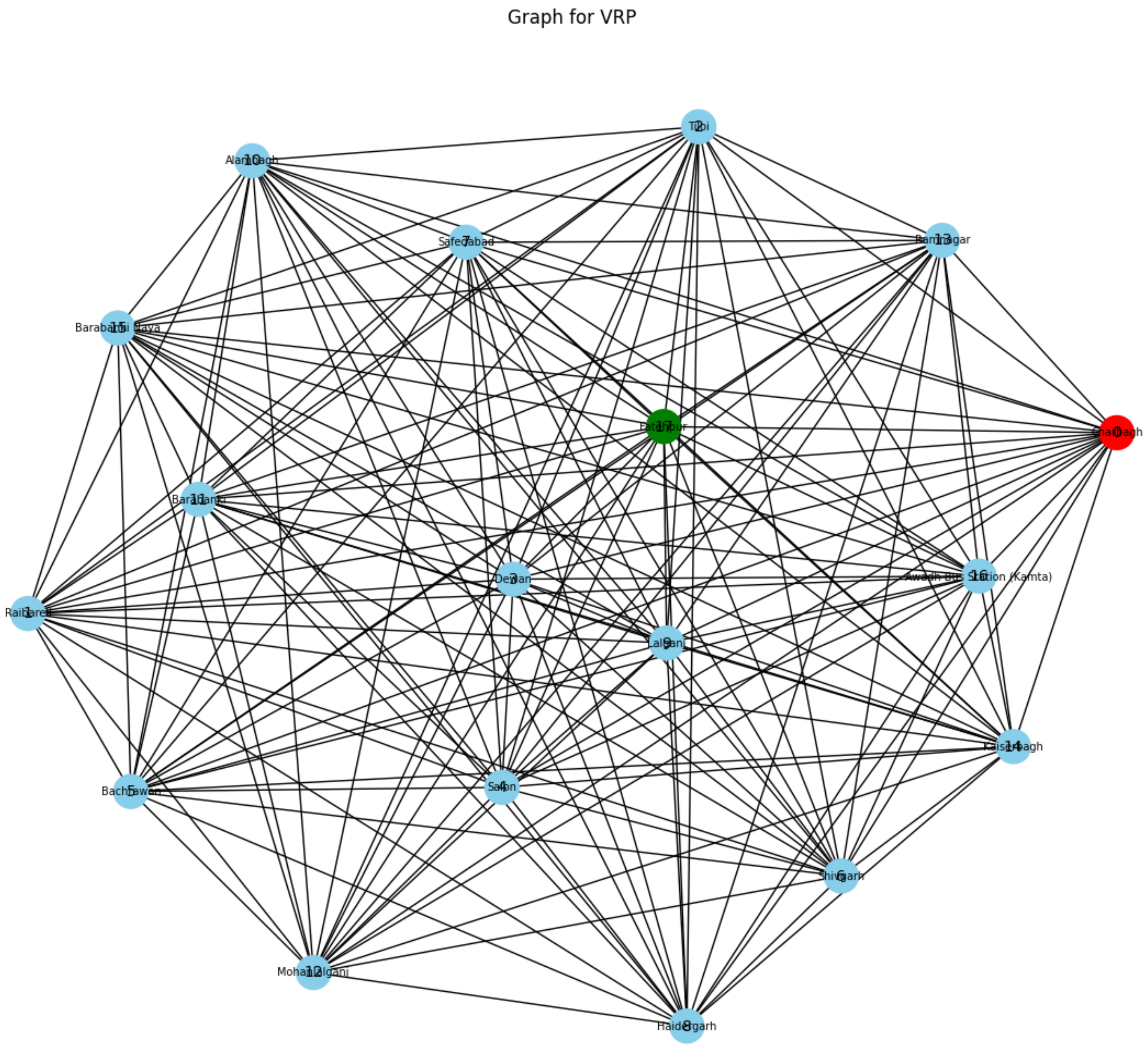
- The objective of this study is to optimize bus routes in Lucknow region using the VRPTW model to minimize total travel distance and operational costs. By leveraging mathematical optimization techniques, we aim to design efficient bus routes that meet passenger demand, adhere to service times, and reduce the number of buses required for operation

Data Description

- Locations: A list of 18 locations in Lucknow region, including starting and ending depots (Charbagh and Fatehpur), bus stops, and important transit points.

Location	**Latitude**	**Longitude**
Charbagh	26.8477	80.9470
Raibareli	26.2325	81.2331
Tiloi	26.1711	81.2155
Dewan	28.6187	79.8087
Salon	26.1522	81.2830
Bachrawan	26.4681	81.3406
Shivgarh	26.2636	81.3070
Safedabad	26.4172	80.7297
Haidergarh	26.7194	81.1133
Lalganj	25.8706	81.7020
Alambagh	26.8462	80.9331
Barabanki	26.9353	81.1951
Mohanlalganj	26.8215	81.0406
Ramnagar	26.5734	81.3752
Kaiserbagh	26.8483	80.9190
Barabanki Naya	26.9255	81.1934
Awadh Bus Station (Kamta)	26.9449	80.9164
Fatehpur	25.9304	80.8133

Possible Routes for Vehicle Routing Problem



Time Windows

- Time windows specifying the opening and closing times for service at each bus stop, excluding depots

Stop Name	Arrival Time Window	
	Open	Close
Charbagh	06:00	07:30
Raibareli	19:00	20:00
Tiloi	19:00	20:30
Dewan	19:30	20:30
Salon	19:00	21:00
Bachrawan	18:00	19:00
Shivgarh	19:00	20:00
Safedabad	17:00	18:00
Haidergarh	13:00	14:30
Lalganj	19:30	20:30
Alambagh	08:00	09:30
Barabanki	15:00	16:30
Mohanlalganj	11:00	12:30
Ramnagar	16:00	17:30
Kaiserbagh	10:00	11:30
Barabanki Naya	14:00	15:30
Awadh Bus Station (Kamta)	12:00	13:30
Fatehpur	21:50	23:30

Demand and vehicle capacity

- Demand: An estimate of the number of passengers expected to board at each bus stop, excluding depots. In our case we have dynamic demand of 10-15 people boarding the bus from each bus stops.
- Vehicle Capacity: The maximum number of passengers that each bus can accommodate. All buses have same capacity of 50 passengers.

Methodology

The VRPTW model is formulated to minimize total travel distance and operational costs while satisfying capacity constraints and time windows for service. The model includes the following components:

- Objective Function: Minimize the total travel distance and operational costs associated with deploying buses.

- Constraints: Ensure that each bus route starts and ends at the designated depots, each bus serves each bus stop exactly once, capacity constraints are met, and service times adhere to specified time windows.

- Optimization Algorithm: The optimization problem is solved using the Gurobi Optimization Software, employing exact optimization algorithms to identify globally optimal solutions within a reasonable computational time frame.

The Model

- The Vehicle Routing Problem with Time Windows (VRPTW) is defined by a fleet of vehicles, V , a set of customers, C, and a directed graph G. Typically, the fleet is considered to be homogeneous, meaning all vehicles are identical. The graph consists of $|C| + 2$ vertices, where the customers are denoted 1, 2, ..., n, and the depot is represented by the vertices 0 ("the starting depot") and n + 1 ("the returning depot"). The set of all vertices, N, includes 0, 1, ..., n + 1, and the set of arcs, A, represents direct connections between the depot and the customers and among the customers. No arcs end at vertex 0 or originate from vertex n + 1. Each arc (i, j), where $i \neq j$, is associated with a cost cij and a time tij , which may include service time at customer i.

Parameters:

- G: Directed graph representing connections between the depot and customers, and among customers.
- cij : Cost associated with arc (i, j).
- tij : Time associated with arc (i, j), including service time at customer i.
- Q: Capacity of each vehicle.
- D: Operational cost of each vehicle.
- di : Demand of customer i.
- ai , bi : Time window for customer i, representing the earliest and latest time for service. • a0, bn+1: Time window for the depot, representing the scheduling horizon.
- V : Set of vehicles.
- C: Set of customers.
- N: Set of all vertices, including depot and customers.

Constraints

Constraints:

1. Each customer is visited exactly once:

$$\sum_{k \in V} \sum_{j \in N} x_{ijk} = 1 \quad \forall i \in C$$

2. Capacity constraint:

$$\sum_{i \in C} d_i \sum_{j \in N} x_{ijk} \leq Q \quad \forall k \in V,$$

3. Leaving depot constraint:

$$\sum_{j \in N} x_{0jk} \leq 1 \quad \forall k \in V$$

4. Returning depot constraint:

$$\sum_{j \in N} x_{i,n+1,k} \leq 1 \quad \forall k \in V$$

5. Time relationship between consecutive vertices:

$$x_{ijk}(s_{ik} + t_{ij} s_{jk}) \leq 0 \quad \forall i, j \in N, \forall k \in V$$

6. Time window constraint:

$$a_i \leq s_{ik} \leq b_i \quad \forall i \in N, \forall k \in V$$

7. Flow conservation at each customer

$$\sum_{i \in N} x_{ihk} - \sum_{j \in N} x_{hjk} = 0 \quad \forall h \in C, \forall k \in V$$

8. Binary decision variable constraint:

$$x_{ijk} \in \{0, 1\} \quad \forall i, j \in N, \forall k \in V$$

Additional Constraints:

- Upper bound on the number of vehicles:

$$\sum_{j \in N} \sum_{k \in V} x_{0jk} \leq |V|$$

Decision variables and Objective

Decision Variables:

- • xijk: Binary decision variable indicating if vehicle k travels directly from vertex i to vertex j.
- • sik: Time at which vehicle k starts servicing customer i.

Objective:

Objective: Minimize the total travel cost:

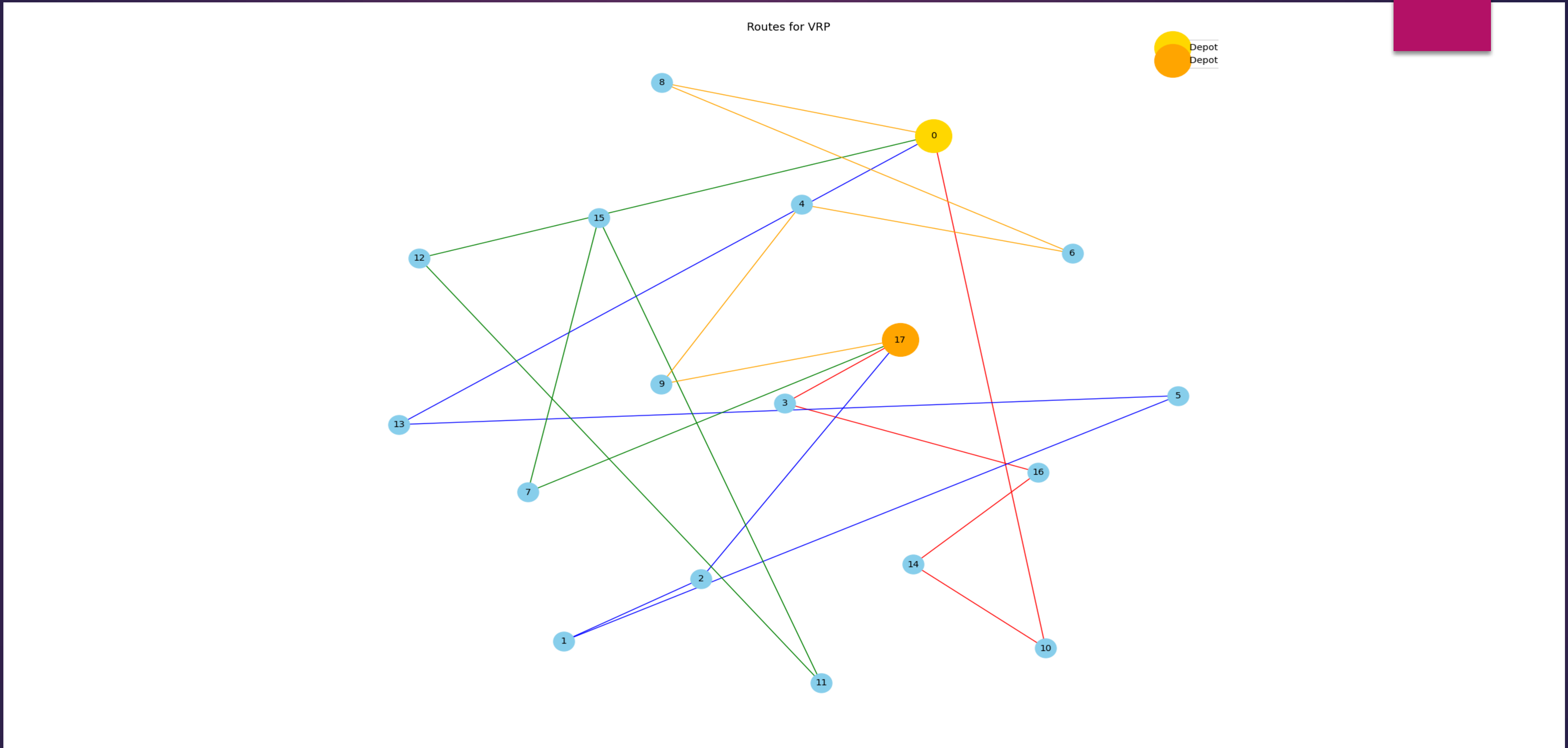
$$\min \sum_{i \in N} \sum_{j \in N} \sum_{k \in V} c_{ij} x_{ijk} + D \sum_{j \in N} \sum_{k \in V} x_{0jk}$$

Results

The optimization model yields optimal bus routes and schedules that minimize total travel distance and operational costs while meeting service requirements. The results demonstrate significant improvements in route efficiency and cost-effectiveness compared to baseline scenarios .The model is solved using Gurobi, a powerful optimization software. The solution provides optimal bus routes and their corresponding travel times. The results include:

- • Routes: A list of stops traversed by each bus, forming individual routes.
- • Buses: The specific bus assigned to each route.
- • Optimal solution found (tolerance 1.00e-04) in 11.85 seconds Best objective 7.621282583518e+04, best bound 7.621282583518e+04, gap 0.0000

Bus	Route (Start Depot → Stops → End Depot)
Bus 0	Charbagh (0) → Alambagh (10) → Salon (14) → Dewan (16) → Tiloi (3) → Fatehpur (17)
Bus 1	Charbagh (0) → Ramnagar (13) → Bachrawan (5) → Raibareli (1) → Tiloi (2) → Fatehpur (17)
Bus 2	Charbagh (0) → Mohanlalganj (12) → Barabanki (11) → Barabanki Naya (15) → Safedabad (7) → Fatehpur (17)
Bus 3	Charbagh (0) → Haidergarh (8) → Shivgarh (6) → Salon (4) → Lalganj (9) → Fatehpur (17)



Routes for VRP
Depot
Depot
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17

Contributions

- Individual project.

Remark

- Future research may explore additional factors such as dynamic demand forecasting and real-time route optimization to further improve system performance and responsiveness.
- Also exact models can be used for small scale problems only. Even for small model as this the solver takes about 11sec to solve . If model become too large then heuristics methods are used .