

Numerical method

In numerical analysis, a **numerical method** is a mathematical tool designed to solve numerical problems. The implementation of a numerical method with an appropriate convergence check in a programming language is called a numerical algorithm.

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Mathematical definition

Let $F(x, y) = 0$ be a well-posed problem, i.e. $F : X \times Y \rightarrow \mathbb{R}$ is a real or complex functional relationship, defined on the cross-product of an input data set X and an output data set Y , such that exists a locally lipschitz function $g : X \rightarrow Y$ called resolvent, which has the property that for every root (x, y) of F , $y = g(x)$. We define **numerical method** for the approximation of $F(x, y) = 0$, the sequence of problems

$$\{M_n\}_{n \in \mathbb{N}} = \{F_n(x_n, y_n) = 0\}_{n \in \mathbb{N}},$$

with $F_n : X_n \times Y_n \rightarrow \mathbb{R}$, $x_n \in X_n$ and $y_n \in Y_n$ for every $n \in \mathbb{N}$. The problems of which the method consists need not be well-posed. If they are, the method is said to be *stable* or *well-posed*.^[1]

Consistency

Necessary conditions for a numerical method to effectively approximate $F(x, y) = 0$ are that $x_n \rightarrow x$ and that F_n behaves like F when $n \rightarrow \infty$. So, a numerical method is called *consistent* if and only if the sequence of functions $\{F_n\}_{n \in \mathbb{N}}$ pointwise converges to F on the set S of its solutions:

$$\lim F_n(x, y + t) = F(x, y, t) = 0, \quad \forall (x, y, t) \in S.$$

When $F_n = F, \forall n \in \mathbb{N}$ on S the method is said to be *strictly consistent*.^[1]

Convergence

Denote by ℓ_n a sequence of *admissible perturbations* of $x \in X$ for some numerical method M (i.e. $x + \ell_n \in X_n \forall n \in \mathbb{N}$) and with $y_n(x + \ell_n) \in Y_n$ the value such that $F_n(x + \ell_n, y_n(x + \ell_n)) = 0$. A condition which the method has to satisfy to be a meaningful tool for solving the problem $F(x, y) = 0$ is *convergence*:

$$\begin{aligned} &\forall \varepsilon > 0, \exists n_0(\varepsilon) > 0, \exists \delta_{\varepsilon, n_0} \text{ such that} \\ &\forall n > n_0, \forall \ell_n : \|\ell_n\| < \delta_{\varepsilon, n_0} \Rightarrow \|y_n(x + \ell_n) - y\| \leq \varepsilon. \end{aligned}$$

One can easily prove that the point-wise convergence of $\{y_n\}_{n \in \mathbb{N}}$ to y implies the convergence of the associated method.^[1]

References

1. Quarteroni, Sacco, Saleri (2000). *Numerical Mathematics* (<http://www.techmat.vgtu.lt/~inga/Files/Quarteroni-SkaitMetod.pdf>) (PDF). Milano: Springer. p. 33.
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