AGENDA

- Hypothesis testing
- Parametric and Non Parametric tests

HYPOTHESIS TESTING

Definition

Hypothesis testing is a form of inferential statistics, it is a statement about a population where, this statement typically is represented by some specific numerical value. In testing a hypothesis, we use a method where we gather data in an effort to gather evidence about the hypothesis.

Typical steps

- Setting up two competing hypotheses.
- Set some level of significance called alpha.
- Calculate a test statistic.
- Calculate probability value (p-value), or find rejection region.
- Make a test decision about the null hypothesis.
- State an overall conclusion.

HYPOTHESIS TESTING: p-VALUE AND TESTS

Interpreting the p-value

p-value answer the questions:

What is the probability of the observed test statistic?

When H_0 *is true?*

Thus, smaller and smaller *p*-values provide stronger and stronger evidence against H_0.

Example: Comparing the population mean.

Which test to choose and when?

Alternate Hypothesis

1. The population parameter is not equal to a certain value. Referred to as a "two-tailed test".

   $H_a : p \neq p_0$, or $H_a : \mu \neq \mu_0$

2. The population parameter is less than a certain value. Referred to as a "left-tailed test"

   $H_a : p < p_0$, or $H_a : \mu < \mu_0$

3. The population parameter is greater than a certain value. Referred to as a "right-tailed test".

   $H_a : p > p_0$, or $H_a : \mu > \mu_0$

Null Hypothesis

The null hypothesis in each case would be:

$H_0 : p = p_0$, or $H_0 : \mu = \mu_0$

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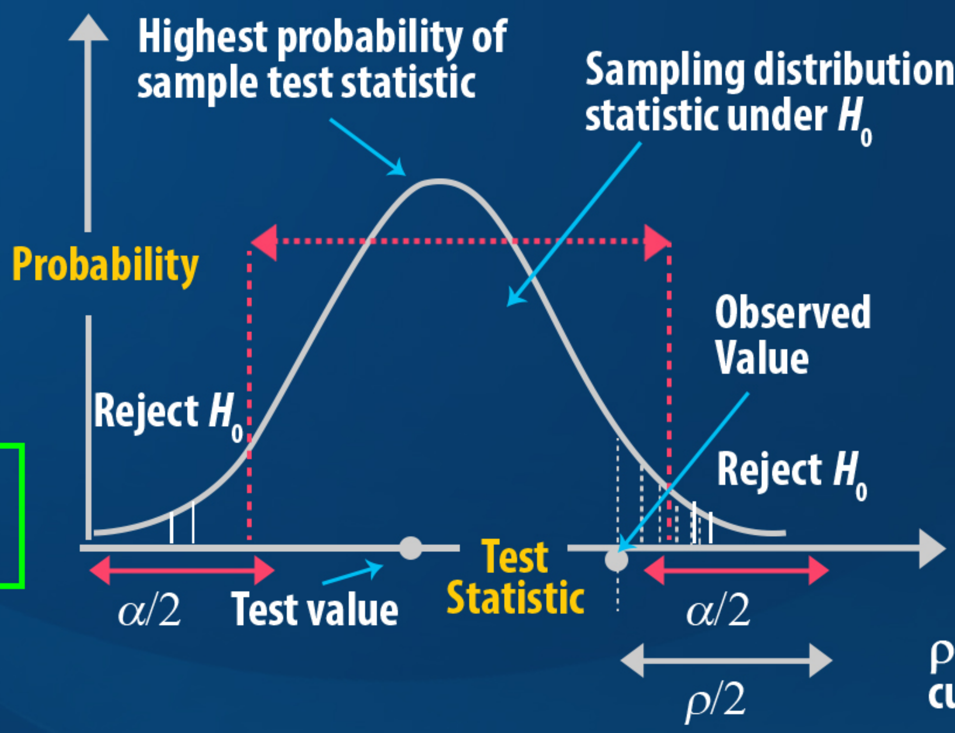
HYPOTHESIS TESTING: CONCEPTS

H_0: Null hypothesis

If $p \leq \alpha$, Reject H_0
If $p > \alpha$, we fail to reject H_0



α: Significance level

P(Reject H_0 | H_0 is true)
= Area under the curve

ρ-value

Probability of finding the observed or more extreme results when the H_0 is true

ρ-value = 2(area under the curve of observed value)*

* symmetry

EXAMPLE

Problem Statement

Blood glucose levels for obese patients have a mean of 100 with a standard deviation of 15. A researcher thinks that a diet high in raw cornstarch will have a positive effect on blood glucose levels. A sample of 36 patients who have tried the raw cornstarch diet have a mean glucose level of 108. Test the hypothesis that the raw cornstarch had an effect or not.

Step 1: State the hypotheses.

The population mean is 100.

$H_0: \mu = 100$

$H_1: \mu > 100$

Step 2: Set up the significance level.

It is not given in the problem so let's assume it as 5% (0.05).

Step 3: Compute Z Score.

$$z = \frac{x - \mu}{\sigma}$$

$\mu =$ Mean

$\sigma =$ Standard Deviation

What the numbers say?

For this set of data: z= (108-100) / (15/√36)=3.20

For this set of data: f

Even though the event is rare: It happened !

p < 108 = .993

Thus, p ≥ 108 is 0.0007.

H_0 is thus rejected.

PARAMETRIC vs NON-PARAMETRIC TESTS

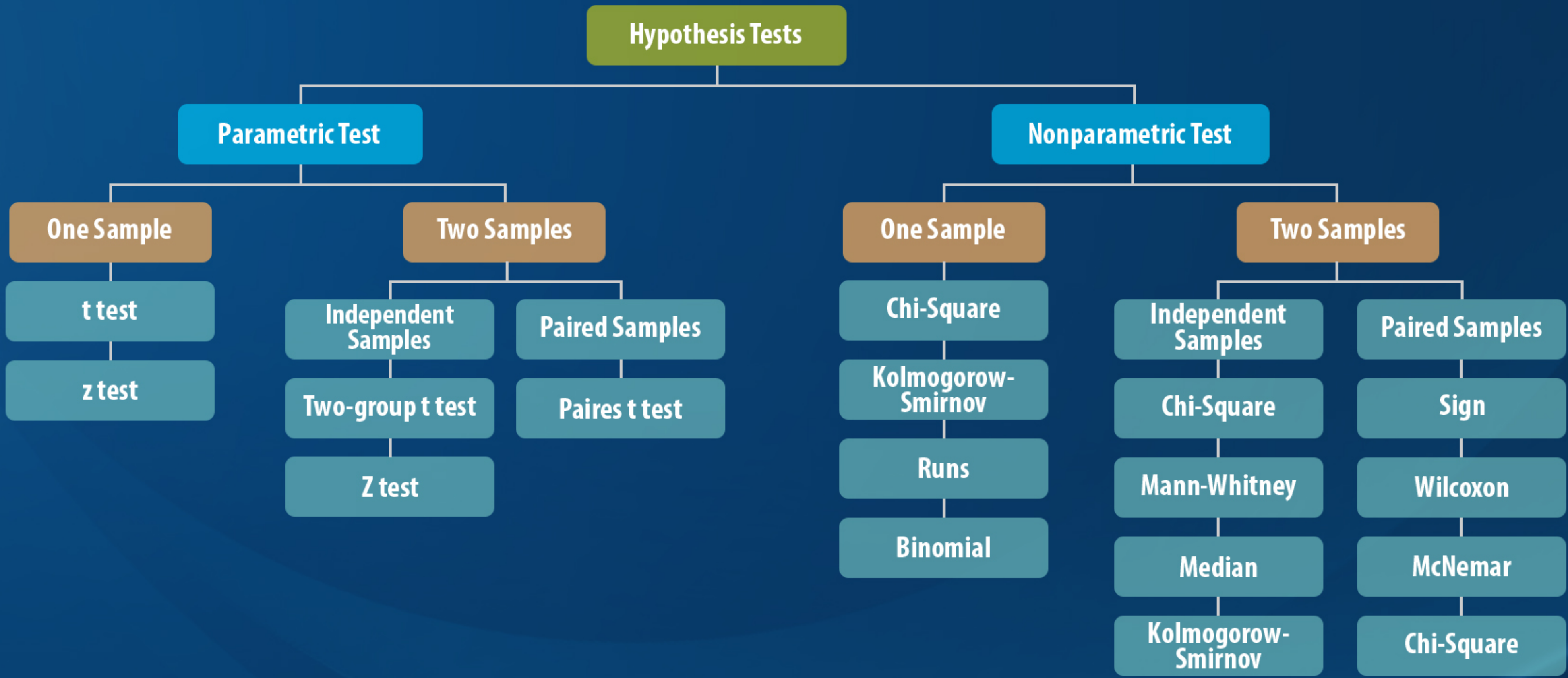
Parametric tests

The parametric test is the hypothesis test which provides generalisations for making statements about the mean of the parent population. A t-test based on Student's t-statistic, which is often used in this regard.

Non Parametric tests

The nonparametric test is defined as the hypothesis test which is not based on underlying assumptions, i.e. it does not require population's distribution to be denoted by specific parameters. The test is mainly based on differences in medians. Hence, it is alternately known as the distribution-free test.

PARAMETRIC vs NON-PARAMETRIC TESTS : BIG PICTURE



PARAMETRIC TEST EXAMPLE I

Problem Statement

A coffee shop relocates to Italy and wants to make sure that all lattes are consistent. They believe that each latte has an average of 4 oz of espresso. If this is not the case, they must increase or decrease the amount. A random sample of 25 lattes shows a mean of 4.6 oz of espresso and a standard deviation of 0.22 oz.

Use alpha = 0.05 and run a one sample t-test to compare with the known population mean.

Step 1: Define Null and Alternate hypothesis.

H_0: mean amount of espresso in a latte = 4 oz

H_a: mean amount of espresso in a latte NOT= 4 oz

Step 2: Computing the test statistic.

t-test = (sample mean – population mean)/[stddev/sqrt(n)]

The sample mean "x" is 4.6 oz

The "mean" is the population mean of 4 oz.

The sample std dev is 0.22 oz

n = 25

df = n – 1 = 24

t -test = (sample mean – population mean)/[stddev/sqrt(n)]

df = n – 1 = 24

=(4.6 – 4) / [0.22/sqrt(25)]

= (0.6)/[0.22/5] = 0.6/0.044 = 13.6

Therefore, the t-test value is 13.6.

PARAMETRIC TEST EXAMPLE II

Step 3: Determine critical values.

Our degrees of freedom for this one sample t-test is :

- df = n – 1 = 25 – 1 = 24
- Our alpha value is .05
- Our test is two-tailed

Therefore, using any t-table, the two "critical values" that represent the cut-off points for rejection are:

$t_c = +/- 2.064$

Step 4: Reject decision.

Result:

13.6 > 2.064

Reject H_0

What does this mean ?

H_a tells us that, there is a significant difference between the amount of espresso in the Italy coffee versus the expected mean.

NON PARAMETRIC TEST: CHI-SQ GOODNESS OF FIT I

About

Chi-Square goodness of fit test is a non-parametric test that is used to find out how the observed value of a given phenomena is significantly different from the expected value.

$$\chi^2 k\text{-}p\ \text{-}1 = \sum_{k} \frac{(f_0 - f_e)^2}{f_e}$$

where

f_0 = **observed frequency**

f_e = **theoretical expected frequency**

k = **number of categories or classes remaining after combining classes**

p = **number of parameters estimated from the data**

NON-PARAMETRIC TEST: CHI-SQ GOODNESS OF FIT II

Context

Employers particularly want to know which days of the week employees are absent in a five day work week. Most employers would like to believe that employees are absent equally during the week.

Given Data

Day of the Week Absent

	Monday	Tuesday	Wednesday	Thursday	Friday
Number of Absences	5	4	2	3	6

Problem Statement

For the population of employees, do the absent days occur with equal frequencies during a five day work week? Test at a 5% significance level.

NON-PARAMETRIC TEST: CHI-SQ GOODNESS OF FIT III

Step 1: Set up Null and Alternate Hypotheses.

Ho: The absent days occur with equal frequencies, that is, they fit a uniform distribution.

Ha: The absent days occur with unequal frequencies, that is, they do not fit a uniform distribution.

Step 2: Compute Chi-Sq Statistic.

Day	Expected Value: E	Observed Value: O	(O-E)	$(O-E)^2$	$\frac{(O-E)^2}{E}$
Mon	4	5	1	1	0.25
Tue	4	4	0	0	0
Wed	4	2	-2	4	1
Thu	4	3	-1	1	0.25
Fri	4	6	2	4	1
			Sum		2.5

Step3:

$P(\chi^2 > 2.5)$ *p* value : 0.6446

We DO NOT reject the NULL Hypotheses