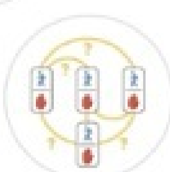
SHA574: Modeling Uncertainty and Risk

What you'll do

- **Calculate marginal value for a binary decision.**
- **Determine optimal values for a repeating, sequential decision.**
- **Build risk aversion into your model.**
- **Calculate utility for a given decision.**
- **Develop and use a Monte Carlo simulation.**
- **Perform a sensitivity analysis.**
- **Use expected utility to accommodate risk.**



Course Description

Incorporating probabilistic calculations into decisions takes them to the next level. The decisions we make are rarely as simple as we would like them to be, and in a competitive environment, the cost of making a haphazard decision can be extreme.

Using probabilistic estimates of future outcomes expressed through probability distributions, you will calculate marginal values for a go/no-go type of decision. From here, this course will guide you in analyzing a more complex set of decisions, where decision makers have the ability to alter course or change decisions as uncertainty–based upon prior decisions–is realized.

The decision model you create allows you to run a Monte Carlo simulation, and the data from that simulation gives you the ability to examine outcomes that vary based on several decisions that may interdependent. This course guides you through modeling situations that are increasingly complex, training you to characterize risk and uncertainty using mathematical structures that you'll use to approximate moderately complex business decisions.

Chris Anderson
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Chris Anderson is a professor at the Cornell School of Hotel Administration. Prior to his appointment in 2006, he was on the faculty at the Ivey Business School in London, Ontario, Canada. His main research focus is on revenue management and service pricing. He actively works with industry, across numerous industry types, in the application and development of revenue management, having worked with a variety of hotels, airlines, rental car and tour companies as well as numerous consumer packaged goods and financial services firms. Anderson's research has been funded by numerous governmental agencies and industry partners, and he serves on the editorial board of the *Journal of Revenue and Pricing Management* and is the regional editor for the *International Journal of Revenue Management*. At the School of Hotel Administration, he teaches courses in revenue management and service operations management.

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Meet Your Class

Using the discussion below, please tell us about yourself; we’re eager to learn more about you as well as your classmates. What do you hope to learn from the course? What is your profession? Where are you located? Please respond in a text format or as a video using the film strip icon that is available once you click "Reply".

(If posting a video response, we recommend that you do not use your cell phone as most do not use Flash software which is required to convert the recording.)

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Module 1: Making One-off and Repeating Decisions

Repeating Decisions

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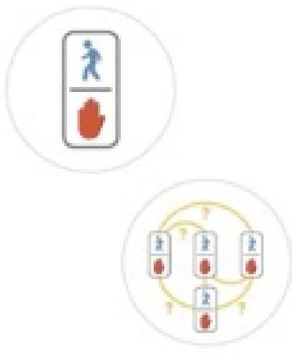
Module Introduction: Making One-off and Repeating Decisions



Decisions often arise as opportunities that may or may not be worth it. Often these are novel situations, and the tendency is to take your best guess, often erring on the side of what seems like the conservative choice. But when you consider the potential costs of not taking an opportunity that may have a significant payoff, it becomes less clear what the conservative option is.

Probabilistic decision making makes a lot of sense, so long as you can find some way of describing how chance affects the outcome of your decision. Often you don't even need historical data to describe your decision in terms of probabilities. In this module, you will examine the basics of go/no-go decision making based on data you've collected or, if no data exists, on theoretical probability distributions. You'll use these methods to perform a marginal analysis that will help you decide whether a given decision is worth it or not.

Watch: Building Uncertainty into Decision Making

In this video, Professor Anderson introduces the idea of using probability to incorporate uncertainty into decision making. He describes the landscape of probabilistic decision making from the more straightforward to the more complex decisions you'll examine in this course.

Video Transcript

Uncertainly plays a role in many of the decisions that we face. Tools like regression or predictive analytics can help us decrease uncertainty. Tools like descriptive statistics or visualizations can help us understand the impacts of uncertainty upon our decision. Our focus now is really going to be how to incorporate that uncertainty into the decision making process itself. To incorporate uncertainty into the decision making process requires the use of probability. Probability helps us describe the chance of future outcomes. Once we know the chance of those future outcomes, we can roll that back and look at the impacts of those future outcomes upon the decisions that we have to make today. We're going to look at four different approaches to build that uncertainty into our decisions.

For simple yes/no or go/no-go type decisions, we're going to weigh the trade-offs between outcomes and their chances of occurring. We're also going to look at a series of incremental decisions where we're going to continue or proceed as long as the incremental impacts are in our favor. We can also look at a sequence of decisions where after a decision is made, uncertainty is realized, and we can adjust subsequent decisions based upon the realization of that uncertainty. And then lastly, we're going to focus on more complex settings where we can't separate that uncertainty out into a series of discrete events. We need to enumerate or simulate all that uncertainty at once to allow us to look at the impacts jointly versus separately. All these approaches hinge upon our ability to describe the likelihood of future events. We're going to use probability to help us describe the likelihood of those future uncertain events. And probability, and a deep understanding of that becomes our first step.

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Watch: Accounting for Dependence in Event Sequences

When predicting an outcome, knowledge is power. Preventative maintenance is based on an awareness that a small fix now reduces the chance of more significant failure later. But to make the justification, you need to be able to connect the decision now with a later decision. Mathematically, this is described as conditional probability, as in "the probability that Y happens is conditional on the event X occurring."

In this video, Professor Anderson provides a basic introduction to sample space and discusses conditional probability. You'll want to pay close attention to the example he describes in the latter part of the video. It's a familiar example of conditional probability with a result that many people find counterintuitive. A solid understanding of the reasoning here will help you as you model problems in which conditional probability is a factor.

Video Transcript

We use probability to quantify the chance that a particular outcome is going to occur, usually in the future. We're going to refer to these outcomes as events. In order to quantify the chance of these outcomes occurring, we need to know the set of all possible outcomes. We're going to refer to this as our sample space. We then describe the probability of events occurring within that sample space, usually as proportions. And so these proportions are going to be between 0 and 1. Event never occurs, it will have a probability of 0. If an event always occurs, it will have a probability of 1. We can use probability of an event to describe the outcomes of two or more events. We can look at a simple well-defined dice example in order to illustrate that. We'll come back to more realistic settings in the near future.

So, what is the probability of getting a 1 on a dice? That's simply our event being 1, and there's a sample space of 6, so it is 1/6. Now we could ask what's the probability of getting two 1s on two separate dice? So we could simply take the 1/6 for the first dice and multiply it by 1/6 for the second dice to get a 1/36. We could also look at it differently where we could say, okay, what are all the possible combinations of different numbers on 2 dice? There's only one way to get two 1s, and then there's 36 other possible combinations. So we have an event of 1 and a sample space of 36, so we would just have 1 over 36.

So we can do these probabilities of multiple events in different ways. Sometimes one method works better than the other. For example, if we were trying to look at the probability of getting a 7 when rolling two dice, that becomes very tricky because it's hard for us to sort of look at all the different ways to get 7. It's much easier to just enumerate all the combinations of getting a 7, which it turns out that there's six different ways to get two dice to sum up to a 7 out of our 36 combinations, to get a probability of 1/6. So sometimes it's easier to calculate those probabilities in one way versus another.

We have to be careful though, because probability is often not as easy or as logical as it seems. A very classic example of that is a game show. In this game show, our host has three doors. Behind one of those doors is a shiny new automobile. And behind the other two doors is sort of a gag prize. In the original form of that game show it was a goat. And so, as a contestant we pick one of those three doors in hopes that the car is behind that door. The host, he opens one of the other two doors. In doing so, he's providing us some new information. Originally, our probability of having the car behind our door is simply one of three doors. And now we would think that because the host has opened one of those doors, that we have a one in two chance, or probability of a half of having a car behind our door.

What's critical here though is that the host is never going to open a door that has the car behind it. He has this private information about where the cars and the goats are. And so, the probability that the car is behind our door versus the host door is now conditioned on that new information. And so originally the host had a two thirds chance of having the car and we had a one third chance of having the car. When the host exposes the goat, his two-third chance all lays now on his remaining door. And so if the host asked us if we would switch our door for his, we should always switch doors. And so here is a classic case where our logic may get in the way of doing the right thing. And so we have to realize that once the host opens that door, the probabilities of where the car are are now dependent versus independent events. Independent events are where probabilities of one event are not impacted by other events.

So, rolling a dice does not impact the outcome of rolling an another dice. Whereas, we have dependent events, where the outcome of a second event may depend in fact on the first event. So think if this is pulling cards from a deck of cards. If I was to pull a red card on my first draw and set it down, the probability of pulling a red card on my second draw is now going to depend upon that first card being a red card because there's fewer red cards left. And so in practice, many of our decisions are going to depend upon prior decisions. And so we're going to have to be cognizant of that when we're thinking about or estimating our probabilities.

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Watch: Estimating Probabilities Using Distributions

No good decisions can be made without some prior knowledge of the possible outcomes of the decision. In the ideal world, we would make decisions with a high degree of predictability based on the data from past decisions that were made under the same conditions. In reality, it's impossible to control for all factors that impact a decision. Still, a good sample of relevant data is a great place to start. But what if you have no data relevant to your decision, or the data you have is a limited or flawed sample?

In this video, Professor Anderson describes the conditions needed for making good decisions based on empirical data. He also addresses the situation when you lack data but still need to make an informed decision. He introduces several different theoretical distributions that you may be able to use to inform a decision when you lack actual data. You will be able to learn more about these distributions on the course page that follows this video.

You may wish to download the Excel workbook referenced in this video and refer to it after watching the video. It is often helpful to look more carefully at the formulas and verify that you understand how each works, or make modifications and see how the results change.

Video Transcript

So, in some settings the process driving our uncertainty is well-defined. This typically occurs in games of chance like rolling the dice, or drawing cards, or flipping coins, where we understand the physics of what's going on in order to estimate those probabilities. More often than not in business, we don't have these simple games of chance driving our success or failure. And so, we don't know all the details of the underlying process which are driving our uncertainty. So we need to look at alternative ways to measure probabilities. We can typically do this in one of two fashions. One is data driven or empirical. Where we are basically going to look at a data sample and calculate probabilities based upon relative frequencies. So counting the number of times a particular event occurred and dividing by the total number of events that happened. This works well if we've collected a sufficiently large sample that has all possible outcomes and the data is not biased by the collection process.

Theoretical distributions on the other hand are not necessarily requiring as much data, but they require a deep understanding of what this sort of data is going to look like? What is the shape or how does the distribution of those outcomes tend to look? So we have to know what the appropriate distribution is in order to rely upon a theoretical distribution function. So let's look at an example and sort of talk about some of the pluses and minuses of each of those. We have some historic sales data, basically daily data of how many items we sold each day. We can take that historic data and look at the different types of outcomes. So the different events are sales of each different level, and then we can calculate the frequency of sales at each of those different levels. And then we could calculate relative frequencies, which basically are those frequencies divided by the total number of observations in our sample.

Those relative frequencies, in essence, are probabilities. We can focus on probabilities of an individual event, or we can focus on cumulative probabilities where we're focused on sales greater than or equal to a particular level, or potentially sales less than or equal to a particular level. Now though that data is probably limited by some of our actions. Specifically, if we only had a certain stock of items, then we're never going to observe sales beyond that highest level that we kept on hand. And so in essence our data is constrained by our actions. And so we can think of our data as being sales data and we're probably trying to estimate future demand, or the probabilities of future demand. And so we're never going to be able to estimate probabilities beyond what we observe. And so this now becomes a really good opportunity to talk about theoretical distributions. Because now we could assume that perhaps sales follow a normal distribution.

A normal distribution is a very common distribution we use in practice. It is basically described by two parameters, the average and the standard deviation of that underlying process. And so now we could look at our data through histograms, we could also overlay probabilities from our normal distribution and see how well they align, we could sort of focus not just on individual probabilities but cumulative probabilities. And we quickly see that our theoretical distribution will allow us to estimate probabilities beyond our max historical sales level. So, we get some of the ability to extrapolate beyond individual observations in the sample itself. So we're going to focus on the use of both empirical and theoretical distributions. We're going to focus on the normal as one of our theoretical distributions, but we're also going to talk about others like the poisson, the exponential, and the binomial.

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Tool: Normal, Exponential, Binomial, and Poisson Distributions

As you have just seen, theoretical probability distributions are useful for informing probability-based go/no-go decisions, especially when no empirical data is available. When you use theoretical distributions, you will need to choose the type of distribution based on your needs and then tailor its parameters for your specific situation.

The information on this page and on the downloadable reference tool linked below will help you make the most appropriate choice of distributions. Each tab shows sample graphs of a distribution with different parameters along with a description of the data type and range, usage, and relevant functions in Excel.

Download the Theoretical Distributions Reference sheet to keep handy for future guidance in making probability-based decisions.

Normal
Exponential
Binomial
Poisson

Watch: Calculating Expected Value for a Binary Decision

Reward comes with risk, and payoff follows (or doesn't) from investment. When making investment or spending decisions with a variable payoff, it's necessary to think in terms of expected value. "If I spend X, what is my average payoff Y?" This average payoff is not necessarily an average. You may only make this decision once! Rather, it's a hypothetical average based on a scenario in which you make this decision—or decisions like it—repeatedly over a period of time.

In this video, Professor Anderson discusses marginal value in the context of a simple gamble with two possible outcomes. He then extends that notion to a sequence of repeating decisions. Assuming the rules remain constant throughout the sequence of decisions, the value of a sequence of *unconnected* decisions is trivial to calculate based on the probability of success with a single decision.

You may wish to download the Excel workbook referenced in this video and refer to it after watching the video. It is often helpful to look more carefully at the formulas and verify that you understand how each works, or make modifications and see how the results change.

Video Transcript

So given that we can use probabilities to describe the likelihood of future outcomes, we can now use those probabilities to help us assess simple yes/no, go/no-go type decisions. The simplest of that we can think about is gambling, and we can ask ourselves is gambling worth it, statistically speaking? So what if we had a gamble and it cost us $10 to play and if we won we received $100, but we would only win 50 percent of the time, or the probability of 0.5. That means if we played a hundred times, it would cost us $1,000 to play, we would win 50 of those 100 times on average, returning us $5,000. So our $10 bet, on average, is going to return us 0.5 times 100, or $50. So a very profitable bet. Versus an alternative gamble which still cost us $10, still returned $100 if we won, but now we only win one one hundredth of the time. And so now our $10 bet is going to return us only $1 if we win. So not profitable at all, and not nearly as profitable as our first bet. So we can generalize these two gambles when we can think about weighing off the probability of winning times our payout versus our investment. So given that we get a chance to play many times, as long as our average payout, right, the payout times the probability of receiving it is greater than our investment, that would be a go-type decision.

So let's look at a slightly more complicated example. So, the Ontario Lottery and Gaming Commission proposed this baseball-based bet. And so, for a particular baseball game, the commission would pit two opposing players against each other. One of those players would be slightly better, would get on base slightly more often, and he would be designated the favored and the other would be designated the underdog. And so basically, the nature of the bet was, you had to pick one of these players and they would decide whether or not your pick was correct or not depending upon how many bases that individual player traversed. So, if the favored traversed more bases, he would be victorious, if the underdog traversed more bases, he would be victorious. If they got the same number of bases, then basically a tie, well that tie would go to the underdog, right? And so the nature of the bet was, it wasn't just about picking one game, you had to pick five games and you had to get all five correct. If you picked all five games, you picked the right player in all five of those games, then your $1 bet would return you $19.

And so curious about this bet, we collected some data. We collected data over the first two months of the baseball season, and we looked at the number of games where the favorite had the most bases, the number of games where the underdog had the most bases, and then the number of games where they tied. If we summed up the number of times the underdog had the most bases plus the number of times that they tied, and divided that by the total number of games, that's the percentage of the time that if we bet on the underdog, we would win, or that's the probability that if we bet on the underdog we would win.

Now, we have to pick five of those bets not just one. So the probability of getting five of those correct would simply be that probability to the power of five. Now, so that's the probability that if we bet on the underdog five times we would win, if we won our $1 bet would return us $19. So that probability times 19 is actually much larger than a dollar, and so now we have this positive return from our investment. So we could participate in this gamble.

Now, participation in this gamble is only going to work if we're going to get multiple times to play and if in the future the underdog performs similarly to the way they have performed in the past. And so we can look at the fact that we got to play over the remaining part of the season, so over the next four months. We had over 1,000 different games to bet on, and it turns out that the fraction of the time that if we bet on the underdog that we would win was very similar to the first two months of our sample. And so it turns out under this setting, where the probability process is consistent and we get lots of times to play, that our expected value rule is going to work very well.

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Watch: Extending Marginal Value for Repeating Decisions

In the previous example, we looked at a single decision: to bet or not to bet. In the example, we were placing this bet repeatedly over a sequence of days, but the probabilities and payouts were constant from bet to bet. We will now look at situations where we make a sequence of similar go/no-go decisions, but in this case, the probabilities of payouts will change with subsequent decisions. Think of advertising spend, stocking decisions, or manufacturing production levels. The law of diminishing returns usually applies in each of these cases. So finding the ideal stopping point for your investment becomes critical for maximizing profit.

In this video, Professor Anderson works through a simple stocking example to illustrate how incremental or marginal value can inform repeating decision problems.

You may wish to download the Excel workbook referenced in this video and refer to it after watching the video. It is often helpful to look more carefully at the formulas and verify that you understand how each works, or make modifications and see how the results change.

Video Transcript

We can now extend our go/no-go approach into a sequence of decisions, versus just a single no go decision. In essence, we're going to incrementally decide whether or not to go or no-go, as long as that expected value is still positive or our probability times our payout is greater than our cost to participate. So let's look at a class of problems, where we have to pre-commit to a decision. And that's, in this case, going to be a capacity decision, prior to observing demand. Right? So let's look at newspapers where I have to order my papers today, prior to seeing demand tomorrow. But this approach can apply to any situation where I have to make that decision in advance of that uncertain demand. And so we're trying to weigh the trade-off between not having enough stuff to sell, versus having stuff that I can't sell because today's paper is of no use tomorrow.

So we have some data to describe potential demand. So we've been operating our newsstand for 100 days. We've sold between 0 and 10 papers each of those days. So, I have the frequency that I've sold a certain demand level. I can look at relative frequencies to calculate probabilities, and then I can look at cumulative probabilities to talk about a range of outcomes. So I could talk about the probability of selling exactly one paper or I could talk about the probability of selling one or more papers. So if I had only one paper, then the probability of selling one or more papers times my revenue, let's assume our revenue is a $1.50, that probability times $1.50 is greater than our cost, in this case 90 cents. So in our go/no-go framework, that would be a go to have that paper.

We could now sort of, continue to apply this process and look at the second paper, should I have the second paper, should I have the third paper? Right? So if we look at the fourth paper, the probability that demand is greater than or equal to four, times the 1.5, is still bigger than our 90 cents. Whereas, when we look at the fifth paper that probability times 1.5 is less than 90 cents. So all of a sudden our go/no-go decision became a no-go because that fifth paper was not profitable. And so you could think of these incremental decisions that we're making as evaluating the incremental or marginal profit of each of those stocking decisions.

So if I have zero papers I make nothing, but if I have one paper, I almost make that full 60 cents for that paper. As I expand my inventory to that second paper, I don't quite make as much as the first paper but its still contribution positive. And what we should realize is that sum of the marginal profit from no papers, plus the marginal profit from one paper, plus the marginal profit from two papers is now our cumulative or total profit. And so as we continue to sum those marginal profits up through the fourth paper, that becomes our maximum total profit. As we add that fifth paper, which has an incremental negative return, then our total profit starts to decrease. Right? So we can use this incremental no-go type framework to look at what is the appropriate number of papers to stock in a piece-wise fashion in order to maximize our total profit. Right? So our decision of what to do never calculates total profit but the total profit is an outcome of making these incremental go/no-go type decisions.

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Watch: Using Expected Marginal Value to Analyze Trade-offs

Sometimes we're faced with making a decision that benefits one aspect of the operation with a potential cost to another part of the operation. Market cannibalization between product lines is one example of this. As complexity creeps into decision making, our analysis tool set needs to respond to keep up with complexity. As a first basic step, it is essential to identify clearly the key performance indicators (KPIs) that describe the outcomes most important to us. Often, this allows us to reduce a complex set of decisions, each with its own outcome, to a simpler set of higher-level decisions.

In this video, Professor Anderson works through an example in which expected marginal value is used to manage resources that have interacting constraints. When the marginal value of a resource drops below a threshold, it opens up alternatives for using that resource.

You may wish to download the Excel workbook referenced in this video and refer to it after watching the video. It is often helpful to look more carefully at the formulas and verify that you understand how each works, or make modifications and see how the results change.

Video Transcript

So now let's look at ways to generalize that incremental no-go type decision to a set of more elaborate-type problems. So let's look at a rental car firm. This rental car firm has two types of cars, standard and luxury. We've been in operation for a while renting cars and we're trying to sort of increase profitability, most likely by changing the size of our fleet. So we have some historic information on how many cars were rented each day. We can look at the frequencies of renting each number of cars. Then we can take those frequencies into relative frequencies and talk about the probability of renting 10 cars versus 11 cars, etc.

So we can look at demand across different inventory sizes. Now, each car has some monthly associated fixed costs. So we have to lease the car and then we have to insure that car in order to put it on the lot. If we take those monthly fixed costs and we divide that by 30, you basically have that car's daily fixed costs. Now when that car is out for rent it generates some revenue, it has some variable cost. That revenue minus its variable costs is its daily revenue per rental. And now we can basically look at our incremental go/no-go type analogy to decide how many cars that we should have on the lot. Where basically our incremental revenue is the probability that demand is greater than or equal to a certain number of cars times that daily revenue. If that is greater than our daily fix cost, then that would be a profitable car to keep on the lot. And so we can continue to expand the number of cars we keep as long as that daily incremental revenue is greater than that daily cost. And we could do that for both types of standard and luxury cars.

But what we realize now as we shrink our fleets and we make it a little bit smaller that there are several days where we're probably going to run out of cars because demand is in fact quite often greater than the number of cars we've stocked. And so one approach might be to, when we run out of standard cars, is perhaps to allow standard car seeking customers access to luxury cars but at standard prices. We probably don't want to give everybody who was looking for a standard car after we ran out of them access to luxury cars because we might actually run out of luxury cars for those who are actually willing to pay luxury prices.

So just like we used our incremental marginal approach to look at how many cars to stock, we can also use it to look at how many cars to allow to be upgraded. Now we can still look at revenues and cost, but our revenues are going to be fixed at this daily standard car revenue because you can think of the individual at the counter willing to hand you over his rental fee for that standard car. What's uncertain here is the potential luxury revenue. If I was to reject that upgrade, keep that car for a luxury customer, would I in fact have a luxury customer who needed that car? So what's the probability that car is needed as a luxury car times the daily net luxury revenue? If that expected daily luxury revenue is greater than my standard car revenue, then I would say no to that free upgrade. But as long as the free upgrade is higher in revenue than the expected luxury car revenue, I would allow that car to be upgraded. And so we can continue to move through our number of allowable upgrades up until the point where we're better off keeping that car as a luxury car versus allowing it to be upgraded as a standard car.

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Course Project, Part One—Perform a Marginal Analysis

You've seen how expected value is used to make one-off and repeating decisions, and how marginal value can be used to explore alternatives for constrained resources. In this first part of your two-part course project, you will apply what you have learned to a fictitious scenario, considering a decision similar to the one you just encountered with the rental car firm. You will be performing Excel spreadsheet calculations to support making a decision that maximizes profit. Completion of all parts of this project is a course requirement.

Instructions:

1. Download the course project document and the Part One Excel workbook template.
2. Complete Part One of the course project.
3. Save your work.
4. Submit your completed project document and the Part One Excel workbook for grading and credit.

Do not hesitate to contact your instructor if you have any questions about the project. You will add to this document as the course proceeds and will submit it to the course instructor at the end of the course.

Before you begin:

Before starting your work, please review the **rubric** (a list of evaluative criteria) for this assignment. Also review eCornell's policy regarding **plagiarism** (the presentation of someone else's work as your own without source credit).

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Determining a Distribution and KPIs

Instructions:

You are required to participate meaningfully in all discussions in this course.

Discussion topic:

In this module, we have examined several scenarios in which you would use expected value or expected marginal value to analyze a key decision. In this discussion, you and your classmates will consider decisions from your own work or life situation.

Consider what challenging decisions from your work or nonwork experience you have made or need to make that involve a significant degree of uncertainty. What data would you need to build a probability model? Alternatively, if it's unlikely that data will be available, what theoretical probability distribution would you use and why?

To participate in this discussion:

Create a post in which you:

Describe a decision in your workplace or outside work

Explain which aspects of the decision involve uncertainty and what that uncertainty looks like
Identify the variables for which you would need data to calculate probabilities *OR* identify a theoretical distribution and justify its use
Identify the KPIs on which you would need to focus your analysis, and indicate how they are linked to the uncertainty in your decision

Click the **Reply** button to post a comment or reply to another comment. Please consider that this is a professional forum; courtesy and professional language and tone are expected. Before posting, please review eCornell's policy regarding **plagiarism** (the presentation of someone else's work as your own without source credit).

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Module Wrap-up: Making One-off and Repeating Decisions

Almost all decision-making situations involve some sort of uncertainty about the outcome. It is critical to rigorously incorporate this uncertainty into the decision-making process. In this module, you saw how using the expected value of future uncertain events can help you make decisions when faced with uncertainty. You created a single decision rule that helped you decide when to continue with a decision based on whether the average or expected payout of continuing was at least as high as the cost. Several examples in this module hint at the broad applicability of this technique.

You saw how this simple decision rule could be extended to describe a sequence of go/no-go decisions, where payouts changed with subsequent decisions. This still involved calculating expected values of uncertain outcomes, but these became expected marginal values, and payouts were calculated in incremental steps.

Next, we looked at a different kind of sequential decision making in which the uncertainty from earlier decisions is realized before having to make later decisions.

Module 2: Adjusting and Accounting for Risk

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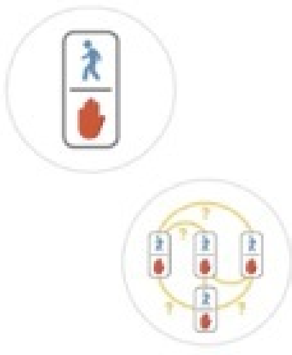
Module Introduction: Adjusting and Accounting for Risk



Knowing that uncertainty is a given in decision making, it's natural to think about how we might stack the odds in our favor. The more information we have about the possible outcomes, for instance, the better prepared we'll be to make the decision.

In this module, you will examine the role that conditional probability can play in improving outcomes. Recall that conditional probability is the chance of an event occurring given that another event has already occurred. So when we make a sequence of decisions over time, we can use conditional probability to suppose the earlier decision has already been made. In this way, we can adjust our assessment of uncertainty over time.

You will look at the specific case of testing before a decision. You'll see how to calculate the monetary value of a test that could improve understanding of your decision and its outcomes. You will also examine ways to account for sensitivity to uncertainty more explicitly. You'll see how utility functions can help you to adjust values to acknowledge risk aversion.

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Watch: Making Decisions in a Sequence

Often we can say something about the uncertainty we face in decision making. For example, we may be able to say something is high or low risk, or we may even be able to use empirical data to give a percentage likelihood for two outcomes from a given course of action. But quite commonly we have to incorporate two distinct sources of uncertainty, or we need to consider a sequence of decision scenarios where the outcome of an earlier decision impacts the risk parameters of a later decision.

In this video, Professor Anderson introduces two examples in which an initial decision is connected indirectly with two or more final outcomes. In the one example, the first decision is a purposeful decision about performing a test to get information. In the other example, the first decision is a business decision but the second is in reality more of an accounting for market forces. From these examples, you can see how the branching nature of nested decision making can lead to complexity. In subsequent videos, you will see how probability and expected value can be used to manage this complexity.

You may wish to download the Excel workbook referenced in this video and refer to it after watching the video. It is often helpful to look more carefully at the formulas and verify that you understand how each works, or make modifications and see how the results change. *Manipulating this Excel workbook requires that you have a third-party add-in (TreePlan) installed, and this add-in has a license fee associated with it. You are not required to install or license this add-in to complete this course.*

Note*:* In this video, decision flow is intentionally presented visually in two slightly different ways. One visualization is more of an operational flowchart approach, while the other is a traditional decision tree. One may be more familiar than the other, but you should understand that both are valid ways of visualizing decision chains. Because a decision tree app is available as an add-in for Excel, there will be a greater emphasis on decision trees in this course.

Video Transcript

So sometimes we're making decisions in the face of uncertainty. We have more than two outcomes. Or we may have to make two or more decisions where those subsequent decisions are made after the uncertainty is realized from that first decision. So think of a product launch. I may decide to do some test marketing in advance of the full launch. And before I have to decide whether or not I'm going to do the full launch, I would have results from the test marketing. So first I have to decide whether or not to test market and then based upon the results of that, I will decide whether or not I should do that launch.

Similarly, we can be drilling for oil and I might decide to do some seismic testing. And then based upon the results of that seismic testing, I may then decide whether or not I should drill or not drill. So, Exxon is trying to decide whether or not it should drill a particular oil well. If it decides to drill it's going to incur some costs associated with drilling. If it actually discovers oil, then it's going to have a revenue stream resulted with discovering oil. If it doesn't discover oil, then basically there's no revenue stream from that dry well. In this particular area, about 70% of the wells are dry and 30% of them actually hit oil. And so we can visualize our decision and its timing, right?

So we have to decide to drill or not to drill. We incur an expense if we drill, after we've drilled then we are either going to hit oil or not. We're going to hit oil 30% of the time and if we hit oil, then we're going to have the revenue stream minus our costs of drilling as our net proceeds. If we don't hit oil, then we have the cost of drilling as our net proceeds. And then we can look at, on average, if we do this numerous times, what would be our expected value. Right, so that would be the 30% of the proceeds of heating oil plus 70% of the proceeds of not hitting oil. And then when we're deciding whether or not to drill or not, we can weigh off those expected proceeds versus not drilling. And deciding to drill if drilling proceeds were higher than not drilling.

And so let's look at a little more complicated example. So we have a manufacturing firm, and it's having to decide on staffing. And so basically it anticipates growing demand. And so it can meet that growing demand in one of three ways. It could simply use overtime, it could add more workers, so it could increase its workforce, and it could actually add a second shift. Right, and so each of those different ways to expand our work force have different sort of economies of scale associated with those. What our firm doesn't know is what future demand is going to be and what is the demand for that workforce.

There's a small probability that future demand is going to be low. There is a fairly large probability that demand will be medium and then a moderate probability that demand will be high. We can outline our decision, right? So the three different ways we could expand our workforce. And then we could look at, so given that decision to use overtime, then what would be our proceeds if demand was low, medium or high. And then based upon those proceeds and the associated probabilities of those three outcomes, we can calculate on average our expected proceeds or our expected, you know, state of nature. From deciding whether or not to pursue overtime versus increase our workforce, or add a second shift. And whichever of those is in our favor, whichever of those is the highest net proceeds, that would logically be the path of capacity expansion most desirable.

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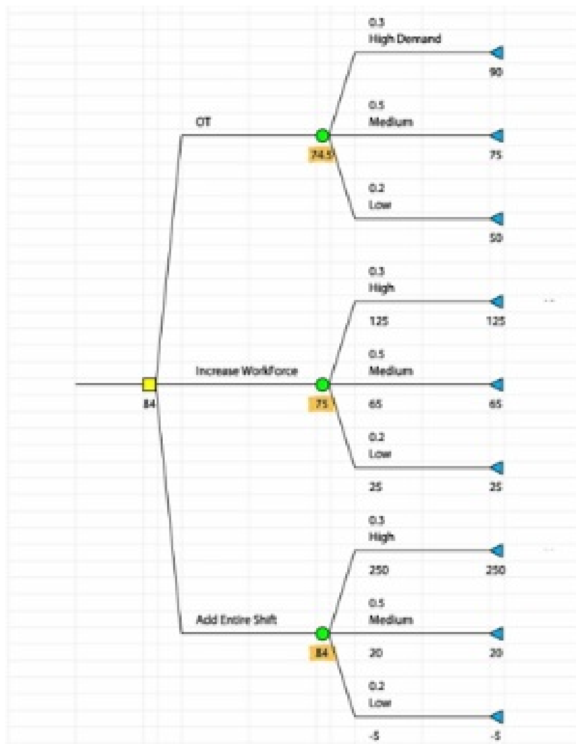
Read: Creating Decision Trees in Excel for Decision Analysis



TreePlan is an add-in tool for Excel that allows you to create a decision tree that supports decision analysis. The tool automatically calculates values of outcomes based on inputs of probabilities and costs. In order for TreePlan or similar decision analysis tools to have value, you need to work with a sequence of two or more decisions in which the outcome of one decision has an impact on any subsequent decisions.

To see how you can use TreePlan in Excel, download the Create a Decision Tree step-by-step how-to guide.

Note: TreePlan is a third-party add-in for Excel that requires you to pay a licensing fee to Treeplan Software. However, you do not need to install or *license TreePlan to complete this course. If you plan to use TreePlan on a routine basis, you will need to purchase a license. Alternatives to TreePlan may exist.*

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Watch: Assigning Value to Perfect and Imperfect Information

This video focuses on the hypothetical scenario of perfect information. The concept of perfect information is based on a thought experiment in which you can perform a test that eliminates all uncertainty around a future decision. Imagine being able to walk into a car dealership and knowing with absolute certainty the minimum sale price the sales manager would accept. While this does not happen in real life, it's useful to consider for this reason: if the perfect test has an associated cost, it will only be worthwhile paying for that test if the potential benefit outweighs that cost. In essence, this gives you a maximum value for any test you might perform to gather information that will reduce uncertainty before a decision is made.

In this video, Professor Anderson uses the oil exploration scenario to examine perfect information and imperfect information in the context of decision trees. He demonstrates how to use TreePlan or other decision analysis tools to identify the most equitable path through a branching sequence of potential choices.

You may wish to download the Excel workbook referenced in this video and refer to the "oil full" tab after watching the video. If you have purchased the TreePlan add-in or are working with a trial version, you will be able to make modifications and see how the results change.

Note: The material in this video may be challenging to grasp all at once. With practice, you will come to understand the approach better. For now, if you can come to terms with the idea that perfect information as an upper limit for the value of testing, you should be in pretty good shape.

Video Transcript

So when we're making decisions in the face of uncertainty, one of the things we might ask is, is it worth it for us to try and reduce that uncertainty? So can we reduce some of that uncertainty prior to having to make that decision? So as a marketer, I might conduct a test market or maybe send out a survey in advance of launching my product in an effort to sort of gauge what future demand is, right? So that test marketing or that survey is basically helping me reduce uncertainty. Or as an oil company, I might conduct a seismic test prior to actually drilling the well to sort of refine my estimates of whether or not I think there's oil under the ground.

So let's continue that thought. So we're looking at drilling for oil. I have sort of an idea of the probability that if I do drill, that I will discover oil. And so we might ask ourselves, what if we had perfect information? So what if we could say if we knew ahead of time that there was actually going to be oil there, would that in fact save us money? So if I knew there was going to be oil, you know there's oil 30% of the time, and I decided to drill, then I would have the associated net proceeds. If I decided not to drill, then there would be no proceeds and obviously if the well was dry, if I knew it was going to be dry and I decided to drill, that would still cost me some money. And if I didn't drill, that wouldn't cost me anything.

So what we can do now is what we call calculate the expected value of this perfect information and it turns out that if we had a crystal ball then we can compare the expected value of knowing ahead of time whether or not there was oil versus our original case of drilling for oil, and the difference between those is the value of perfect information. And so now I have this upper bound of how much I might want to spend before on my seismic test in order to reduce that uncertainty. So, now no test is perfect. So now we have this seismic test but it's slightly imperfect. So we know that 30% of the time there is oil under the ground. If I was to conduct my seismic test and turn back a positive result for oil, well because the test is not perfect, that means that some of the time there is oil but some of the times there's not oil even though the test was positive. Whereas if the test was negative, there still might be oil. And more often than not, when the test was negative it was dry.

So we could calculate these conditional probabilities. What's the probability that there's oil given that we had a positive test? So, now we have this new information. So if I decide to do the test and it turns out to be positive, now I have a refined estimate of whether or not in fact there's oil there. And so we can think about this as our tree again. So we can think about this where now our first decision is test or no test. The test comes back it's positive or negative. So if we decided to test and the test was positive, we know what fraction of the time the test had positive results. And then if we decided to drill and incur the costs associated with drilling, then we can now estimate the probability that if the test was positive and I drilled that I struck oil.

That new probability of striking oil is considerably higher than the original 30% because this is conditioned on actually having a positive test result. And so, now we could sort of calculate our states of nature. So we can look at all the things that could happen, right? And how much money would be in the bank on each of those different points in time. And then we could roll those back to our decision points and then we can simply use our expected value rules. Which is greater, that's the path I'm going to go down. And then we can continue to roll that back all the way to our original test or no test. And now we can look at the expected value of deciding to test and we see that the difference between not expected value and not testing, that is actually the value of the test itself. And we'll see that that difference between the value of testing and not testing is not the same extreme as perfect information, but it tells us basically the value of that test and whether or not we want to spend more money on actually getting a better test.

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Watch: Accounting for Risk Aversion

Two decisions that have equivalent outcomes in terms of expected values are not always treated the same by decision makers. People will sometimes have an irrationally inconsistent reaction to two equivalent decisions. The basis for this inconsistency is sometimes based on risk-averse behavior. While it would be nice to think we could just apply probability and let the numbers speak for themselves, the reality is that we cannot always escape the human factor in our decisions.

In this video, Professor Anderson explores risk aversion and introduces the idea of a utility function as a tool to incorporate risk aversion and other human factors into decision making. Expected utility calculations allow us to restore decision making to a numerically based process without ignoring factors that lie outside the realm of pure probability.

Video Transcript

Up until this point, we've been using expected values to help us make our decisions. And in essence, expected values as a decision making tool assumes that we're going to make similar types of decisions over and over again. So I may hit oil once and not the next time, or even not even the time after that, but on average I'm going to hit it in sufficiently enough time to make money. But what if we don't make these decisions numerous times or what if there is catastrophic risk associated with failure such that we don't get a subsequent opportunity to drill another well? So expected values, in essence, assume that our returns are linear in monetary reward. So $1,000 is twice as beneficial as $500, right? Whereas, for most of us we have decreasing utility in increasing financial awards. And so it's not simply linear but the actual scale is important.

So you could think about it visually. So if I had a graph and on the X-axis was your financials state of mind and on our Y-axis was utility then in our expected value world, in our expected value world things are linear, that they're just constantly increasing as our reward increases. But for most of us there is this big return mentally from going negative to positive. And then as I become more and more financially stable then we have this increasing utility, but at some point it starts to level off and the incremental changes in utility start to decrease. So these non-linear utilities are inconsistent with our expected value approach.

So a classic example that we often face is something like insurance. So you're trying to insure your house against catastrophic loss from Mother Nature. And so your $350,000 house has a one in a thousand chance of being destroyed by fire but we purchase home insurance for $500. In our expected value world, we would basically only agree to pay $350 for that insurance, right? So implicitly in that decision we have these non linear returns with risk associated with losing our house. And so we can still capitalize on our expected value approach, we're just going to use utilities versus dollars. We're going to take, instead of calculating the expected value of the outcome, we're going to calculate its expected utility where we translate dollar values into utilities.

So if we're looking at drilling for oil and we're trying to calculate the expected value of drilling, instead of looking at oil and no oil and the associated states of financial nature, we're going to look at drilling versus not drilling. And then when we look at drilling we're going to evaluate having oil and not having oil and we're going to look at the expected utility, where we translate our dollar value into this utility value, and then we're going to make these expected value trade-offs but based upon utility versus simply dollars.

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Watch: Calculating Utility

Utility functions are a nice idea, but how exactly do you define a utility function?

In this video, Professor Anderson introduces a simple method for developing a utility curve, and he explains how to use that curve to obtain a utility value for any probabilistically calculated expected value.

Video Transcript

So before we can use utilities, and our expected value approach, in essence turning it into an expected utility approach, we need to have some way to translate dollars into utilities. In essence, we have to create some sort of function or a curve in order to evaluate the utilities of certain monetary values. There's basically three common ways to do that. The first of those is what's referred to as certainty equivalence. And you can think of this as a game, this game is settled by a coin flip. Heads you have a certain reward, tails you have a different reward. And then we simply ask participants, "What would be the dollar value where you'd be equally happy taking the money and running versus playing this game settled by a coin flip?"

We could think of a second approach which is referred to as the lottery example where it's a very similar structure but now the three payouts are specified, and the user has to specify the probability of the game winning and losing for when they're indifferent to playing the game or not participating. And then the third approach is actually just assigning curves or functions, typically an exponential utility curve, to the underlying utility. So, let's go through our certainty equivalence example so we get a sense of the dynamics as how we might go about assigning these utilities.

So we might start with this coin flip which has our max payout of a million and our min payout of zero. Let's assign utility of zero to a payout of zero and utility of one to a payout of a million. And then we ask participants, "When are you willing to- when are you indifferent to this game in some fixed payout?" If our participant said $300,000, now we can estimate the utility of 300,000 because it's simply just the midpoint of the utilities of those two other payouts, so in this case 0.5. And we could do a subsequent round where now the payouts of the coin flip are 300,000 and a million. And we ask the user, when are they willing to participate in this coin flip. And if they said 500,000, then we could calculate the utility of 500,000 as being 0.75. And so, we could go through this for many different values of financial reward and calculate different points along our utility function. So, once we have this utility function, then we can interpolate utility of any monetary value between what we initially started with as zero utility and our maximum utility.

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Read: Three Ways to Generate Utility Curves

In a previous video, Professor Anderson described two of three common methods for generating utility curves. This page summarizes all three methods and describes them in slightly greater detail. On each tab, it is assumed you are looking at a game with two distinct payouts for winning and losing.

Certainty equivalence and lottery methods are typically used with individuals, but when utility is being considered at the company level, it is more typical to apply a mathematical function to generate utilities.

Certainty Equivalence
Lottery Method
Functional Form

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Tool: Certainty Equivalence Method

Certainty Equivalence Method
Tool for Creating a Utility Curve

The Certainty Equivalence Method tool linked from this page is an Excel sheet that allows you to develop a utility curve and estimate utility using the certainty equivalence method. Certainty equivalence is one of the more commonly used methods for generating a utility curve, perhaps due to its ease of use.

Recall that in the certainty equivalence method you are always looking at a binary choice between:

An event with two outcomes (a coin flip, essentially)
An alternative to the coin flip event

The expected value of the coin flip is based on the probability of the two outcomes. You begin by identifying the payout amount for which you consider the two alternatives equally valuable. This first equivalence is the payout that has a utility of 0.5 and can be graphed as ($value_{payout}$, 0.5). You then repeat this process to add points to the graph.

The supplied tool allows you to generate 5 points on the utility curve and can be modified to allow for more than 5 points. Once your curve has been generated, you can use it to estimate the utility of any value in your range. The accuracy of your estimate depends on the number of points in your curve.

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Activity: Experiment with Risk Taking

Two Gambles Worksheet

Whether or not you accept the premise that people sometimes make irrational, inconsistent decisions, there is no substitute for experiencing this yourself. In this activity, you will get some first-hand experience with risk-averse behavior and asymmetric decision making.

In order to complete this activity, you will need to engage with 5 to 10 individuals outside this class. You will need only a few minutes of each participant's time, and overall you should not devote more than an hour to completing this activity. Essentially what you will be doing is asking each of your willing participants to play two games involving chance outcomes and simple decisions made by them.

What they will not know (and you must not tell them) is that the expected value of the two games will be exactly the same in both cases. What you should see is that at least some of the participants will express bias against one presentation of the risk—

that is, some will take the certain payout for one the games but will choose to gamble on the chance outcome in the other game. As an interesting follow-up, you might consider asking participants who chose inconsistently why they feel one gamble was better than the other.

Because you will be asked to summarize your experience in a class discussion, you should use the worksheet to keep track of your results. You can download this worksheet from the link above.

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Inconsistent Decision Making

Instructions:

You are required to participate meaningfully in all discussions in this course.

Discussion topic:

In the experiment with risk-taking activity, you should have seen that at least some participants in the games were inconsistent in their choice between risk and the option that provided a clearly defined outcome. Why do you think this was? In which game did you see the risk-averse behavior?

To participate in this discussion:

Create a post in which you describe the results of your experiment. Be sure to indicate how many participants you had, what proportion of them were inconsistent, and how many of these inconsistent players showed risk-averse behavior for each of the games. As reminder, game #1 involved a payout and game #2 involved what we might call a penalty.

In addition to describing your results, address the following questions in your post:

What rationale can you offer for the behavior you observed?
If you asked any of the participants for their rationale, what noteworthy responses (if any) did you receive?
What other (non-gambling) examples in life or work can you think of that lead to similarly inconsistent choices being made?

Use the **Reply** button to post a comment or reply to another comment. Please consider that this is a professional forum; courtesy and professional language and tone are expected. Before posting, please review eCornell's policy regarding **plagiarism** (the presentation of someone else's work as your own without source credit).

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Module Wrap-up: Adjusting and Accounting for Risk

In this module, you moved away from the comparatively simple decisions in the first module and on to more complex decision scenarios. Using decision analysis (with decision trees), you explored the role uncertainty plays when making a sequence of decisions over time. As you make these decisions one after the other, the uncertainty connected to prior decisions is realized before we need to make subsequent decisions—e.g. you know the results of the test market before you need to decide whether to do a full national product launch. You saw how decision analysis enables you to build the realization of uncertainty into your initial decision through the roll-back process—e.g. you can decide whether or not you should perform test marketing with a clear understanding of how testing would impact the profit/loss prospects of launching (or not launching) the product. In essence, decision analysis allows you to adjust for uncertainty over time.

You also explored utility as a way to directly accommodate risk preference into uncertain decisions. In your experiment with risk-based games, you experienced risk aversion and the inconsistent decision making that can result. You saw how you can adjust for these inconsistencies in your expected value frameworks using expected utility in place of expected values.

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Module 3: Using Monte Carlo Simulation for Nuanced Decision Making

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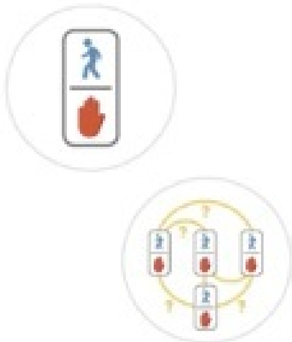
Module Introduction: Using Monte Carlo Simulation for Nuanced Decision Making



In the previous module, you examined techniques that can help you see the best decision or sequence of decisions in the face of uncertainty unfolding over time. These techniques fall under the term prescriptive analytics because they define a best decision given probabilities, costs, and payouts.

In this module, we return to the descriptive spreadsheet-based models like those built at the beginning of the course. As you've seen, these models are helpful for one-off decisions or a sequence of decisions in which the outcome probabilities do not change over time. What we'll do now, though, is look at how a simulation can be built on top of a probability-based model such that we can identify a best decision given our projected outcomes. As before, this best decision will be calculated in terms of key performance indicators (KPIs). Among the benefits of this simulation-based approach is the flexibility with which we can frame simulations. This allows us to build in the detail necessary to reflect the logic of our situation accurately while accounting for the random nature of real-world outcomes.

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Watch: Developing a Simulation

It's natural as decision makers to focus on and remember decisions we have made—especially ones that we think of as particularly good or particularly bad. But from the perspective of others—in our company, our community, or in society—a lot more attention is paid to outcomes. Did we meet our sales targets or deadlines? Is our customer satisfaction rating better or worse than it was last month?

With simulation, the focus is appropriately on KPIs. A simulation is essential for measuring our decisions against what we (and others) ultimately care about. The power of simulation is in the shift of focus, but also in the fact that it can accommodate an arbitrary number of decisions.

In this video, Professor Anderson lays out the conceptual framework for creating a simulation and considers an example in which our decision-making power is elevated well beyond what is possible with expected value decision making.

Video Transcript

Expected value and marginal value approaches allow us to tackle decisions where we have one form of uncertainty impacting that decision. Decision analysis allows us to tackle a sequence of decisions where each of those decisions has a single form of uncertainty and that uncertainty is realized before we have to move on to that next decision. But in many settings, we have multiple sources of uncertainty acting all at one time and we need to be able to make decisions giving those multiple forms of uncertainty. We can't isolate each of those forms of uncertainty. And so, simulation is a tool that allows us to tackle decisions given there is multiple forms of uncertainty acting at one point in time.

Now, simulation is not going to make decisions for us, but it's going to allow us to evaluate numerous key performance indicators as a function of the decisions that we could make. So expected value approaches start with probability distributions and calculate the expected values of future outcomes. In simulation, we're not going to calculate the expected value of future outcomes, rather, we're going to use those probability distributions to generate potential future outcomes. And then, given those potential future outcomes, evaluate potential decisions. So we could think of a rental car firm having to decide how many standard or luxury cars to keep in inventory, given that demand for each of those cars is uncertain. And the firm may allow some of the standard car seeking consumers to upgrade to luxury cars if they run out of standard cars. So the demand for standard and luxury cars is linked by these upgrades.

So we could use simulation to now generate say, 100 or 1,000 realizations of potential demand for standard cars and potential demand for luxury cars, and then look at the revenues that result from a set of stocking decisions. So simulation is not going to tell us how many cars to stock, but if we give our simulation our stalking decisions, it could tell us the average revenue. It could tell us how many customers were served, how many customers were not served, etc. Any type of performance indicator that is of concern to you.

So for our rental car example, our simulation would start with some data. That data would describe the frequencies or probabilities of demand across certain levels of sales. We could translate that probability distribution into a histogram and then we could translate that histogram into a pie chart. And then, all we're doing with simulation, is taking that pie chart and thinking about it as a spinner where we have a dial that we're going to spin around this pie chart and wherever that needle points to, that's a potential realization of tomorrow's demand. The sizes of the pies are proportionate to the probabilities of each of those outcomes. So as we spin the wheel, it will generate future demand that looks like historic demand. And so, we might spin that wheel for standard cars, we might spin that wheel for luxury cars. And then, given a set of inventory decisions, calculate our profit or our loss demand. So we can do that spinning process over and over again and look at how our profit is impacted by those individual stocking decisions.

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Watch: Generating Random Inputs

You saw, via simulation, how future outcomes can be thought of as sections of a spinner that are proportional to their relative likelihood of occurring.

In this video, Professor Anderson continues with the spinner analogy and introduces the concept of uniform random variables. He also discusses how, in practice, to generate and use the uniform random variable in building the virtual spinner that is the basis of our simulation. Excel-specific techniques related to random values will be described in even more detail on the next page.

Video Transcript

At the heart of simulation is our ability to generate potential future values that look like those historic observations, or look like we think those historic observations look like. And so we don't actually have our computer spin the spinner, what we have it do is generate what we refer to as Uniform (0,1) Random Variables. So basically, every computer has this ability to generate these numbers, numbers between 0 and 1, such that every number between 0 and 1 is equally likely to be generated. So that's why we refer to it as uniform (0,1) random variables. You could think of those uniform (0,1) random variables in the context of our spinner. So the top of our spinner corresponds to zero, and as we generate this random uniform (0,1) random variable, our dial rotates around the spinner, and going all the way back to 1 at the top. So we could generate that random number, our spinner would go a certain way around the wheel, and we would translate that to the corresponding level of rental car demand.

We could do another spin, and that would correspond to a different level of rental car demand. So, those fractions around the wheel are defined by our data. So we're using these empirical probability distributions. We can also use theoretical probability distributions. So we think demand follows a normal distribution or an exponential distribution. All we have to have is the ability to translate that random (0,1) random variable to the corresponding say, normally distributed random variable.

So let's look at a hypothetical example. We're looking at a snowfall dependent promotion, and so as a function of snowfall, there will be certain rebate levels. And so, we think that snowfall follows a normal distribution with some mean and standard deviation. So, given I know the mean and the standard deviation, I could use Excel's NORMDIST function to calculate the probability that it's going to snow less than or equal to some certain number.

And that would return this probability by definition between 0 and 1.

Now, I could generate this random number between 0 and 1, and then I could now ask Excel to return the corresponding normally distributed random variable that has that mean and standard deviation. We do that with this norm inverse function, so it's the inverse of the normal distribution function. So norm inverse, I give it the probability, or this uniform (0,1) random variable, I specify the mean and the standard deviation, and it returns the corresponding value. And so whether we have an empirical data driven probability distribution, or a theoretical understanding of what our future variables will look like, we can use this uniform random number to generate potential values from those distributions.

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Read: Generating Random Variables in Excel

A lookup table of outcomes and probabilities is efficient for generating random variables based on empirical data.

Any theoretical distribution that can be inverted can be easily simulated in Excel.

Simulation requires the generation of potential future outcomes and the evaluation of KPIs across decisions. The KPIs in essence evaluate the impact of these potential future values upon the decisions. We can generate these potential values via two main approaches: empirical/data-driven probability distributions and theoretical probability distributions. Here we will explore in detail the functions in Excel useful for generating potential future outcomes.

Empirical/Data-Driven Random Variables

Let's assume we have a sample and have summarized that sample via relative frequencies, or perhaps we fully understand the process behind our uncertain outcomes (e.g. coin flips or dice rolling). Either way, we have a set of outcomes and their associated probability of occurring. The table shown here lists our 6 outcomes: A, B, C, 1, 2, and 3, with probabilities 0.1, 0.15, 0.05, 0.2, 0.25, and 0.25, respectively.

Outcome	Probability	Index
A	0.1	0
B	0.15	0.1
C	0.05	0.25
1	0.2	0.3
2	0.25	0.5
3	0.25	0.75

The first step in generating representative potential future values is to create a lookup index, which is a function of the probabilities. The index starts at 0 and increases by adding the outcome probabilities, for example, 0 + P[A] = 0.1 and 0.1 + P[B] = 0.25. Now we generate a uniform random variable, =RAND(), and use =LOOKUP to find this value in the Index and return the corresponding Outcome from the same row. So, our function in Excel would be =LOOKUP(RAND(), C2:C7, A2:A7). If RAND() returned a 0.23, this number is bigger than 0.1 but less than 0.25, so it would return outcome B. If RAND() was 0.8, it would return a 3—i.e. if our spinner rotates between 10% and 25% (10% plus 15%) of the way around the wheel, then our outcome is a B. If it spins 80% around (i.e. past the 75), then our outcome is a 3.

Deriving Random Variables from Theoretical Distributions

When using theoretical distributions to generate potential future outcomes, we don't need the table or index functions; we simply use mathematical expressions or built-in formulas to generate the values. We discuss a few common distributions below.

Uniform Distributions

Uniform distributions are those where all outcomes are equally likely (i.e. a flat histogram). The RAND() function in Excel generates uniform random variables between 0 and 1. Therefore =10*RAND() generates uniform random variables between 0 and 10, and =5+5*rand() between 5 and 10. If we want uniform discrete random variables, then we also need to use the =INT() function in Excel. For example, =INT(RAND()*100) generates discrete uniform random variables between 0 and 99 since INT(99.999999)=99.

Normal Distributions

In Excel, =NORMDIST(X, μ,σ,1) returns the probability of an observation being less than or equal to X. The function =NORMINV(p,μ,σ) is the inverse of NORMDIST—i.e.

NORMDIST(0.5, 1, 2, 1) = 0.4013 whenever NORMINV(0.4013, 1, 2,) = 0.5. In essence, if you give NORMINV a probability, it returns the X that corresponds to that probability. This makes simulation easy since =RAND() is our probability. =NORMINV(RAND(), 1, 2,) will simulate random variables that follow a normal distribution with a mean of 1 and standard deviation of 2.

Exponential Distributions

The Excel function =EXPONDIST(X, 1/μ,1) calculates the probability for exponentially distributed variables, but Excel does not have a built-in function to invert exponentially distributed variables. However, it is easy to generate this inverse ourselves with =–LN(RAND())*μ, which generates exponentially distributed random variables with a mean of μ.

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Watch: Connecting Simulation Outputs to KPIs

In this video, Professor Anderson returns to the rental car example to illustrate just how a simulation is built in Excel. Pay careful attention to the steps he goes through, and make sure you have a sense of the decisions and intent (if not the actual formulas used) as he puts each building block of the simulation in place. You will have the opportunity to build a simulation yourself later in the course, and you may find it useful to return to this video to review the process after your first attempt to go through simulation building yourself.

Note that in this video probabilities are based on empirical data. Any simulation you build based on empirical data will follow a similar process. However, if you are using a theoretical distribution, your method will vary slightly.

You may wish to download the Excel workbook referenced in this video and refer to the "car rental simulation" tab after watching the video. It is often helpful to look more carefully at the formulas and verify that you understand how each works, or make modifications and see how the results change.

Video Transcript

So looking at our rental car simulation model, at the top of that model we have our three decisions. How many standard luxury cars to put on the lot, as well as how many free upgrades to allow. Then we have our net revenue from renting each of these car types, as well as the daily fix costs for each car in order to put it on the lot. As a function of those decisions and parameters, we can calculate a series of KPIs, whether those KPIs are profit or potential lost demand. We can estimate profit through generation of a series of rental car demands. We generate those rental car demands by first generating a series of uniform (0,1) random variables. And then, using our lookup functions to translate those uniform (0,1) variables into the associated demand.

Our look up tables are based upon historic data. That historic data is translated into probabilities and then cumulative probabilities. And then, we look up those uniform random variables across those cumulated probabilities translating those to realize rental car demands. We can take those realized rental car demands and estimate sales. Our standard car sales will be a function of how many standard cars are available for rent and that standard rental car demand. Now, if it so happens that demand is larger than our cars available, then we have some upgrade opportunities, but we can't upgrade more consumers than we've pre-specified. When we look at our luxury cars, we have to keep in mind that luxury cars available for sale may be impacted by the number of free upgrades that we've already allowed.

So once we have our sales across our two car types, we can calculate our revenue based upon our net revenue for rental and our profit by subtracting out the fixed cost associated with keeping those cars on the lot. We could calculate that profit, say for a thousand realizations of rental car demand, and then we could average that profit across all 1,000 realizations. Now, we can look at how those averages of those KPIs are impacted by our three decisions. Ideally, finding the combination of number of cars to stock and free upgrades such that, say, we've maximized our average daily profit.

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Determining Simulation Parameters

Now that you have begun to see how simulation works, it's time to check your understanding. In order to set up a simulation properly, you need to be able to identify appropriately the inputs and outputs of the simulation. The questions in this quiz are the questions you should be asking yourself as you begin to think about how you might frame your own decision in a way that allows you to solve it with simulation.

In this quiz, you will answer questions related to a fictitious scenario involving Bitcoin.

Read the following scenario carefully and either copy the scenario into another window or take notes before proceeding to the quiz questions.

Scenario: As the CFO of a global manufacturer, you have been asked to investigate if you should allow payments to be made using Bitcoin, as this would simplify exchange across numerous currencies. The downside of using Bitcoin is its volatility. Recently, Bitcoin value has been swinging up and down very quickly. If you decide to use Bitcoin, you will save about 1% on every transaction because you do not need to pay currency exchange fees (e.g. to convert

US dollars to euros).

You have collected some data on Bitcoin prices, and you have used this data to see how large minute-over-minute swings in prices are. You then looked at the distribution of these price swings and are now trying to assess the potential risk of using Bitcoins to receive payment from international customers. To gauge the impact, you are looking at ten upcoming transactions and are wondering the potential impact of using Bitcoins to facilitate these transactions.

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Watch: Maximizing or Minimizing KPIs Using n-Way Tables

Sometimes we're faced with making a decision that benefits one aspect of our operation with a potential cost to another part of the operation. Market cannibalization between product lines is one example of this. As complexity creeps into decision making, our analysis tool set needs to respond to keep up. As a first basic step, it is essential to identify clearly the KPIs that describe the outcomes most important to us. Often, this allows us to reduce a complex set of decisions, each with its own outcome, to a simpler set of higher-level decisions.

In this video, Professor Anderson works through an example in which several related decisions must be made in a way that optimizes profit. He introduces the use of an Excel table (a 3-way table, in this case) to maximize the profit KPI.

You may wish to download the Excel workbook referenced in this video and refer to the "car rental sim with 3-way table" tab after watching the video. It is often helpful to look more carefully at the formulas and verify that you understand how each works, or make modifications and see how the results change.

Use the Create an n-way table Excel step-by-step instructions to guide you in making your own analysis tables in Excel.

Video Transcript

So in our rental car simulation, we're using a table to enumerate average profit across different combinations of our three decisions of how many cars to stock, across each type, and our number of free upgrades. As we generate a new sample, so as we recalculate our spreadsheet, we'll notice that the profit changes a little bit, but hopefully, the set of decisions that are maximizing that average profit are the same. If we re-evaluate our spreadsheet, and we generate another thousand set of customers and our decision changes, so we move from one set of number of cars to stock to a different one, all of a sudden we see that our decision is a function of the sample. And so that's a really good indicator that our sample is not large enough, or not representative enough. And so we need to look at a larger sample, such that when we recalculate that larger sample, that yes the profit changes a little bit, but we always have the same decision on how many cars to stock.

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Watch: Using Common Random Variables in Simulation Models

As you set up your simulation, you will need to consider carefully how you generate the random variables you'll be using. There are several options here. Will you put a =RAND() function in each cell that has uncertain values? Can you generate just one random variable and refer to that cell throughout your simulation? Or is neither of these the right approach? The choices you make in how you generate and use random numbers does matter, and it needs to reflect the realities of the scenario you're trying to describe.

In this video, Professor Anderson considers a scenario in which random values generated in one part of the simulation model are used in several other parts of the model. In this case, the scenario is a five year model with two distinct random occurrences for any given year. As a result, this simulation model requires 5 × 2 = 10 random variables.

You may wish to download the Excel workbook referenced in this video and refer to it after watching the video. It is often helpful to look more carefully at the formulas and verify that you understand how each works, or make modifications and see how the results change.

Video Transcript

In our rental car simulation, we had to generate a fairly large sample, such that our decisions were independent of the sample itself. What we're going to look at now is another technique that helps us sort of limit the sensitivity of the decision to the sample. And we're going to do that through what's called common random variables. Our example is set in an airline. So, Ontario Airlines is looking to insure its fleet of aircraft. The typical insurance policy lasts about five years, and insures their crafts against both incidental damage as well as major crashes. Now, there's a very low chance of any individual plane crashing. But given that each plane takes off and lands multiple times per day and flies almost everyday of the year, then given that we have several planes, there's a lot of potential crash events in any given year. And so, there is a fairly reasonable probability that the airline might have one or two crashes in any given year. And so, now we want to look at is the impact of those crashes upon their costs under a series of different insurance policies.

Now, to simplify our model and ensure that we don't need to have too large of a sample, we're going to use what's referred to as common random variables, and what that really means is, is the same set of planes in the same set of crash events are going to be evaluated across each of our four policies. So, in our spreadsheet-based simulation, we're going to have a block of cells, one cell for each of five years. Those cells are going to simply have a uniform random variable. And we're going to use those cells to determine whether or not there was a crash or more than one crash in any given year. We're going to have another block of cells also with a set of uniform random variables, and we're going to use that set of cells to quantify any incidental damage that might occur from maybe, perhaps, hitting a bird while we're in flight.

And so the logic is there is our spreadsheet just split out into these blocks. We have these 10 sets of uniform random variables which we're going to drag down and copy for 70, for several different years. And then what we're going to have is a set of blocks for each of our insurance policies. And given those incidental damage or crash events, we can calculate the cash flows from each of those four policies given the same set of planes are going through each of those policies. So we're looking at the same planes, in essence going through each of these four different decisions in order to simplify the size of the sample we need to look at.

Watch: Applying Common Random Variables across Cases

In this video, Professor Anderson continues with the airline insurance scenario and shows how the simulation model is used to inform an actual decision.

As you watch the video or look more closely at the supplied spreadsheet, you may notice that crash data is generated using two blocks of five columns. The left block of five columns includes just random numbers. To the right of this block, five more columns use cell references to fetch those random numbers to generate crash data. Given this approach, you may wonder why the incidental value calculations (the third block of 5 columns) have the =RAND( ) function built directly into the formula rather than drawing from an external set of random numbers. The reason is that the random numbers used to generate simulated crash data are used in two places within the formula. Within each instance of the formula, *both instances of the random value must be the same value.* Using the =RAND( ) function twice in the crash cells would generate two different random numbers, which would not work for the formula. By contrast, each of the formulas for incidental damage requires only one instance of a random number.

You may wish to download the Excel workbook referenced in this video and refer to it after watching the video. This is the same Excel workbook that is linked in the previous page.

Video Transcript

So in our airline simulation, we basically have two different sources of damages. Catastrophic failure, or crashes, as well as incidental damage. Each of those can occur on an annual basis for 10 sources of damage across our five years. We're going to generate 10 columns of uniform random variables to model those 10 sources of damages. For the first block of five, those are going to correspond to potential crash events. And we can take those uniform random variables and look up whether or not we had zero, one, or two crashes in each of those five years. For the next block of five sets of random variables, we can transform those uniform (0,1) random variables into incidental damages that range from $1 to $5 million dollars, uniformly distributed.

Now that I have those 10 sources of damages, I can sum up annual damages for each of the five years to basically have our potential costs, had we chose not to insure. Then we can take those potential costs and run those through each of our four policies. Our policies are going to require us to pay a premium. And then, each of those policies is going to cover some of those damages, and we'll have an annual cost associated with each of the policies across the five years. Some of the policies have refunds, which are great. We can take the costs across each of the five years for each policies and using the NPV function, discount those back to a net present value back into today's dollars. Once we have net present values for each of our four policies, then we can describe those net present values across a series of realizations of plane crashes.

So if we simulated a thousand different potential scenarios, we can look at our descriptives across those thousand scenarios. Given that we're focused on risk, not just average costs, we can talk about the average cost, the standard deviation, as well as the min and max of our expenses under each of those four policies, as well as self or no insurance. We can rule out some of the policies because we noticed that policy one costs more and is more risky than policy three. So we can set it aside right away. We noticed that policy two starts to look very efficient because it's only slightly more expensive than self-insurance, but has half of the downside risk because it's max cost is half of that of self-insurance and also has half of the standard deviation. So simulation starts to enumerate or illuminate some of the insight of these four policies and really narrows our focus down. Using common random variables, or the same set of crashes going through each of the four policies, ensures that we are doing an apples to apples comparison from policy one, to two, to three, etc. So, greatly simplifying how many scenarios we need to look at.

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Watch: Using Simulation Models for Atypical KPIs

In order to make a good simulation model, you will need to identify all the key variables in a scenario and connect them in the right way to your KPIs. Sometimes this is straightforward. Other times, less so. With practice, it should become easier to translate your scenario into an Excel workbook that reflects the randomness and connections between your situation's inputs and outputs. As long as you have a good understanding of your situation, you should be able to create an accurate model.

In this video, Professor Anderson explores an example related to making a staffing decision. You may find it challenging on your first viewing to follow the exact logic that underlies the model described and some of its details. For now, you shouldn't worry about that. The main takeaway here is that there are purposeful choices linking the randomness with the model and that the KPIs are specific (% of customers exceeding a wait time threshold) while being based on a broader, high-level goal (excellent customer satisfaction).

You may wish to download the Excel workbook referenced in this video and refer to it after watching the video. It is often helpful to look more carefully at the formulas and verify that you understand how each works, or make modifications and see how the results change. The "two service reps 100 rows" runs the simulation with a small data set, while the other two tabs involve substantially more data. As a result, this is a larger Excel file (>2MB). It may load and refresh slowly for you.

Video Transcript

So one of the advantages of simulation is the flexibility it allows us in choosing what sort of key performance indicators we want to choose. So we can have a whole myriad of different types of KPIs. So let's illustrate some of those with a simple example. So we're looking at staffing our call center and we're trying to decide if we should be moving from having just two customer service representatives, to three customer service representatives. Now, given what individuals call our call center, they're okay with a little wait, and they're even okay if they have to wait. What becomes critical is that people don't like to wait very long. And so one of our KPIs might be the fraction of consumers who call that have to wait, say, more than a minute to reach an operator. Right? So yes we could keep track of average wait, we could keep track of the percent that wait, but we're really focused on what fraction wait longer than some critical level.

Now for our call center model, there's really just two major levels of uncertainty, right? When people call, and then how long it takes to handle their call. So in our simulation, we're going to have two random variables, two uniform random variables, uniform (0,1). And then we're going to translate each of those into the respective uncertainties, one will do time between arrivals. Where we're going to assume the time between arrivals follows an exponential distribution. And similarly, we're going to assume that service time follows an exponential distribution. So we're going to have these four columns which we use to generate this randomness, and then we're going to have this block of logic that we use to determine what fraction of people wait. So each customer, we can determine whether or not they wait, and then we can determine what fraction wait more than a critical level. We'll do that for a two customer service rep world, as well as three CSR world.

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Watch: Developing the Logic for Your Simulation Model

In this video, Professor Anderson tackles the logic underlying the call center service scenario. Try to follow along with the logic and the way it is expressed in the Excel formulas here. Chances are you will encounter a simulation model with at least this level of complexity, and there are lessons to be taken from every model-building example you encounter.

You may wish to download the Excel workbook referenced in this video and refer to it after watching the video. This is a larger Excel file (>2MB). It may load and refresh slowly for you. This is the same Excel workbook that is linked in the previous page.

Video Transcript

So conceptually, simulation is broken down into really three pieces. Generating random variables, working out the logic of our model, and then calculating our key performance indicators. Quite often that middle part, the logic of our model, is the hardest part to work through. So let's think about staffing our call center. There's really just two pieces of uncertainty. Right? When the customers arrive, and how long does it take them to be serviced. So we can generate two sets of uniform random variables, one for each of those, and then we can generate our time between subsequent arrivals under the assumption that time between arrivals follows a exponential distribution with a specific mean. Similarly, we can generate service times, again assuming service times follow an exponential distribution, with a specified mean. So once we have the arrival time or the time between arrivals for each customer and their service time, then we can work on our mechanics.

So let's start our shift at time zero. The first customer arrives at zero plus the time between adjacent arrivals. Customer two arrives his time after between arrival, plus when the first customer arrives. So we can have this running clock of when people get there. Now we have to ask, well when did you talk to an operator, when were you served? So basically we can't talk to an operator unless the operator is free, so we have to see which of the servers was first available. And then if that time is sooner than when we arrived, we entered the server when we, sort of, arrived or made a call. If that first server was not free until after we had arrived, then we can't enter the server until it was free. And the difference between those two was how long we had to wait for an operator.

Once we know that a particular operator is handling a call because they were the first available operator, then that operator will be available for the next call at that time, plus that customer service time. Right? So we can keep track of when each of the CSRs, which one is free, and then when they become free, follow completion of service, and we can calculate how long each customer waits. Then we can go back as a function of that wait by customer, calculate our KPIs, average wait, percent waiting, or what fraction are waiting say greater than a minute.

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Watch: Contrasting Simulation with Other Methods

Hopefully you've gotten a sense of the incredible potential of simulations to model for robust decision making.

Here, Professor Anderson takes a step back and considers the techniques you've examined throughout this course. All of these techniques have value. Each is the best tool for its own job. In this video, Professor Anderson highlights the value of simulation as a technique that handles scenarios that other techniques can't begin to describe.

Video Transcript

When we're making decisions in the face of uncertainty, we can quite often make those decisions based upon expected values, whether those are marginal expected values or expected values through a decision analysis. Those expected values implicitly assume that we're okay with the averages and outcome, and that we're potentially making these types of decisions over and over again, so the pluses and minuses sort of cancel themselves out. We can adjust for some of those pluses and minuses not canceling themselves out through the use of utility. So, instead of making those decisions based upon expected value, we can make them on an expected utility to so help us, sort of, accommodate some risk. One of the nice parts about simulation is, we can relax those expected value assumptions. We can look at very complex scenarios with multiple sources of uncertainty, and more importantly, lots of unique and distinct KPIs that may implicitly account for risk.

So, when we think about our rental car example, yes we could determine how many cars to stock simply based upon expected marginal value, but when we simulate that scenario, we can look at how many cars to stock jointly with an upgrade policy. With given that, if I'm going to allow upgrades, I might want to have more luxury cars and less of our standard cars. We can also look at different key performance indicators where now I can keep track of lost customer demand. We looked at insuring our airline fleet. Sort of by definition, when we're talking about insurance, we're really moving away from expected values and talking about risk. And so, our simulation allowed us to look at the standard deviation as well as the min and max of our expenses across those four policies, not just the average cost. We talked about our customer service representatives and staffing our call center, we could have very unique KPIs where we're focused on very specific service levels. Making sure that only X percent waited more than one minute. So, very generalizable KPIs. Now, of course with simulation we have to enumerate some of these large samples and so, sometimes simulation comes at this computational expense.

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Course Project, Part Two—Make a Simulation-Based Decision

By this point, you have assembled a set of tools and techniques for identifying, describing, and managing uncertainty in decision making. In this part of the course project, you will bring these tools to bear on a fictitious scenario involving multiple random variables. You will use a simulation to perform a cash flow analysis and make decisions based on your projections. Completion of all parts of this project is a course requirement.

Instructions:

1. Open your saved course project document. (If needed, download it again now.)
2. Download the Part Two Excel workbook template.
3. Complete at least sections A and B in Part Two of the course project. (Section C is optional.)
4. Save your work.
5. Once you've finished, review the course project document and your Excel workbook, making any final additions or revisions, and then **submit both files for instructor review using the Submit Assignment button on this page.**

Do not hesitate to contact your instructor if you have any questions about the project.

Before you begin:

Before starting your work, please review the **rubric** (a list of evaluative criteria) for this assignment. Also review eCornell's policy regarding **plagiarism** (the presentation of someone else's work as your own without source credit).

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Module Wrap-up: Using Monte Carlo Simulation for Nuanced Decision Making

As you saw in this module, a model-based simulation can be a very powerful decision-making tool. When you describe your decision in terms of KPIs and look at outcomes in this way, it reframes the decision to reflect your goals without sacrificing the benefit of the historical perspective. A simulation is often about not just one decision but about several decisions that are connected. A simulation gives you freedom to experiment with decision parameters to find your best outcome.

You generated uniform random variables and incorporated them in a simulation. Based on that simulation, you were able to look at your KPIs and determine what decisions would best help you meet your goals. This method can be applied to many different kinds of scenarios, with the only requirements that you understand the logic of the situation and reflect it in the way you structure your simulation. With practice, you should find you have increasingly facility with building simulations based on your decision-making scenarios.

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Read: Thank You and Farewell

Chris Anderson
Professor
School of Hotel Administration
SC Johnson College of Business, Cornell University

Congratulations on completing *Modeling Uncertainty and Risk*. I hope your work in this course has built on your understanding of probability based decision making and given you new tools that you'll find valuable for describing and managing uncertainty in the decisions you make.

From all of us at Cornell University and eCornell, thank you for participating in this course.

Sincerely,

Chris Anderson