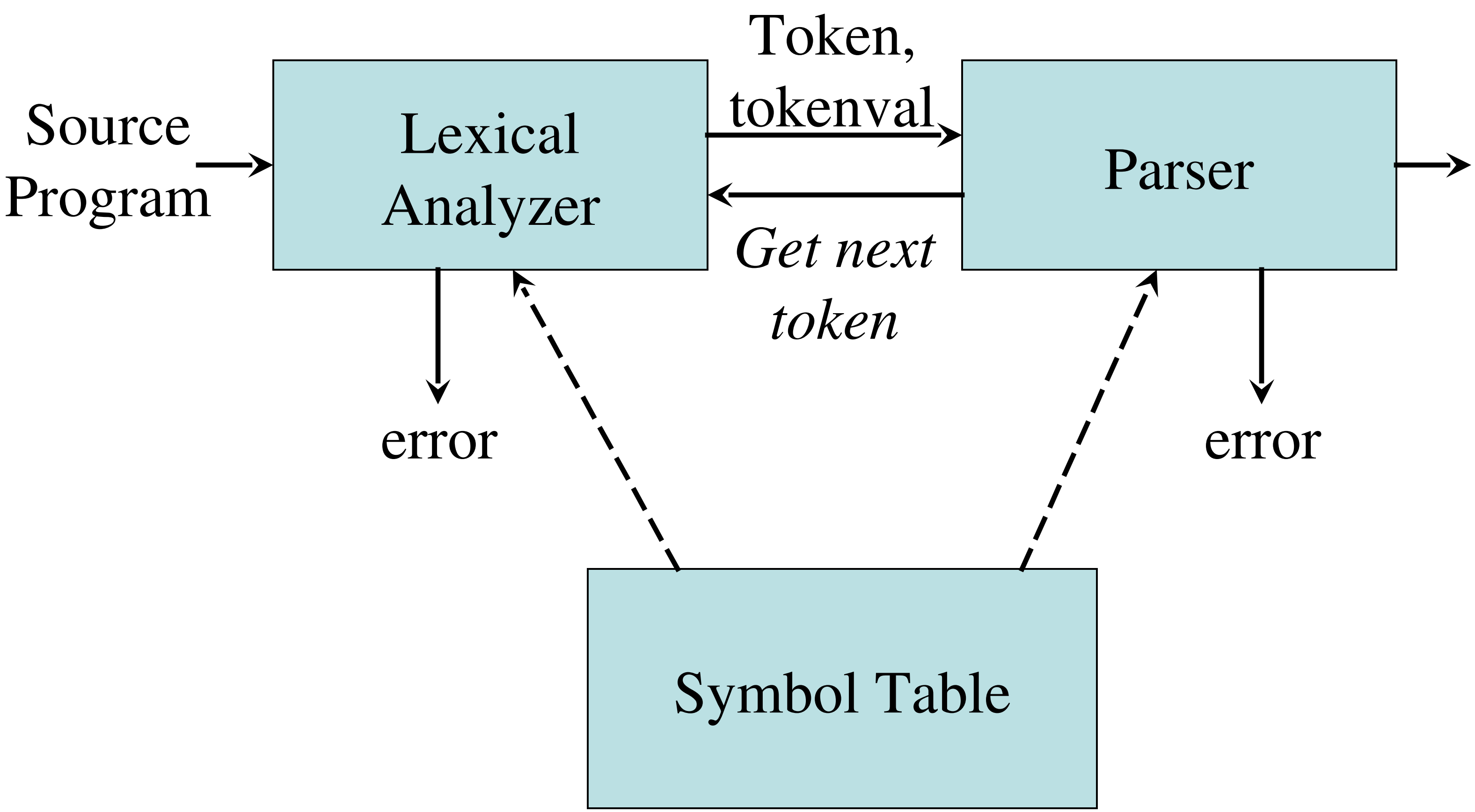
Lexical Analysis and Lexical Analyzer Generators

Chapter 3

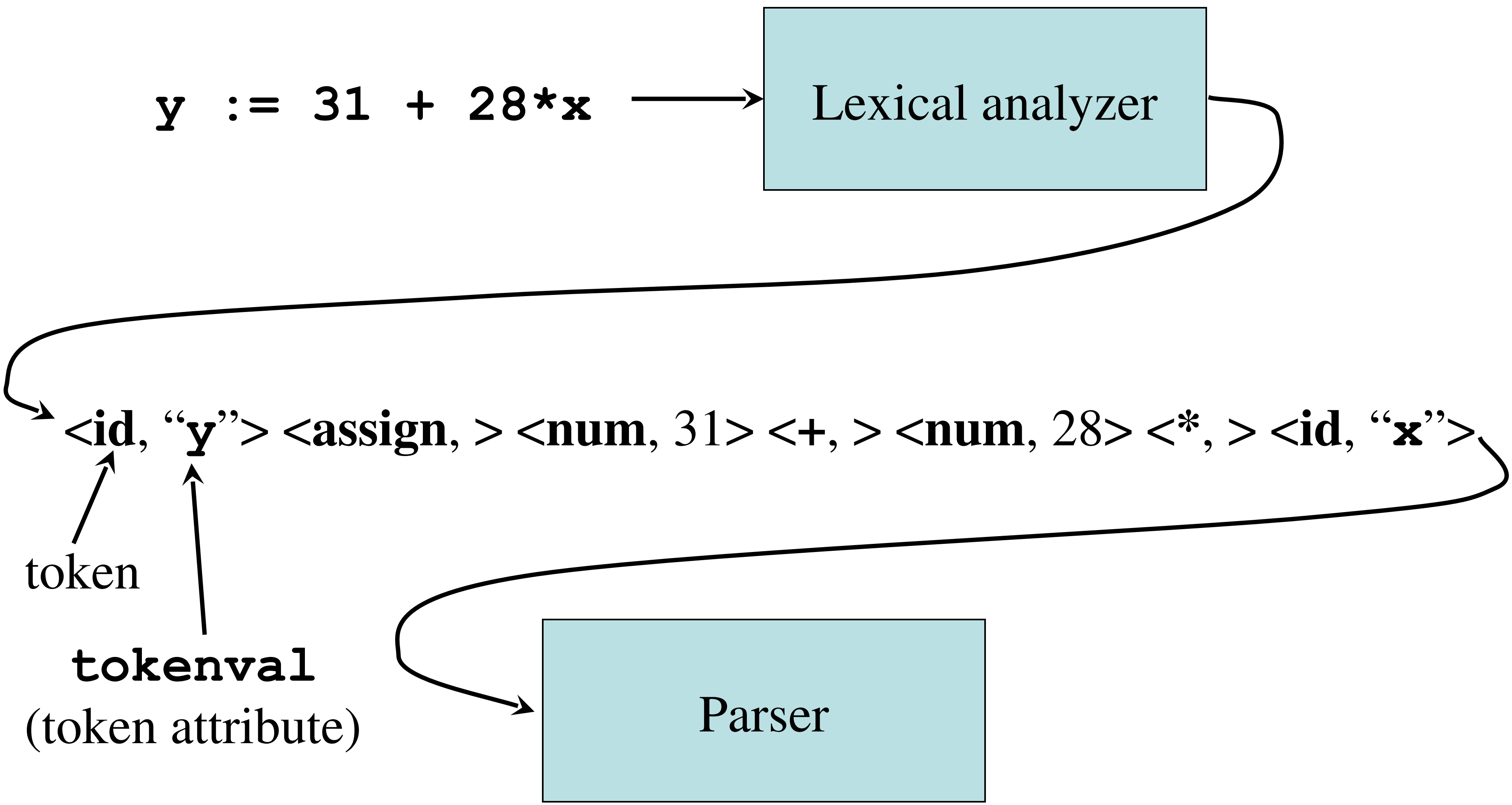
The Reason Why Lexical Analysis is a Separate Phase

- Simplifies the design of the compiler
  - LL(1) or LR(1) parsing with 1 token lookahead would not be possible (multiple characters/tokens to match)
- Provides efficient implementation
  - Systematic techniques to implement lexical analyzers by hand or automatically from specifications
  - Stream buffering methods to scan input
- Improves portability
  - Non-standard symbols and alternate character encodings can be normalized (e.g. trigraphs)

Interaction of the Lexical Analyzer with the Parser



Attributes of Tokens



Tokens, Patterns, and Lexemes

- A *token* is a classification of lexical units
  - For example: **id** and **num**
- *Lexemes* are the specific character strings that make up a token
  - For example: **`abc`** and **`123`**
- *Patterns* are rules describing the set of lexemes belonging to a token
  - For example: "*letter followed by letters and digits*" and "*non-empty sequence of digits*"

Specification of Patterns for Tokens: *Definitions*

- An *alphabet* Σ is a finite set of symbols (characters)
- A *string s* is a finite sequence of symbols from Σ

  $|s|$ denotes the length of string s

  ε denotes the empty string, thus $|\varepsilon| = 0$
- A *language* is a specific set of strings over some fixed alphabet Σ

Specification of Patterns for Tokens: *String Operations*

- The *concatenation* of two strings x and y is denoted by xy
- The *exponentation* of a string s is defined by

$$s^0 = \varepsilon$$
$$s^i = s^{i-1}s \quad \text{for } i > 0$$

note that $s\varepsilon = \varepsilon s = s$

Specification of Patterns for Tokens: *Language Operations*

- *Union*
  $L \cup M = \{s \mid s \in L \text{ or } s \in M\}$
- *Concatenation*
  $LM = \{xy \mid x \in L \text{ and } y \in M\}$
- *Exponentiation*
  $L^0 = \{\varepsilon\}; \quad L^i = L^{i-1}L$
- *Kleene closure*
  $L^* = \cup_{i=0,\ldots,\infty} L^i$
- *Positive closure*
  $L^+ = \cup_{i=1,\ldots,\infty} L^i$

Specification of Patterns for Tokens: *Regular Expressions*

- Basis symbols:
  - ε is a regular expression denoting language $\{\varepsilon\}$
  - $a \in \Sigma$ is a regular expression denoting $\{a\}$
- If r and s are regular expressions denoting languages $L(r)$ and $M(s)$ respectively, then
  - $r \mid s$ is a regular expression denoting $L(r) \cup M(s)$
  - rs is a regular expression denoting $L(r)M(s)$
  - r^* is a regular expression denoting $L(r)^*$
  - (r) is a regular expression denoting $L(r)$
- A language defined by a regular expression is called a *regular set*

Specification of Patterns for Tokens: *Regular Definitions*

- Regular definitions introduce a naming convention:

  $d_1 \to r_1$

  $d_2 \to r_2$

  …

  $d_n \to r_n$

  where each r_i is a regular expression over

  $\Sigma \cup \{d_1, d_2, \ldots, d_{i-1}\}$

- Any d_j in r_i can be textually substituted in r_i to obtain an equivalent set of definitions

Specification of Patterns for Tokens: *Regular Definitions*

- Example:

  $$\textbf{letter} \to \texttt{A} \mid \texttt{B} \mid \ldots \mid \texttt{Z} \mid \texttt{a} \mid \texttt{b} \mid \ldots \mid \texttt{z}$$
  $$\textbf{digit} \to \texttt{0} \mid \texttt{1} \mid \ldots \mid \texttt{9}$$
  $$\textbf{id} \to \textbf{letter} \; (\; \textbf{letter} \mid \textbf{digit} \;)^*$$

- Regular definitions are not recursive:

  $$\textbf{digits} \to \textbf{digit digits} \mid \textbf{digit} \qquad \textit{wrong!}$$

Specification of Patterns for Tokens: *Notational Shorthand*

- The following shorthands are often used:

$$r^+ = rr^*$$

$$r? = r \mid \varepsilon$$

$$[\mathbf{a}\text{-}\mathbf{z}] = \mathbf{a} \mid \mathbf{b} \mid \mathbf{c} \mid \ldots \mid \mathbf{z}$$

- Examples:
  digit → [**0**-**9**]
  num → **digit**$^+$ (**. digit**$^+$)? (**E** (**+** | **-**)? **digit**$^+$)?

Regular Definitions and Grammars

Grammar

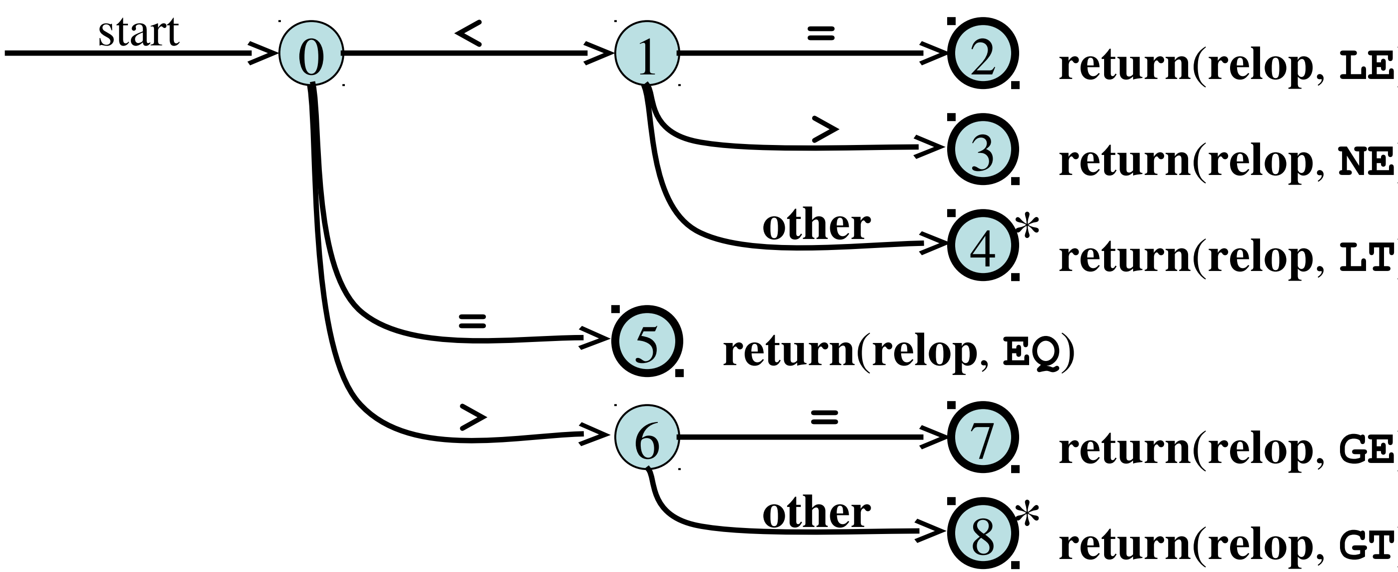
$$
\begin{aligned}
\mathit{stmt} &\rightarrow \textbf{if}\ \mathit{expr}\ \textbf{then}\ \mathit{stmt} \\
&\mid\ \textbf{if}\ \mathit{expr}\ \textbf{then}\ \mathit{stmt}\ \textbf{else}\ \mathit{stmt} \\
&\mid\ \varepsilon \\
\mathit{expr} &\rightarrow \mathit{term}\ \textbf{relop}\ \mathit{term} \\
&\mid\ \mathit{term} \\
\mathit{term} &\rightarrow \textbf{id} \\
&\mid\ \textbf{num}
\end{aligned}
$$

Regular definitions

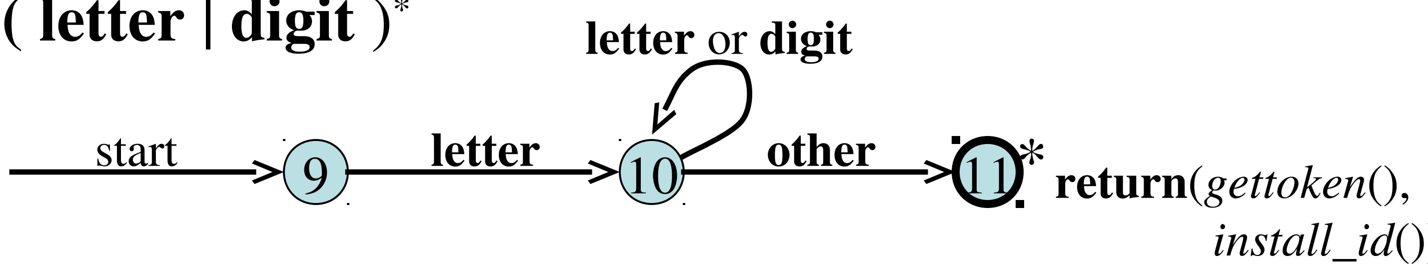
$$
\begin{aligned}
\textbf{if} &\rightarrow \texttt{if} \\
\textbf{then} &\rightarrow \texttt{then} \\
\textbf{else} &\rightarrow \texttt{else} \\
\textbf{relop} &\rightarrow \texttt{<} \mid \texttt{<=} \mid \texttt{<>} \mid \texttt{>} \mid \texttt{>=} \mid \texttt{=} \\
\textbf{id} &\rightarrow \textbf{letter}\ (\ \textbf{letter} \mid \textbf{digit}\ )^* \\
\textbf{num} &\rightarrow \textbf{digit}^+\ (.\ \textbf{digit}^+)?\ (\ \textbf{E}\ (\textbf{+} \mid \textbf{-})?\ \textbf{digit}^+\ )?
\end{aligned}
$$

Coding Regular Definitions in *Transition Diagrams*

relop → **<** | **<=** | **<>** | **>** | **>=** | **=**



id → **letter** (**letter** | **digit**)*



Coding Regular Definitions in Transition Diagrams: Code


```
token nexttoken()
{ while (1) {
    switch (state) {
    case 0: c = nextchar();
      if (c==blank || c==tab || c==newline) {
        state = 0;
        lexeme_beginning++;
      }
      else if (c=='<') state = 1;
      else if (c=='=') state = 5;
      else if (c=='>') state = 6;
      else state = fail();
      break;
     case 1:
      …
     case 9: c = nextchar();
      if (isletter(c)) state = 10;
      else state = fail();
      break;
     case 10: c = nextchar();
      if (isletter(c)) state = 10;
      else if (isdigit(c)) state = 10;
      else state = 11;
      break;
     …
```

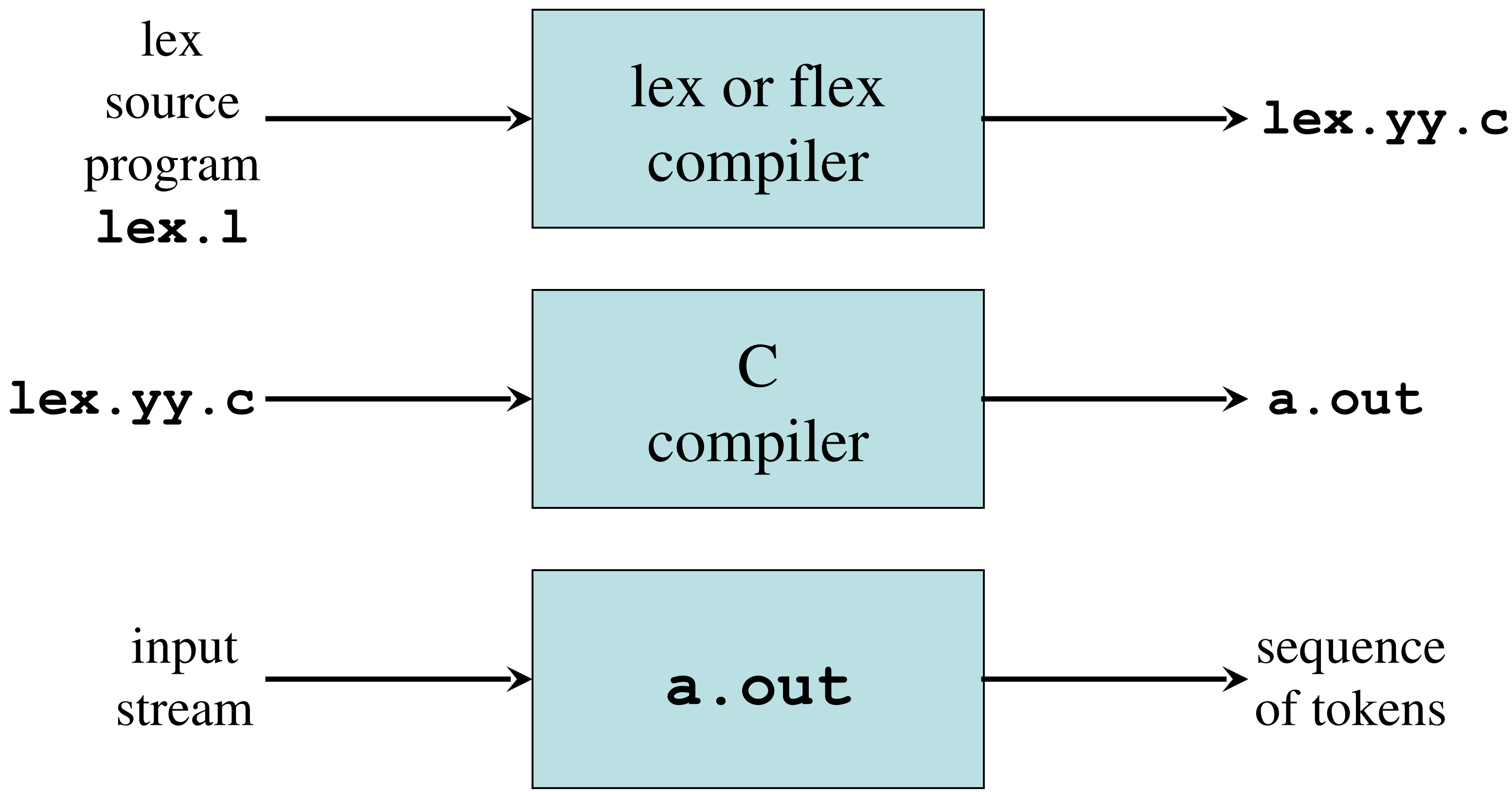
Decides the next start state to check

```
int fail()
{ forward = token_beginning;
  swith (start) {
  case  0: start =  9; break;
  case  9: start = 12; break;
  case 12: start = 20; break;
  case 20: start = 25; break;
  case 25: recover(); break;
  default: /* error */
  }
  return start;
}
```

The Lex and Flex Scanner Generators

- *Lex* and its newer cousin *flex* are scanner generators
- Systematically translate regular definitions into C source code for efficient scanning
- Generated code is easy to integrate in C applications

Creating a Lexical Analyzer with Lex and Flex



Lex Specification

- A *lex specification* consists of three parts:

  regular definitions, C declarations in `%{ %}`
  `%%`
  translation rules
  `%%`
  user-defined auxiliary procedures

- The *translation rules* are of the form:

  p_1 { $action_1$ }
  p_2 { $action_2$ }
  …
  p_n { $action_n$ }

Regular Expressions in Lex

x	match the character **x**
\\.	match the character **.**
"*string*"	match contents of string of characters
.	match any character except newline
^	match beginning of a line
$	match the end of a line
[xyz]	match one character **x**, **y**, or **z** (use **** to escape **-**)
[^xyz]	match any character except **x**, **y**, and **z**
[a-z]	match one of **a** to **z**
r*****	closure (match zero or more occurrences)
r**+**	positive closure (match one or more occurrences)
r**?**	optional (match zero or one occurrence)
$r_1 r_2$	match r_1 then r_2 (concatenation)
$r_1 \mid r_2$	match r_1 or r_2 (union)
(r **)**	grouping
$r_1 \backslash r_2$	match r_1 when followed by r_2
{d**}**	match the regular expression defined by d

Example Lex Specification 1

Translation rules

```
%{
#include <stdio.h>
%}
%%
[0-9]+  { printf("%s\n", yytext); }
.|\n    { }
%%
main()
{ yylex();
}
```

Contains the matching lexeme

Invokes the lexical analyzer

```
lex spec.l
gcc lex.yy.c -ll
./a.out < spec.l
```

Example Lex Specification 2

Translation rules

Regular definition

```
%{
#include <stdio.h>
int ch = 0, wd = 0, nl = 0;
%}
delim      [ \t]+
%%
\n         { ch++; wd++; nl++; }
^{delim}   { ch+=yyleng; }
{delim}    { ch+=yyleng; wd++; }
.          { ch++; }
%%
main()
{ yylex();
  printf("%8d%8d%8d\n", nl, wd, ch);
}
```

Example Lex Specification 3

Translation rules

Regular definitions

```
%{
#include <stdio.h>
%}
digit      [0-9]
letter     [A-Za-z]
id         {letter}({letter}|{digit})*
%%
{digit}+   { printf("number: %s\n", yytext); }
{id}       { printf("ident: %s\n", yytext); }
.          { printf("other: %s\n", yytext); }
%%
main()
{ yylex();
}
```

Example Lex Specification 4

```
%{ /* definitions of manifest constants */
#define LT (256)
…
%}
delim      [ \t\n]
ws         {delim}+
letter     [A-Za-z]
digit      [0-9]
id         {letter}({letter}|{digit})*
number     {digit}+(\.{digit}+)?(E[+\-]?{digit}+)?
%%
{ws}       { }
if         {return IF;}
then       {return THEN;}
else       {return ELSE;}
{id}       {yylval = install_id(); return ID;}
{number}   {yylval = install_num(); return NUMBER;}
"<"        {yylval = LT; return RELOP;}
"<="       {yylval = LE; return RELOP;}
"="        {yylval = EQ; return RELOP;}
"<>"       {yylval = NE; return RELOP;}
">"        {yylval = GT; return RELOP;}
">="       {yylval = GE; return RELOP;}
%%
int install_id()
…
```

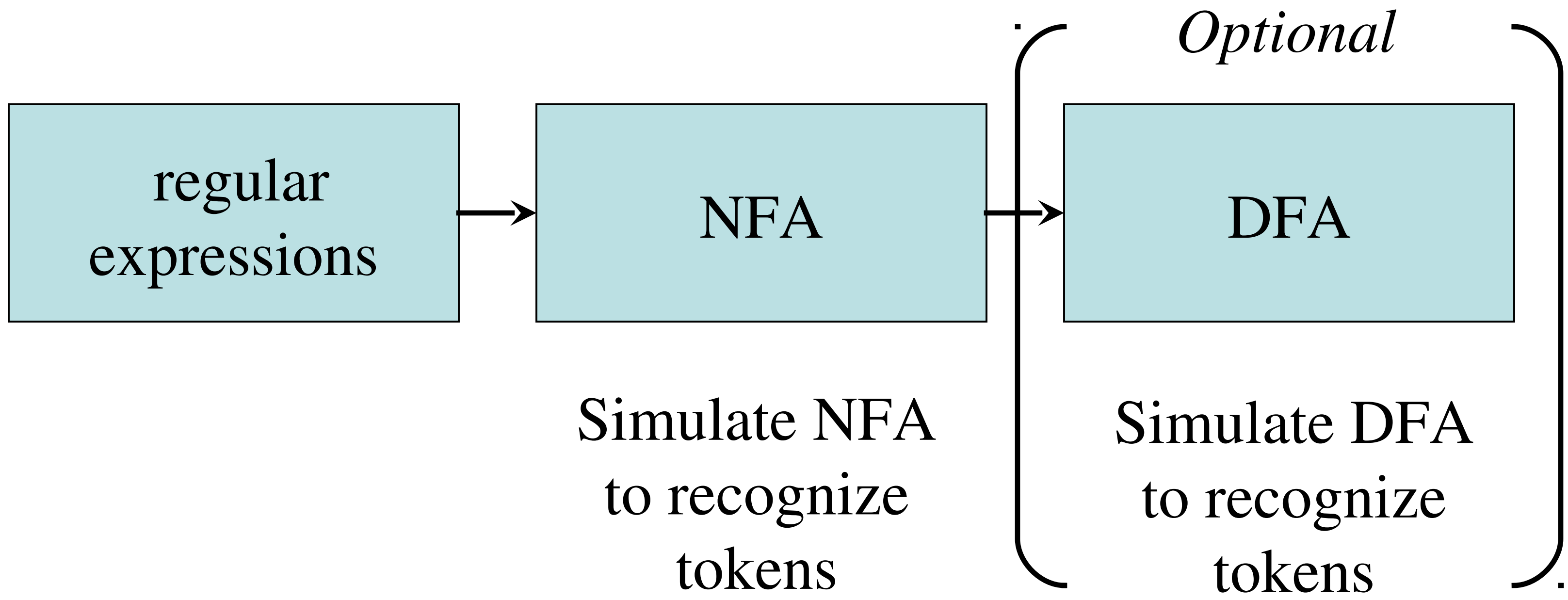
Return token to parser

Token attribute

Install **`yytext`** as identifier in symbol table

Design of a Lexical Analyzer Generator

- Translate regular expressions to NFA
- Translate NFA to an efficient DFA



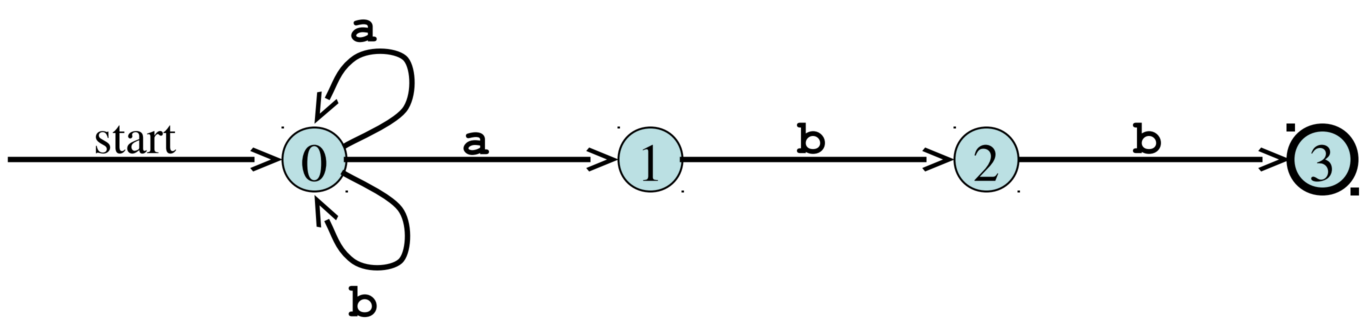
Nondeterministic Finite Automata

- An NFA is a 5-tuple (S, Σ, δ, s_0, F) where

  S is a finite set of *states*
  Σ is a finite set of symbols, the *alphabet*
  δ is a *mapping* from $S \times \Sigma$ to a set of states
  $s_0 \in S$ is the *start state*
  $F \subseteq S$ is the set of *accepting* (or *final*) *states*

Transition Graph

- An NFA can be diagrammatically represented by a labeled directed graph called a *transition graph*



$S = \{0,1,2,3\}$

$\Sigma = \{\mathbf{a},\mathbf{b}\}$

$s_0 = 0$

$F = \{3\}$

Transition Table

- The mapping δ of an NFA can be represented in a *transition table*

$\delta(0,\mathbf{a}) = \{0,1\}$
$\delta(0,\mathbf{b}) = \{0\}$
$\delta(1,\mathbf{b}) = \{2\}$
$\delta(2,\mathbf{b}) = \{3\}$

$\longrightarrow$

State	*Input* **a**	*Input* **b**
0	{0, 1}	{0}
1		{2}
2		{3}

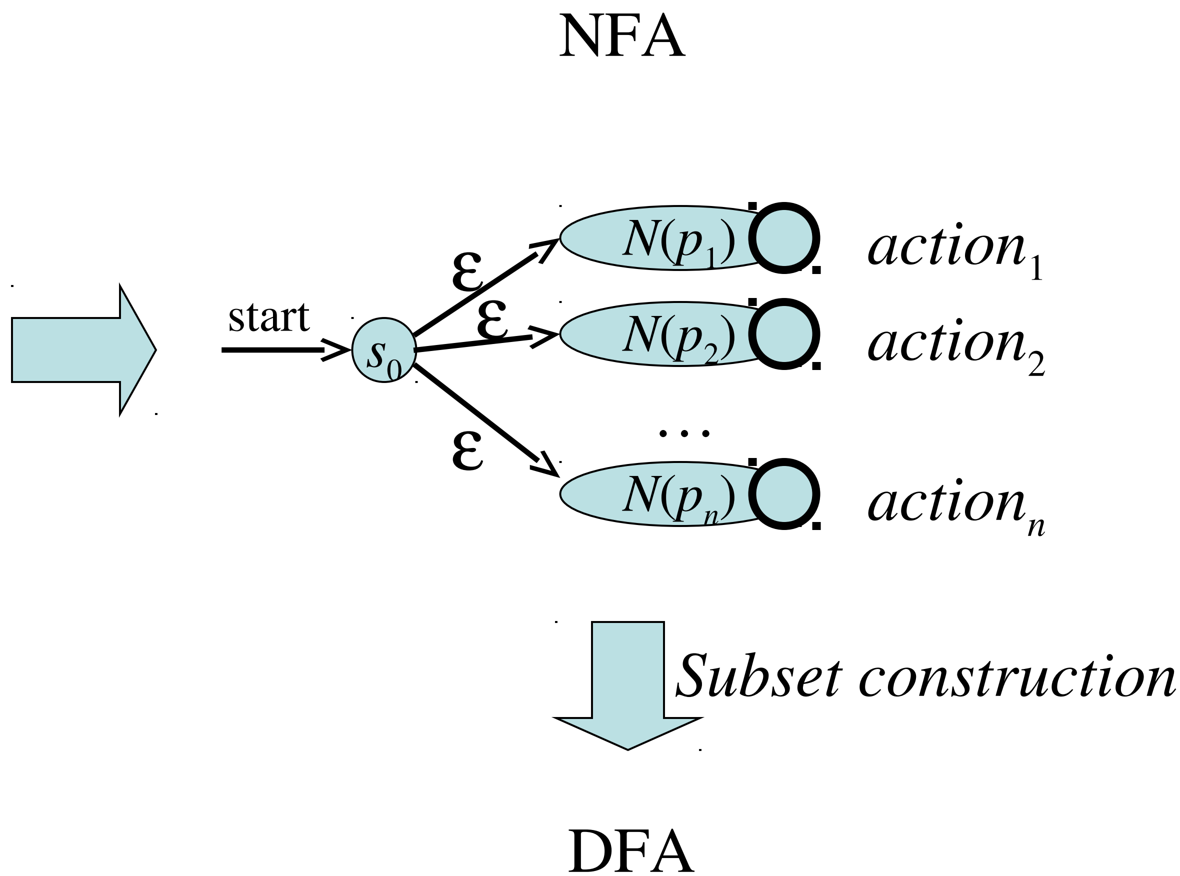
The Language Defined by an NFA

- An NFA *accepts* an input string *x* if and only if there is some path with edges labeled with symbols from *x* in sequence from the start state to some accepting state in the transition graph
- A state transition from one state to another on the path is called a *move*
- The *language defined by* an NFA is the set of input strings it accepts, such as (**`a`**|**`b`**)***`abb`** for the example NFA

Design of a Lexical Analyzer Generator: RE to NFA to DFA

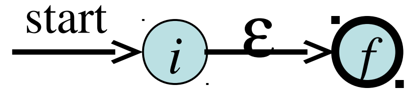
Lex specification with regular expressions

p_1	$\{ \ action_1 \ \}$
p_2	$\{ \ action_2 \ \}$
…	
p_n	$\{ \ action_n \ \}$

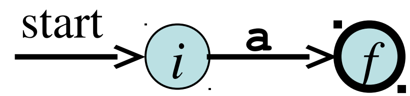


From Regular Expression to NFA (Thompson's Construction)

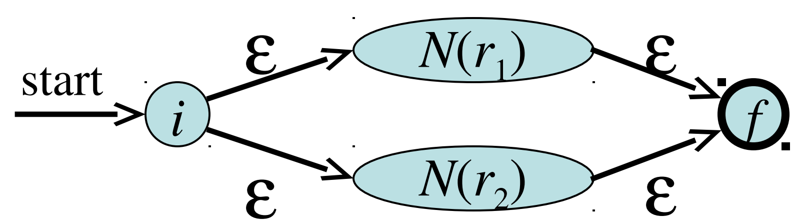
ε



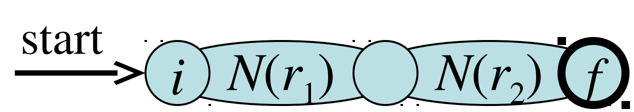
a



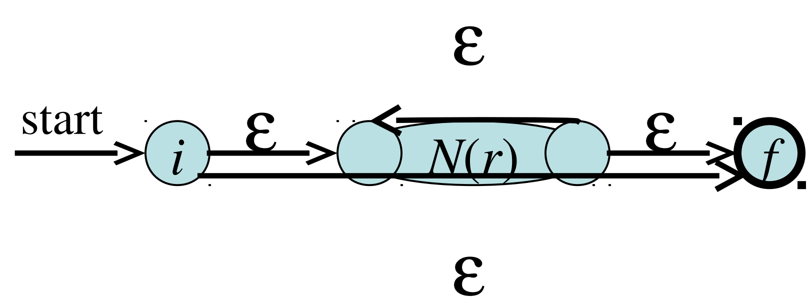
$r_1 \mid r_2$



$r_1 r_2$

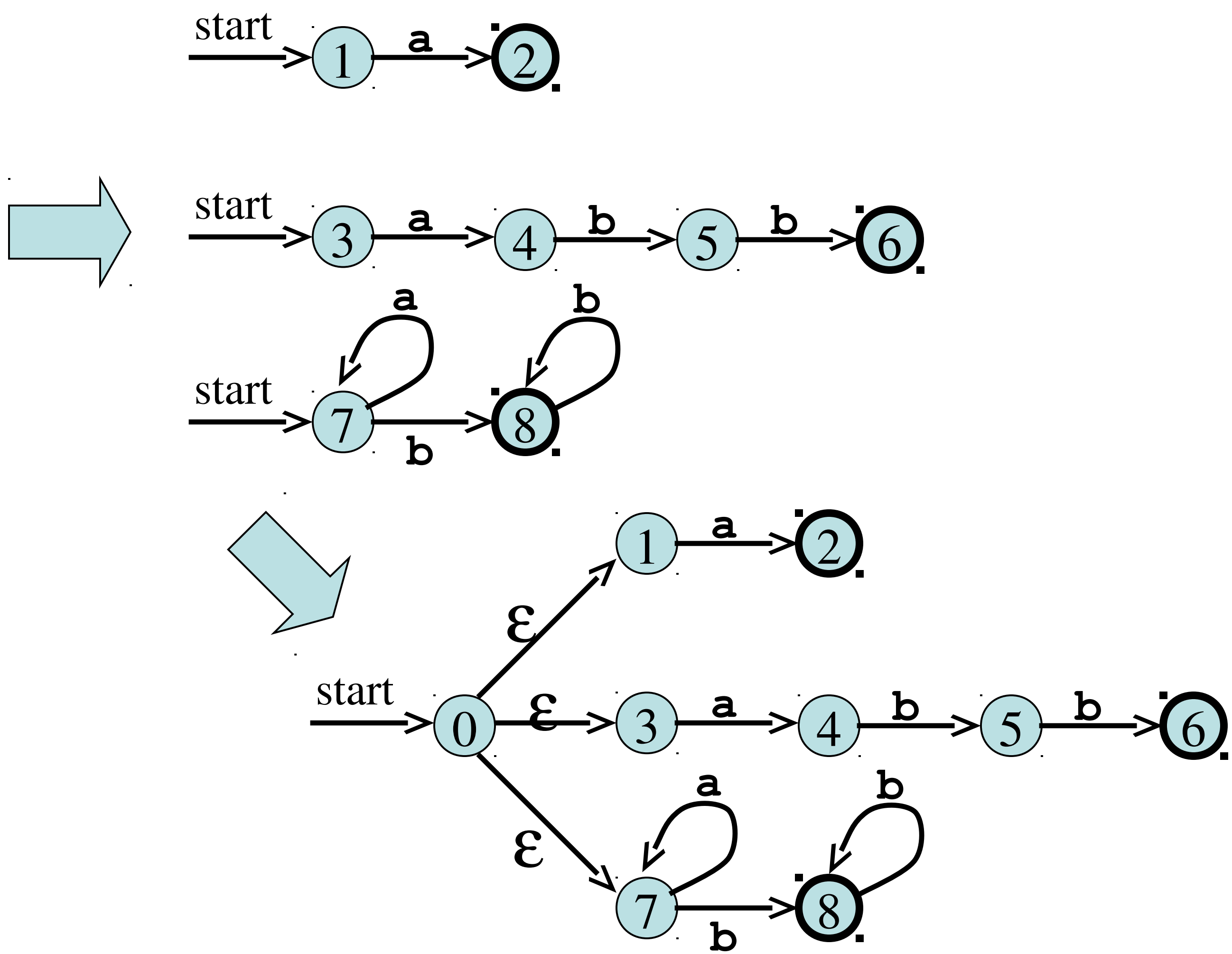


r^*

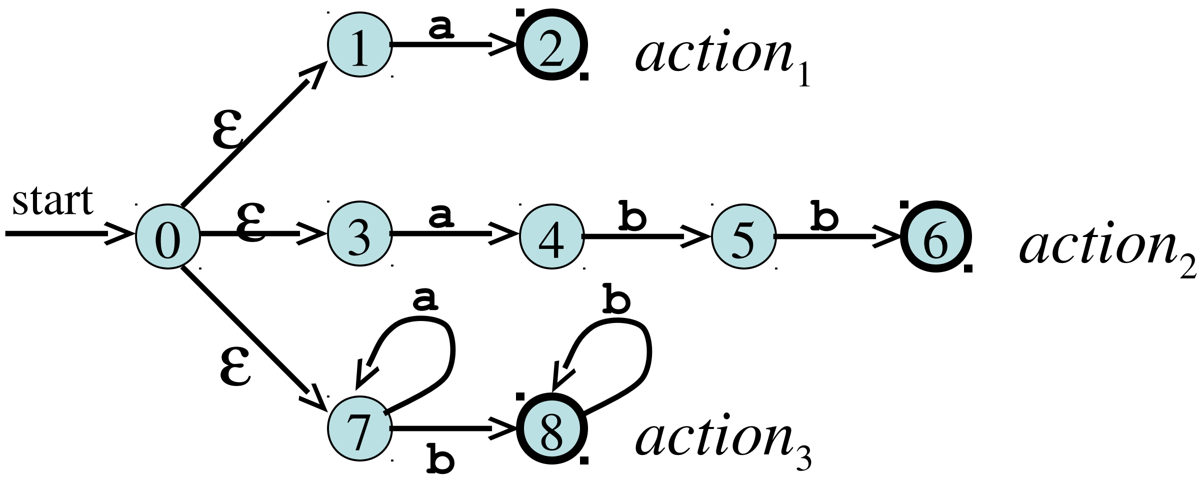


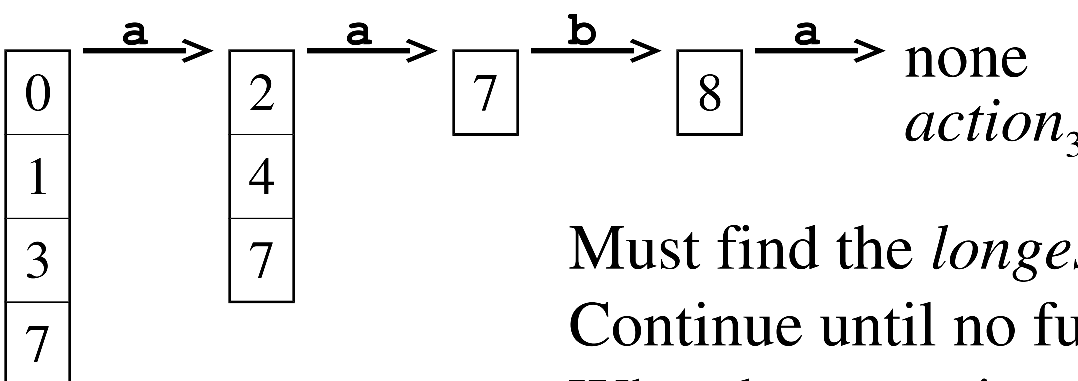
Combining the NFAs of a Set of Regular Expressions

a	{ $action_1$ }
abb	{ $action_2$ }
a*b+	{ $action_3$ }



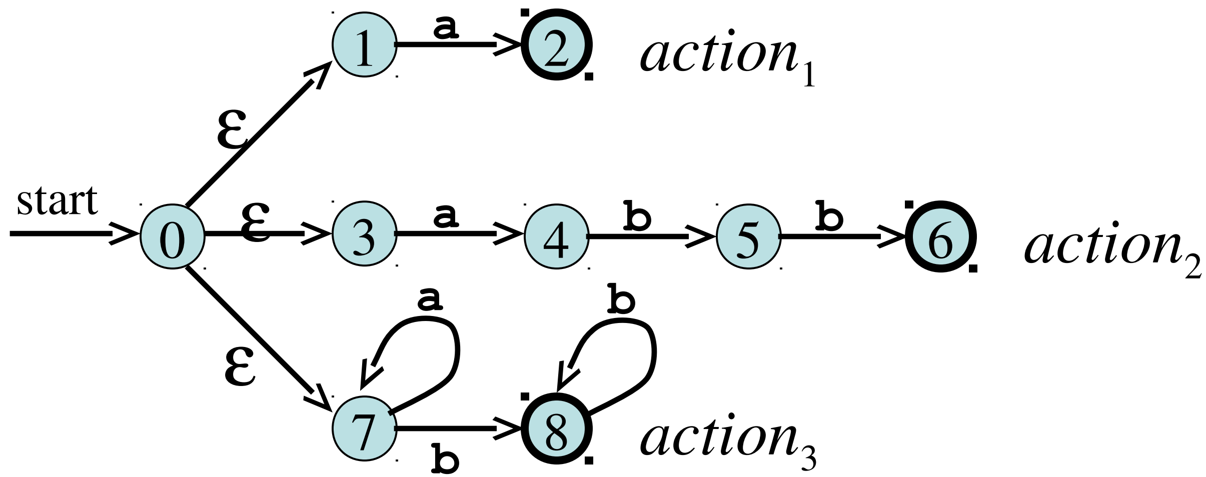
Simulating the Combined NFA Example 1

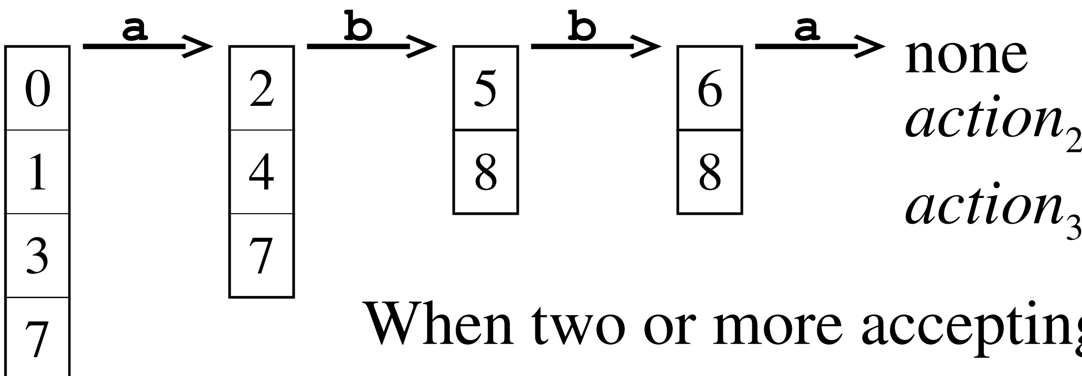




Must find the *longest match*:
Continue until no further moves are possible
When last state is accepting: execute action

Simulating the Combined NFA Example 2





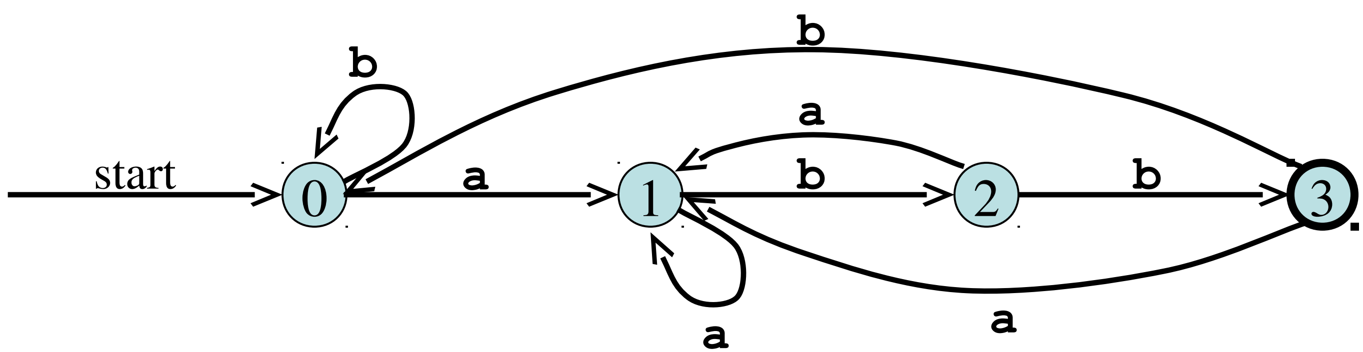
When two or more accepting states are reached, the first action given in the Lex specification is executed

Deterministic Finite Automata

- A *deterministic finite automaton* is a special case of an NFA
  - No state has an ε-transition
  - For each state *s* and input symbol *a* there is at most one edge labeled *a* leaving *s*
- Each entry in the transition table is a single state
  - At most one path exists to accept a string
  - Simulation algorithm is simple

Example DFA

A DFA that accepts (**a** | **b**)***abb**



Conversion of an NFA into a DFA

- The *subset construction algorithm* converts an NFA into a DFA using:

  $\varepsilon\text{-}closure(s) = \{s\} \cup \{t \mid s \rightarrow_\varepsilon \ldots \rightarrow_\varepsilon t\}$

  $\varepsilon\text{-}closure(T) = \cup_{s \in T}\, \varepsilon\text{-}closure(s)$

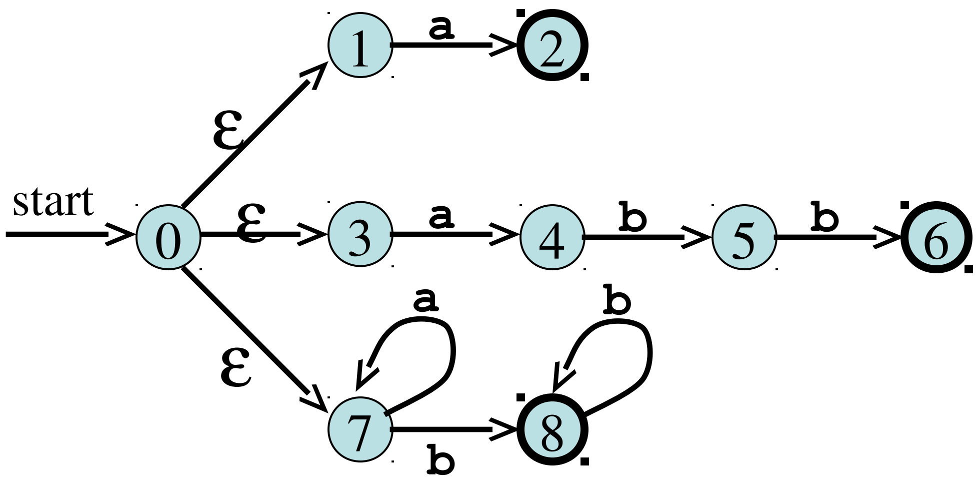
  $move(T,a) = \{t \mid s \rightarrow_a t \text{ and } s \in T\}$

- The algorithm produces:

  Dstates is the set of states of the new DFA consisting of sets of states of the NFA

  Dtran is the transition table of the new DFA

ε-*closure* and *move* Examples



$\varepsilon\text{-}closure(\{0\}) = \{0,1,3,7\}$

$move(\{0,1,3,7\},\mathbf{a}) = \{2,4,7\}$

$\varepsilon\text{-}closure(\{2,4,7\}) = \{2,4,7\}$

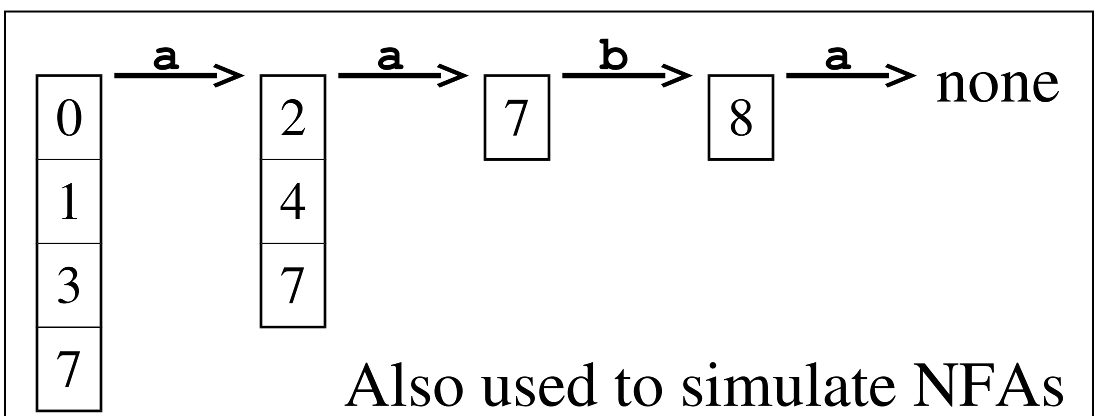
$move(\{2,4,7\},\mathbf{a}) = \{7\}$

$\varepsilon\text{-}closure(\{7\}) = \{7\}$

$move(\{7\},\mathbf{b}) = \{8\}$

$\varepsilon\text{-}closure(\{8\}) = \{8\}$

$move(\{8\},\mathbf{a}) = \emptyset$



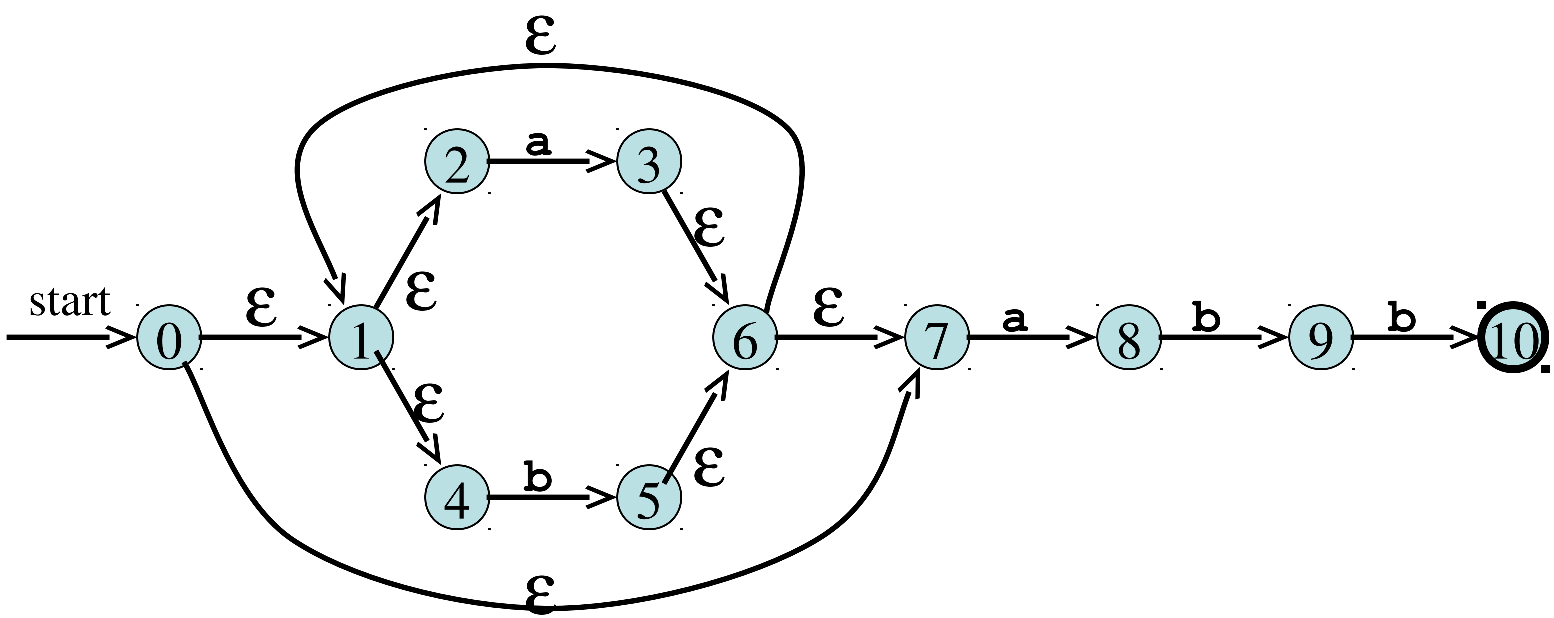
Simulating an NFA using ε-*closure* and *move*

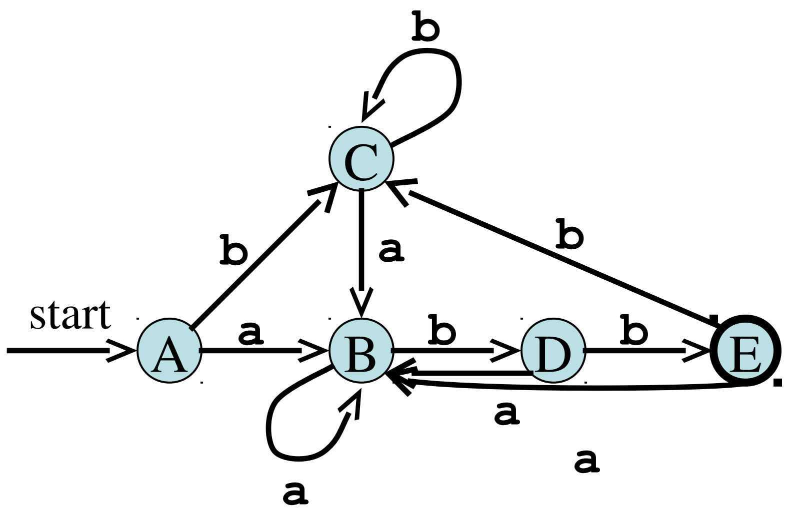
$S := \varepsilon\text{-}closure(\{s_0\})$
$S_{prev} := \varnothing$
$a := nextchar()$
while $S \neq \varnothing$ **do**
    $S_{prev} := S$
    $S := \varepsilon\text{-}closure(move(S,a))$
    $a := nextchar()$
end do
if $S_{prev} \cap F \neq \varnothing$ **then**
    execute *action in* S_{prev}
    return "yes"
else **return** "no"

The Subset Construction Algorithm

Initially, ε-*closure*(s_0) is the only state in *Dstates* and it is unmarked
while there is an unmarked state T in *Dstates* **do**
    mark T
    for each input symbol $a \in \Sigma$ **do**
        U := ε-*closure*(*move*(T,a))
        if U is not in *Dstates* **then**
            add U as an unmarked state to *Dstates*
        end if
        Dtran[T,a] := U
    end do
end do

Subset Construction Example 1





Dstates

A = {0,1,2,4,7}

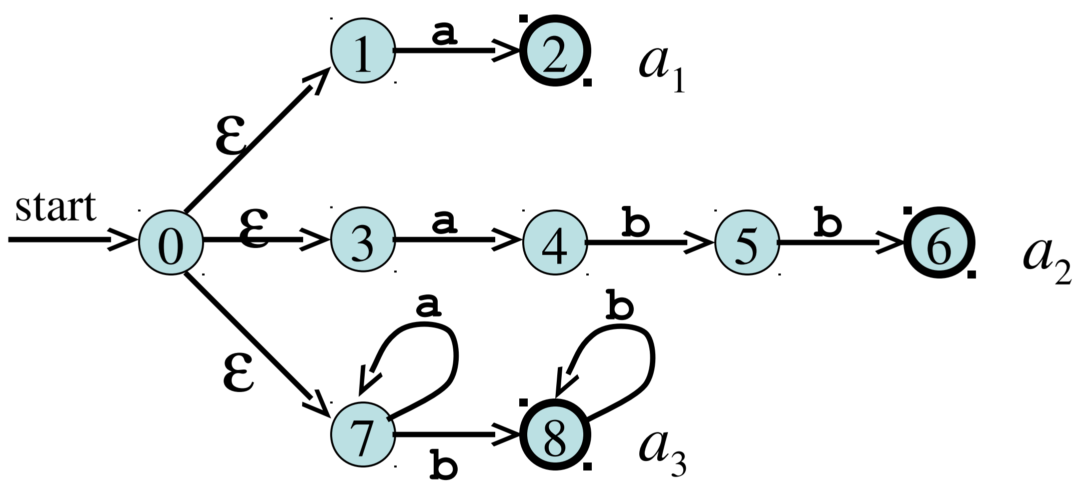
B = {1,2,3,4,6,7,8}

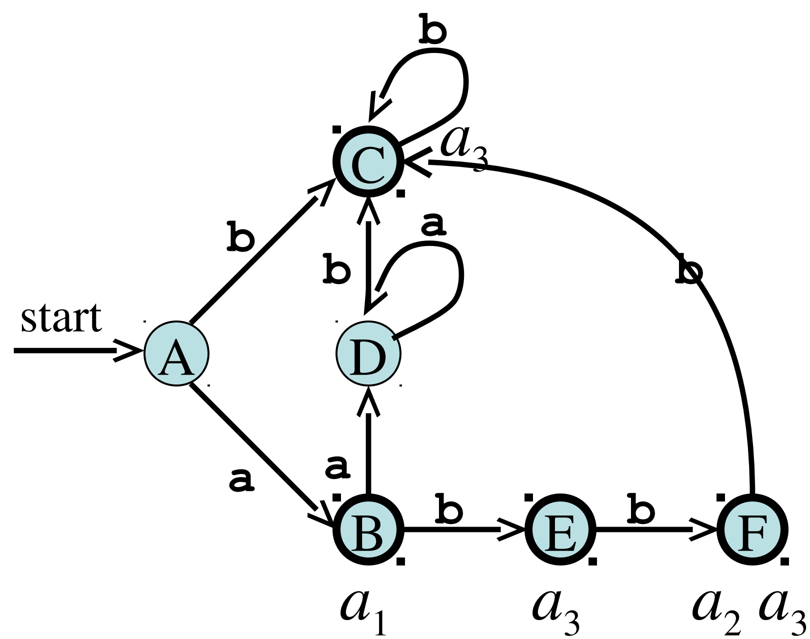
C = {1,2,4,5,6,7}

D = {1,2,4,5,6,7,9}

E = {1,2,4,5,6,7,10}

Subset Construction Example 2





Dstates

A = {0,1,3,7}

B = {2,4,7}

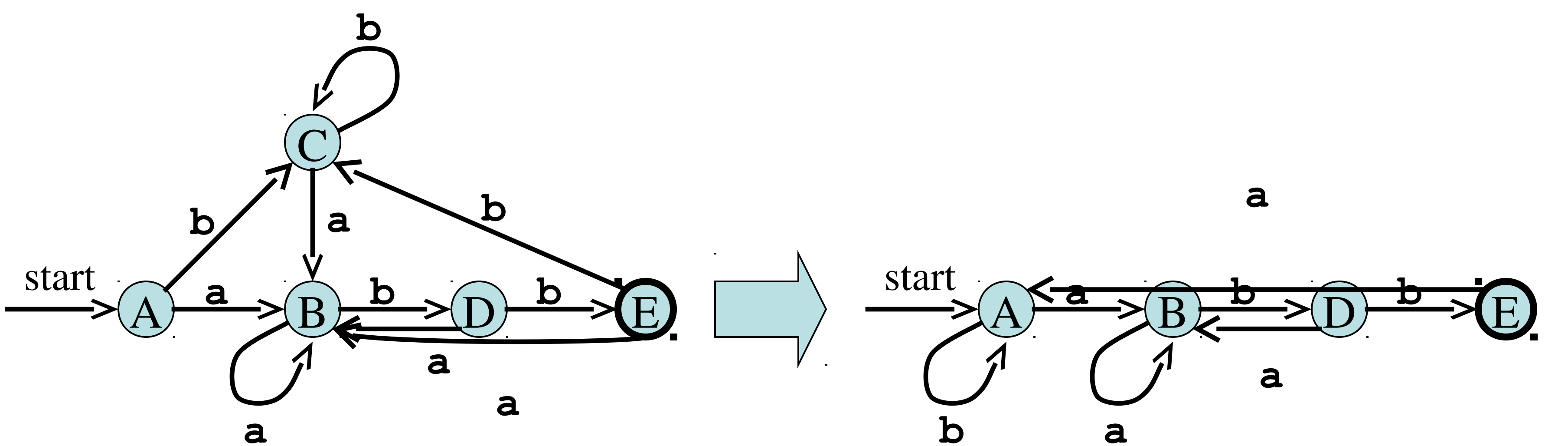
C = {8}

D = {7}

E = {5,8}

F = {6,8}

Minimizing the Number of States of a DFA



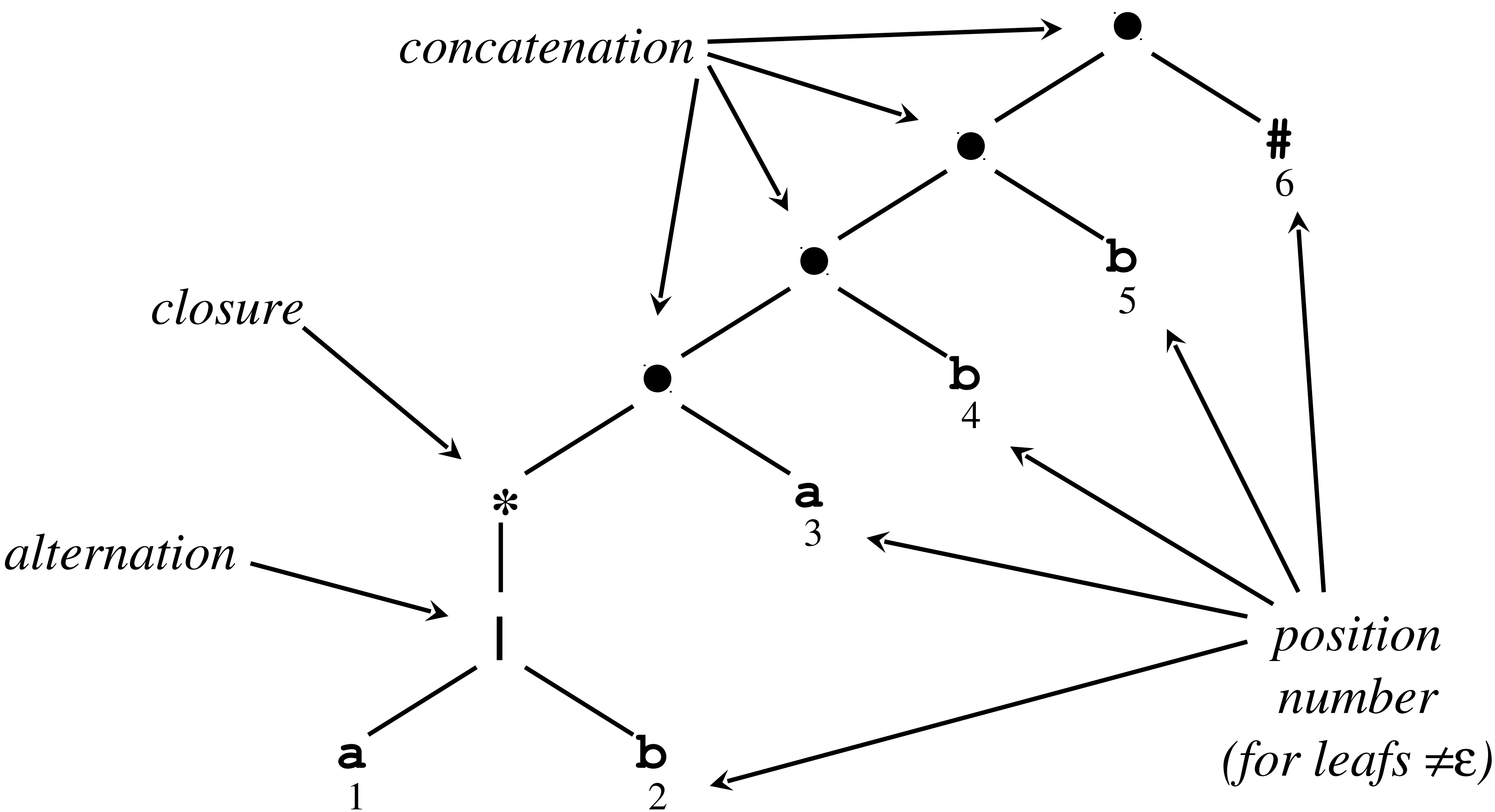
From Regular Expression to DFA Directly

- The "*important states*" of an NFA are those without an ε-transition, that is if $move(\{s\},a) \neq \emptyset$ for some a then s is an important state
- The subset construction algorithm uses only the important states when it determines ε-$closure(move(T,a))$

From Regular Expression to DFA Directly (Algorithm)

- Augment the regular expression *r* with a special end symbol # to make accepting states important: the new expression is *r*#
- Construct a syntax tree for *r*#
- Traverse the tree to construct functions *nullable*, *firstpos*, *lastpos*, and *followpos*

From Regular Expression to DFA Directly: Syntax Tree of (a|b)*abb#



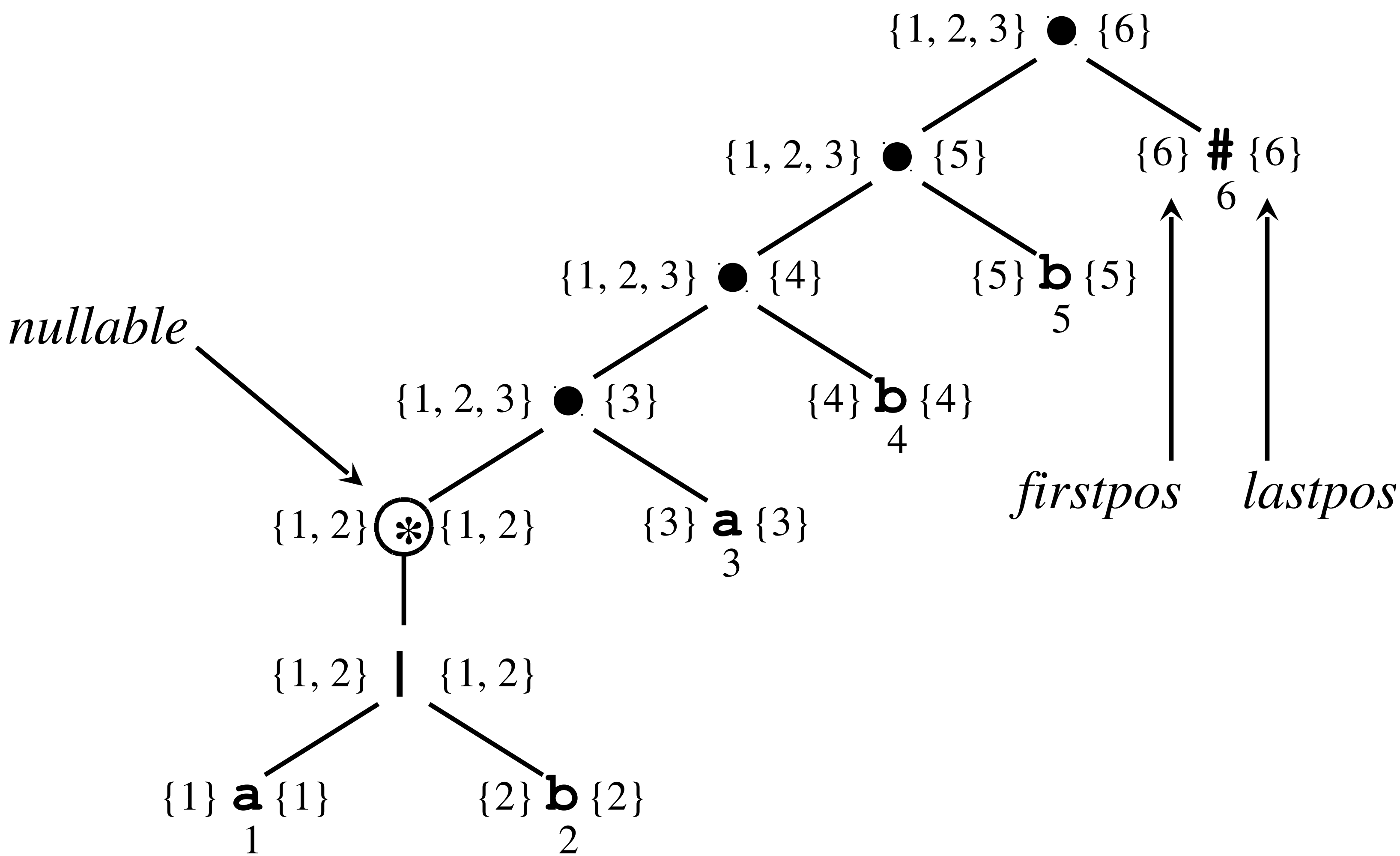
From Regular Expression to DFA Directly: Annotating the Tree

- *nullable*(n): the subtree at node n generates languages including the empty string
- *firstpos*(n): set of positions that can match the first symbol of a string generated by the subtree at node n
- *lastpos*(n): the set of positions that can match the last symbol of a string generated be the subtree at node n
- *followpos*(i): the set of positions that can follow position i in the tree

From Regular Expression to DFA Directly: Annotating the Tree

Node n	$nullable(n)$	$firstpos(n)$	$lastpos(n)$
Leaf ε	true	$\varnothing$	$\varnothing$
Leaf i	false	$\{i\}$	$\{i\}$
| / \ c_1 c_2	$nullable(c_1)$ or $nullable(c_2)$	$firstpos(c_1)$ $\cup$ $firstpos(c_2)$	$lastpos(c_1)$ $\cup$ $lastpos(c_2)$
• / \ c_1 c_2	$nullable(c_1)$ and $nullable(c_2)$	**if** $nullable(c_1)$ **then** $firstpos(c_1) \cup$ $firstpos(c_2)$ **else** $firstpos(c_1)$	**if** $nullable(c_2)$ **then** $lastpos(c_1) \cup$ $lastpos(c_2)$ **else** $lastpos(c_2)$
* | c_1	true	$firstpos(c_1)$	$lastpos(c_1)$

From Regular Expression to DFA Directly: Syntax Tree of (a|b)*abb#



From Regular Expression to DFA Directly: *followpos*

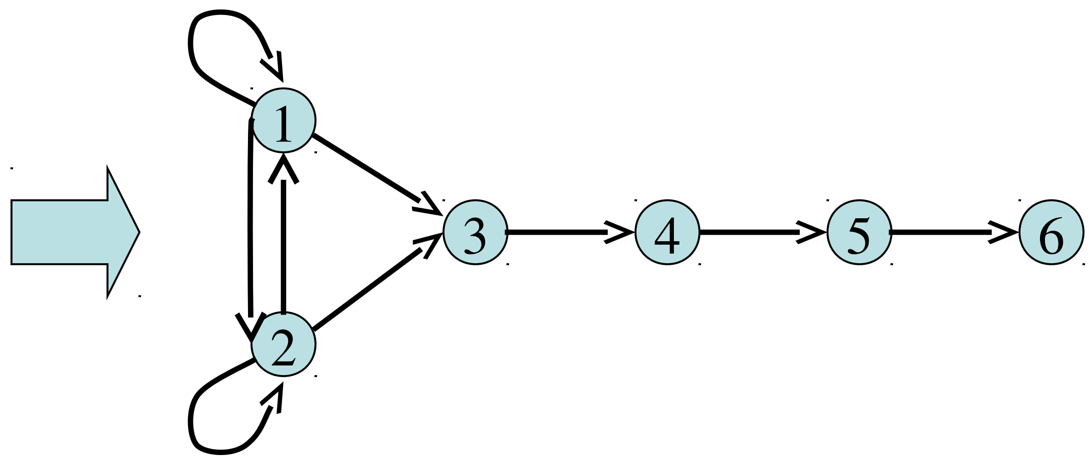
for each node n in the tree **do**
    if n is a cat-node with left child c_1 and right child c_2 **then**
        for each i in $lastpos(c_1)$ **do**
            $followpos(i) := followpos(i) \cup firstpos(c_2)$
        end do
    else if n is a star-node
        for each i in $lastpos(n)$ **do**
            $followpos(i) := followpos(i) \cup firstpos(n)$
        end do
    end if
end do

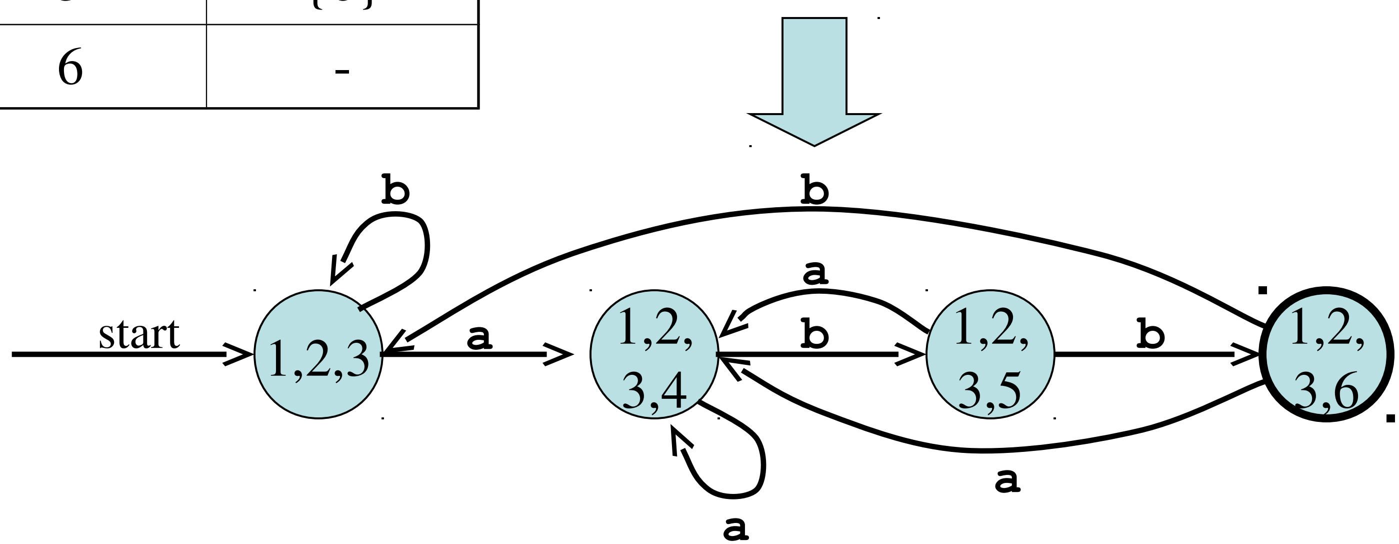
From Regular Expression to DFA Directly: Algorithm

s_0 := *firstpos*(*root*) where *root* is the root of the syntax tree
Dstates := {s_0} and is unmarked
while there is an unmarked state T in *Dstates* **do**
    mark T
    for each input symbol $a \in \Sigma$ **do**
        let U be the set of positions that are in *followpos*(p)
            for some position p in T,
            such that the symbol at position p is a
        if U is not empty and not in *Dstates* **then**
            add U as an unmarked state to *Dstates*
        end if
        Dtran[T,a] := U
    end do
end do

From Regular Expression to DFA Directly: Example

Node	*followpos*
1	{1, 2, 3}
2	{1, 2, 3}
3	{4}
4	{5}
5	{6}
6	-





Time-Space Tradeoffs

Automaton	*Space (worst case)*	*Time (worst case)*
NFA	$O(\lvert r \rvert)$	$O(\lvert r \rvert \times \lvert x \rvert)$
DFA	$O(2^{\lvert r \rvert})$	$O(\lvert x \rvert)$