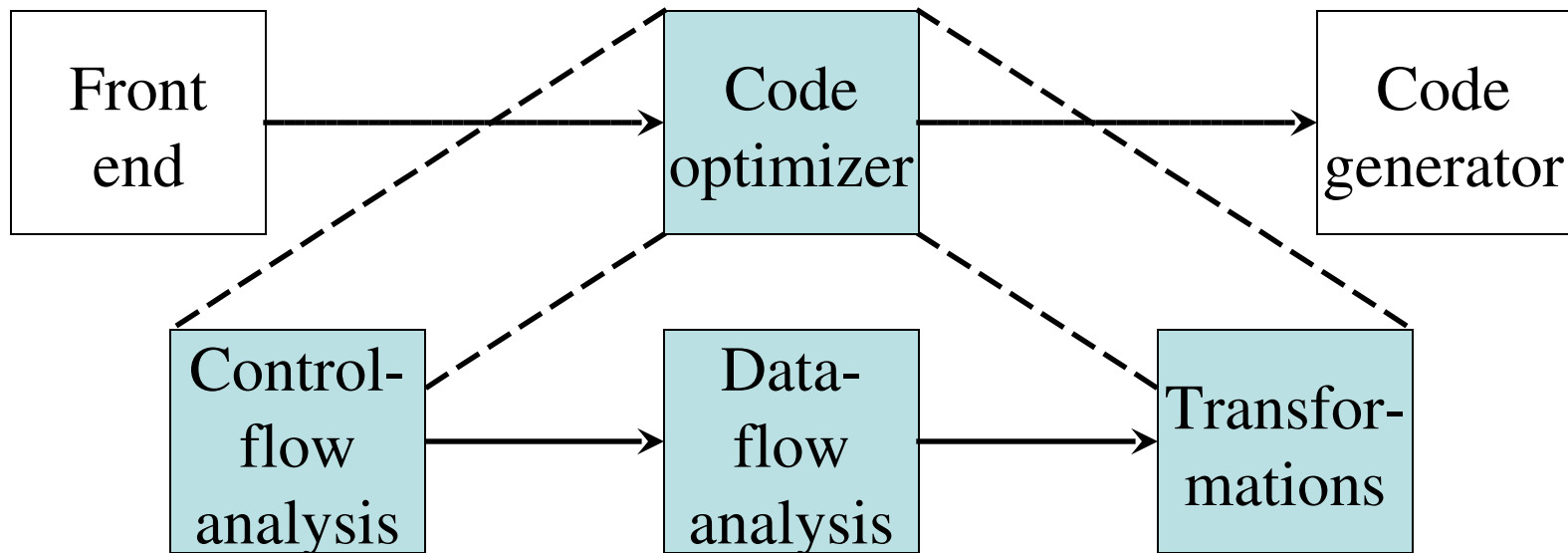


# Code Optimization

## Chapter 10

# The Code Optimizer

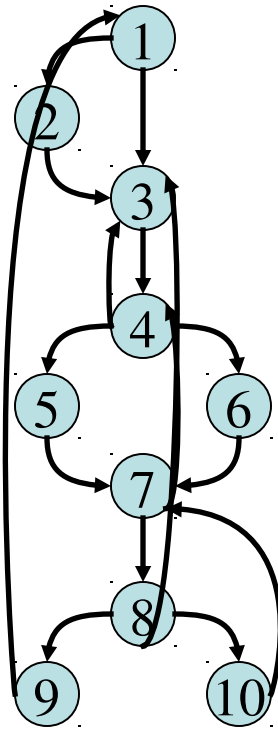
- Control flow analysis: CFG (Ch. 9)
- Data-flow analysis
- Transformations



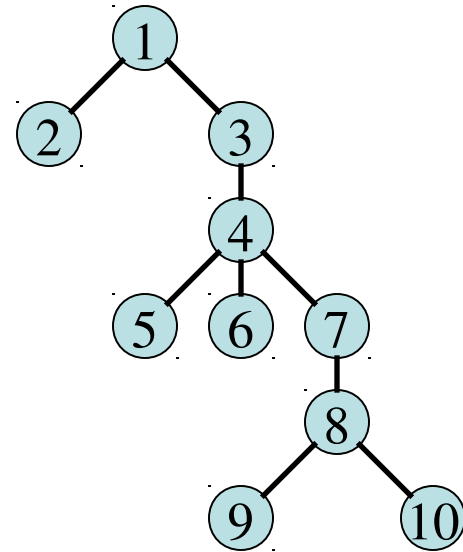
# Determining Loops in Flow Graphs: Dominators

- Dominators:  $d \text{ dom } n$ 
  - Node  $d$  of a CFG *dominates* node  $n$  if *every* path from the initial node of the CFG to  $n$  goes through  $d$
  - The loop entry dominates all nodes in the loop
- The *immediate dominator*  $m$  of a node  $n$  is the last dominator on the path from the initial node to  $n$ 
  - If  $d \neq n$  and  $d \text{ dom } n$  then  $d \text{ dom } m$

# Dominator Trees



CFG

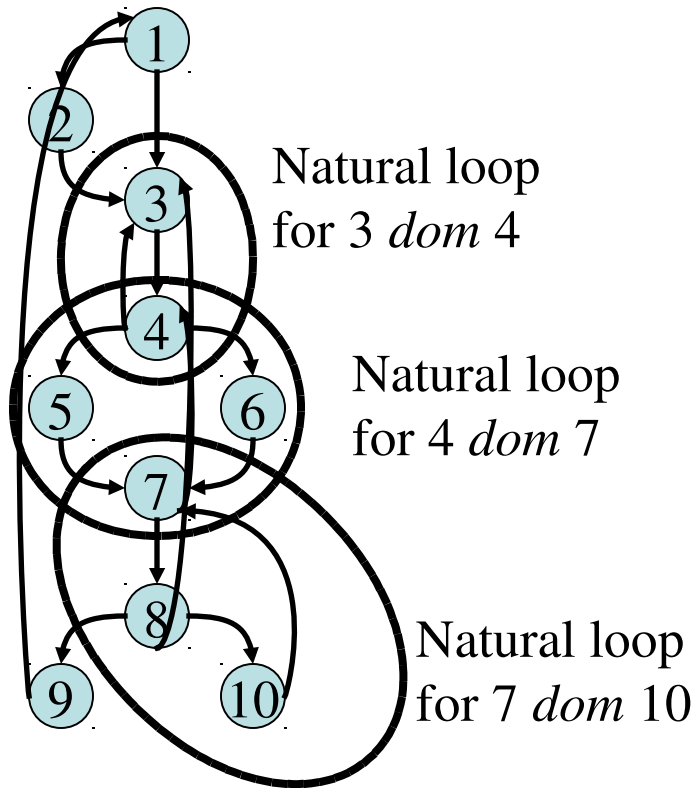


Dominator tree

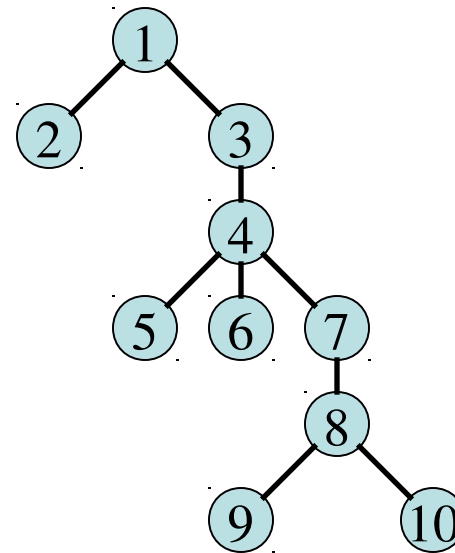
# Natural Loops

- A *back edge* is an edge  $a \rightarrow b$  whose head  $b$  dominates its tail  $a$
- Given a back edge  $n \rightarrow d$ 
  - The *natural loop* consists of  $d$  plus the nodes that can reach  $n$  without going through  $d$
  - The *loop header* is node  $d$
- Unless two loops have the same header, they are disjoint or one is nested within the other
  - A nested loop is an *inner loop* if it contains no other loops

# Natural Inner Loops Example



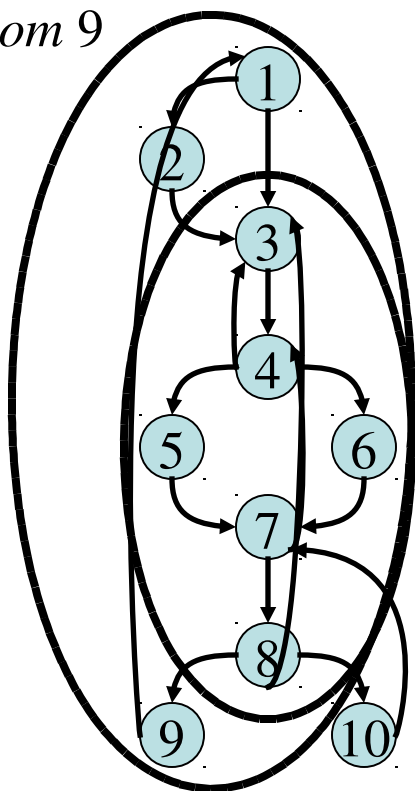
CFG



Dominator tree

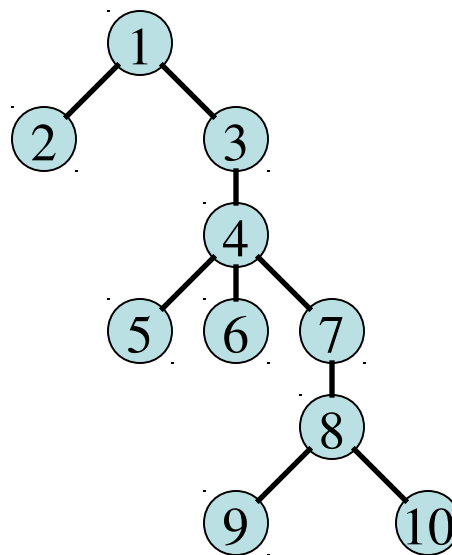
# Natural Outer Loops Example

Natural loop  
for 1 *dom* 9



CFG

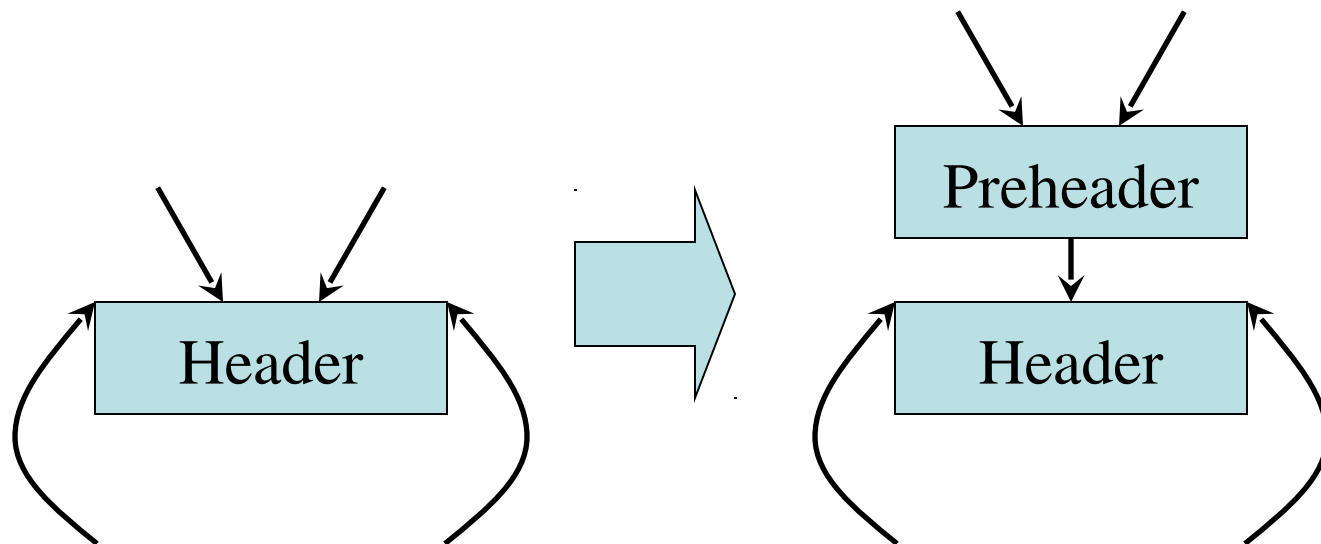
Natural loop  
for 3 *dom* 8



Dominator tree

# Pre-Headers

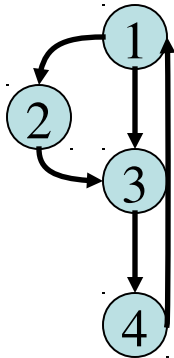
- To facilitate loop transformations, a compiler often adds a *preheader* to a loop
- Code motion, strength reduction, and other loop transformations populate the preheader



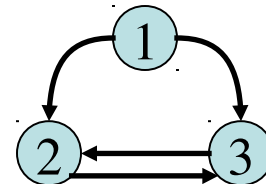


# Reducible Flow Graphs

- *Reducible graph* = disjoint partition in forward and back edges such that the forward edges form an acyclic (sub)graph



Example of a  
reducible CFG



Example of a  
nonreducible CFG

# Global Data-Flow Analysis

- To apply global optimizations on basic blocks, *data-flow information* is collected by solving systems of *data-flow equations*
- Suppose we need to determine the *reaching definitions* for a sequence of statements  $S$

$$out[S] = gen[S] \cup (in[S] - kill[S])$$

B1: 

d1: i := m-1
d2: j := n



B2: 

d3: j := j-1
--------------



B3: 

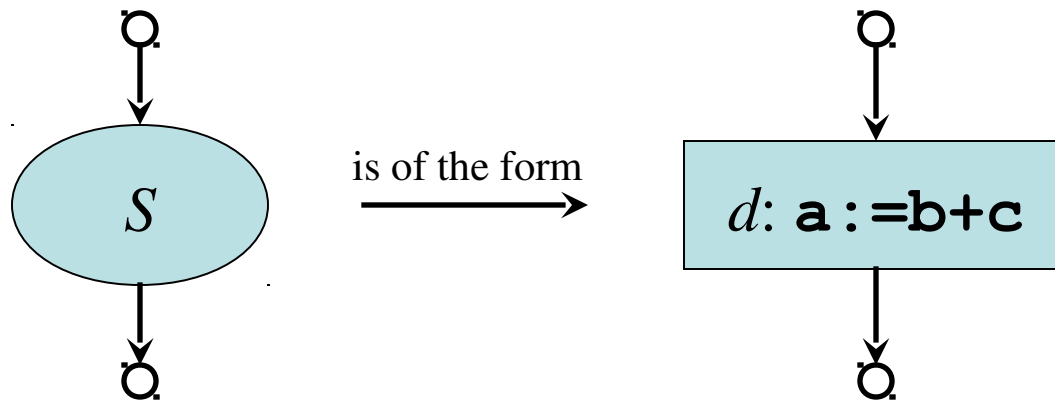
--

$$out[B1] = gen[B1] = \{d1, d2\}$$

$$out[B2] = gen[B2] \cup \{d1\} = \{d1, d3\}$$

d1 reaches B2 and B3 and  
d2 reaches B2, but not B3  
because d2 is killed in B2

# Reaching Definitions



Then, the data-flow equations for  $S$  are:

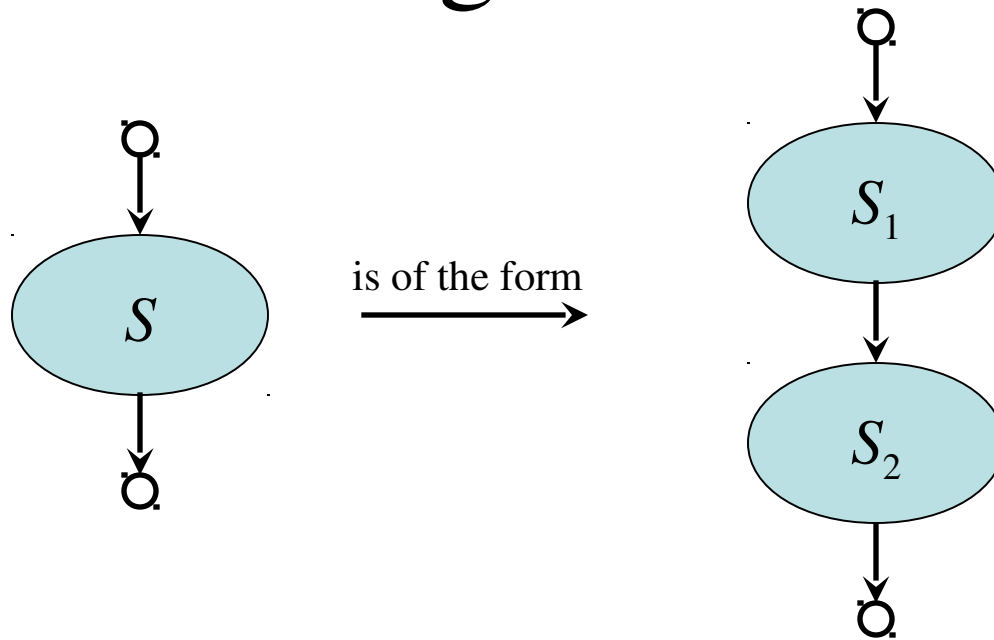
$$gen[S] = \{d\}$$

$$kill[S] = D_a - \{d\}$$

$$out[S] = gen[S] \cup (in[S] - kill[S])$$

where  $D_a$  = all definitions of  $\mathbf{a}$  in the region of code

# Reaching Definitions



$$gen[S] = gen[S_2] \cup (gen[S_1] - kill[S_2])$$

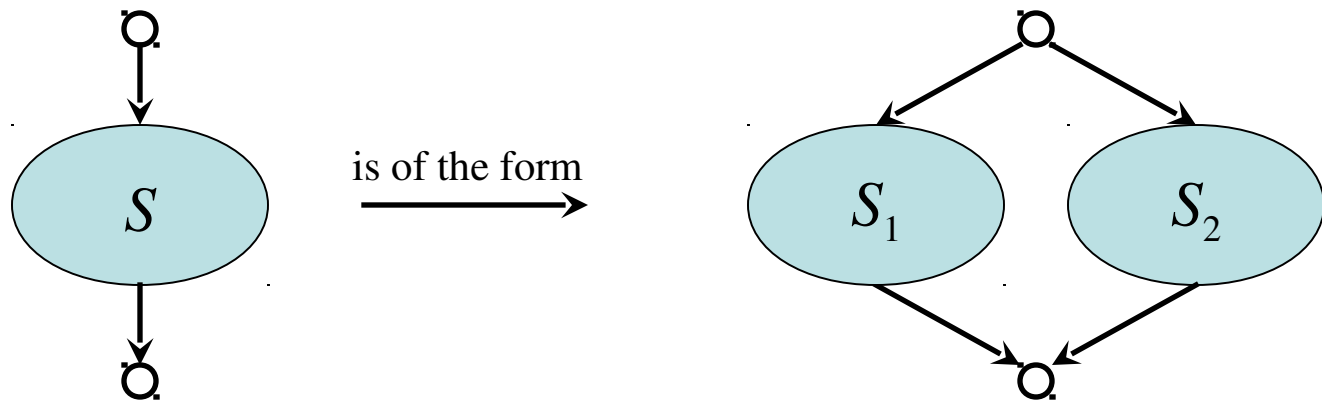
$$kill[S] = kill[S_2] \cup (kill[S_1] - gen[S_2])$$

$$in[S_1] = in[S]$$

$$in[S_2] = out[S_1]$$

$$out[S] = out[S_2]$$

# Reaching Definitions



$$gen[S] = gen[S_1] \cup gen[S_2]$$

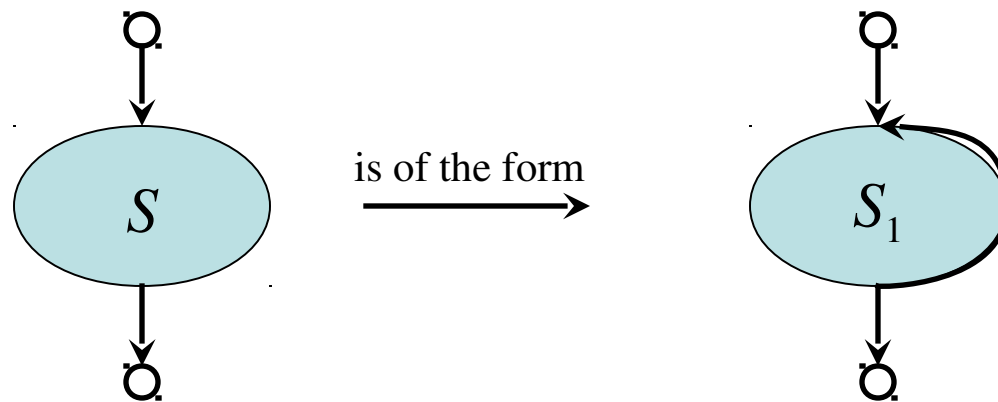
$$kill[S] = kill[S_1] \cap kill[S_2]$$

$$in[S_1] = in[S]$$

$$in[S_2] = in[S]$$

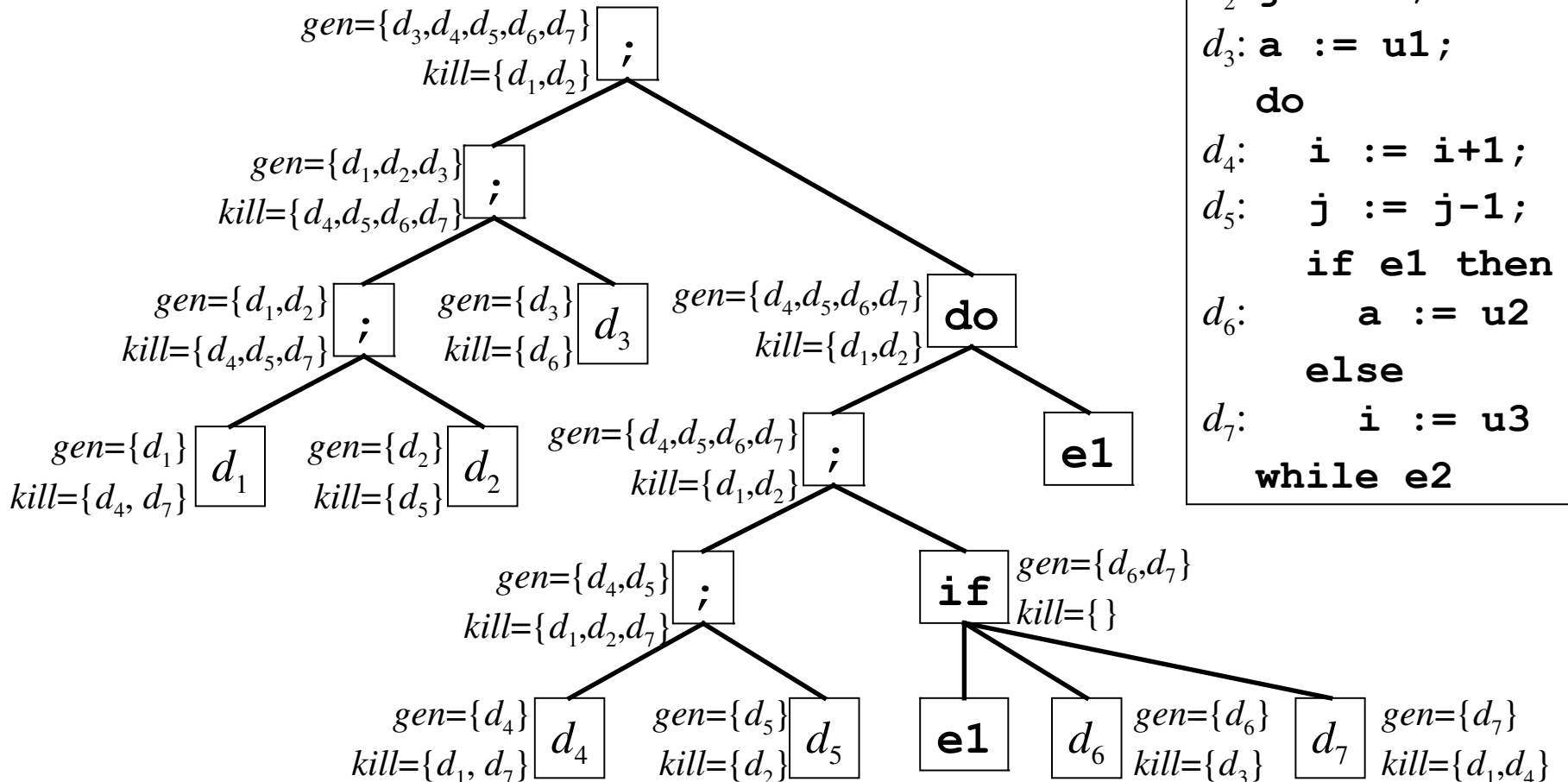
$$out[S] = out[S_1] \cup out[S_2]$$

# Reaching Definitions

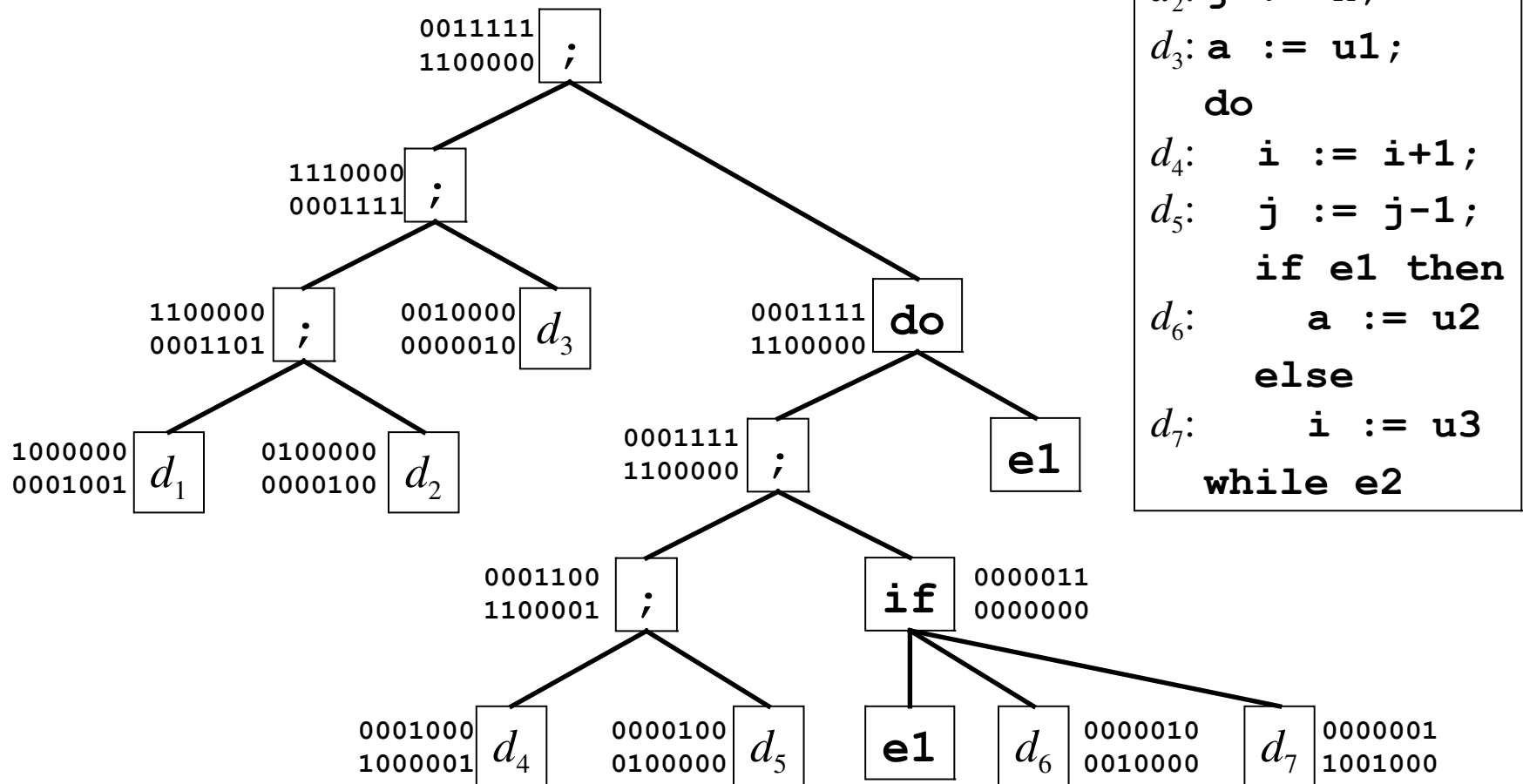


$$\begin{array}{ll} gen[S] & = gen[S_1] \\ kill[S] & = kill[S_1] \\ in[S_1] & = in[S] \cup gen[S_1] \\ out[S] & = out[S_1] \end{array}$$

# Example Reaching Definitions



# Using Bit-Vectors to Compute Reaching Definitions

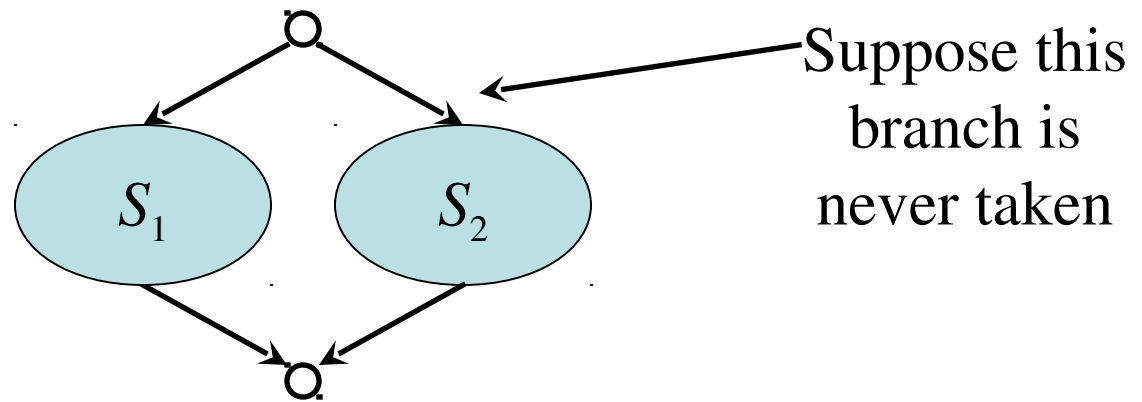




# Accuracy, Safeness, and Conservative Estimations

- *Conservative*: refers to making safe assumptions when insufficient information is available at compile time, i.e. the compiler has to guarantee not to change the meaning of the optimized code
- *Safe*: refers to the fact that a superset of reaching definitions is safe (some may have been killed)
- *Accuracy*: the larger the superset of reaching definitions, the less information we have to apply code optimizations

# Reaching Definitions are a Conservative (Safe) Estimation



Estimation:

$$gen[S] = gen[S_1] \cup gen[S_2]$$

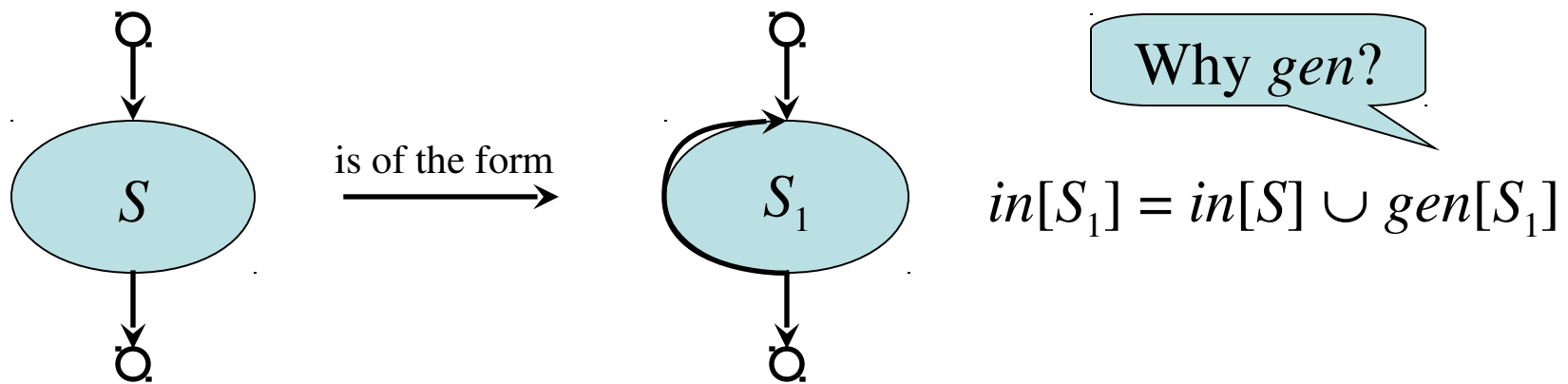
$$kill[S] = kill[S_1] \cap kill[S_2]$$

Accurate:

$$gen'[S] = gen[S_1] \subseteq gen[S]$$

$$kill'[S] = kill[S_1] \supseteq kill[S]$$

# Reaching Definitions are a Conservative (Safe) Estimation



The problem is that

$$in[S_1] = in[S] \cup out[S_1]$$

makes more sense, but we cannot solve this directly, because  $out[S_1]$  depends on  $in[S_1]$

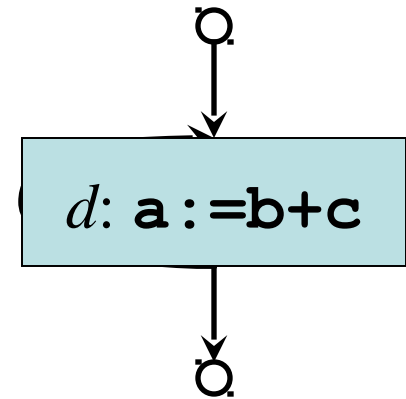
# Reaching Definitions are a Conservative (Safe) Estimation

We have:

$$(1) \text{ in}[S_1] = \text{in}[S] \cup \text{out}[S_1]$$

$$(2) \text{ out}[S_1] = \text{gen}[S_1] \cup (\text{in}[S_1] - \text{kill}[S_1])$$

Solve  $\text{in}[S_1]$  and  $\text{out}[S_1]$  by estimating  $\text{in}^1[S_1]$  using safe but approximate  $\text{out}[S_1] = \emptyset$ , then re-compute  $\text{out}^1[S_1]$  using (2) to estimate  $\text{in}^2[S_1]$ , etc.



$$\text{in}^1[S_1] =_{(1)} \text{in}[S] \cup \text{out}[S_1] = \text{in}[S]$$

$$\text{out}^1[S_1] =_{(2)} \text{gen}[S_1] \cup (\text{in}^1[S_1] - \text{kill}[S_1]) = \text{gen}[S_1] \cup (\text{in}[S] - \text{kill}[S_1])$$

$$\text{in}^2[S_1] =_{(1)} \text{in}[S] \cup \text{out}^1[S_1] = \text{in}[S] \cup \text{gen}[S_1] \cup (\text{in}[S] - \text{kill}[S_1]) = \text{in}[S] \cup \text{gen}[S_1]$$

$$\text{out}^2[S_1] =_{(2)} \text{gen}[S_1] \cup (\text{in}^2[S_1] - \text{kill}[S_1]) = \text{gen}[S_1] \cup (\text{in}[S] \cup \text{gen}[S_1] - \text{kill}[S_1])$$

$$= \text{gen}[S_1] \cup (\text{in}[S] - \text{kill}[S_1])$$

Because  $\text{out}^1[S_1] = \text{out}^2[S_1]$ , and therefore  $\text{in}^3[S_1] = \text{in}^2[S_1]$ , we conclude that