

# Canonical LR(1) & LALR(1) Parser

# Definition of LR(1)

- Two-component element of the form

$$[A \rightarrow \alpha.\beta, u]$$

where 1st component is marked production

$A \rightarrow \alpha.\beta$ , called the core of the item

$u$  is a lookahead character belongs to the set  $V \cup \{\epsilon\}$ .



# Validity

- An LR(1) item  $[A \rightarrow \alpha.\beta, u]$  is *valid* for *viable prefix*  $\lambda$ , if there exists a *rightmost derivation*

$$S \xrightarrow[R]{*} \phi A t \xrightarrow[R]{} \phi \alpha \beta t$$

Where  $\lambda = \phi \alpha$  is the viable prefix and  $u$  is the 1st symbol of  $t$ , or  $\epsilon$  if  $t = \epsilon$ .

# Example

**G** → **S**  
**S** → **E=E**  
**S** → **f**  
**E** → **T**  
**E** → **E+T**  
**T** → **f**  
**T** → **T\*f**

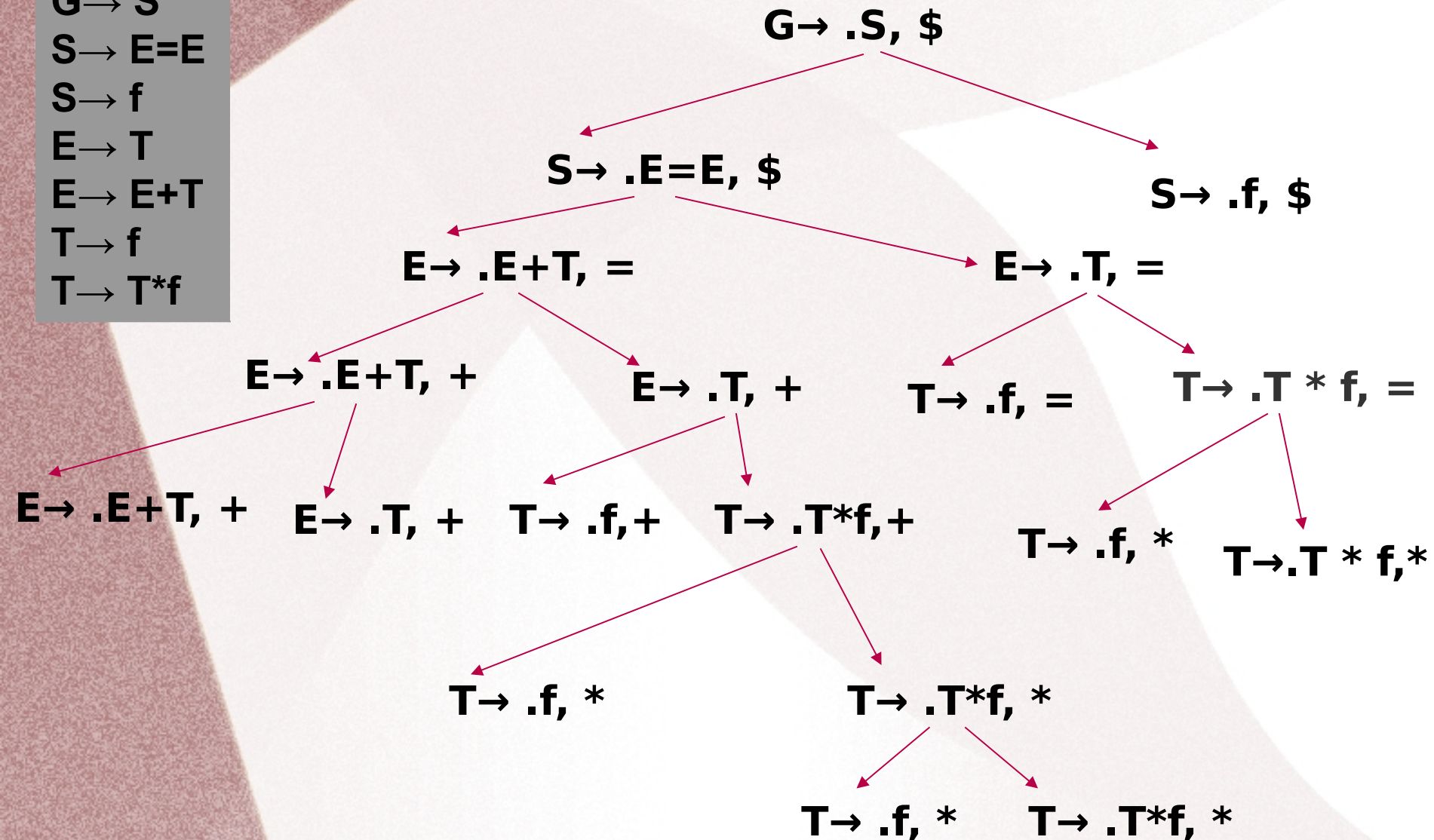
State-0 (I0) :

**G** → **.S** , \$  
**S** → **.E=E** , \$  
**S** → **.f** , \$  
**E** → **.T** , = +  
**E** → **.E+T** , = +  
**T** → **.f** , + \* =  
**T** → **.T\*f** , + \* =



# Closure of a LR(1) example

$G \rightarrow S$   
 $S \rightarrow E=E$   
 $S \rightarrow f$   
 $E \rightarrow T$   
 $E \rightarrow E+T$   
 $T \rightarrow f$   
 $T \rightarrow T*f$



# Example

**G** → S  
**S** → E=E  
**S** → f  
**E** → T  
**E** → E+T  
**T** → f  
**T** → T\*f

State-0 (I0) :

**G** → .S , \$  
**S** → .E=E , \$  
**S** → .f , \$  
**E** → .T , = +  
**E** → .E+T , = +  
**T** → .f , + \* =  
**T** → .T\*f , + \* =



# Example

State1 (I1) : from state 0 on S

$G \rightarrow S. , \$$

State0 (I0) :

$G \rightarrow .S , \$$

$S \rightarrow .E=E , \$$

$S \rightarrow .f , \$$

$E \rightarrow .T , = +$

$E \rightarrow .E+T , = +$

$T \rightarrow .f , + * =$

$T \rightarrow .T*f , + * =$

State2 (I2) : from state 0 on E

$S \rightarrow E. = E , \$$

State3 (I3) : from state 0 on f

$S \rightarrow f. , \$$

$T \rightarrow f. , = + *$

# Example

State4 (I4) : from state 0 on T

$E \rightarrow T. , = +$

$T \rightarrow T.*f , + * =$

State6 (I6) : from state 2 on +

$E \rightarrow E+.T , = +$

$T \rightarrow .f , = + *$

$T \rightarrow .T*f , = + *$

State5 (I5) : from state 2 on =

$S \rightarrow E=.E , \$$

$E \rightarrow .T , \$ +$

$T \rightarrow .f , \$ + *$

$T \rightarrow .T*f , \$ + *$

$E \rightarrow .E+T , \$ +$

State7 (I7) :

From state 4 on \*

$T \rightarrow T*.f , = + *$



# Example

State8 (I8) :

From 5 on E

$S \rightarrow E=E. , \$$

$E \rightarrow E.+T , \$ +$

State9 (I9) :

From state 5 on T

$E \rightarrow T. , \$ +$

$T \rightarrow T.*f , \$ + *$

State10 (I10) :

From state 5 on f

$T \rightarrow f. , \$ + *$

State11 (I11) :

From state 6 on T

$T \rightarrow T.*f , = + *$

$E \rightarrow E+T. , = +$

State12 (I12) :

From state 6 on f

$T \rightarrow f. , = + *$

# Example

State13 (I13) :  
From state 7 on f

$T \rightarrow T*f. , = + *$

State14 (I14) :  
From state 8 on +

$E \rightarrow E+.T, \$ +$

$T \rightarrow .T*f, \$ + *$

$T \rightarrow .f, \$ + *$

State15 (I15) :  
From state 9 on \*

$T \rightarrow T*.f , \$ + *$

State16 (I16) :  
From state 14 on T

$E \rightarrow E+T., \$ +$

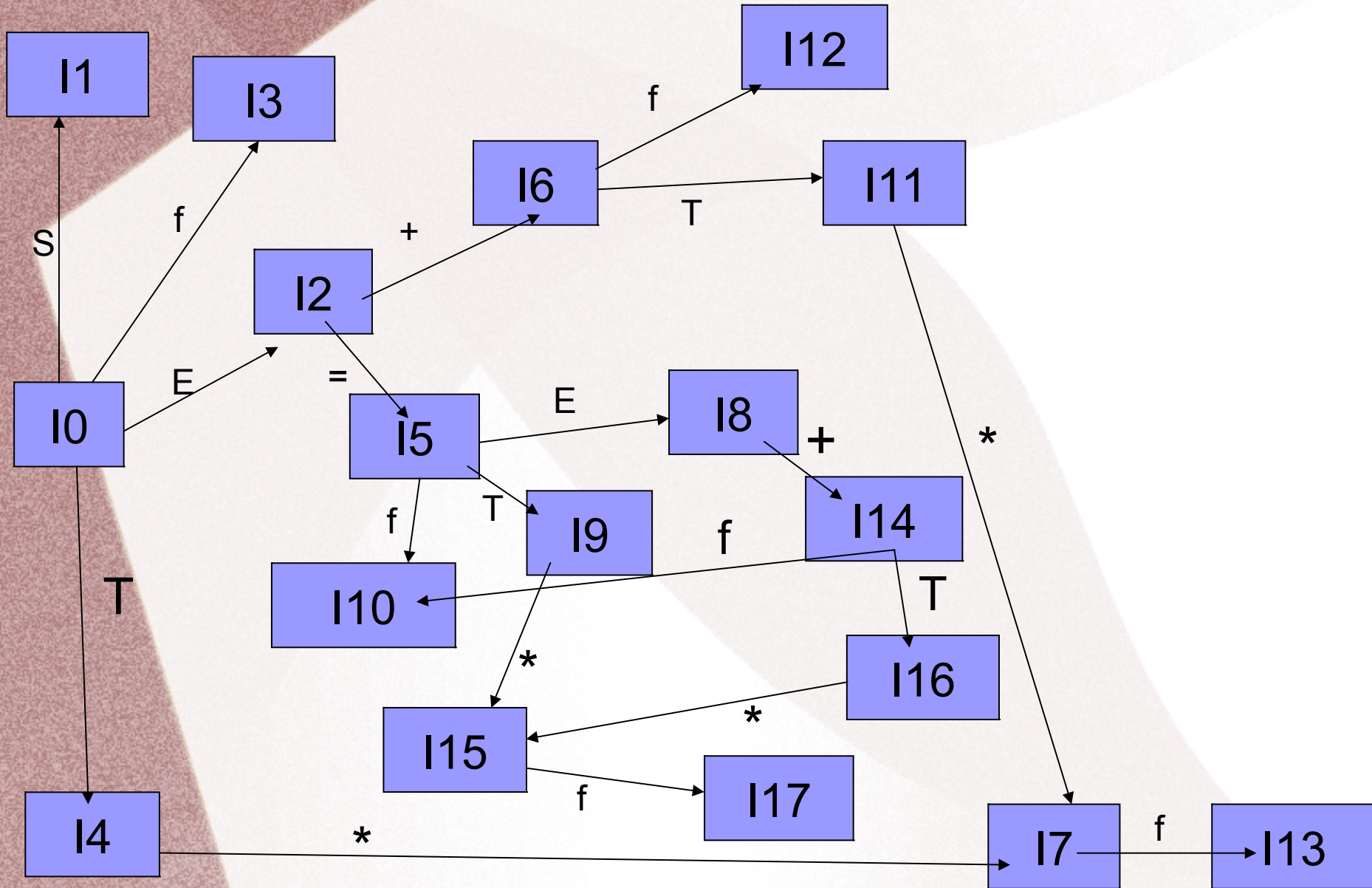
$T \rightarrow T.*f, \$ + *$

State17 (I17) :  
From state 15 on f

$T \rightarrow T*f. , \$ + *$



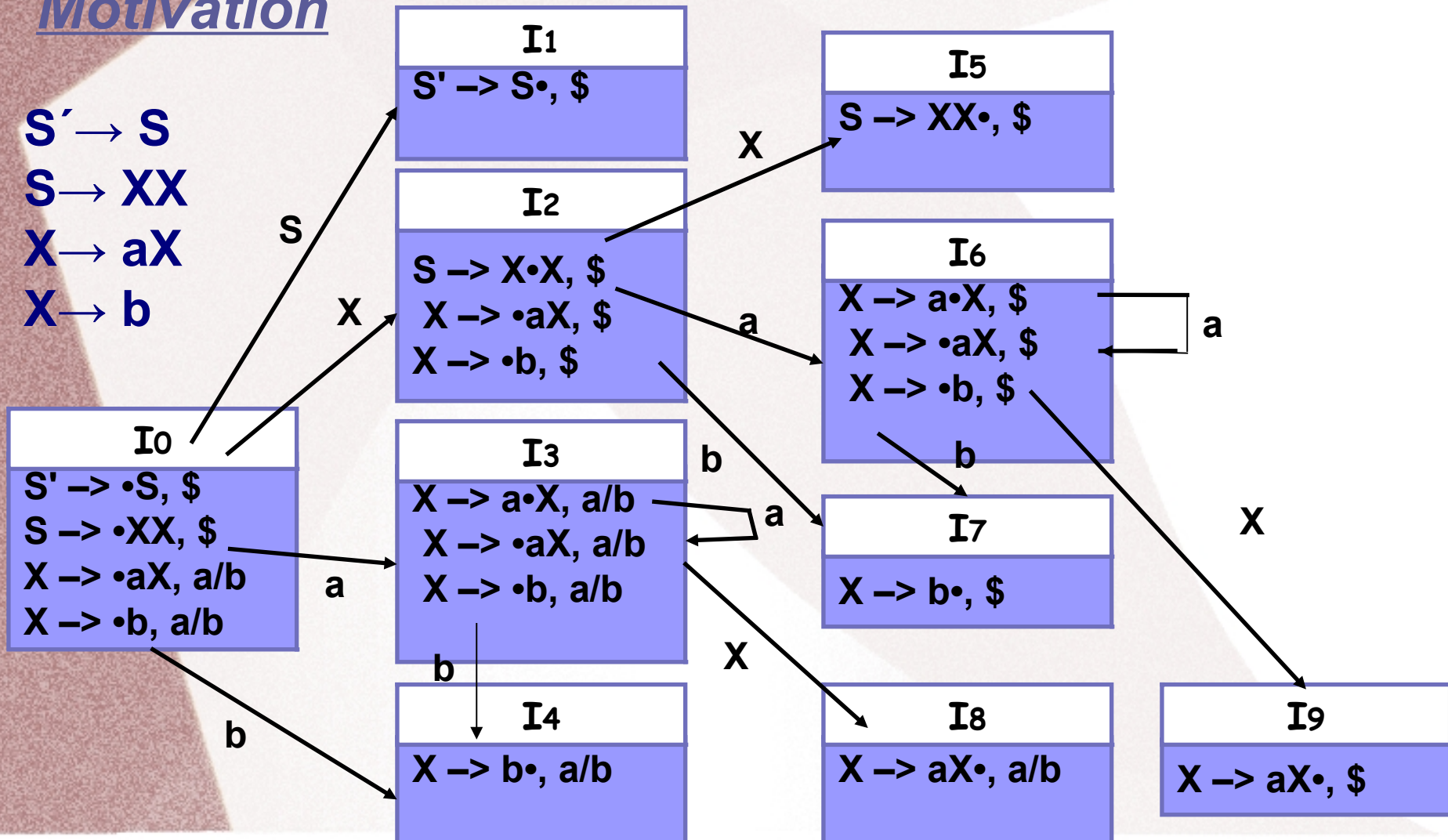
# Finite Control of LR(1) parser



# LALR parsing

## Motivation

$S' \rightarrow S$   
 $S \rightarrow XX$   
 $X \rightarrow aX$   
 $X \rightarrow b$





# LALR parsing

## After Merging:

I<sub>36</sub>:  $X \rightarrow a \cdot X, a/b/\$$

$X \rightarrow \cdot aX, a/b/\$$

$X \rightarrow \cdot b, a/b/\$$

I<sub>47</sub>:  $X \rightarrow b \cdot, a/b/\$$

I<sub>89</sub>:  $X \rightarrow aX \cdot, a/b/\$$

# LALR parsing

## Example:

$S' \rightarrow S$   
 $S \rightarrow Bbb \mid aab \mid bBa$   
 $B \rightarrow a$

I0:  $S' \rightarrow \cdot S, \$$   
 $S \rightarrow \cdot Bbb, \$$   
 $S \rightarrow \cdot aab, \$$   
 $S \rightarrow \cdot bBa, \$$   
 $B \rightarrow \cdot a, b$

I1:  $S' \rightarrow S \cdot, \$$

I2:  $S \rightarrow B \cdot bb, \$$

I3:  $S \rightarrow a \cdot ab, \$$   
 $B \rightarrow a \cdot, b$

....



# LALR merge conflict

## Shift-Reduce conflict:

1. If LR (1) has shift-reduce conflict then LALR will also have it.
2. If LR (1) does not have shift-reduce conflict LALR will also not have it.
3. Any shift-reduce conflict which can be removed by LR (1) can also be removed by LALR.
4. If SLR has shift-reduce conflict then LALR may or may not remove it.
5. SLR and LALR tables for a grammar always have same number of states.

Hence, LALR parsing is the most suitable for parsing general programming languages. The table size is quite small as compared to LR (1), and by carefully designing the grammar it can be made free of conflicts.



# LALR merge conflict

## Reduce-Reduce conflict: Example

$S' \rightarrow S$

$S \rightarrow aBc$

$S \rightarrow bCc$

$S \rightarrow aCd$

$S \rightarrow bBd$

$B \rightarrow e$

$C \rightarrow e$

I0:  $S' \rightarrow \cdot S, \$$   
 $S \rightarrow \cdot aBc, \$$   
 $S \rightarrow \cdot bCc, \$$   
 $S \rightarrow \cdot aCd, \$$   
 $S \rightarrow \cdot bBd, \$$

I1:  $S' \rightarrow S \cdot, \$$

I2:  $S \rightarrow a \cdot Bc, \$$   
 $S \rightarrow a \cdot Cd, \$$   
 $B \rightarrow \cdot e, c$

I3:  $S \rightarrow b \cdot Cc, \$$   
 $S \rightarrow b \cdot Bd, \$$   
 $C \rightarrow \cdot e, c$   
 $B \rightarrow \cdot e, d$

I4:  $S \rightarrow aB \cdot c, \$$

I5:  $S \rightarrow aC \cdot d, \$$

I6:  $B \rightarrow e \cdot, c$   
 $C \rightarrow e \cdot, d$

I7:  $S \rightarrow bC \cdot c, \$$

I8:  $S \rightarrow bB \cdot d, \$$

I9:  $B \rightarrow e \cdot, d$   
 $C \rightarrow e \cdot, c$

I10:  $S \rightarrow aBc \cdot, \$$

I11:  $S \rightarrow aCd \cdot, \$$

I12:  $S \rightarrow bCc \cdot, \$$

I13:  $S \rightarrow bBd \cdot, \$$