Assignment 2

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Question (9) $\Rightarrow$ Solve the differential equation :

$$\frac{dy}{dx} = \frac{x+y+2}{2(x+y)-1} \tag{1}$$

Solution:

$$let \qquad x+y=t \tag{2}$$

differentiate (i) w.r.t to x

$$1+\frac{dy}{dx} = \frac{dt}{dx} \tag{3}$$

$$\frac{dy}{dx} = \frac{dt}{dx} - 1 \tag{4}$$

now substituting the value of $\frac{dy}{dx}$ in question

$$\frac{dt}{dx} - 1 = \frac{t+2}{2t-1} \tag{5}$$

$$\frac{dt}{dx} = \frac{t+2}{2t-1} + 1 \tag{6}$$

$$\frac{dt}{dx} = \frac{t+2+2t-1}{2t-1} \tag{7}$$

$$\frac{dt}{dx} = \frac{3t+1}{2t-1} \tag{8}$$

$$\frac{2t-1}{3t+1}dt = dx \tag{9}$$

$$\int \frac{2t-1}{3t+1}dt = \int dx \tag{10}$$

$$\int \left(\frac{2}{3} - \frac{5}{3(3t+1)}\right)dt = \int dx \tag{11}$$

$$\int \frac{2}{3}dt - \int \frac{5}{3(3t+1)}dt = \int dx \tag{12}$$

$$\frac{2}{3}\int dt - \frac{5}{3}\int \frac{1}{3t+1}dt = \int dx \tag{13}$$

$$\frac{2}{3}t - \frac{5}{3}\frac{\ln 3t+1}{3} = x \tag{14}$$

$$\frac{2}{3}t - \frac{5(\ln 3t+1)}{9} = x \tag{15}$$

putting the value of t in equation (15)

$$\frac{2}{3}(x+y) - \frac{5(\ln(3(x+y)+1)}{9} = x \tag{16}$$