Assignment 2

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Question: $(9) \Rightarrow$ Solve the differential equation :

$$\frac{dy}{dx} = \frac{x+y+2}{2(x+y)-1}$$

Solution:

$$let x + y = t (1)$$

differentiate (1) w.r.t to x

$$1 + \frac{dy}{dx} = \frac{dt}{dx} \tag{2}$$

$$\frac{dy}{dx} = \frac{dt}{dx} - 1\tag{3}$$

now substituting the value of $\frac{dy}{dx}$ in question

$$\frac{dt}{dx} - 1 = \frac{t+2}{2t-1}$$
 (4)

$$\frac{dt}{dx} = \frac{t+2}{2t-1} + 1 \tag{5}$$

$$\frac{dt}{dx} = \frac{t+2+2t-1}{2t-1}$$
 (6)

$$\frac{dt}{dx} = \frac{3t+1}{2t-1} \tag{7}$$

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$$\frac{2t-1}{3t+1}dt = dx \tag{8}$$

$$\int \frac{2t-1}{3t+1} dt = \int dx \tag{9}$$

$$\int \left(\frac{2}{3} - \frac{5}{3(3t+1)}\right) dt = \int dx \tag{10}$$

$$\int \frac{2}{3}dt - \int \frac{5}{3(3t+1)}dt = \int dx$$
 (11)

$$\frac{2}{3} \int dt - \frac{5}{3} \int \frac{1}{3t+1} dt = \int dx \tag{12}$$

$$\frac{2}{3}t - \frac{5\ln(3t+1)}{3} = x + C \tag{13}$$

$$\frac{2}{3}t - \frac{5\ln(3t+1)}{9} = x + C \tag{14}$$

putting the value of t from (1) in equation (14)

$$\frac{2}{3}(x+y) - \frac{5\ln(3(x+y)+1)}{9} = x + C$$
 (15)

here C is constant of integration