Assignment 2

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Question: (ex 2.16 d) Consider the following difference equation

$$y[n] - \frac{1}{4}y[n-1] - \frac{1}{8}y[n-2] = 3x[n] \qquad (1)$$

Find the paricular solution to the difference equation when

$$x[n] = \left(\frac{1}{2}\right)^n u[n]$$

Solution: Taking the z-transform to the both side , we find that,

$$Y(z)(1 - \frac{1}{4}z^{-1} - \frac{1}{8}z^{-2}) = 3X(z) \qquad (2)$$

and Therfore

$$H(z) = \frac{Y(z)}{X(z)} \qquad (3)$$

$$= \frac{3}{1 - \frac{1}{4}z^{-1} - \frac{1}{8}z^{-2}} \qquad (4)$$

$$= \frac{3}{(1 + \frac{1}{4}z^{-1})(1 - \frac{1}{2}z^{-1})} \qquad (5)$$

$$= \frac{1}{1 + \frac{1}{4}z^{-1}} + \frac{2}{1 - \frac{1}{2}z^{-1}} \qquad (6)$$

$$Y(z) = H(z)X(z) \qquad (7)$$

$$= \frac{1}{1 - \frac{1}{2}z^{-1}} \frac{3}{(1 + \frac{1}{4}z^{-1})(1 - \frac{1}{2}z^{-1})} \qquad (8)$$

$$= \frac{1/3}{1 + \frac{1}{4}z^{-1}} + \frac{2}{1 - \frac{1}{2}z^{-1}} + \frac{2/3}{1 + \frac{1}{4}z^{-1}} \qquad (9)$$

Taking inverse z-transform

$$y[n] = \frac{1}{3}\left(\frac{1}{4}\right)^n u[n] + 4(n+1)\left(\frac{1}{2}\right)^{n+1} u[n+1] + \frac{2}{3}\left(\frac{1}{2}\right)^n u[n] \qquad (10)$$