Q1) Use the method of separation of variables to find the solution of the telegraph equation

$$u_{tt} + au_t + bu = c^2 u_{xx}, \quad 0 < x < l, \quad t > 0$$
$$u(x,0) = f(x),\ u_t(x,0) = 0$$
$$u(0,t) = u(l,t) = 0, \quad t > 0$$

Q2) The torsional oscillation of a shaft of circular cross section is governed by the partial differential equation

$$\theta_{tt} = a^2 \theta_{xx}, \quad 0 < x < l, \quad t > 0$$

where $\theta(x, t)$ is the angular displacement of the cross section and a is a physical constant. The ends of the shaft are fixed elastically, that is,

$$\theta_x(0,t) - h\theta(0,t) = 0, \quad \theta_x(l,t) - h\theta(l,t) = 0$$

Determine the angular displacement if the initial angular displacement is $f(x)$.

Q3) The temperature distribution $\theta(x, t)$ isn a rod of length l satisfies the differential equation

$$\theta_t = \kappa \theta_{xx}, \quad 0 < x < l, \quad t > 0$$

(a) Find the temperature distribution if the faces are insulated, and the initial temperature distribution is given by $x(1 - x)$
(b) Find the temperature distribution if the length of the rod is π, one end of which is kept at zero temperature and the other end of which loses heat at a rate proportional to the temperature at that end $x = \pi$. The initial temperature distribution is given by $f(x) = x$.

Q4) For a function $f(x)$, the Fourier Series is given by

$$f(x) \approx \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + b_n \sin nx$$

Without actually finding the Fourier Series, guess the value of the coefficients if
(a) $f(x) = \cos x \sin x$ (b) $f(x) = \cos^3 x$ (c) $f(x) = \sin^3 x + \cos^2 x$

Q5) Plot the function $f(x) = \begin{cases} \dfrac{2x}{\pi}, & 0 \le x \le \dfrac{\pi}{2} \\[2ex] \dfrac{2}{\pi}(\pi - x), & \dfrac{\pi}{2} \le x \le \pi \end{cases}$

(a) How will you extend the function to make it periodic? What will be the period of the function?
(b) Find the Fourier Series of $f(x)$.
(c) Use the Fourier Series to show that the sum of the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$ is approximately $\dfrac{\pi^2}{8}$.