

# CS 224n Assignment #2: Written Assignment

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(a)  $-\sum_{w \in Vocab} y_w \log(\hat{y}_w) = \sum_{w=o} \log(\hat{y}_w) = \log(\hat{y}_o).$

(b)

$$\begin{aligned} \frac{\partial}{\partial \mathbf{v}_c} \mathbf{J}_{\text{naive-softmax}}(\mathbf{v}_c, o, \mathbf{U}) &= \frac{\partial}{\partial \mathbf{v}_c} \left( -\log \left( \frac{\exp(\mathbf{u}_o^\top \mathbf{v}_c)}{\sum_{w \in Vocab} \exp(\mathbf{u}_w^\top \mathbf{v}_c)} \right) \right) \\ &= \frac{\partial}{\partial \mathbf{v}_c} \left( \log \left( \sum_{w \in Vocab} \exp(\mathbf{u}_w^\top \mathbf{v}_c) \right) - \log(\exp(\mathbf{u}_o^\top \mathbf{v}_c)) \right) \\ &= \frac{\partial}{\partial \mathbf{v}_c} \log \left( \sum_{w \in Vocab} \exp(\mathbf{u}_w^\top \mathbf{v}_c) \right) - \frac{\partial}{\partial \mathbf{v}_c} \mathbf{u}_o^\top \mathbf{v}_c \\ &= \frac{\sum_{x \in Vocab} \exp(\mathbf{u}_x^\top \mathbf{v}_c) \mathbf{u}_x}{\sum_{w \in Vocab} \exp(\mathbf{u}_w^\top \mathbf{v}_c)} - \mathbf{u}_o \\ &= \sum_{x \in Vocab} \frac{\exp(\mathbf{u}_x^\top \mathbf{v}_c)}{\sum_{w \in Vocab} \exp(\mathbf{u}_w^\top \mathbf{v}_c)} \mathbf{u}_x - \mathbf{u}_o \\ &= \sum_{x \in Vocab} \hat{\mathbf{y}}_x \mathbf{u}_x - \sum_{x \in Vocab} \mathbf{y}_x \mathbf{u}_x \\ &= \sum_{x \in Vocab} \mathbf{u}_x (\hat{\mathbf{y}}_x - \mathbf{y}_x) \\ &= \mathbf{U}(\hat{\mathbf{y}} - \mathbf{y}). \end{aligned}$$

(c)

$$\begin{aligned} \frac{\partial}{\partial \mathbf{u}_w} \mathbf{J}_{\text{naive-softmax}}(\mathbf{v}_c, o, \mathbf{U}) &= \frac{\partial}{\partial \mathbf{u}_w} \log \left( \sum_{w \in Vocab} \exp(\mathbf{u}_w^\top \mathbf{v}_c) \right) - \frac{\partial}{\partial \mathbf{u}_w} \mathbf{u}_o^\top \mathbf{v}_c \\ &= \frac{\exp(\mathbf{u}_w^\top \mathbf{v}_c)}{\sum_{w \in Vocab} \exp(\mathbf{u}_w^\top \mathbf{v}_c)} \mathbf{v}_c - \mathbf{y}_w \mathbf{v}_c \\ &= \hat{\mathbf{y}}_w \mathbf{v}_c - \mathbf{y}_w \mathbf{v}_c \\ &= \mathbf{v}_c (\hat{\mathbf{y}}_w - \mathbf{y}_w). \end{aligned}$$