CS 224n Assignment #2: Written Assignment

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(a)
$$-\sum_{w \in Vocab} y_w \log(\hat{y}_w) = \sum_{w=o} \log(\hat{y}_w) = \log(\hat{y}_o).$$

(b)

$$\frac{\partial}{\partial \boldsymbol{v}_{c}} \boldsymbol{J}_{\text{naive-softmax}}(\boldsymbol{v}_{c}, o, \boldsymbol{U}) = \frac{\partial}{\partial \boldsymbol{v}_{c}} \left(-\log \left(\frac{\exp(\boldsymbol{u}_{o}^{\top} \boldsymbol{v}_{c})}{\sum_{w \in Vocab} \exp(\boldsymbol{u}_{w}^{\top} \boldsymbol{v}_{c})} \right) \right) \\
= \frac{\partial}{\partial \boldsymbol{v}_{c}} \left(\log \left(\sum_{w \in Vocab} \exp(\boldsymbol{u}_{w}^{\top} \boldsymbol{v}_{c}) \right) - \log \left(\exp(\boldsymbol{u}_{o}^{\top} \boldsymbol{v}_{c}) \right) \right) \\
= \frac{\partial}{\partial \boldsymbol{v}_{c}} \log \left(\sum_{w \in Vocab} \exp(\boldsymbol{u}_{w}^{\top} \boldsymbol{v}_{c}) \right) - \frac{\partial}{\partial \boldsymbol{v}_{c}} \log \left(\exp(\boldsymbol{u}_{o}^{\top} \boldsymbol{v}_{c}) \right) \\
= \frac{\partial}{\partial \boldsymbol{v}_{c}} \log \left(\sum_{w \in Vocab} \exp(\boldsymbol{u}_{w}^{\top} \boldsymbol{v}_{c}) \right) - \frac{\partial}{\partial \boldsymbol{v}_{c}} \boldsymbol{u}_{o}^{\top} \boldsymbol{v}_{c} \\
= \frac{\sum_{x \in Vocab} \exp(\boldsymbol{u}_{x}^{\top} \boldsymbol{v}_{c}) \boldsymbol{u}_{x}}{\sum_{w \in Vocab} \exp(\boldsymbol{u}_{w}^{\top} \boldsymbol{v}_{c})} - \boldsymbol{u}_{o} \\
= \sum_{x \in Vocab} \frac{\exp(\boldsymbol{u}_{x}^{\top} \boldsymbol{v}_{c})}{\sum_{w \in Vocab} \exp(\boldsymbol{u}_{w}^{\top} \boldsymbol{v}_{c})} \boldsymbol{u}_{x} - \boldsymbol{u}_{o}.$$