# Nest Site Selection in Small Swarms of Honeybees: A Computational Study

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***Abstract*— Various animals from whales to honeybees tend to work in groups and employ various decentralized strategies to complete their task. Even with limited actuation, computation and sensing, they are able to engage in challenging tasks efficiently, accurately, and robustly in often uncertain environments. A mathematical model is constructed to represent the distributed dynamics and control of efficient decision-making in honeybee swarms that must choose between two alternative hive sites. We design distributed multi-agent networks of sizes three and four in order to demonstrate the behavior of decision-making with certain preferences. Bifurcation diagrams are used to study the steady-state behavior of hive selection by varying a control parameter related to stop signaling cross-inhibition. We show that even for trivially small network sizes we can still achieve robustness and value-sensitivity in terms of decision-making behavior—something that has been already shown for significantly larger networks. The transient behavior of the model is also characterized using a discrete nonlinear dynamical system model, as well as a brief foray into the adaptive control aspects of hive site selection.**

## I. INTRODUCTION

Nest site selection in honeybees can involve hundreds of bees scouting out several locations for a suitable new hive. The decision-making process is not centralized but distributed amongst multiple (numbering in the hundreds) agents (scout bees) each forming their own opinions as to which hive site is the best to relocate to. Eventually, these individual opinions or preferences coalesce into a single decision for the whole hive [6]. What is unique about this particular multi-agent system is that total consensus is not necessarily the goal in reaching a decision. There is a contest among the scouts for which hive site is optimal and a mechanism exists within the scout bees to discourage or inhibit other scouts from promulgating their own preferences [7]. Seely et al. (2000) have previously shown through rigorous testing that bee swarms implement the best-of-N decision rule not perfectly but were accurate enough, where N is the number of potential new hive sites that are not transient so that the considerations amongst these N is done simultaneously [2]. Furthermore, Seeley notes that bee swarms exhibit bounded rationality–they lack complete knowledge of available nesting sites–in their decision-making. A decision is typically reached after at least a dozen or so hive sites are examined for their suitability according to at least six properties, some of which are listed here as criteria for selection.

According to Seely et al. (2006), a quorum (sufficient number) not a consensus is perhaps the mode in which the dancing scouts operate when reaching a decision, however consensus is still necessary for the swarm when flying to its new hive. The quorum mechanism is also buttressed by more recent work by Pais et al. (2013) in which the quorum value is variable [4]. The criteria used in arriving at such a decision can involve factors such as the height of the candidate site above the ground, the size of the opening of the hive, and the overall space provided within the hive. Upon their return to the hive, the scout bees perform a waggle dance for their fellow bees in order to convey information about a particular hive site and its suitability. The waggle dance encodes information about how far and in what direction the candidate site is as well as the aforementioned characteristics regarding criteria for selection. Some scouts may send inhibitory stop signals (to reduce waggle dancing) to other scouts in order to break a deadlock (indecision or no overall preference) amongst

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the scouts and reach a decision in cases where expedience is demanded and accuracy less so. However, sometimes it is useful to remain in deadlock when promising sites are less forthcoming and waiting before reaching a decision is actually the best course of action [7]. This ability to wait for information on better alternatives is due to the hive's sensitivity to absolute and relative values of the alternatives being considered [4].

### *A. Pitchfork Bifurcation*

In the case of two alternatives that are equally appealing the behavior of the swarm exhibits a pitchfork bifurcation where the parameter being varied corresponds to the stop signal inhibition employed by scout bees to break deadlock. For smaller values of this parameter, referred to as *u*, a deadlock persists and the hive remains at its original location. As *u* is increased, the dynamics of the decision-making reaches a singularity point (typically around one) at which the deadlock is broken and the bees either arrive at one of two decisions with equal probability (a stable steady-state) or remain in deadlock (an unstable steady-state). A pitchfork bifurcation is the result of varying the control parameter. The singularity or critical point is inversely proportional to the average value of alternative sites [1]. Therefore the more enticing the alternative sites the smaller is this singularity (and the less effort it takes to break deadlock).

Reina et al. (2017) explore the dynamics of honeybee hive site selection for the more general case of N alternatives [8]. In order to formulate the general case the *u* parameter (stop signaling cross-inhibition) is replaced with a parameter "describing the relative frequencies with which individual group members engage in independent discovery and abandonment behaviors" [8]. In this paper, we will explore only the binary case (N=2) in which two alternative hive sites are simultaneously under consideration.

### *B. Persistent Unfolding*

The introduction of asymmetry in the quality of the sites (or preferences) causes the pitchfork to unfold into a persistent bifurcation diagram. In the case of larger networks, there exist four of these persistent diagrams two of which are simply reflections of the other two.

### *C. Multi-Agent Based Dynamical Systems Model*

The state of each agent is represented as $x_i \in \mathbb{R}$ where $i \in 1...N$ where N represents the number of agents (not to be confused with number of alternative hive sites). The state corresponds to the opinion each agent (honeybee) holds at a particular time. Given a binary decision (two alternatives), if alternative X is favored then $x_i > 0$ and if alternative Y is favored then $x_i < 0$ for a particular agent i. If neither is favored or the agent is neutral then $x_i = 0$. The strength of the opinion is defined as $|x_i|$ and the average opinion is defined as $y = \frac{1}{N}\sum_{i=1}^{N} x_i(t)$. The disagreement vector is defined at steady state to be $\delta = |y_{ss}| - \frac{1}{N}||x_{ss}||_1$. Deadlock is defined as either $\delta \neq 0$ or $x_{ss} = 0$ where $x_{ss}$ is the vector of steady-state values of the opinions of all the agents. We assume the networks of bees are static and strongly connected where an edge from agent i to agent j signifies that agent i can measure the opinion of agent j and that the weight of this edge signifies how much importance agent i places on the opinion of agent j. This of course can be encoded in an adjacency matrix A. We also define a degree matrix D and the Laplacian matrix L = D - A. The rate of change of a particular agent's opinion is given by:

$$\frac{dx_i}{dt} = -u_I d_i x_i + \sum_{j=1}^{N} u_S a_{ij} S(x_j) + \nu_i$$

The $\nu_i$ term encodes an external preference imposed on the agent (that is, not originating from any of the other agents). It can assume three values $\nu_i \in \{v_X, 0, -v_Y\}$ where $v_X, v_Y \in \mathbb{R}^+$ The $u_I$ term represents the inertia that prevents rapid opinion development and $u_S$ represents the strength of social effort–the larger it is the more importance is placed on the opinions of other agents [1]. The function S(x) is a sigmoidal function that caps the influence of the opinions of the other agents at some amount. Using a timescale change suggested by Gray et al (2017) $s = u_I t$ we can derive the following form for the dynamics:

$$\dot{x} = -Dx + uAS(x) + \beta$$

where $\beta_i \in \{\beta_X, 0, \beta_Y\}$. The new control parameter is $u = u_S/u_I$. With $\beta = 0$ (the uninformed case) we should retrieve the pitchfork bifurcation (symmetrical). This pitchfork bifurcation lies along the consensus manifold. Linearizing about $x = 0$ and $u = 1$ with $\beta = 0$ yields $\dot{x} = -Lx$ which is the relation for linear consensus dynamics. In the paper by Gray et. al (2017) they rigorously prove using manifold theory that the dynamics exhibits a one-dimensional invariant manifold tangent to the consensus manifold at the origin. On this manifold, the pitchfork bifurcation occurs. In short, the theorem proved in the paper by Gray et al. provides guarantees about stability, proves the existence of a symmetric pitchfork singularity developing at $u = 1$ and $x = 0$. The steady-state branches bifurcating from the pitchfork are solved using $y - uS(y) = 0$ where y is the average opinion. For $\beta \neq 0$ the theorem also describes the unfolding of the pitchfork into some asymmetrical form of bifurcation. The important point is that the pitchfork is central to the discussion of the dynamical behavior of this multi-agent system and the unfolding cases arise when this symmetry is broken. Gray et al. also state that only four persistent bifurcation diagrams can be observed upon breaking symmetry (universal unfolding) of the pitchfork lending the dynamics a sense of robustness [1]. These four persistent cases are can be observed with strongly connected graphs. Even a slight deviation from the symmetrical state can cause unfolding of the pitchfork and break the deadlock. The proof of the theorem recounted in [1] is quite involved and the reader is referred to Gray et al (2017) in case more thorough explanation and detail is desired.

## II. Methods

We examine networks comprised of three and four nodes (or agents). Steady state of the dynamical system modeling decision-making is solved for using the fsolve() command in Matlab. Transient behavior is also examined using an Euler scheme to propagate the states forward in time until a steady-state is reached. Because fsolve() only solves for one solution (essentially it is solving for $0 = -Dx + uAS(x) + \beta$) other global optimization routines were experimented with to include MultiStart within the Global Optimization Toolbox in Matlab where several (about 100) random initial conditions can be assigned to the problem to test for multiple local minima. Neither of these approaches yielded complete solutions to the bifurcation problem (i.e. finding all roots). Therefore, it was decided to use a brute force approach to find all solutions by searching a grid of points. For this purpose, we made use of cluster computing nodes available at Georgia Institute of Technology's PACE-ICE cluster. The PACE-ICE cluster offers 13 nodes that have the Dual Xeon Gold 6226 processors with 24 cores per node and various other types of nodes with GPU capability. Several cores were allocated for each job and the runtimes varied from seconds to minutes depending on the scenario and the size of the network. It is not clear how solutions were derived for larger networks (N=12) shown in the paper by Gray et al. and attempts to contact the authors were unsuccessful.

Bifurcation diagrams were constructed for the symmetrical pitchfork case as well as the robust persistent bifurcation diagrams that result from breaking symmetry by varying the control parameter $u$ from a value of .25 to 2 in order to capture the singularity that occurs at $u = 1$. Plots were generated for varying levels of $\beta$ in order to recover all possibilities (of universal unfolding) as outlined earlier. For the N=3 size network, an exhaustive search for all unfolding cases was pursued. For N=4 less cases needed to be considered as all possible persistent bifurcations were recovered early in the study, obviating the need for an exhaustive search. Network topologies are shown below:

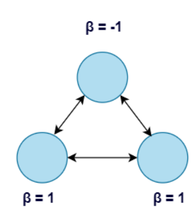


Fig. 1: N=3 Network (strongly connected)

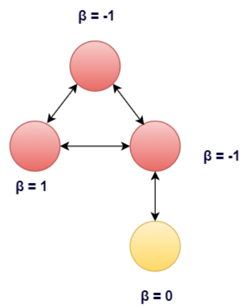


Fig. 2: N=4 Network (strongly connected)

## III. Results

### A. *Steady State Behavior*

Bifurcation diagrams at steady-state are shown now for the N=3 network:

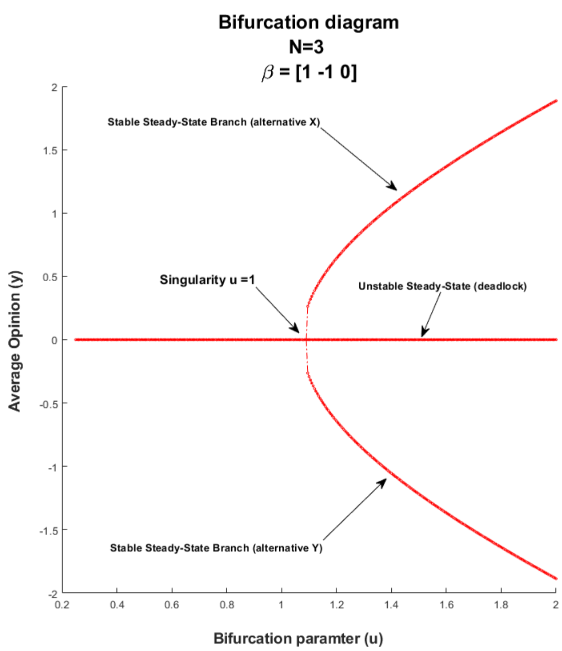


Fig. 3: N=3 Bifurcation: Pitchfork
$\beta = 1, -1, 0$

Note that in the paper by Gray et al (2017) there are four persistent unfolding bifurcation types but two are just mirror images of the other two simply by reversing signs on the $\beta$'s so it was deemed unnecessary to show those cases here.

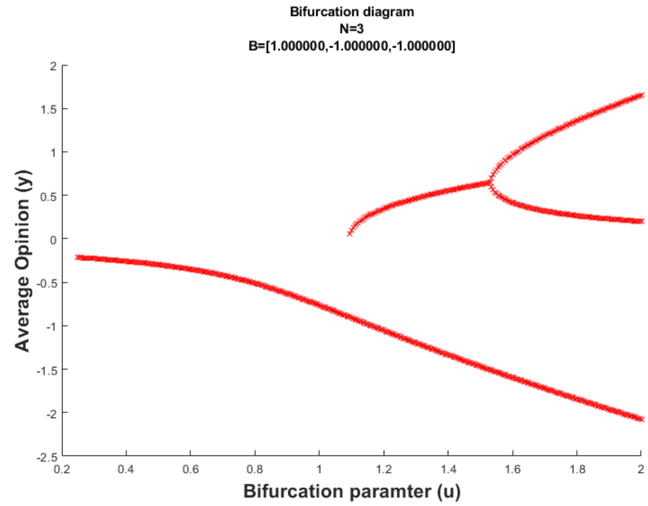


Fig. 4: N=3 Bifurcation: Unfolding Type I
$\beta = 1, -1, -1$

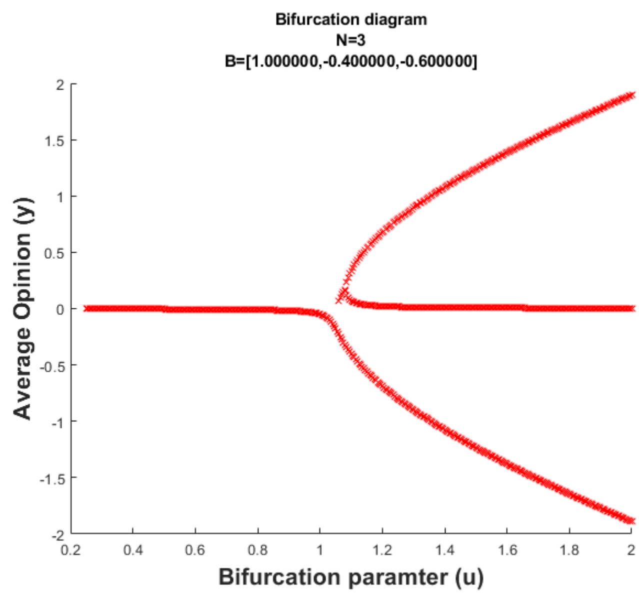


Fig. 5: N=3 Bifurcation: Unfolding Type II
$\beta = 1, -.4, -.6$

Now we turn to the N=4 network and reveal the pitchfork case, unfolding type I and type II cases (not surprisingly it is very similar to the N=3 case) [Note: lower resolution is used here to speed up computation]:

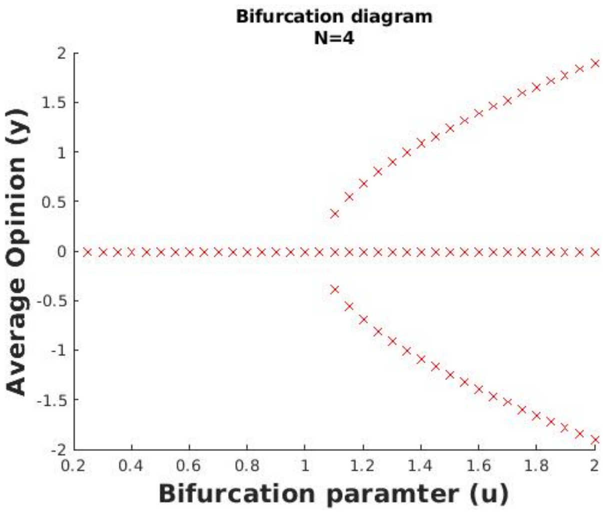


Fig. 6: N=4 Bifurcation: Pitchfork

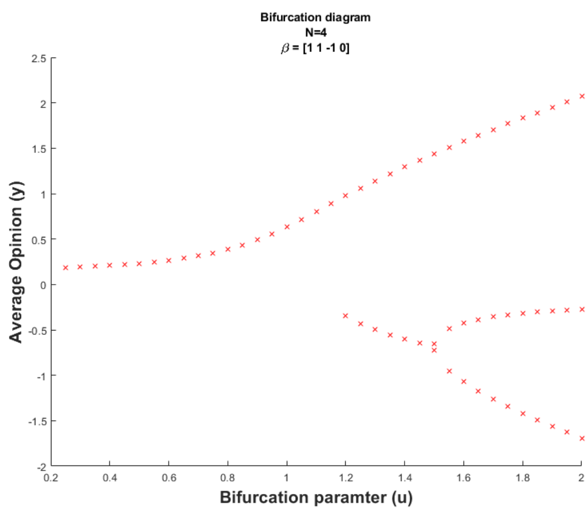


Fig. 7: N=4 Bifurcation: Unfolding Type I

## B. Transient Behavior

The brute force approach to solving the steady-state solution is computationally expensive and is obviously not suitable for implementation on real time robotic systems. To overcome this, we discretized the bifurcation dynamics for faster consensus among multiple agents. Figure 9 illustrates the graph for a bifurcation simulation with 12 agents. The whole network has three completely connected subgroups. A grey arrow indicates a directed edge from every node in one subcomponent to another. The bifurcation unfolding for this network is illustrated in Figure 10. $\beta$ values chosen for the simulation are [-5,5,0] for the groups 1,2, and 3 respectively. This bifurcation unfolding is

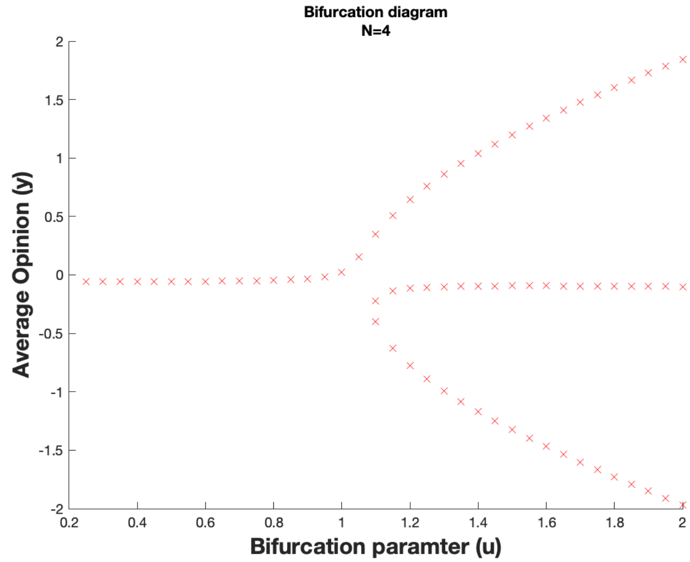


Fig. 8: N=4 Bifurcation: Unfolding Type II

very similar to the Type II unfolding seen in the N=4 network.

Next, we observe how these agents achieve consensus for some fixed control parameter $u$. Each agent's opinion is updated and plotted for each time step. Figure 11 depicts the evolution of an agent's opinion or preference over time for the network shown in Figure 9 with $u = 2$. The dashed black line represents average opinion over time.

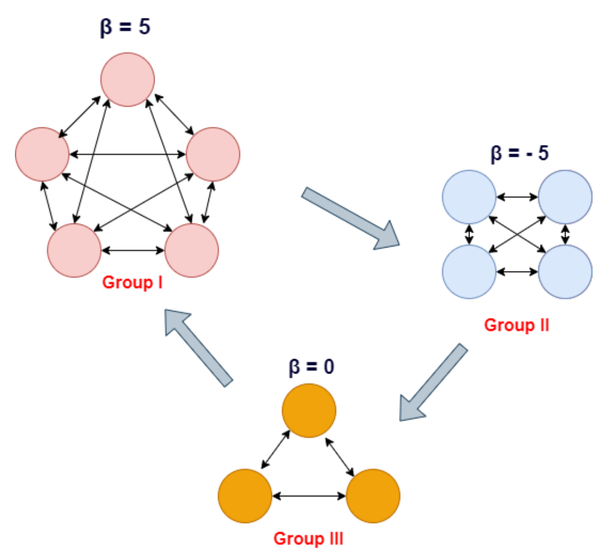


Fig. 9: Network With 12 agents For Bifurcation Simulation

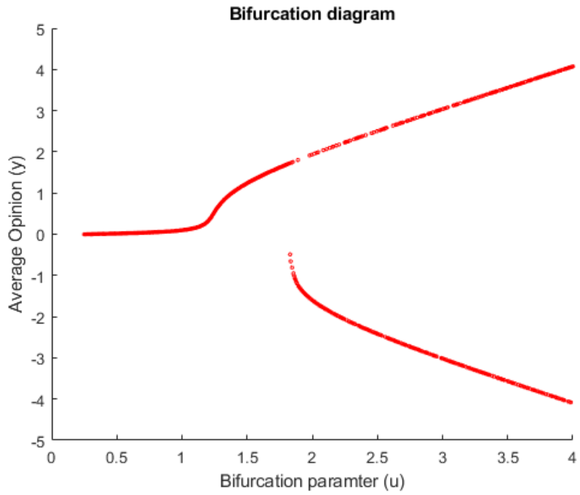


Fig. 10: Bifurcation Unfolding For the Figure 9 Network

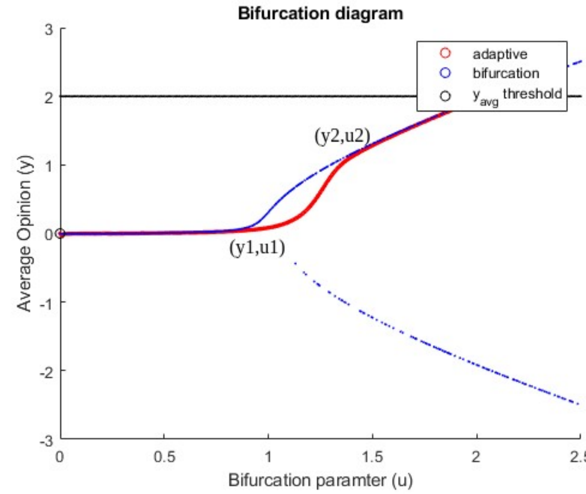


Fig. 12: Adaptive Dynamics For the Graph Shown in Figure 9.

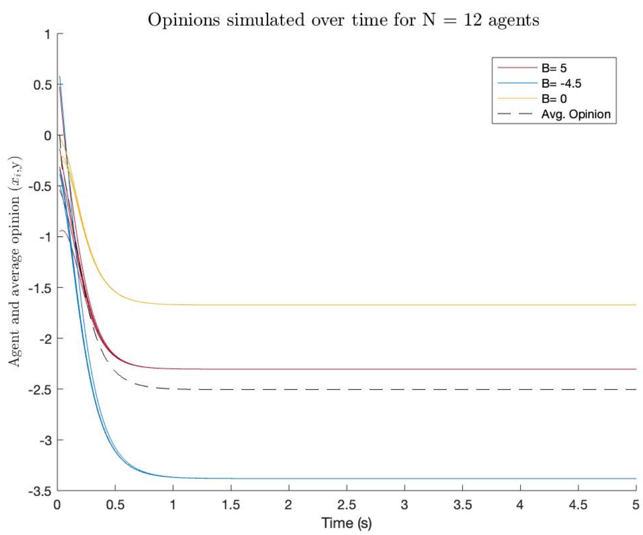


Fig. 11: Agent opinions For Figure 9 Network Plotted Against Time

### C. Adaptive Control

We extend our analysis to include adaptive control as well. This is extremely useful as the value of social effort parameter needed to break the deadlock may not be known to agents. The adaptive feedback in this case will allow agents to adapt their social effort parameter to reach consensus. Gray et al. introduce an agent-based feedback model that drives the magnitude of opinion of the group $|y|$ to a prescribed threshold value $y_{th} > 0$, thereby ensuring consensus. The two phase dynamic model is given by:

$$\dot{x} = -Dx + UAS(x) + \beta$$

$$\dot{u} = \epsilon(y_{th}^2 - y^2)1_N$$

where $U = diag(u_1, u_2, .....u_N)$ and $\epsilon$ is a non-zero control parameter. The adaptive control is implemented for the Fig. 9 network and the plot is illustrated in Fig. 12. The red lines indicates the simulated trajectories for average opinion versus $u$ using adaptive control with $y_{th} = 2$ whereas the blue lines indicate bifurcation unfolding without any feedback. For the adaptive trajectory, we start with $u = 0$ and $y_{intial} = 0$. The average opinion remains zero until the control parameter reaches a threshold value $u1$ and starts to increase until it reaches $u2$, at which point it jumps to the upper branch of bifurcation and continues to slide along it until it reaches $y_{th}$

## IV. Discussion

### A. Steady State Behavior

It is clear from the figures shown earlier that for smaller—even somewhat trivial—networks the theorem proved in [1] appears to hold. It was hypothesized that due to the simplicity of the smaller networks (especially N=3) the richness of behavior exhibited by larger networks would not be inherent in the smaller ones, that is, the smaller swarms would display less robustness and value-sensitivity.

As shown earlier in the results, even trivially small networks inherit these desirable properties. The emergence of complex and intelligent behaviors in self-organizing systems such as bee swarms would suggest a minimum number in flocking to occur before this very phenomenon begins to manifest. Perhaps the full repertoire of emergent properties and functions is lacking in these smaller networks, yet with respect to nest-site selection, the multi-agent mathematical models presented here appear to lend credibility to the notion that even a small swarm may in fact be capable of making accurate decisions when looking for a new home.

### B. Transient Behavior

The bifurcation framework for this study was discretized in time and implemented for larger networks. As depicted in Figure 11, agents are able to break deadlock in less than 2 seconds for the Figure 9 network. This of course is not reflective of actual consensus dynamics among members of a bee swarm as studies by Seeley et. al have shown consensus being achieved on the order of hours or even days.The adaptive feedback in the network can be used to vary u in time so as to break deadlock when conditions are most favorable.

## V. Conclusion

In summary, we have shown that even trivially small swarms can manifest robustness and value sensitivity when selecting a new hive site. This was proven mathematically by the theorem given in Gray et al., yet it seemed to conflict with the notion that emergent properties are only apparent in sizeable populations. Not all emergent properties may manifest in such small populations but the fact that some do is still remarkable nonetheless.

## References


[1] Gray, R.; Franci, A.; Vaibhav, S.; Ehrich, N. (2018). "Multiagent Decision-Making Dynamics Inspired by Honeybees". *IEEE Transactions On Control Of Network Systems*, Vol 5, No 2.

[2] Seeley, T.D.; Buhrman, S.C. (2001). "Nest-site Selection in Honeybees: how well do swarms implement the 'best-of-N' decision rule?". *Behav Ecol Sociobiol* Vol 49, pg 416-427

[3] Leonard, N.E. (2013). "Multi-Agent System Dynamics: Bifurcation and Behavior of Animal Groups". *9th IFAC Symposium on Nonlinear Control Systems*

[4] Pais D.; Hogan P.M.; Schlegel T.; Franks N.R.; Leonard N.E.; Marshall J.A.R. (2013). "A Mechanism for Value-Sensitive Decision-Making". *PLoS ONE 8(9): e73216*

[5] Parrish J.K.; Edelstein-Keshet L. (1999). "Complexity, Pattern, and Evolutionary Trade-Offs in Animal Aggregation". *Science Vol 284, Issue 5411*

[6] Seeley, T.D.; Visscher P.K.; Passino K.M. (2006). "Group Decision Making in Honey Bee Swarms". *American Scientist*, Vol 94

[7] Seeley, T.D. et al (2012). "Stop Signals Provide Cross Inhibition in Collective Decision-Making by Honeybee Swarms". *Science*, Vol 335.

[8] Andreagiovanni R; Marshall J.A.R; Trianni V.; Bose T. (2017). "Model of the best-of-N nest-site selection process in honeybees". *Physical Review E* Vol 95.