# Homework 7

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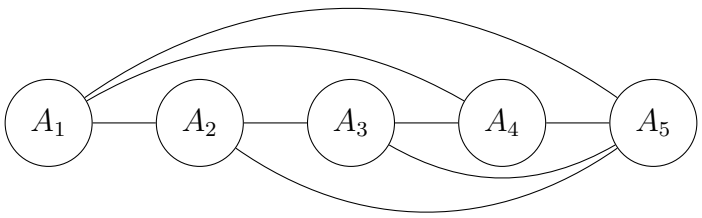
**Problem 1.** The *intersection graph* of a collection of sets $A_1, A_2, \ldots, A_n$, is the graph that has a vertex for each of these sets and has an edge connecting the vertices representing two sets if these sets have a nonempty intersection.

Construct the intersection graph of these collections of sets.

1. Collection of sets $A_i, i = 1..5$
$A_1 = \{0, 2, 4, 6, 8\}$
$A_2 = \{0, 1, 2, 3, 4\}$
$A_3 = \{1, 3, 5, 7, 9\}$
$A_4 = \{5, 6, 7, 8, 9\}$
$A_5 = \{0, 1, 8, 9\}$

Intersection Table and Graph

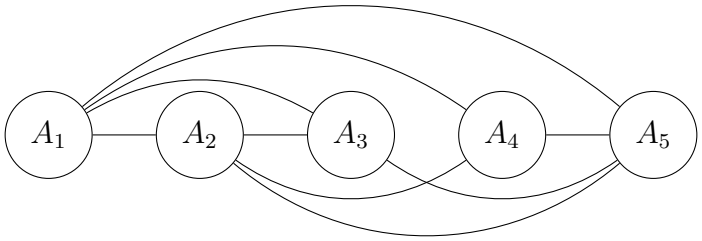
| | $A_1$ | $A_2$ | $A_3$ | $A_4$ | $A_5$ |
|---|---|---|---|---|---|
| $A_1$ | - | {0,2,4} | $\emptyset$ | {6,8} | {0,8} |
| $A_2$ | {0,2,4} | - | {1,3} | $\emptyset$ | {0,1} |
| $A_3$ | $\emptyset$ | {1,3} | - | {5,7} | {1,9} |
| $A_4$ | {6,8} | $\emptyset$ | {5,7} | - | {8,9} |
| $A_5$ | {0,8} | {0,1} | {1,9} | {8,9} | - |



2. Collection of sets $A_i, i = 1..5$
$A_1 = \{\ldots, -4, -3, -2, -1, 0\}$
$A_2 = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$
$A_3 = \{\ldots, -6, -4, -2, 0, 2, 4, 6, \ldots\}$
$A_4 = \{\ldots, -5, -3, -1, 1, 3, 5, \ldots\}$
$A_5 = \{\ldots, -6, -3, 0, 3, 6, \ldots\}$

Intersection Table and Graph

| | $A_1$ | $A_2$ | $A_3$ | $A_4$ | $A_5$ |
|---|---|---|---|---|---|
| $A_1$ | - | $A_1$ | $\{\ldots, -2, 0\}$ | $\{\ldots, -3, -1\}$ | $\{\ldots, -3, 0\}$ |
| $A_2$ | $A_1$ | - | $A_3$ | $A_4$ | $A_5$ |
| $A_3$ | $\{\ldots, -2, 0\}$ | $A_3$ | - | $\emptyset$ | $\{\ldots, -6, 0, 6, \ldots\}$ |
| $A_4$ | $\{\ldots, -3, -1\}$ | $A_4$ | $\emptyset$ | - | $\{\ldots, -9, -3, 3, 9, \ldots\}$ |
| $A_5$ | $\{\ldots, -3, 0\}$ | $A_5$ | $\{\ldots, -6, 0, 6, \ldots\}$ | $\{\ldots, -9, -3, 3, 9, \ldots\}$ | - |

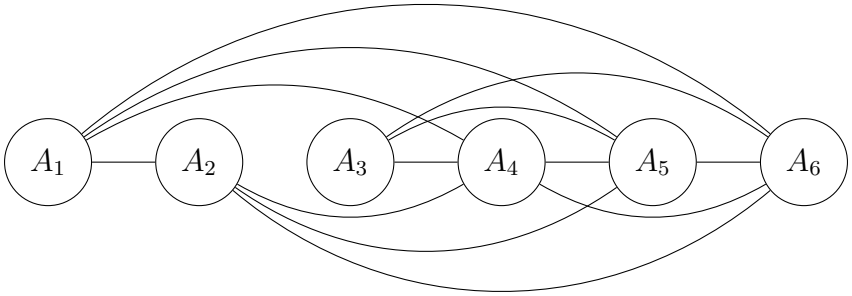



3. Collection of sets $A_i, i = 1..5$
$A_1 = \{x | x < 0\}$
$A_2 = \{x | -1 < x < 0\}$
$A_3 = \{x | 0 < x < 1\}$
$A_4 = \{x | -1 < x < 1\}$
$A_5 = \{x | -1 < x\}$
$A_6 = \mathbb{R}$

Intersection Table

| | $A_1$ | $A_2$ | $A_3$ | $A_4$ | $A_5$ | $A_6$ |
|---|---|---|---|---|---|---|
| $A_1$ | - | $A_2$ | $\emptyset$ | $A_2$ | $A_2$ | $A_1$ |
| $A_2$ | $A_2$ | - | $\emptyset$ | $A_2$ | $A_2$ | $A_2$ |
| $A_3$ | $\emptyset$ | $\emptyset$ | - | $A_3$ | $A_3$ | $A_3$ |
| $A_4$ | $A_2$ | $A_2$ | $A_3$ | - | $A_4$ | $A_4$ |
| $A_5$ | $A_2$ | $A_2$ | $A_3$ | $A_4$ | - | $A_5$ |
| $A_6$ | $A_1$ | $A_2$ | $A_3$ | $A_4$ | $A_5$ | - |

Intersection Graph

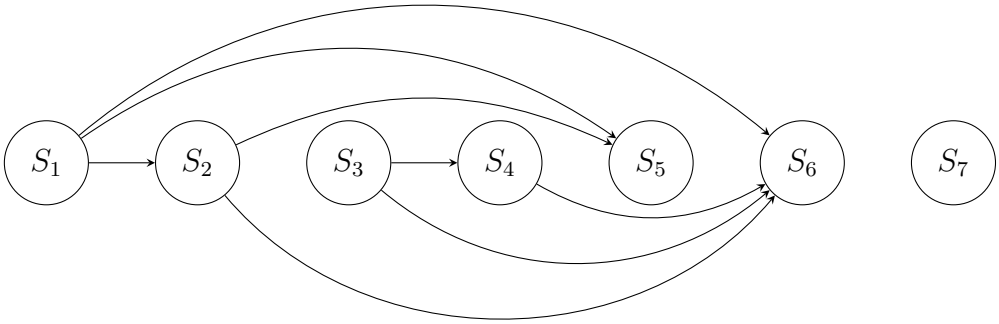



**Problem 2.** Consider the following program:
$S_1 : x := 0;$
$S_2 : x := x + 1;$
$S_3 : y := 2;$
$S_4 : z := y;$
$S_5 : x := x + 2;$
$S_6 : y := x + z;$
$S_7 : z := 4;$

1. Show the different *data dependencies* between all statements, including those that are not direct. Recall that there is a data dependency between two statements $S_i$ and $S_j$ if and only if $S_i$ computes the value of a variable that is used by $S_j$.

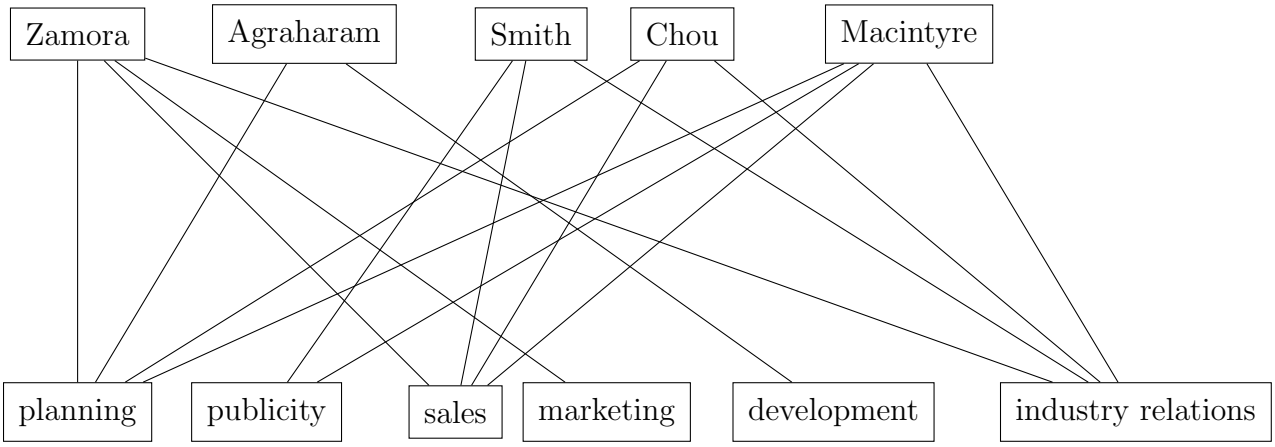
$S_1 : \emptyset$
$S_2 : \{S_1\}$
$S_3 : \emptyset$
$S_4 : \{S_3\}$
$S_5 : \{S_1, S_2\}$
$S_6 : \{S_1, S_2, S_3, S_4, S_5\}$
$S_7 : \emptyset$

2. Construct the precedence graph for the above program.



**Problem 3.** Suppose that a new company has five employees: Zamora, Agraharam, Smith, Chou, and Macintyre. Each employee will assume one of six responsibilities, namely planning, publicity, sales, marketing, development, and industry relations. Each employee is capable of doing one or more of these jobs. Zamora could do planning, sales, marketing, or industry relations. Agraharam could do planning or development. Smith could do publicity, sales, or industry relations. Chou could do planning, sales, or industry relations. Macintyre could do planning, publicity, sales, or industry relations.

1. Model the capabilities of these employees using a bipartite graph.




2. Find an assignment of responsibilities such that each employee is assigned a responsibility.
Since only Zamora can do marketing, he is assigned marketing.
Likewise, Agraharam is assigned development.
Let Smith handle publicity.
Let Chou handle planning.
Macintyre can then handle sales.