Discussion 12

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Problem statement. Feel free to answer one of the following questions:

1. Discuss the direct proof method. Give an illustrative example.
2. Discuss the indirect proof (or proof by contraposition) method. Give an illustrative example.

Problem 2. An indirect proof can take two forms, a proof by contraposition or a proof by contradiction. A proof by contraposition rearranges the original implication into its contrapositive form. Namely $P \to Q$ becomes $\neg Q \to \neg P$. Since the contrapositive is logically equivalent to the original, one can then proceed with a direct proof of the contrapositive and thereby prove the original.

Example: For any integer n, if n^2 is odd, then n is odd
Contrapositive: For any integer n, if n is even, then n^2 is even
Since n is even, let $n = 2k$ for some integer k, then $n^2 = (2k)^2 = 4k^2 = 2(2k^2)$, so n^2 must be even. $\square$

A proof by contradiction starts assuming the original premise implies the negation of the original conclusion. Then from the negated conclusion, derive an inconsistency with the original premise, thereby arriving at a contradiction and proving the original. Namely if the original was $P \to Q$, then assume $P \to \neg Q$, and from $\neg Q$ derive $\neg P$.

Example: For any integer n, if n^2 is odd, then n is odd
Assume for contradiction: For any integer n, if n^2 is odd, then n is even.
Since n is even, let $n = 2k$ for some integer k, then $n^2 = (2k)^2 = 4k^2 = 2(2k^2)$, so n^2 must be even. But we assumed n^2 is odd, and n^2 cannot be both odd and even, therefore contradiction. $\square$