Discussion 1

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Problem statement. Feel free to answer one of the following questions:
1. Define the concept of algorithm. Give an illustrative example.
2. Define the concept of program. What is the difference between algorithm and program?

Problem 1. In *The Art of Computer Programming* by Donald Knuth, he defines an algorithm as a "set of rules that gives a sequence of operations for solving a specific type of problem" with "five important features":

The first feature is *finiteness*, where the algorithm must terminate in a finite number of operations.

The second feature is *definiteness*, where each operation must be precisely and unambiguously defined.

The third feature is *input*, where the algorithm can take in zero or more quantities of information to process.

The fourth feature is *output*, where the algorithm has to produce one or more quantities of information as the result.

The fifth and final feature is *effectiveness*, which is stated in the book as "operations must all be sufficiently basic that they can in principle be done exactly and in a finite length of time by someone using pencil and paper".

The illustrative example given in the book is Euclid's greatest common divisor algorithm:

Algorithm E (*Euclid's algorithm*). Given two positive integers m and n, find their *greatest common divisor*, that is, the largest positive integer that evenly divides both m and n.
E1. [Find remainder.] Divide m by n and let r be the remainder. (We will have $0 \le r < n$.)
E2. [Is it zero?] If $r = 0$, the algorithm terminates; n is the answer.
E3. [Reduce.] Set $m \leftarrow n$, $n \leftarrow r$, and go back to step El.

This algorithm meets all 5 of the features:

1. Since m and n are strictly decreasing after every iteration, and since a decreasing sequence of positive integers must terminate in finite time, the algorithm is finite.
2. Although the algorithm is given in English (with all it's ambiguities), the choice of words like *positive*, *integer*, and *remainder* are fairly specific in mathematical meaning to avoid misunderstanding.
3. The inputs for the algorithm are m and n.
4. The output of the algorithm is n, once the algorithm terminates.
5. Since positive integers can be represented on paper in a finite form and can be divided using long division in finite time, a person in theory can perform all the steps of the algorithm from start to completion.

Example execution of Algorithm E with input $m = 119$ and $n = 544$

Iteration	m	n	r
1	119	544	119
2	544	119	68
3	119	68	51
4	68	51	17
5	51	17	0