Teamwork 1

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Problem statement. In a hallway, there are three doors which allow access to three different rooms. These three doors are numbered 1, 2, and 3. Furthermore, there is a poster on each door that has a claim. One of the rooms is empty while the other two rooms contain a crazy person and a murderer, respectively. The poster on the door of the room of the crazy person says the truth and the one on the door of the murderer lies. However, we do not know whether the poster on the door of the empty room says the truth. Here are the three posters:

Poster 1: Room 3 is empty
Poster 2: The murderer is in room 1
Poster 3: This room is empty

Problem 1. In which rooms are the crazy person and the murderer located?

The Crazy Person is located in Room 1, and the Murderer is located in Room 2.

Problem 2. Explain your answer.

Proof. Assume that Poster 1 is telling the truth, then Room 3 is empty which also means that Poster 3 is telling the truth. This would imply that Room 1 has the Crazy Person, and therefore Room 2 must have the Murderer. Poster 2 would therefore be a lie implying the Murderer is not in Room 1. None of this is contradictory so we save this as a possible solution.

Now Assume that Poster 1 is lying, then Room 3 is not empty which also means that Poster 3 is lying. This would imply that Room 3 must contain the Murderer, and since the Crazy Person cannot lie, he must also not be in Room 1, and the only available room is Room 2. But Room 2 says that the Murderer is in Room 1, but we already established that the Murderer must be in Room 3, so this would also be a lie, so this is not a solution.

Since Poster 1 must either be telling the truth or lying, these two cases are all we need to show. □

Problem statement 2. Now, let us assume that the three posters are as follows:

Poster 1: The murderer is in room 1
Poster 2: Room 3 is empty
Poster 3: This room is empty

Problem 3. Check whether there is any solution. If so, show your solution. Otherwise, explain why.

There is no solution.

Proof. Assume Poster 1 is telling the truth, then it would contain the Murderer, but the Murderer always lies, so assumption has failed.

Assume poster 1 is lying, then the Murderer is not in Room 1, so he must be in either Room 2 or Room 3. But since the Crazy Person cannot lie, he cannot be in Room 1 and must be in either Room 2 or Room 3. Therefore Room 1 must be Empty. However, given that the Crazy Person has to tell the truth and is either in Room 2 or Room 3, one of those posters must be true, but we already know the Empty Room is Room 1, while Poster 2 claims the Empty Room is Room 3 and Poster 3 claims that the Empty Room is also Room 3. So we have a contradiction.

Since Poster 1 must either be telling the truth or lying, these two cases are all we need to show, and both lead to no solution. □