Homework 3

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CSEN 5336 Analysis of Algorithms

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Problem 1. Perform the following $UNION$ operation on the disjoint-set forest shown in figure 1.
$UNION(f, m)$
Apply both the union by rank and path compression heuristics to improve the running time while doing the UNION operation. Given the rank of root node 'a' is 3 and the rank of root node 'h' is 2 (the picture shows 'h' has rank 3, might be a mistake).

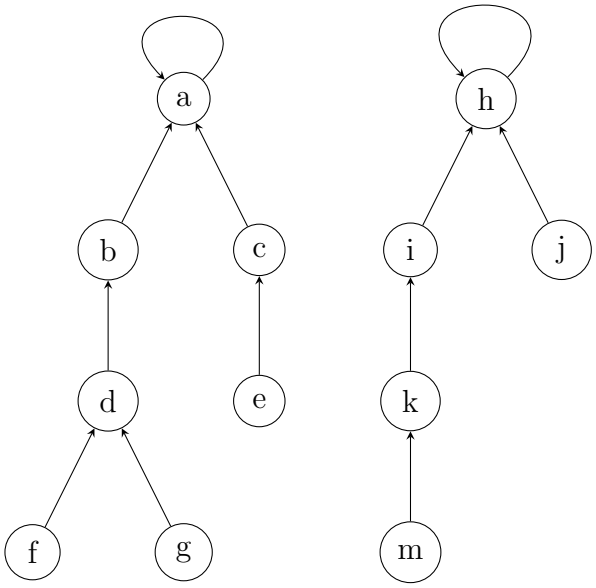
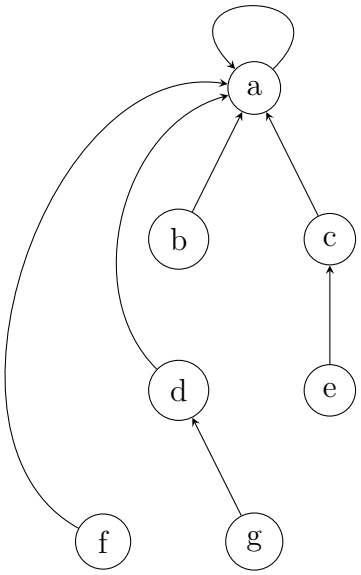


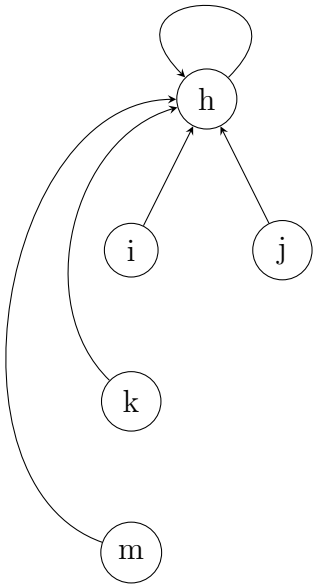
Figure 1: Disjoint-Set Forest

$UNION(f, m) \rightarrow LINK(FIND\_SET(f), FIND\_SET(m))$

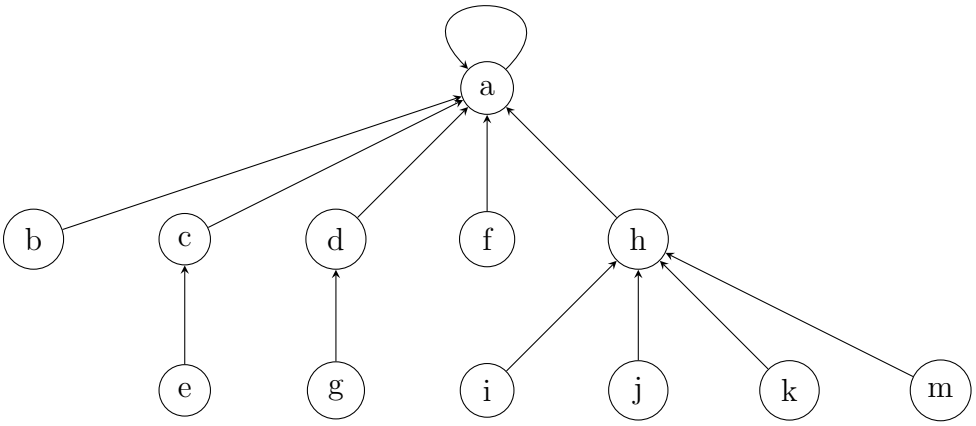
$FIND\_SET(f)$:



$FIND\_SET(m)$:



$LINK(FIND\_SET(f), FIND\_SET(m))$:



Problem 2. Extract the minimum node from the Fibonacci heap shown in figure 2.

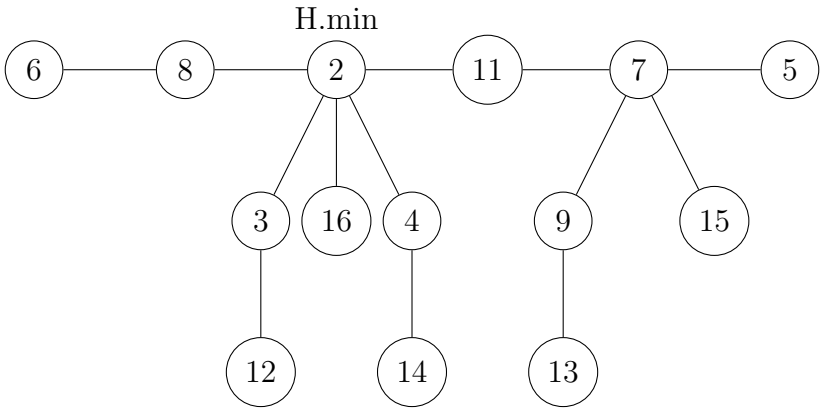
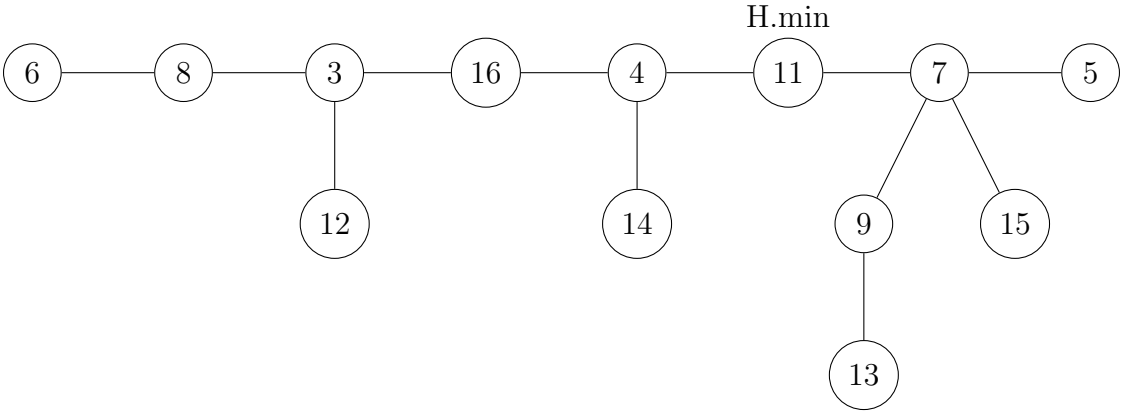
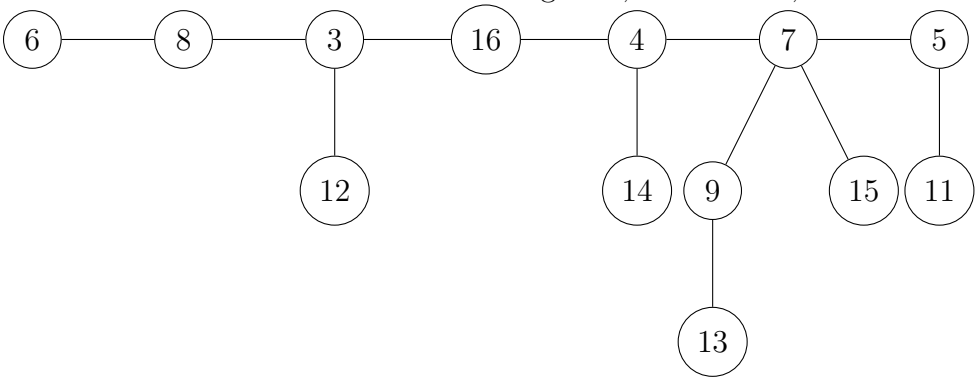


Figure 2: Fibonacci Heap

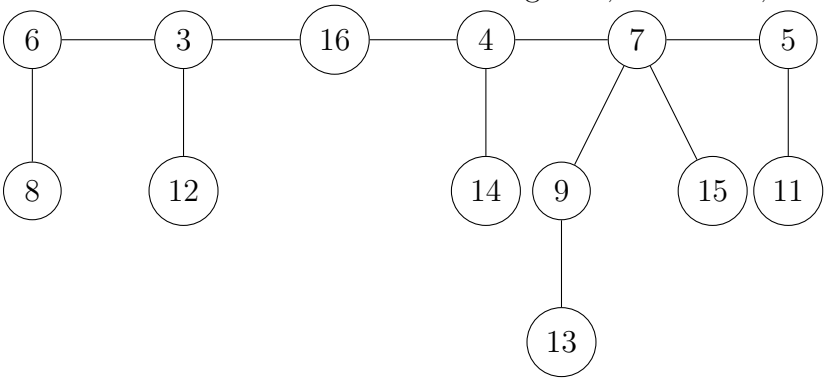
After removing node 2 (previous H.min) and making roots out of each of its children:



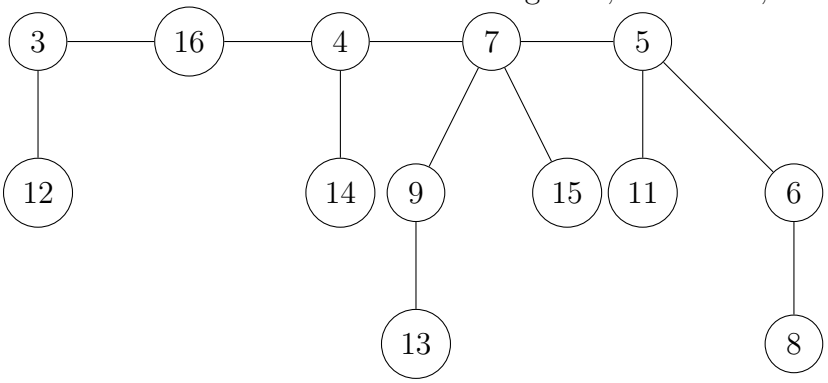
Since both node 11 and node 5 have degree 0, and $5 < 11$, link 11 to 5:



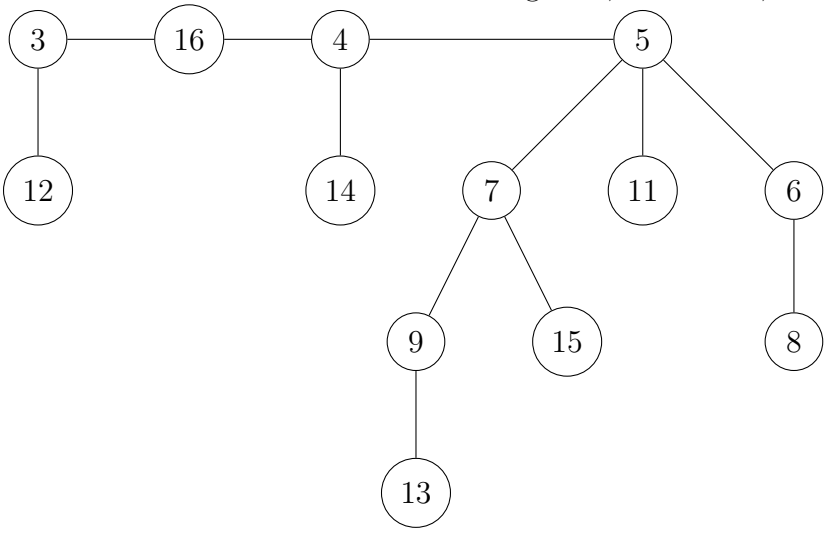
Since both node 6 and node 8 have degree 0, and $6 < 8$, link 8 to 6:



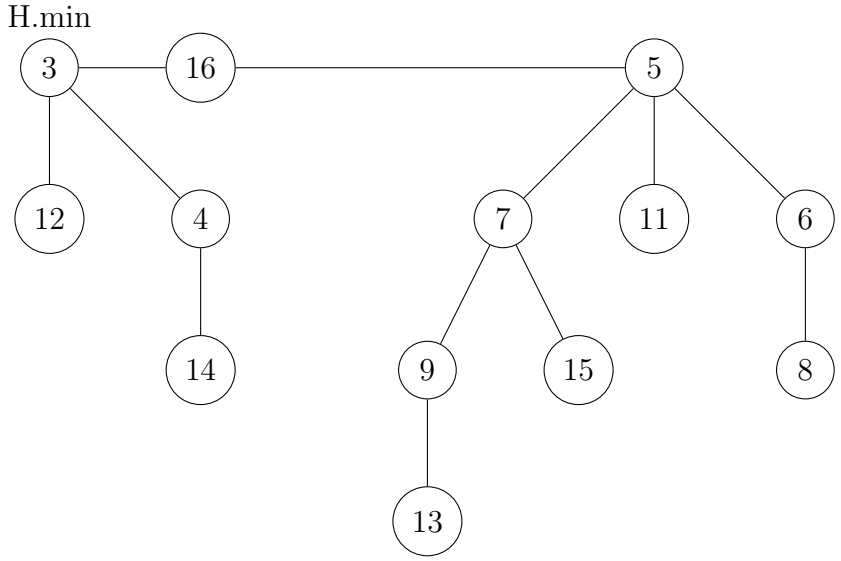
Since both node 6 and node 5 have degree 1, and $5 < 6$, link 6 to 5:



Since both node 7 and node 5 have degree 2, and $5 < 7$, link 7 to 5:



Since both node 3 and node 4 have degree 1, and $3 < 4$, link 4 to 3, no more root nodes with same degree, so node 3 becomes new H.min:



Problem 3. Perform Depth-First Search using Stack data structure on the (un)directed graph shown in figure 3.

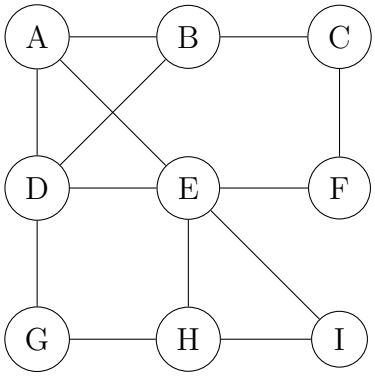
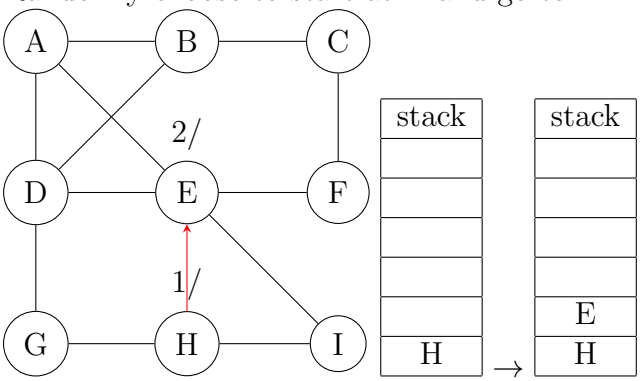
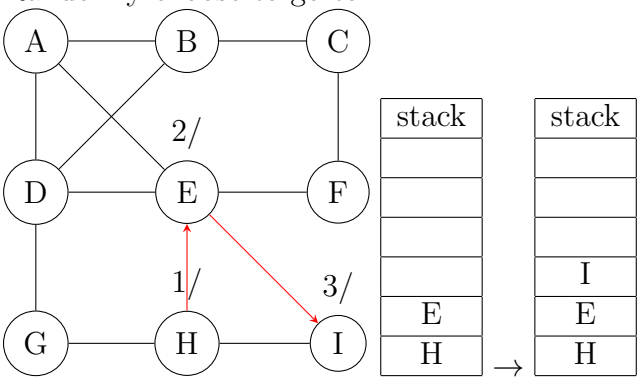


Figure 3: (Un)Directed Graph

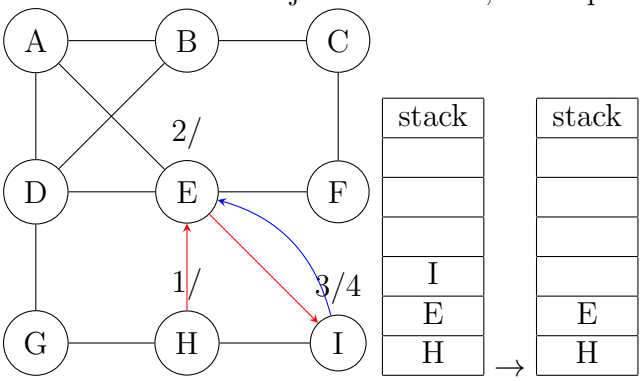
Randomly choose to start at H and go to E



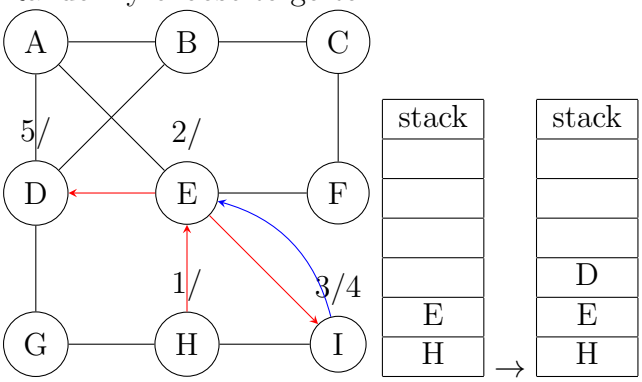
Randomly choose to go to I



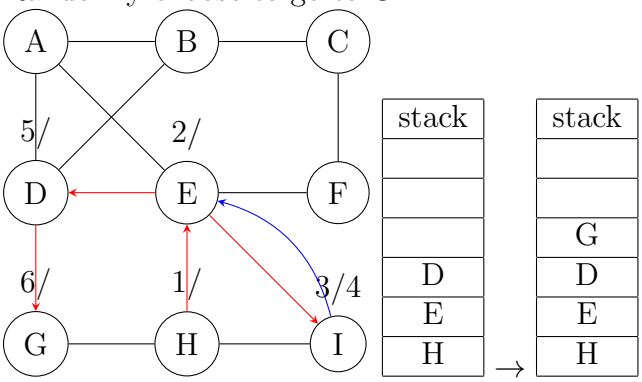
I has no unvisited adjacent vertices, finish processing and pop from stack



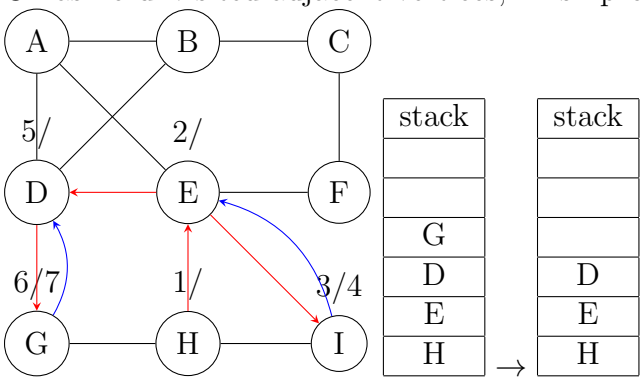
Randomly choose to go to D



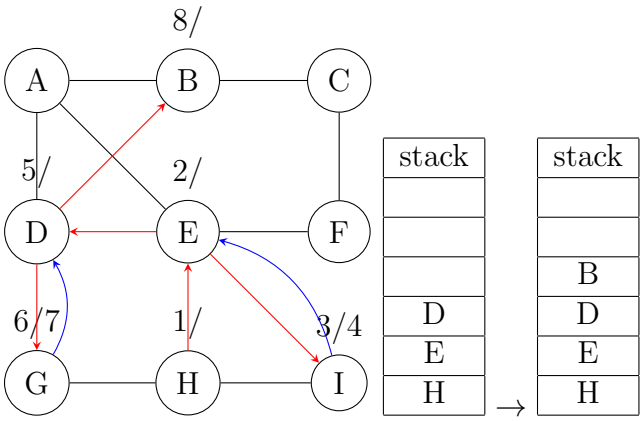
Randomly choose to go to G



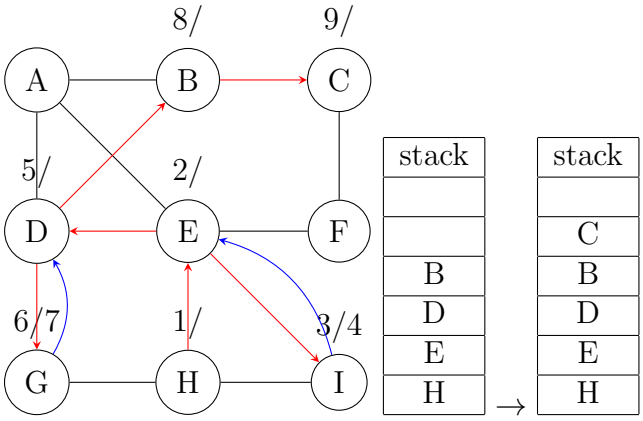
G has no unvisited adjacent vertices, finish processing and pop from stack



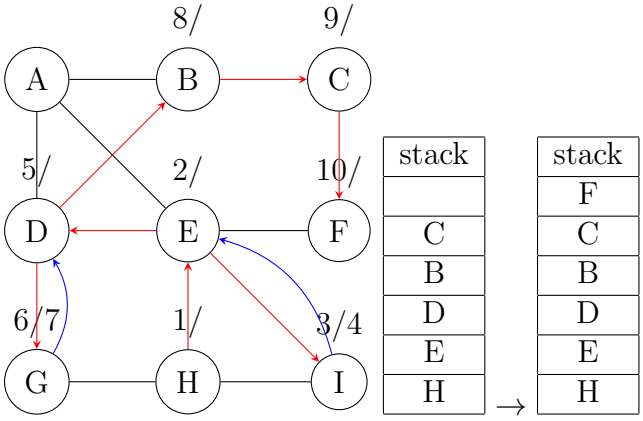
Randomly choose to go to B



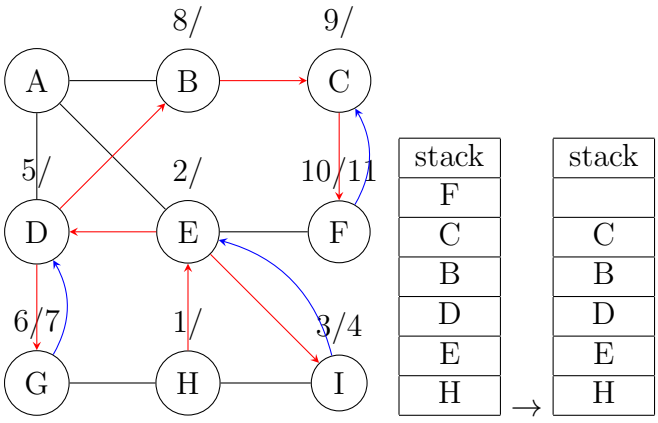
Randomly choose to go to C



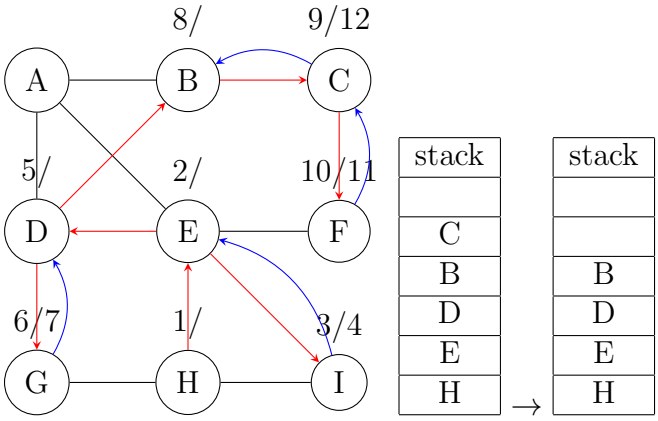
Randomly choose to go to F



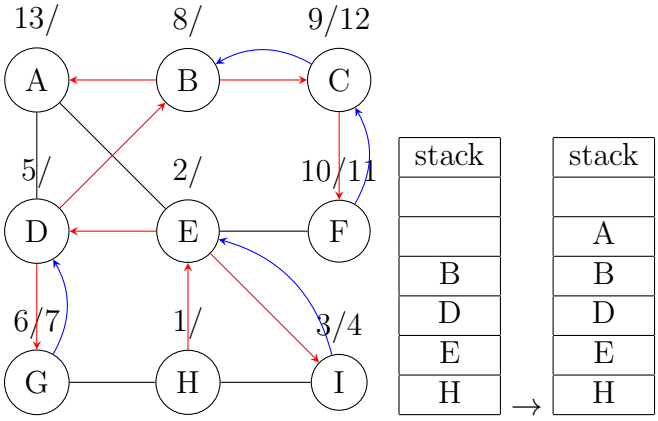
F has no unvisited adjacent vertices, finish processing and pop from stack



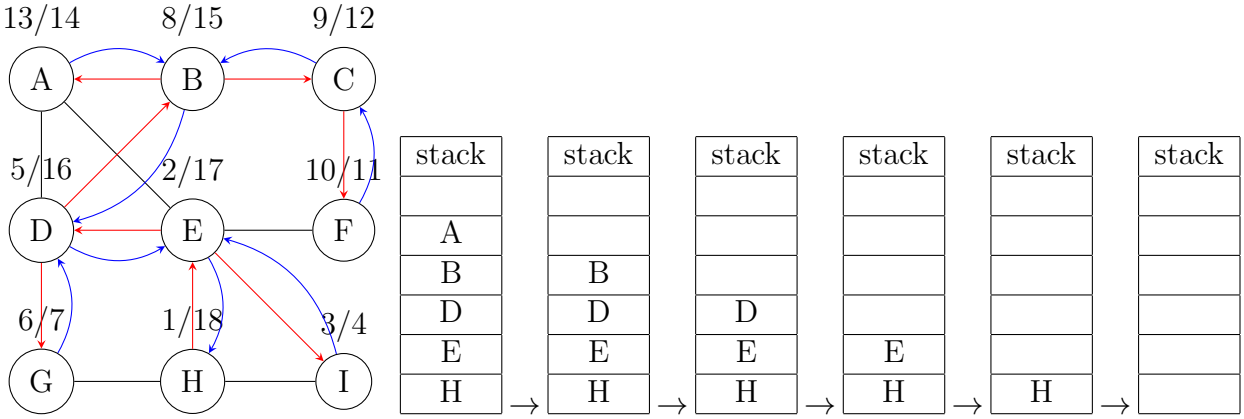
C has no unvisited adjacent vertices, finish processing and pop from stack



Randomly choose to go to A



All nodes visited, finish processing and pop until stack empy



Problem 4. Using Prim's algorithm find the minimum spanning tree for the graph shown in figure 4

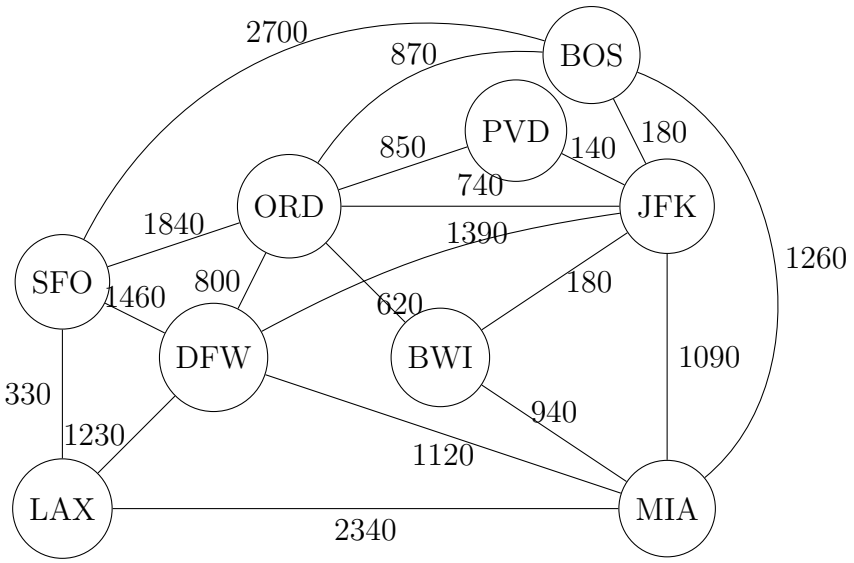
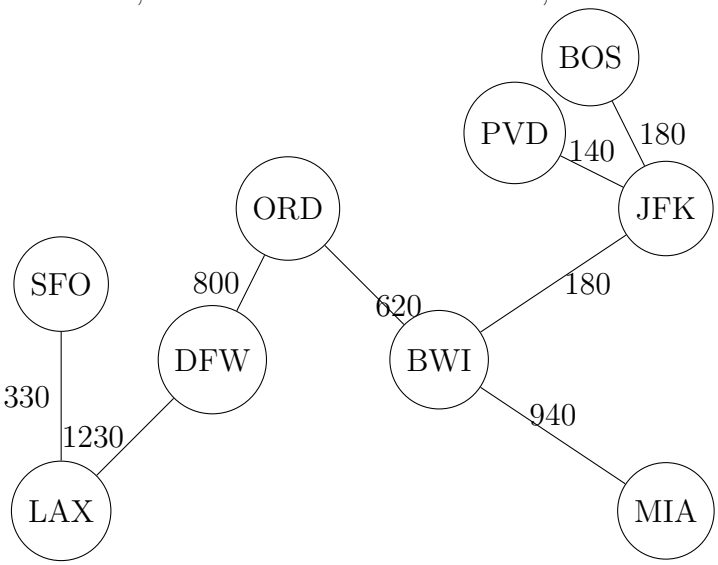
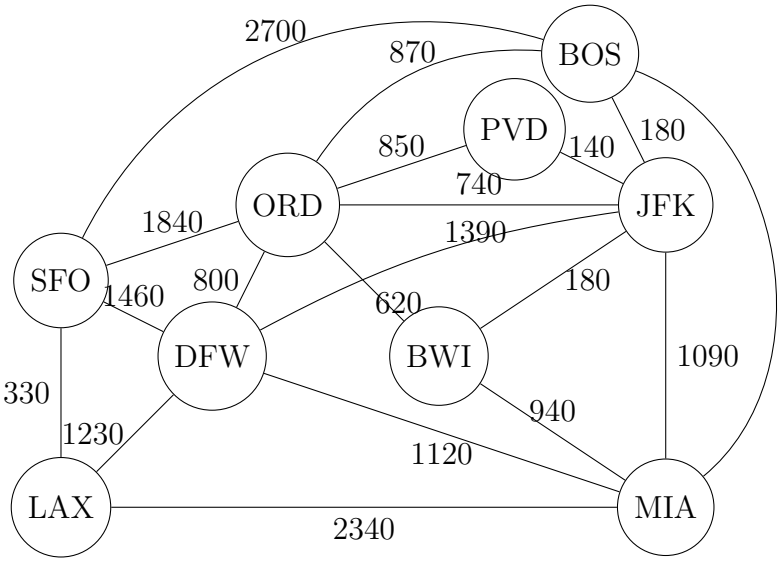


Figure 4: Undirected Graph for Problem 4 and 5

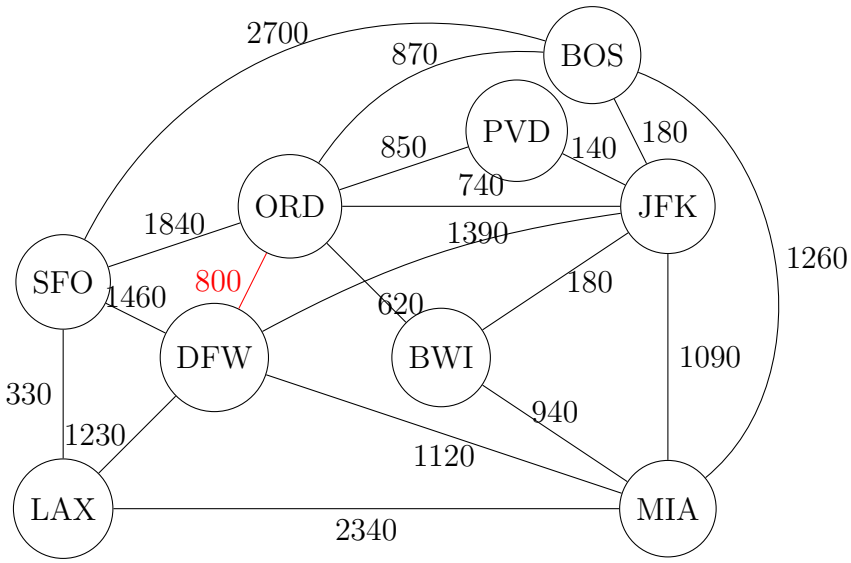
Start at SFO, add 330 to LAX. From LAX, add 1230 to DFW.
From DFW, add 800 to ORD. From ORD, add 620 to BWI.
From BWI, add 180 to JFK. From JFK, add 140 to PVD.
From JFK, add 180 to BOS. From BWI, add 940 to MIA.



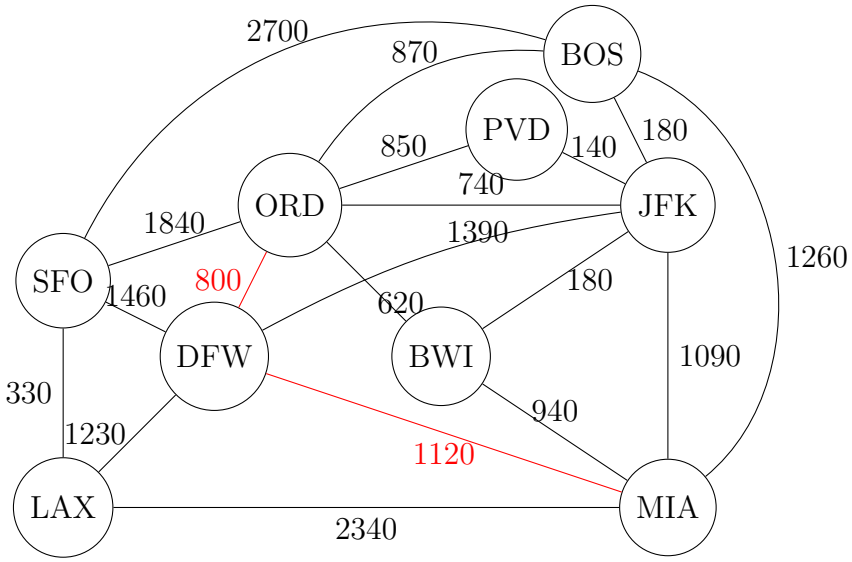
Problem 5. Using Dijkstra's algorithm find the shortest paths from the source vertex DFW to all the other vertices for the graph shown in figure 4. Note that Dijkstra's algorithm can be applied both on directed and undirected graph.



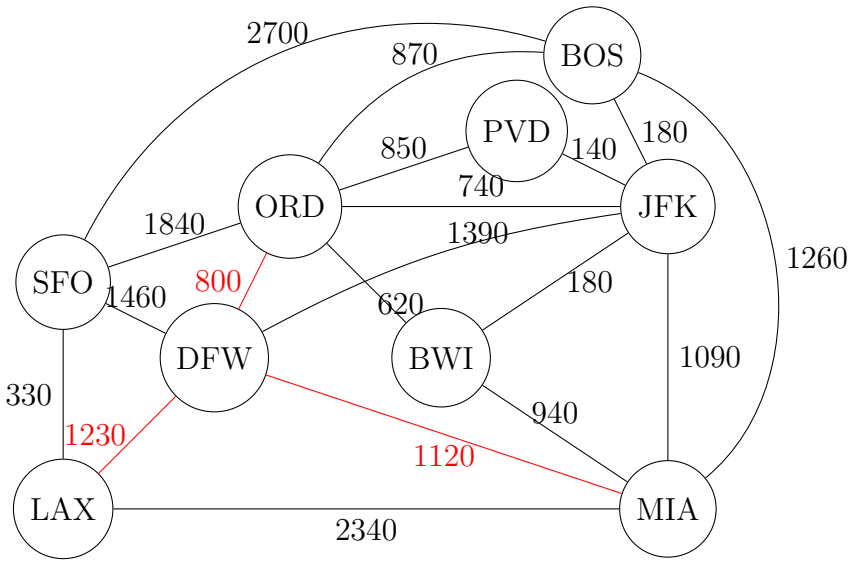
vertex v	$v.d$	$v.\pi$
DFW	0	nil
SFO	∞	nil
LAX	∞	nil
ORD	∞	nil
BOS	∞	nil
PVD	∞	nil
JFK	∞	nil
BWI	∞	nil
MIA	∞	nil



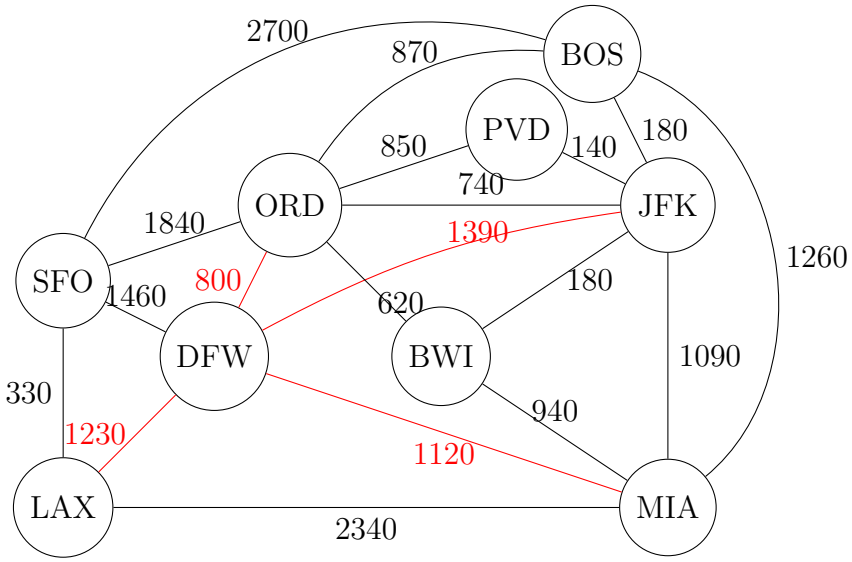
vertex v	$v.d$	$v.\pi$
DFW	0	nil
SFO	∞	nil
LAX	∞	nil
ORD	800	DFW
BOS	∞	nil
PVD	∞	nil
JFK	∞	nil
BWI	∞	nil
MIA	∞	nil



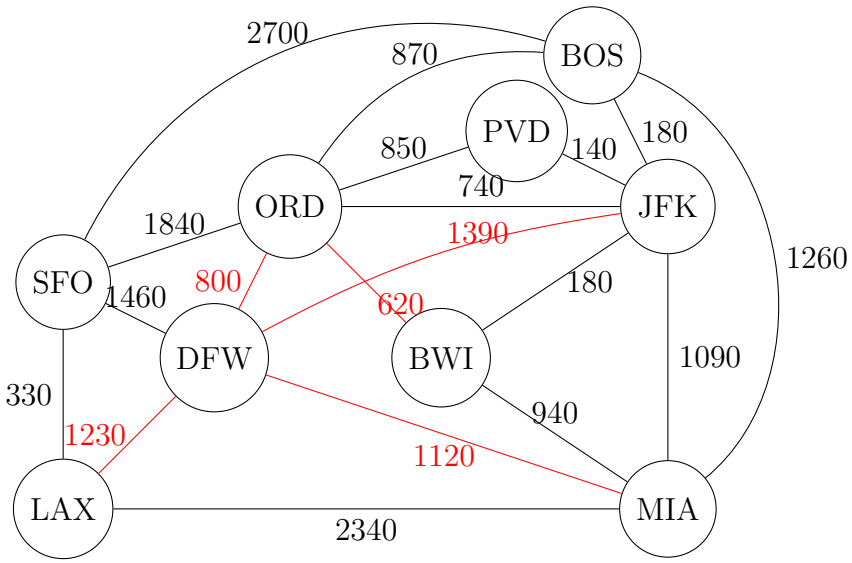
vertex v	$v.d$	$v.\pi$
DFW	0	nil
SFO	∞	nil
LAX	∞	nil
ORD	800	DFW
BOS	∞	nil
PVD	∞	nil
JFK	∞	nil
BWI	∞	nil
MIA	1120	DFW



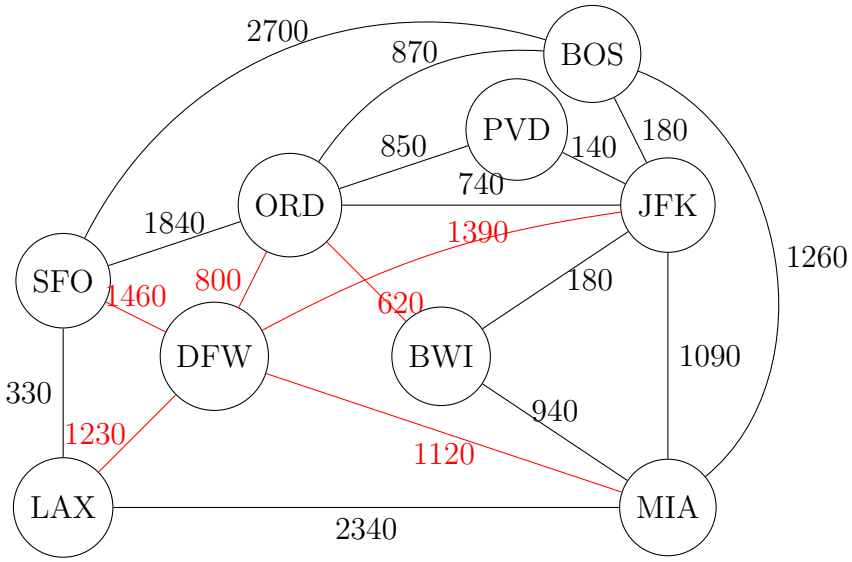
vertex v	$v.d$	$v.\pi$
DFW	0	nil
SFO	∞	nil
LAX	1230	DFW
ORD	800	DFW
BOS	∞	nil
PVD	∞	nil
JFK	∞	nil
BWI	∞	nil
MIA	1120	DFW



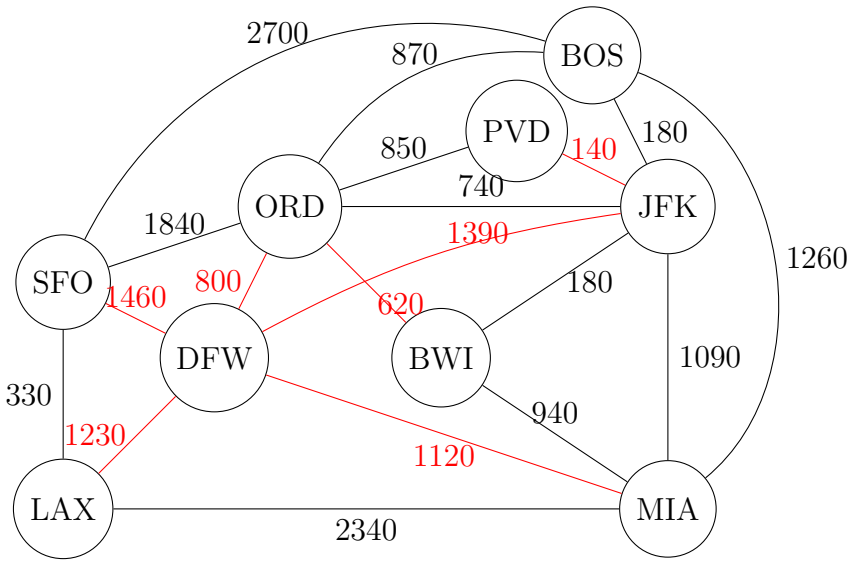
vertex v	$v.d$	$v.\pi$
DFW	0	nil
SFO	∞	nil
LAX	1230	DFW
ORD	800	DFW
BOS	∞	nil
PVD	∞	nil
JFK	1390	DFW
BWI	∞	nil
MIA	1120	DFW



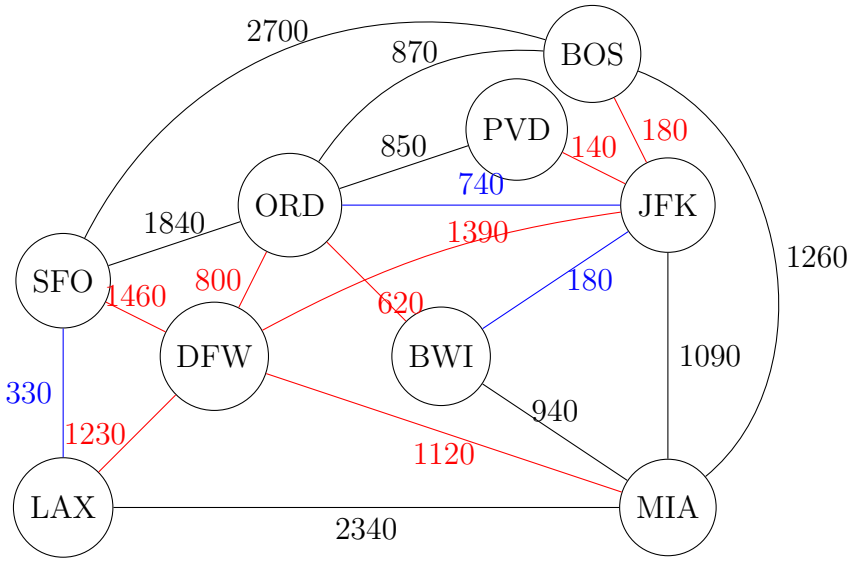
vertex v	$v.d$	$v.\pi$
DFW	0	nil
SFO	∞	nil
LAX	1230	DFW
ORD	800	DFW
BOS	∞	nil
PVD	∞	nil
JFK	1390	DFW
BWI	1420	ORD
MIA	1120	DFW



vertex v	$v.d$	$v.\pi$
DFW	0	nil
SFO	1460	DFW
LAX	1230	DFW
ORD	800	DFW
BOS	∞	nil
PVD	∞	nil
JFK	1390	DFW
BWI	1420	ORD
MIA	1120	DFW



vertex v	$v.d$	$v.\pi$
DFW	0	nil
SFO	1460	DFW
LAX	1230	DFW
ORD	800	DFW
BOS	∞	nil
PVD	1530	JFK
JFK	1390	DFW
BWI	1420	ORD
MIA	1120	DFW



vertex v	$v.d$	$v.\pi$
DFW	0	nil
SFO	1460	DFW
LAX	1230	DFW
ORD	800	DFW
BOS	1570	JFK
PVD	1530	JFK
JFK	1390	DFW
BWI	1420	ORD
MIA	1120	DFW