Homework 4

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EEEN 5338 Digital and DSP Based Control

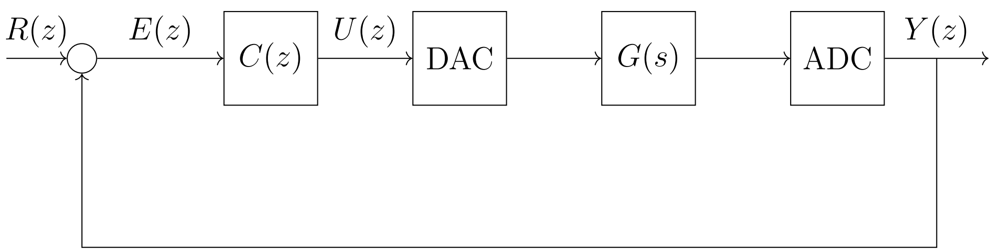
November 1, 2023

Problem 1. For the following unity feedback system with

$$G(s) = \frac{s+8}{s+5}$$

which is digitally controlled with a sampling period of 0.02 seconds and a controller transfer function

$$C(z) = \frac{0.35z}{z-1}$$



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(a) Find the z-transfer function for the analog subsytem with DAC and ADC.

$$\frac{G(s)}{s} = \frac{s+8}{s(s+5)} = \frac{A}{s} + \frac{B}{s+5} \implies s+8 = (s+5)A + sB$$

$$s = 0 \implies 8 = 5A \implies A = \frac{8}{5} = 1.6$$

$$s = -5 \implies 3 = -5B \implies B = -\frac{3}{5} = -0.6$$

$$\implies \frac{G(s)}{s} = \frac{1.6}{s} - \frac{0.6}{s+5}$$

$$G_{ZAS}(z) = (1 - z - 1)\mathcal{Z}\left\{\frac{G(s)}{s}\right\}$$

$$= \frac{z-1}{z}\left(\frac{1.6z}{z-1} - \frac{0.6z}{z - e^{-5(0.02)}}\right)$$

$$= 1.6 - \frac{0.6(z-1)}{z - e^{-0.1}}$$

using Wolfram Alpha to simplify the algebra:

$$G_{ZAS}(z) = \frac{z - 0.84774}{z - 0.904837}$$

(b) Find the closed-loop transfer function and characteristic equation.

transfer function:

$$G_{cl}(z) = \frac{C(z)G_{ZAS}(z)}{1 + C(z)G_{ZAS}(z)}$$
$$= \left(\frac{0.35z(z - 0.84774)}{(z-1)(z-0.904837)}\right) / \left(1 + \frac{0.35z(z - 0.84774)}{(z-1)(z-0.904837)}\right)$$

using Wolfram Alpha to simplify the algebra:

$$G_{cl}(z) = \frac{0.259259(z - 0.84774)z}{z^2 - 1.63077z + 0.67025}$$

characteristic equation:

$$1 + C(z)G_{ZAS}(z) = 0$$
$$z^2 - 1.63077z + 0.67025 = 0$$

(c) Find the steady-state error due to a sampled unit step and a sampled unit ramp. Comment on the effect of the controller on steady-state error.

$$C(z)G_{ZAS}(z) = L(z)$$
$$\implies L(z) = \frac{0.35z(z - 0.84774)}{(z-1)(z-0.904837)}$$
$$\implies \text{system is type 1}$$

The steady state error for a sampled unit step is 0 for type 1 and higher.
The steady state error for a sampled unit ramp is:

$$e(\infty) = \frac{T}{(z-1)L(z)|_{z=1}} = \frac{0.02}{\frac{0.35(1-0.84774)}{1-0.904837}} = 0.035714$$

If the controller has additional pole(s) at unity, the type of the system can be increased to make the steady-state error 0 for the unit ramp.

Problem 2. Use MATLAB for the following system with a sampling period of 0.05 seconds:

$$G(s) = \frac{10(s+2)}{s(s+5)}$$

(a) Obtain the transfer function for the system with sampled input and output.

```
>> g=zpk(-2,[0,-5],10)

g =

  10 (s+2)
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  s (s+5)

Continuous-time zero/pole/gain model.

>> gd=c2d(g,.05,'imp')

gd =

  0.5 z (z-0.9115)
  ----------------
  (z-1) (z-0.7788)

Sample time: 0.05 seconds
Discrete-time zero/pole/gain model.
```

(b) Obtain the transfer function for the system with DAC and ADC.

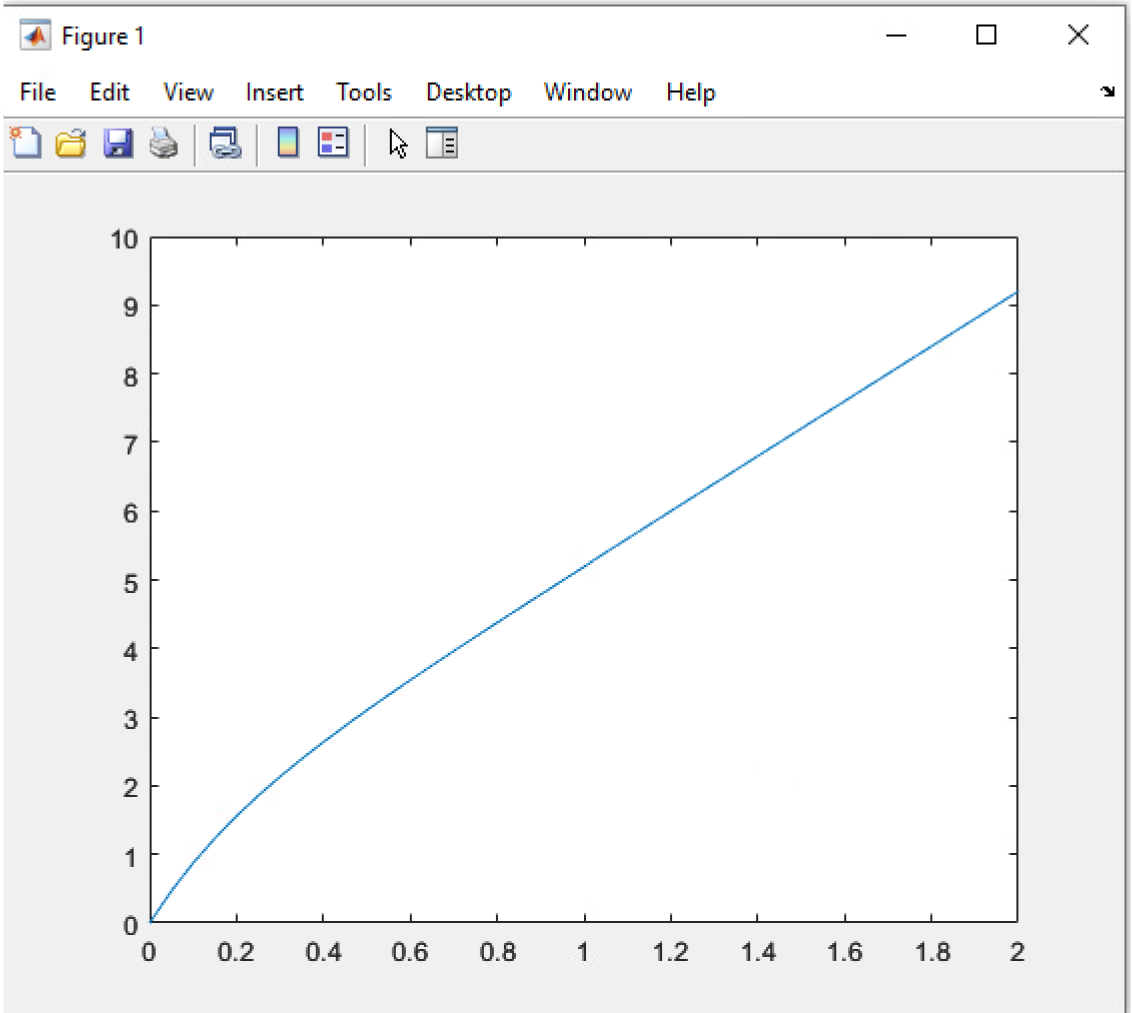
```
>>  gd = c2d(g,.05,'zoh')

gd =

  0.46544 (z-0.905)
  -----------------
  (z-1) (z-0.7788)

Sample time: 0.05 seconds
Discrete-time zero/pole/gain model.
```

(c) Obtain the unit step response of the system with sampled output and analog input.

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>> [y,t]=step( tf([10,20],[1,5,0]),[0:0.05:2]);
>> plot(t, y)
>>
```

(d) Obtain the poles of the system in (a), (b), and the output of (c) and comment on the differences between them.

Both (a) and (b) have poles at 1 and 0.7788. (c) has an additional pole at 1 from the unit step.