Homework 6

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EEEN 5338 Digital and DSP Based Control

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Problem 1. A lead compensator is designed for a unity-feedback system whose plant transfer function is:

$$G_p(s) = \frac{100K}{s(s+36)(s+1000)}$$

The design specifications are:
Percent overshoot = 20%
Peak time = 0.1 seconds
$K = 1440$
The lead compensator is:

$$G_c(s) = 2.38\left(\frac{s+25.3}{s+60.2}\right)$$

If the system is to be computer controlled:

1. Find the digital controller, $G_c(z)$, assume: $T = 0.001$ seconds.

$$\text{substitute } s = \frac{2(z-1)}{0.001(z+1)} \text{ into } G_c(s):$$

$$G_c(z) = 2.38\left(\frac{2(z-1)+0.0253(z+1)}{0.001(z+1)}\right)\left(\frac{0.001(z+1)}{2(z-1)+0.0602(z+1)}\right)$$

$$G_c(z) = 2.38\left(\frac{2.0253z-1.9747}{2.0602z-1.9398}\right) = 2.38\left(\frac{2.0253}{2.0602}\right)\left(\frac{z-0.9750}{z-0.9416}\right) = 2.3397\left(\frac{z-0.9750}{z-0.9416}\right)$$

2. Find the z-transform of the plant and the zero-order hold with $T = 0.001$ seconds.

```
>> g = zpk([], [0, -36, -1000], 144000)

g =

       1.44e+05
  -----------------
  s (s+36) (s+1000)

Continuous-time zero/pole/gain model.

>> gd = c2d(g, 0.001, 'zoh')

gd =

  1.8848e-05 (z+0.2028) (z+2.944)
  ------------------------------
    (z-1) (z-0.9646) (z-0.3679)

Sample time: 0.001 seconds
Discrete-time zero/pole/gain model.
```

Expanding the expression leads to:

$$G_p(z) = \frac{1.8848 \times 10^{-5} z^2 + 5.9310 \times 10^{-5} z + 1.1253 \times 10^{-5}}{z^3 - 2.3325 z^2 + 1.6874 z - 0.3549}$$

3. Revise the MATLAB code (as per our class discussion in the examples from Lecture Notes 6) to solve and plot the step response for this system. Make sure to submit the code, the results, and the plot.

```
T=0.001;
numgcs=2.38*[1  25.3];
dengcs=[1 60.2];
'Gc(s) in expanded form'
Gcs=tf(numgcs,dengcs)
'Gc(s) in factored form'
Gcszpk=zpk(Gcs)
'Gc(z) in expanded form via Tustin Transformation'
Gcz=c2d(Gcs,T,'tustin')
'Gc(z) in factored form vai Tustin Transformation'
Gczzpk=zpk(Gcz)
numgps=144000;
dengps=poly([0 -36 -1000]);
'Gp(s) in expanded form'
Gps=tf(numgps,dengps)
'Gp(s) in factored form'
Gpszpk=zpk(Gps)
'Gp(z) in expanded form'
Gpz=c2d(Gps,T,'zoh')
'Gp(z) in factored form'
Gpzzpk=zpk(Gpz)
Gez=Gcz*Gpz;
'Ge(z) = Gc(z)Gp(z) in factored form'
Gezzpk=zpk(Gez)
'z-1'
zm1=tf([1 -1],1,T)
zm1Gez=minreal(zm1*Gez,0.00001);
'(z-1)Ge(z) for finding steady-state error'
zm1Gezzpk=zpk(zm1Gez)
Kv=(1/T)*dcgain(zm1Gez);
'T(z)=Ge(z)/(1+Ge(z))'
Tz=feedback(Gez,1)
step(Tz,0:T:2);
```

Figure 1: MATLAB code

```
ans =

'Gc(s) in expanded form'


Gcs =

2.38 s + 60.21
--------------
 s + 60.2

Continuous-time transfer function.


ans =

'Gc(s) in factored form'


Gcszpk =

2.38 (s+25.3)
-------------
(s+60.2)

Continuous-time zero/pole/gain model.


ans =

'Gc(z) in expanded form via Tustin Transformation'


Gcz =

2.34 z - 2.281
--------------
z - 0.9416

Sample time: 0.001 seconds
Discrete-time transfer function.


ans =

'Gc(z) in factored form vai Tustin Transformation'


Gczzpk =

2.3397 (z-0.975)
----------------
 (z-0.9416)

```

```
Sample time: 0.001 seconds
Discrete-time zero/pole/gain model.


ans =

'Gp(s) in expanded form'


Gps =

       144000
-----------------------
s^3 + 1036 s^2 + 36000 s

Continuous-time transfer function.


ans =

'Gp(s) in factored form'


Gpszpk =

  1.44e+05
----------------
s (s+1000) (s+36)

Continuous-time zero/pole/gain model.


ans =

'Gp(z) in expanded form'


Gpz =

1.885e-05 z^2 + 5.931e-05 z + 1.125e-05
---------------------------------------
z^3 - 2.333 z^2 + 1.687 z - 0.3549

Sample time: 0.001 seconds
Discrete-time transfer function.


ans =

'Gp(z) in factored form'


Gpzzpk =

```

```
1.8848e-05 (z+2.944) (z+0.2028)
-------------------------------
(z-1) (z-0.9646) (z-0.3679)

Sample time: 0.001 seconds
Discrete-time zero/pole/gain model.


ans =

'Ge(z) = Gc(z)Gp(z) in factored form'


Gezzpk =

4.4097e-05 (z+2.944) (z-0.975) (z+0.2028)
-----------------------------------------
(z-1) (z-0.9646) (z-0.9416) (z-0.3679)

Sample time: 0.001 seconds
Discrete-time zero/pole/gain model.


ans =

'z-1'


zm1 =

z - 1

Sample time: 0.001 seconds
Discrete-time transfer function.


ans =

'(z-1)Ge(z) for finding steady-state error'


zm1Gezzpk =

4.4097e-05 (z+2.944) (z-0.975) (z+0.2028)
-----------------------------------------
  (z-0.9646) (z-0.9416) (z-0.3679)

Sample time: 0.001 seconds
Discrete-time zero/pole/gain model.


ans =

'T(z)=Ge(z)/(1+Ge(z))'
```

```


Tz =

4.41e-05 z^3 + 9.577e-05 z^2 - 0.000109 z - 2.566e-05
-----------------------------------------------------
 z^4 - 3.274 z^3 + 3.884 z^2 - 1.944 z + 0.3341

Sample time: 0.001 seconds
Discrete-time transfer function.
```

Listing 1: MATLAB results

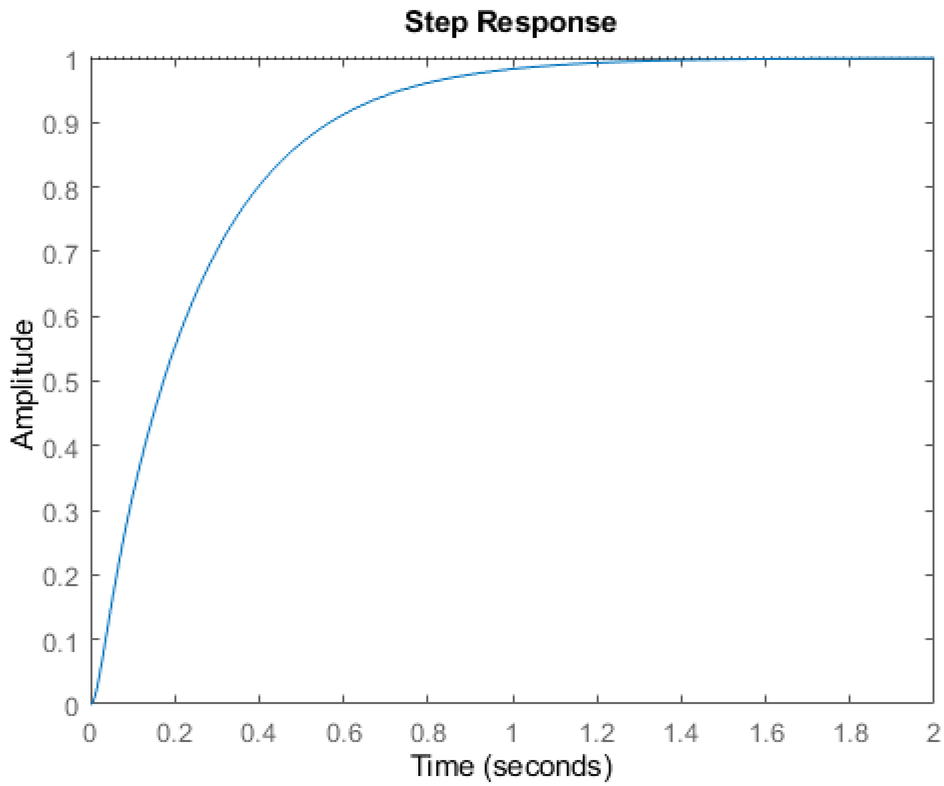

Figure 2: MATLAB plot