Homework 1

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EEEN 5338 Digital and DSP Based Control

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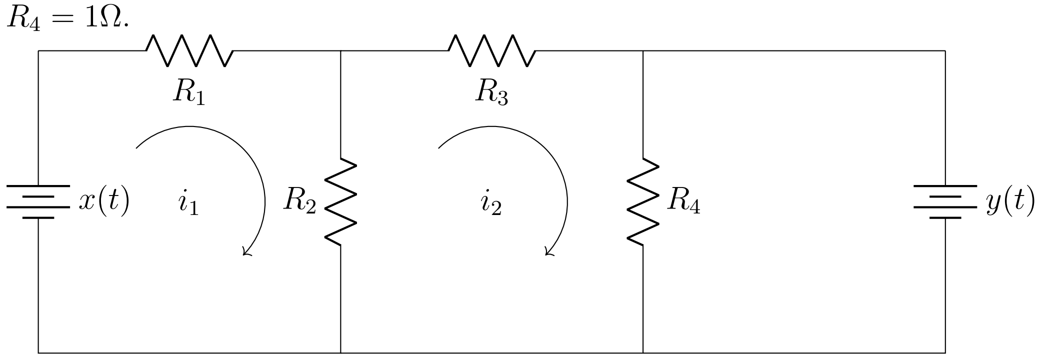
Problem 1. Find the math model for the following system. Assume: $R_1 = R_2 = R_3 = R_4 = 1\Omega$.



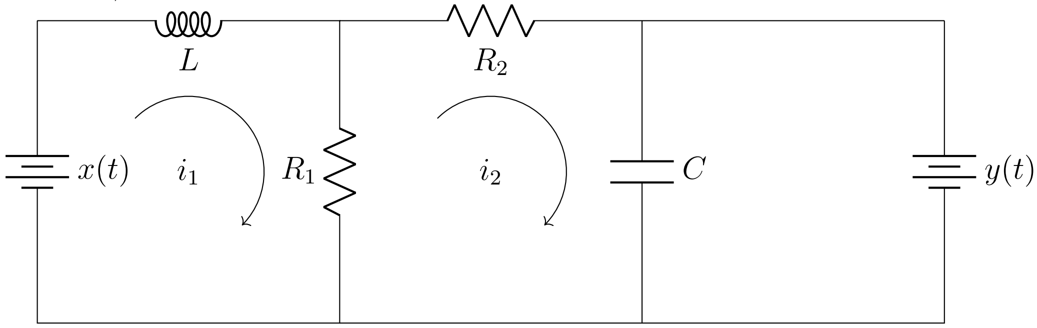
$$x = R_1 i_1 + R_2(i_1 - i_2) = 2i_1 - i_2 \tag{1}$$

$$R_2(i_2 - i_1) + R_3 i_2 + R_4 i_2 = 3i_2 - i_1 = 0 \implies 3i_2 = i_1 \tag{2}$$

$$y = R_4 i_2 = i_2 \tag{3}$$

If we put everything in terms of i_2, we have $x = 5i_2$ and since $y = i_2$, $x = 5y$.

Problem 2. Find the math model for the following system. Assume: $R_1 = R_2 = 1\Omega$, $L = 1H$, $C = 1F$.



$$x = L\frac{di_1}{dt} + R_1(i_1 - i_2) = \frac{di_1}{dt} + i_1 - i_2 \tag{4}$$

$$\begin{aligned} R_1(i_2 - i_1) + R_2 i_2 + \frac{1}{C}\int i_2 dt &= 0 \\ \implies \frac{2di_2}{dt} - \frac{di_1}{dt} + i_2 &= 0 \\ \implies \frac{di_1}{dt} &= \frac{2di_2}{dt} + i_2 \end{aligned} \tag{5}$$

$$\begin{aligned} y &= \frac{1}{C}\int i_2 dt \\ \implies \dot{y} &= i_2 \end{aligned} \tag{6}$$

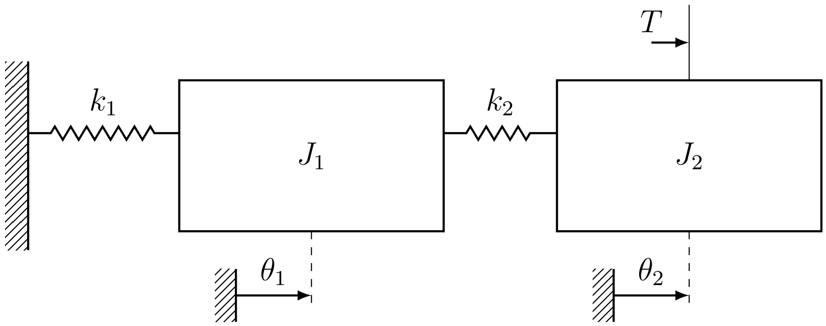
Using (5) and (6), we have

$$\frac{di_1}{dt} = 2\ddot{y} + \dot{y} \tag{7}$$

Taking the derivative of (4) and substituting in the results from (6) and (7), we have

$$\begin{aligned} \dot{x} &= \frac{d^2 i_1}{dt^2} + \frac{di_1}{dt} - \frac{di_2}{dt} \\ \implies \dot{x} = 2\,\dddot{y} + \ddot{y} + 2\ddot{y} + \dot{y} - \ddot{y} &= 2\,\dddot{y} + 2\ddot{y} + \dot{y} \\ \implies x &= 2\ddot{y} + 2\dot{y} + y \end{aligned} \tag{8}$$

Problem 3. Find the math model for the following system. (Note: LaTeX doesn't have a good way of drawing rotational mechanical systems, so I am drawing an equivalent translational one instead).



$$\begin{aligned} \sum T &= J\alpha \\ -k_1\theta_1 + k_2(\theta_2 - \theta_1) &= J_1\ddot{\theta}_1 \\ -k_2(\theta_2 - \theta_1) + T &= J_2\ddot{\theta}_2 \end{aligned} \tag{9}$$

$$\begin{aligned} J_1\ddot{\theta}_1 + (k_1 + k_2)\theta_1 - k_2\theta_2 &= 0 \\ \implies \theta_2 &= \frac{J_1\ddot{\theta}_1}{k_2} + \frac{(k_1 + k_2)\theta_1}{k_2} \end{aligned} \tag{10}$$

$$J_2\ddot{\theta}_2 - k_2\theta_1 + k_2\theta_2 = T \tag{11}$$