Final

Mengxiang Jiang
EEEN 5338 Digital and DSP Based Control

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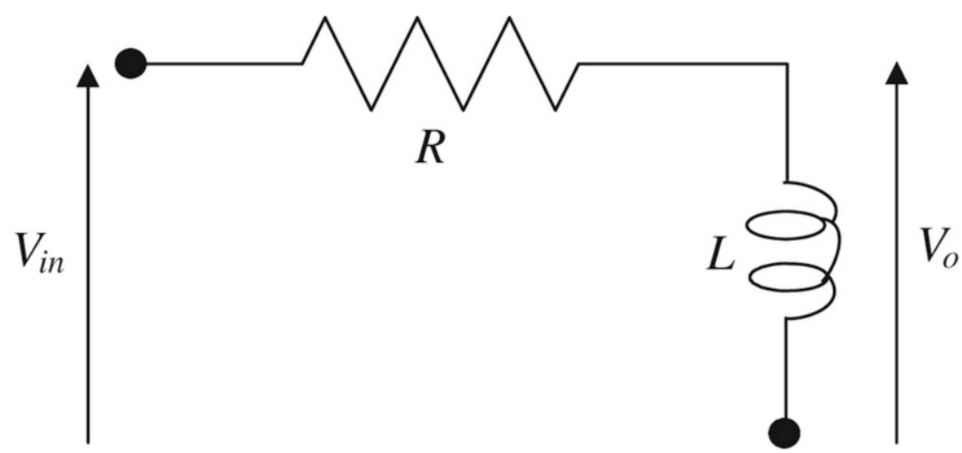
Problem 1. Find the z-domain transfer function for ADC, DAC, and the shown series R-L circuit with the inductor voltage as output.



From Example 3.4 of Lecture Notes 3:

Using the voltage divider rule gives:

$$\frac{V_o}{V_{in}} = \frac{Ls}{R + Ls} = \frac{(L/R)s}{1 + (L/R)s} = \frac{\tau s}{1 + \tau s},\ \tau = \frac{L}{R}$$

$$G_{ZAS}(z) = (1 - z^{-1})\mathcal{Z}\left\{\frac{1}{s + 1/\tau}\right\}$$

$$= \frac{z-1}{z} \times \frac{z}{z - e^{-T/\tau}} = \frac{z-1}{z - e^{-T/\tau}}$$

Problem 2. Find the steady-state position error for the digitally controlled DC motor with unity-feedback and make sure to explain your results.

$$G_{ZAS}(z) = \frac{K(z+a)}{(z-1)(z-b)}, \; C(z) = K_c \frac{(z-b)}{(z-c)}$$

$$0 < a, b, c < 1$$

From Example 3.10 of textbook (p. 78):

(a) Due to a sampled unit step.

The loop gain of the system is given by:

$$L(z) = C(z)G_{ZAS}(z) = \frac{KK_c(z+a)}{(z-1)(z-c)}$$

The system is type 1 (only one pole at unity).

Therefore, it has 0 steady-state error for unit step.

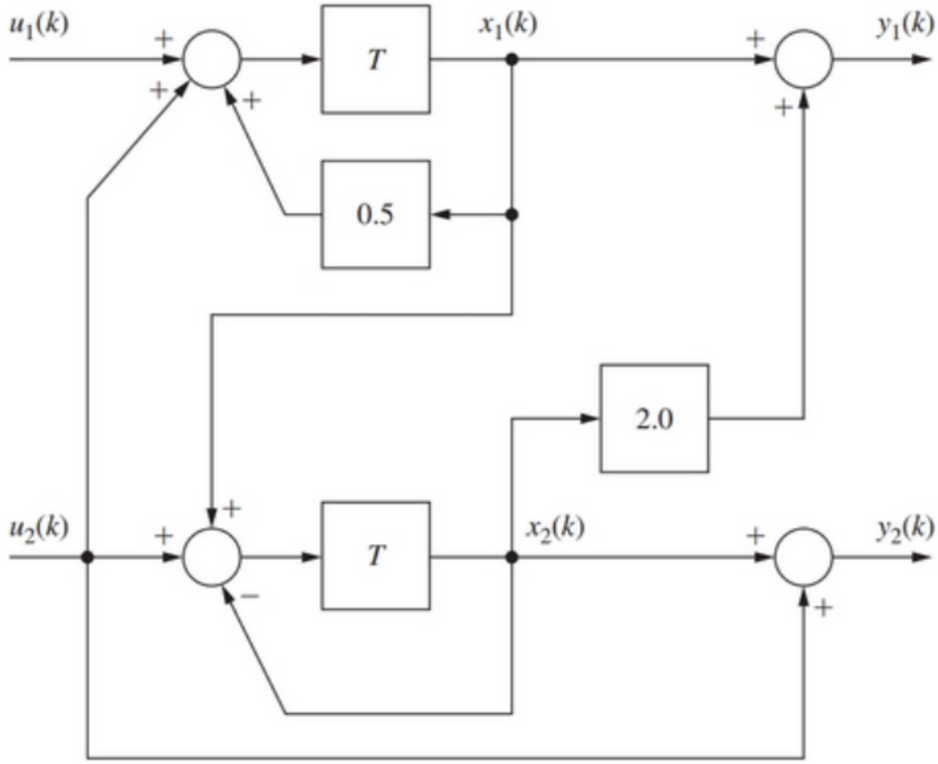
(b) Due to a sampled unit ramp input.

The finite steady-state error for a sampled ramp input is given by:

$$e(\infty) = \frac{T}{(z-1)L(z)|_{z=1}} = \frac{T}{KK_c}\left(\frac{1-c}{1+a}\right)$$

This steady-state error can be reduced by increasing the controller gain and is also affected by the choice of controller pole and zero

Problem 3. Find the state equations given the following simulation diagram:



$$
\begin{aligned}
x_1(k+1) &= T[u_1(k) + u_2(k) + 0.5x_1(k)] \\
x_2(k+1) &= T[x_1(k) + u_2(k) - x_2(k)] \\
y_1(k) &= x_1(k) + 2.0x_2(k) \\
y_2(k) &= x_2(k) + u_2(k)
\end{aligned}
$$

Problem 4. Given

$$G(z) = \frac{N(z)}{D(z)}$$

where

$$D(z) = z^3 - z^2 - 0.2z + 0.1$$

Use the Routh-Hurwitz criterion to find the number of z-plane poles of $G(z)$ inside, outside, and on the unit circle. Is the system stable?
From Example 4 of Lecture Note 5A:

Use the bilinear transformation equation for $D(z) = 0$:

$$z = \frac{s+1}{s-1}$$

We get:

$$s^3 - 19s^2 - 45s - 17 = 0$$

Table 1: Routh Table

s^3	1	-45
s^2	-19	-17
s^1	-45.89	0
s^0	-17	0

One sign change in the first column $\Longrightarrow$ one root in the R.H.P and two roots in the L.H.P in the s-plane.
$\Longrightarrow$ G(z) has one pole outside the unit circle, no poles on the unit circle, and two poles inside the unit circle
$\Longrightarrow$ system is unstable because of the pole outside the unit circle.

Problem 5. Find the stable range of the gain K for the unity-feedback digital control system with analog plant

$$G(s) = \frac{K}{s+3}$$

with DAC and ADC if the sampling period is 0.02 seconds.
From Example 4.7 of the textbook (p. 107):
The transfer function for analog subsystem ADC and DAC is:

$$\begin{aligned} G_{ZAS} &= (1 - z^{-1})\mathcal{Z}\left\{\mathcal{L}^{-1}\left[\frac{G(s)}{s}\right]\right\} \\ &= (1 - z^{-1})\mathcal{Z}\left\{\mathcal{L}^{-1}\left[\frac{K}{s(s+3)}\right]\right\} \end{aligned}$$

Using the partial fraction expansion

$$\frac{K}{s(s+3)} = \frac{K}{3}\left[\frac{1}{s} - \frac{1}{s+3}\right]$$

we obtain the transfer function

$$G_{ZAS}(z) = \frac{1.9412 \times 10^{-2} K}{z - 0.9418}$$

For unity feedback, the closed-loop characteristic equation is:

$$1 + G_{ZAS}(z) = 0$$

which can be simplified to

$$z - 0.9418 + 1.9412 \times 10^{-2} K = 0$$

The stability conditions are

$$0.9418 - 1.9412 \times 10^{-2} K < 1$$

$$-0.9418 + 1.9412 \times 10^{-2} K < 1$$

Thus, the stable range of K is

$$-3 < K < 100.03$$