

Homework 6

Mengxiang Jiang

EEEN 5338 Digital and DSP Based Control

November 29, 2023

Problem 1. A lead compensator is designed for a unity-feedback system whose plant transfer function is:

$$G_p(s) = \frac{100K}{s(s+36)(s+1000)}$$

The design specifications are:

Percent overshoot = 20%

Peak time = 0.1 seconds

$K = 1440$

The lead compensator is:

$$G_c(s) = 2.38 \left(\frac{s+25.3}{s+60.2} \right)$$

If the system is to be computer controlled:

1. Find the digital controller, $G_c(z)$, assume: $T = 0.001$ seconds.

substitute $s = \frac{2(z-1)}{0.001(z+1)}$ into $G_c(s)$:

$$G_c(z) = 2.38 \left(\frac{2(z-1) + 0.0253(z+1)}{0.001(z+1)} \right) \left(\frac{0.001(z+1)}{2(z-1) + 0.0602(z+1)} \right)$$
$$G_c(z) = 2.38 \left(\frac{2.0253z - 1.9747}{2.0602z - 1.9398} \right) = 2.38 \left(\frac{2.0253}{2.0602} \right) \left(\frac{z - 0.9750}{z - 0.9416} \right) = 2.3397 \left(\frac{z - 0.9750}{z - 0.9416} \right)$$

2. Find the z -transform of the plant and the zero-order hold with $T = 0.001$ seconds.

```
>> g = zpk([], [0, -36, -1000], 144000)

g =

      1.44e+05
-----
      s (s+36) (s+1000)

Continuous-time zero/pole/gain model.

>> gd = c2d(g, 0.001, 'zoh')

gd =

      1.8848e-05 (z+0.2028) (z+2.944)
-----
      (z-1) (z-0.9646) (z-0.3679)

Sample time: 0.001 seconds
Discrete-time zero/pole/gain model.
```

Expanding the expression leads to:

$$G_p(z) = \frac{1.8848 \times 10^{-5} z^2 + 5.9310 \times 10^{-5} z + 1.1253 \times 10^{-5}}{z^3 - 2.3325 z^2 + 1.6874 z - 0.3549}$$

3. Revise the MATLAB code (as per our class discussion in the examples from Lecture Notes 6) to solve and plot the step response for this system. Make sure to submit the code, the results, and the plot.

```

T=0.001;
numgcs=2.38*[1 25.3];
dengcs=[1 60.2];
'Gc(s) in expanded form'
Gcs=tf(numgcs,dengcs)
'Gc(s) in factored form'
Gcszpk=zpk(Gcs)
'Gc(z) in expanded form via Tustin Transformation'
Gcz=c2d(Gcs,T,'tustin')
'Gc(z) in factored form via Tustin Transformation'
Gczzpk=zpk(Gcz)
numgps=144000;
dengps=poly([0 -36 -1000]);
'Gp(s) in expanded form'
Gps=tf(numgps,dengps)
'Gp(s) in factored form'
Gpszpk=zpk(Gps)
'Gp(z) in expanded form'
Gpz=c2d(Gps,T,'zoh')
'Gp(z) in factored form'
Gpzpk=zpk(Gpz)
Gez=Gcz*Gpz;
'Ge(z) = Gc(z)Gp(z) in factored form'
Gezzpk=zpk(Gez)
'z-1'
zm1=tf([1 -1],1,T)
zm1Gez=minreal(zm1*Gez,0.00001);
'(z-1)Ge(z) for finding steady-state error'
zm1Gezzpk=zpk(zm1Gez)
Kv=(1/T)*dcgain(zm1Gez);
'T(z)=Ge(z)/(1+Ge(z))'
Tz=feedback(Gez,1)
step(Tz,0:T:2);

```

Figure 1: MATLAB code

```

1 ans =
2
3 'Gc(s) in expanded form'
4
5
6 Gcs =
7
8 2.38 s + 60.21
9 -----
10 s + 60.2
11
12 Continuous-time transfer function.
13
14
15 ans =
16
17 'Gc(s) in factored form'
18
19
20 Gcszpk =
21
22 2.38 (s+25.3)
23 -----
24 (s+60.2)
25
26 Continuous-time zero/pole/gain model.
27
28
29 ans =
30
31 'Gc(z) in expanded form via Tustin Transformation'
32
33
34 Gcz =
35
36 2.34 z - 2.281
37 -----
38 z - 0.9416
39
40 Sample time: 0.001 seconds
41 Discrete-time transfer function.
42
43
44 ans =
45
46 'Gc(z) in factored form vai Tustin Transformation'
47
48
49 Gczzpk =
50
51 2.3397 (z-0.975)
52 -----
53 (z-0.9416)
54

```

```

55 Sample time: 0.001 seconds
56 Discrete-time zero/pole/gain model.
57
58
59 ans =
60
61 'Gp(s) in expanded form'
62
63
64 Gps =
65
66      144000
67 -----
68 s^3 + 1036 s^2 + 36000 s
69
70 Continuous-time transfer function.
71
72
73 ans =
74
75 'Gp(s) in factored form'
76
77
78 Gpszp =
79
80      1.44e+05
81 -----
82 s (s+1000) (s+36)
83
84 Continuous-time zero/pole/gain model.
85
86
87 ans =
88
89 'Gp(z) in expanded form'
90
91
92 Gpz =
93
94      1.885e-05 z^2 + 5.931e-05 z + 1.125e-05
95 -----
96 z^3 - 2.333 z^2 + 1.687 z - 0.3549
97
98 Sample time: 0.001 seconds
99 Discrete-time transfer function.
100
101
102 ans =
103
104 'Gp(z) in factored form'
105
106
107 Gpzzpk =
108

```

```

109 1.8848e-05 (z+2.944) (z+0.2028)
110 -----
111 (z-1) (z-0.9646) (z-0.3679)
112
113 Sample time: 0.001 seconds
114 Discrete-time zero/pole/gain model.
115
116
117 ans =
118
119 'Ge(z) = Gc(z)Gp(z) in factored form'
120
121
122 Gezzpk =
123
124 4.4097e-05 (z+2.944) (z-0.975) (z+0.2028)
125 -----
126 (z-1) (z-0.9646) (z-0.9416) (z-0.3679)
127
128 Sample time: 0.001 seconds
129 Discrete-time zero/pole/gain model.
130
131
132 ans =
133
134 'z-1'
135
136
137 zm1 =
138
139 z - 1
140
141 Sample time: 0.001 seconds
142 Discrete-time transfer function.
143
144
145 ans =
146
147 '(z-1)Ge(z) for finding steady-state error'
148
149
150 zm1Gezzpk =
151
152 4.4097e-05 (z+2.944) (z-0.975) (z+0.2028)
153 -----
154 (z-0.9646) (z-0.9416) (z-0.3679)
155
156 Sample time: 0.001 seconds
157 Discrete-time zero/pole/gain model.
158
159
160 ans =
161
162 'T(z)=Ge(z)/(1+Ge(z))'

```

```

163
164
165 Tz =
166
167 4.41e-05 z^3 + 9.577e-05 z^2 - 0.000109 z - 2.566e-05
168 -----
169 z^4 - 3.274 z^3 + 3.884 z^2 - 1.944 z + 0.3341
170
171 Sample time: 0.001 seconds
172 Discrete-time transfer function.

```

Listing 1: MATLAB results

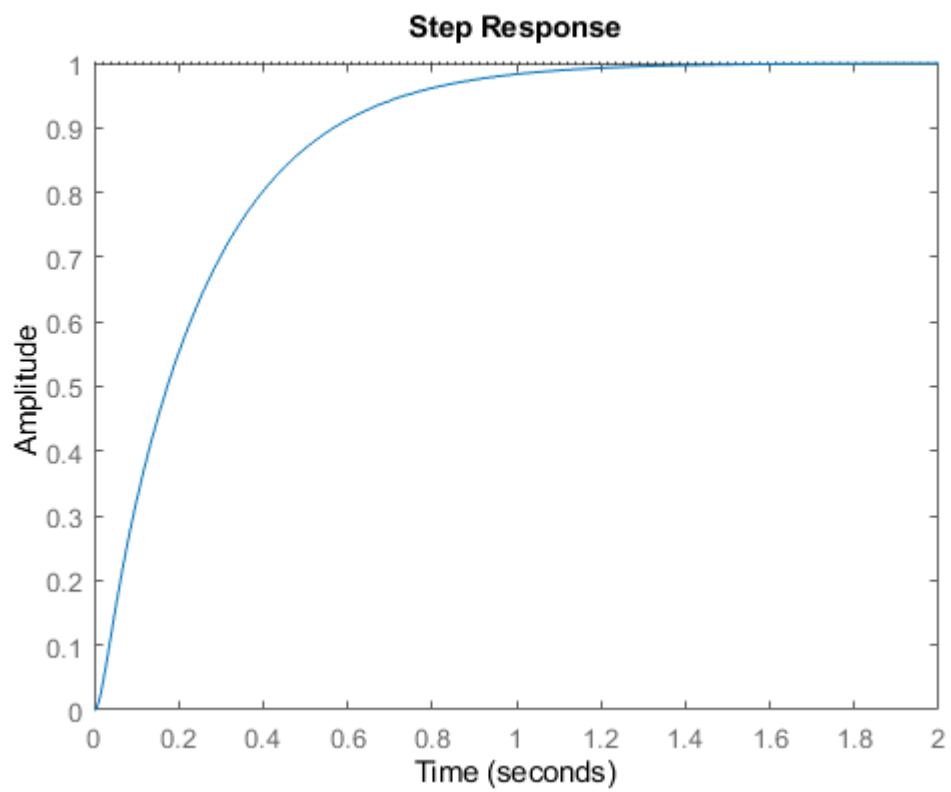


Figure 2: MATLAB plot