Project 3 Writing An Assessment
Week 5: The Product and Quotient Rules and Derivatives of Trig Functions

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Problem 1. Let
$$f(x) = (3x^2 - 2x)\sin x.$$
Compute $f'(x)$.

(a) $(6x-2)\cos x + (3x^2-2x)\sin x$

(b) $(6x-2)\sin x + (3x^2-2x)\cos x$

(c) $(6x-2)\sin x - (3x^2-2x)\cos x$

(d) $(6x-2)\cos x - (3x^2-2x)\sin x$

Solution:
To compute $f'(x)$, we apply the product rule as well as $(\sin x)' = \cos x$:
$$f'(x) = (3x^2-2x)'\sin x + (3x^2-2x)(\sin x)' = (6x-2)\sin x + (3x^2-2x)\cos x.$$
This is choice (b).
This problem is worth 3 points and can be given partial credit as follows:

- 1 point for either writing the product rule or applying it correctly.
- 1 point for computing the derivative of $3x^2 - 2x$ correctly.
- 1 point for computing the derivative of $\sin x$ correctly.

Problem 2. Let
$$g(x) = \frac{\tan x}{x}.$$
Compute $g'(x)$.

(a) $\dfrac{x\sec^2 x - \tan x}{x^2}$

(b) $\dfrac{\sec^2 x - \tan x}{x}$

(c) $\dfrac{x\sec^2 x + \tan x}{x^2}$

(d) $\dfrac{x\tan^2 x - \sec^2 x}{x^2}$

Solution:
To compute $g'(x)$, we apply the quotient rule as well as $(\tan x)' = \sec^2 x$:
$$g'(x) = \frac{(\tan x)' \cdot x - \tan x \cdot (x)'}{x^2} = \frac{x\sec^2 x - \tan x}{x^2}.$$
This is choice (a).
This problem is worth 3 points and can be given partial credit as follows:

- 1 point for either writing the quotient rule or applying it correctly.
- 1 point for computing the derivative of $\tan x$ correctly.
- 1 point for computing the derivative of x correctly.

Problem 3. (Note: One thing I didn't like about this week's lecture was that $\lim_{x\to 0} \frac{\sin x}{x} = 1$ was given without proof. I think it is important to prove this before it is used in the proof of the derivative of $\sin x$. Here is an alternative way to prove the derivative of $\sin x$ without using this limit.)
One of the most famous identities in mathematics is Euler's formula:

$$e^{ix} = \cos x + i \sin x.$$

This identity also gives us a way to represent the sine and cosine functions as exponential functions by taking advantage of the even and odd properties of the sine and cosine, with $e^{-ix} = \cos(-x) + i\sin(-x) = \cos x - i \sin x$ and adding or subtracting the two equations:

$$\sin x = \frac{e^{ix} - e^{-ix}}{2i}, \quad \cos x = \frac{e^{ix} + e^{-ix}}{2}.$$

Using this representation as well as $\frac{d}{dx}e^{cx} = ce^{cx}$, compute the derivative of $\sin x$.
(The derivative of the exponential function was covered in the previous week.)

Solution:
Use the identity:

$$\sin(x) = \frac{e^{ix} - e^{-ix}}{2i}$$

Differentiate both sides with respect to x:

$$\begin{aligned}
\frac{d}{dx}\sin(x) &= \frac{d}{dx}\left(\frac{e^{ix} - e^{-ix}}{2i}\right) \\
&= \frac{1}{2i}\left(\frac{d}{dx}e^{ix} - \frac{d}{dx}e^{-ix}\right) \\
&= \frac{1}{2i}\left(ie^{ix} + ie^{-ix}\right)
\end{aligned}$$

Factor out i and simplify:

$$\begin{aligned}
\frac{d}{dx}\sin(x) &= \frac{i}{2i}(e^{ix} + e^{-ix}) \\
&= \frac{e^{ix} + e^{-ix}}{2} = \cos(x).
\end{aligned}$$

This problem is worth 3 points and can be given partial credit as follows:

- 1 point for differentiating the exponential functions.
- 1 point for proper algebraic manipulations.
- 1 point for showing that the final answer is the identity for $\cos x$.