

# Project 3 Writing An Assessment

## Week 5: The Product and Quotient Rules and Derivatives of Trig Functions

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**Problem 1.** Let

$$f(x) = (3x^2 - 2x) \sin x.$$

Compute  $f'(x)$ .

- (a)  $(6x - 2) \cos x + (3x^2 - 2x) \sin x$
- (b)  $(6x - 2) \sin x + (3x^2 - 2x) \cos x$
- (c)  $(6x - 2) \sin x - (3x^2 - 2x) \cos x$
- (d)  $(6x - 2) \cos x - (3x^2 - 2x) \sin x$

**Problem 2.** Let

$$g(x) = \frac{\tan x}{x}.$$

Compute  $g'(x)$ .

- (a)  $\frac{x \sec^2 x - \tan x}{x^2}$
- (b)  $\frac{\sec^2 x - \tan x}{x}$
- (c)  $\frac{x \sec^2 x + \tan x}{x^2}$
- (d)  $\frac{x \tan^2 x - \sec^2 x}{x^2}$

**Problem 3.** (Note: One thing I didn't like about this week's lecture was that  $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$  was given without proof. I think it is important to prove this before it is used in the proof of the derivative of  $\sin x$ . Here is an alternative way to prove the derivative of  $\sin x$  without using this limit.)

One of the most famous identities in mathematics is Euler's formula:

$$e^{ix} = \cos x + i \sin x.$$

This identity also gives us a way to represent the sine and cosine functions as exponential functions by taking advantage of the even and odd properties of the sine and cosine, with  $e^{-ix} = \cos(-x) + i \sin(-x) = \cos x - i \sin x$  and adding or subtracting the two equations:

$$\sin x = \frac{e^{ix} - e^{-ix}}{2i}, \quad \cos x = \frac{e^{ix} + e^{-ix}}{2}.$$

Using this representation as well as  $\frac{d}{dx} e^{cx} = ce^{cx}$ , compute the derivative of  $\sin x$ . (The derivative of the exponential function was covered in the previous week.)

**Problem 1 Solution.** To compute  $f'(x)$ , we apply the product rule as well as  $(\sin x)' = \cos x$ :

$$f'(x) = (3x^2 - 2x)' \sin x + (3x^2 - 2x)(\sin x)' = (6x - 2) \sin x + (3x^2 - 2x) \cos x.$$

This is choice (b).

This problem is worth 3 points and can be given partial credit as follows:

- 1 point for either writing the product rule or applying it correctly.
- 1 point for computing the derivative of  $3x^2 - 2x$  correctly.
- 1 point for computing the derivative of  $\sin x$  correctly.

**Problem 2 Solution.** To compute  $g'(x)$ , we apply the quotient rule as well as  $(\tan x)' = \sec^2 x$ :

$$g'(x) = \frac{(\tan x)' \cdot x - \tan x \cdot (x)'}{x^2} = \frac{x \sec^2 x - \tan x}{x^2}.$$

This is choice (a).

This problem is worth 3 points and can be given partial credit as follows:

- 1 point for either writing the quotient rule or applying it correctly.
- 1 point for computing the derivative of  $\tan x$  correctly.
- 1 point for computing the derivative of  $x$  correctly.

**Problem 3 Solution.** Use the identity:

$$\sin(x) = \frac{e^{ix} - e^{-ix}}{2i}$$

Differentiate both sides with respect to  $x$ :

$$\begin{aligned} \frac{d}{dx} \sin(x) &= \frac{d}{dx} \left( \frac{e^{ix} - e^{-ix}}{2i} \right) \\ &= \frac{1}{2i} \left( \frac{d}{dx} e^{ix} - \frac{d}{dx} e^{-ix} \right) \\ &= \frac{1}{2i} (ie^{ix} + ie^{-ix}) \end{aligned}$$

Factor out  $i$  and simplify:

$$\begin{aligned} \frac{d}{dx} \sin(x) &= \frac{i}{2i} (e^{ix} + e^{-ix}) \\ &= \frac{e^{ix} + e^{-ix}}{2} = \cos(x). \end{aligned}$$

This problem is worth 3 points and can be given partial credit as follows:

- 1 point for differentiating the exponential functions.
- 1 point for proper algebraic manipulations.
- 1 point for showing that the final answer is the identity for  $\cos x$ .