Printed Name: ______________________________

STAT 610 Exam I
Friday, 29 September 2023

Instructions.

- You have 50 minutes.
- You may use a stand-alone calculator, but no other resources.
- Please write your solutions on separate sheets of paper, and *return them in order* with this sheet on top.
- You may use any results from class or homework as long as you explain clearly the result you are using (by name or description).
- To ensure credit, explain or show all of your steps.
- There are 50 points total.

1. (10 points) Consider the following continuous cdf for a random variable W.

$$F_W(x) = \begin{cases} 0 & \text{if } x \le 0, \\ x/3 & \text{if } 0 \le x \le 1, \\ 1/3 & \text{if } 1 \le x \le 2, \\ 2x/3 - 1 & \text{if } 2 \le x \le 3, \\ 1 & \text{if } 3 \le x. \end{cases}$$

   Find the probability density function (pdf) and $\mathsf{E}(W)$.

2. (10 points) A population of 1000 school districts consists of 150 districts with 1 high school, 350 with 2 high schools, 300 with 3 high schools, and 200 with 4 high schools.

   A researcher selects two districts at random, without replacement. Given that the two districts have the <u>same number</u> of high schools, what is the chance they have x high schools, for each $x = 1, 2, 3, 4$? Computable expressions suffice.

3. Suppose T has gamma(2,1) distribution.

   (a) (10 points) Determine the cumulative distribution function (cdf), for all real values, and provide an equation that the median must solve. (Do not try to solve it.)

   (b) (10 points) Find the pdf for $Y = T^{1/3}$.

4. (10 points) Derive the moment generating function (mgf) for the geometric(p) pmf. (Recall that $\sum_{k=1}^{\infty} a^k = a/(1-a)$ for $|a| < 1$.)

(distribution formulas on next page)

Some common probability density functions and probability mass functions are provided below.

beta(α, β) $f(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}$, for $0 < x < 1$, $\alpha > 0$, $\beta > 0$.

binomial(n, p) $f(x) = \binom{n}{x} p^x (1-p)^{n-x}$, for $x = 0, 1, \ldots, n$; $0 < p < 1$.

exponential(β) $f(x) = \frac{1}{\beta} \mathrm{e}^{-x/\beta}$, for $x > 0$; $\beta > 0$.

gamma(α, β) $f(x) = \frac{1}{\beta^\alpha \Gamma(\alpha)} x^{\alpha-1} \mathrm{e}^{-x/\beta}$, for $x > 0$; $\alpha > 0$, $\beta > 0$.

geometric(p) $f(x) = p(1-p)^{x-1}$, for $x = 1, 2, \ldots$; $0 < p < 1$.

hypergeometric(n, N, M) $f(x) = \binom{M}{x}\binom{N-M}{n-x} / \binom{N}{n}$, for $x = 0, 1, \ldots, n$; $n, M, N > 0$.

negative binomial(k, p) $f(x) = \binom{k+x-1}{k-1} p^k (1-p)^x$, for $x = 0, 1, 2, \ldots$; $0 < p < 1$, $k > 0$.

normal(μ, σ^2) $f(x) = \frac{1}{\sqrt{2\pi}\sigma} \mathrm{e}^{-(x-\mu)^2/2\sigma^2}$, for $-\infty < x < \infty$; $-\infty < \mu < \infty$, $\sigma^2 > 0$.

Poisson(λ) $f(x) = \frac{\lambda^x}{x!} \mathrm{e}^{-\lambda}$, for $x = 0, 1, 2, \ldots$; $\lambda > 0$.

uniform(a, b) $f(x) = \frac{1}{b-a}$, for $a < x < b$; $a < b$.

Weibull(γ, β) $f(x) = \frac{\gamma}{\beta} x^{\gamma-1} \mathrm{e}^{-x^\gamma/\beta}$, for $x > 0$; $\gamma > 0$, $\beta > 0$.