Assignment 3

Partial solutions for selected problems

1. Exer. 1.47. Check the conditions in Theorem 2.6 of the notes. As both are continuous, the nondecreasing property can be shown by getting nonnegative derivatives (which you must do anyway to get the pdfs).

   For the requested probability, use the cdfs since you already know them and that they are continuous: $\mathsf{P}(1 \le X \le 2) = F(2) - F(1)$.

   (b) $f(x) = \frac{e^{-x}}{(1+e^{-x})^2}$. $\mathsf{P}(1 \le X \le 2) = 0.14974$.

2. (b) Compute

$$\mathsf{P}(X_1 = X_2 = 1 | X_1 = X_2) = \frac{\mathsf{P}(X_1 = 1, X_2 = 1, X_3 = 8)}{\sum_{i=0}^{5} \mathsf{P}(X_1 = i, X_2 = i, X_3 = 10 - 2i)} = \cdots = 0.37815.$$

   (d) No, because (compute, using the respective binomial pmfs) $\mathsf{P}(X_1 = 1) = 0.38742$, $\mathsf{P}(X_2 = 1) = 0.12106$ and

$$\mathsf{P}(X_1 = 1)\mathsf{P}(X_2 = 1) \ne \mathsf{P}(X_1 = 1, X_2 = 1).$$

3. (a) is a continuous cdf,

   (b) is not a cdf,

   (c) is a mixture: continuous except for a single jump of size $\frac{3}{16}$ at $t = \frac{1}{2}$, and

   (d) is a discrete cdf with a pmf that is positive at $0, \frac{1}{2}, \frac{3}{4}, 1$.

4. $F_V(v) = \mathsf{P}(U \le v^2) = v^2$ and $f_V(v) = 2v$ for $0 \le v \le 1$.

5. Use $\{T_2 \le t\} \subset \{T_1 \le t\}$, as subsets of the sample space. [Another way to think about it is the logic statement $T_2 \le t \implies T_1 \le t$.]

6. Exer. 2.1. (c) $f_Y(y) = 15y^{3/2}(1 - y^{1/2})^2$ for $0 < y < 1$.

7. (a) For $y \ge 0$, $F_Y(y) = \mathsf{P}(-y \le X \le y) = F_X(y) - F_X(-y)$ (using the fact that F_X is continuous), and $f_Y(y) = f_X(y) + f_X(-y)$ (note the plus). Of course, $f_Y(y) = 0$ for $y < 0$. [Check: $f_Y(y)$ is nonnegative and integrates to 1.]

8. (a) $c = 4$.

   (b) $F_X(x) = 1 - (1 + 2x)e^{-2x}$. [This is the gamma$(2, \frac{1}{2})$ cdf. Check: $F_X(0) = 0$ since the rv is positive and $F_X(x) \to 1$ as $x \to \infty$.]

   (d) $F_Y(y) = F_X(e^y)$ so $f_Y(y) = e^y f_X(e^y)$ (or use Corollary 2.12 in the notes), then go from there.