STAT610 Assignment 4
due Friday, 26 September 2025

Please Note. This assignment involves a lot of calculus, so plan accordingly.

1. Let Y have the Poisson(λ) distribution with pmf
$$f_Y(y) = \frac{\lambda^y e^{-\lambda}}{y!} \quad y = 0, 1, 2, \ldots.$$
Find $\mathsf{E}(Y)$ and $\mathsf{E}(Y(Y-1))$, and use these to get $\mathsf{Var}(Y)$. (Recall the binomial example in class.)

2. Find $\mathsf{E}(X)$ for the random variable in Problem 8 of Assignment 2.

3. Chapter 2 Exercise 4. (Laplace distribution.) Also, add
   (d) Find $\mathsf{E}(X)$, $\mathsf{Var}(X)$ and the standard deviation of X.

4. Let $f(x) = 4xe^{-2x}$ for $x > 0$, and $f(x) = 0$ otherwise. (Recall Problem 8 of Assignment 3.) Now find $\mathsf{E}(X)$ and $\mathsf{Var}(X)$. Hint: Theorem 2.16 in the notes.

5. Let T have pdf $f_T(v) = 60v^3(1-v)^2$, $0 < v < 1$.
   (a) Verify that f_T is indeed a pdf.
   (b) Find the mean of T.
   (c) Find the pdf for $R = T/(1-T)$.
   (d) Use the pdf for T to find $\mathsf{E}(R)$ (recalling what Theorem 2.17 in the notes says).
   (e) What does Jensen's inequality say about the relationship between $\mathsf{E}(R)$ and $\mathsf{E}(T)/(1-\mathsf{E}(T))$? Confirm by evaluating both.

6. Prove Theorem 2.21 in the notes.

7. Let W be a positive random variable with finite mean μ_W.
   (a) Suppose $\alpha > 1$. Use Jensen's inequality to show that $\mathsf{E}(W^\alpha) > \mu_W^\alpha$. (Note: this is valid even if the left-hand side is infinite.)
   (b) Suppose $\alpha < 0$ and show that $\mathsf{E}(W^\alpha) > \mu_W^\alpha$.
   (c) Now suppose $0 < \alpha < 1$. What is true in this case? Hint: if $g(x)$ is concave then $-g(x)$ is convex.

8. Let $f(y) = \frac{\lambda^y \mathrm{e}^{-\lambda}}{y!}$ for $y = 0, 1, 2, \ldots$ be the Poisson(λ) pmf, where $\lambda > 0$. (Recall Problem 1 above.) Now show that the mgf is $M(t) = \mathrm{e}^{\lambda(\mathrm{e}^t - 1)}$, and use the mgf to get the mean and variance.

9. Let X_m have pdf $\frac{x^{m-1}\mathrm{e}^{-x/\beta}}{\beta^m(m-1)!}$ for $m = 1, 2\ldots$ and $x > 0$, $\beta > 0$.
   (a) Show that the mgf for X_m is $M_{X_m}(t) = (1-\beta t)^{-m}$ for $t < 1/\beta$. Hint: use a linear change of variables.
   (b) Use the mgf to derive the mean and variance.
   (c) Show that for any m, n, $M_{X_m}(t)M_{X_n}(t) = M_{X_{n+m}}(t)$. As we shall see, this has a probability interpretation in terms of sums of independent random variables.