Statistics 610 Assignment 1
due Friday, 5 September

1. Chapter 1 Exercises 4, 5, 6.

2. Here is a sample space: $\mathcal{S} = \{a, b, c\}$.

   (a) Explicitly provide the σ-algebra of all subsets.

   (b) Suppose one may only observe whether the outcome is a or not. Explicitly provide the smallest relevant σ-algebra. Hint: what events may be obtained by complements, unions and intersections, starting only with $\{a\}$?

3. For each of the following, be sure to explicitly provide the sample space and the event indicated.

   (a) I hand out 4 pieces of candy at random, each to a child. There are three children and some will get more than others. Find the probability that each child gets at least one piece of candy. Find the probability that one child gets it all.

   (b) The local Chevy dealer has 5 trucks to give away: two Silverados (S_1 and S_2) and three Tahoes (T_1, T_2 and T_3). Suppose they randomly select a truck, give it to a lucky customer, and then randomly select another for a second lucky customer. Find the probability that x Silverados are selected, where $x = 0, 1, 2$.

4. Chapter 1 Exercise 10. Use the case $n = 2$ (Thm. 1.1.4.d in the book) and mathematical induction to solve this. (See Exercise 9 for a general case; but here I want you to demonstrate that you can do mathematical induction.)

5. (See Example 1.4, Slide 20, in the notes.) The lifetime X of a cell phone battery, inspected under stress, satisfies $\mathsf{P}(X > t) = \mathrm{e}^{-t/10}$ for all $t > 0$.

   (a) Show that, if $0 \le a < b$ then $\mathsf{P}(a < X \le b) = \frac{1}{10}\int_a^b \mathrm{e}^{-t/10}\mathrm{d}t$.

   (b) Is there a similar integral representation $\mathsf{P}(a < X \le b) = \int_a^b g(t)\mathrm{d}t$ valid even if $a < 0 < b$? Hint: use an indicator function – it should not depend on either a or b.

   (c) Let $a \uparrow b$ to find $\mathsf{P}(X = b)$.

6. Suppose a communications system is designed with K circuits so that if one is in use another can be chosen. Assume j circuits are in use and a circuit is chosen at random (without replacement) until a circuit not in use is selected.

   (a) What is the chance that at least two selections are required?

   (b) What is the chance that exactly two selections are required?

   (c) What is the chance that exactly i selections are required, for $i = 1, 2, \ldots$?

7. Quordle is a daily word game in which you have 9 tries to guess four 5-letter words. Suppose the four words are selected at random (without replacement) from a dictionary of 2309 5-letter words (which is what a similar game has).

   (a) How many possible 4 word combinations are there?

   (b) Find the number of ways that k daily games have no words in common, and use that to find the probability that at least one word is repeated. This is similar to the birthday problem discussed in class, except the sampling method is different.

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(c) Find the probability that at least one word is repeated for $k = 2, 3, \ldots, 20$, using R. Note: choose(n,i) gives the binomial coefficient $\binom{n}{i}$ and lchoose(n,i) is the logarithm of that.

(d) What is the smallest number of daily games for which the probability that at least one word is repeated is at least 0.50? (Later we will see that this is the *median* number of games to be played until a word is repeated.)