STAT610 Assignment 3
due Friday, 19 September 2025

Note: you may use a computer or calculator to do the numerical computations and for the plots.

1. Chapter 1 Exercise 47(b,c). Add the following for each.

   ii. Find the pdf or pmf, whichever is appropriate.

   iii. Find the probability between 1 and 2, inclusive.

2. Suppose we sample 10 individuals at random and with replacement from a population that has three categories C_1, C_2, C_3 with respective proportions 10%, 30% and 60%. Let X_i be the number in the sample from C_i, $i = 1, 2, 3$. Then (recall Example 1.11 on Slide 31 in the notes)

$$\mathsf{P}(X_1 = i, X_2 = j \text{ and } X_3 = k) = \frac{10!}{i!j!k!}\, 0.1^i\, 0.3^j\, 0.6^k, \quad \text{if } i + j + k = 10,\ i \geq 0,\ j \geq 0,\ k \geq 0.$$

   (a) Determine the probability that X_1 and X_2 are both equal 1 (keeping in mind what X_3 must be).

   (b) Find the probability that $X_1 = X_2$ by splitting the event into disjoint cases according to their common value, and then get the conditional probability that $X_1 = X_2 = 1$, given $X_1 = X_2$. (Recall Problem 2(b) of Assignment 2.)

   (c) Call C_1 "Success" and C_1^c "Failure" to argue that X_1 has binomial$(10, 0.1)$ distribution. Draw a similar conclusion about X_2.

   (d) Are the events $\{X_1 = 1\}$ and $\{X_2 = 1\}$ independent? Explain.

3. Determine whether the following functions are cdfs or not, explaining your reasoning. If it is a cdf, is it discrete, continuous or a mixture of the two? You may assume the interval given for the variable is the ostensible support (no probability outside that interval). It may help to plot the functions.

   (a) $F(t) = t(2 - t)$ for $t \in [0, 1]$.

   (b) $F(t) = t(2 - t)$ for $t \in [0, 2]$.

   (c) $F(t) = t(2 - t)1_{[0,1/2)}(t) + \frac{t+7}{8}1_{[1/2,1]}(t)$ for $t \in [0, 1]$.

   (d) $F(t) = \frac{1}{5}1_{[0,\infty)}(t) + \frac{1}{4}1_{[1/2,\infty)}(t) + \frac{1}{2}1_{[3/4,\infty)}(t) + \frac{1}{20}1_{[1,\infty)}(t)$ for $t \in [0, 1]$.

4. Let U have uniform(0,1) distribution (see Example 2.6 in the notes). Let $V = \sqrt{U}$. Show that V is a continuous random variable and find its cdf. Find the pdf and plot both the cdf and the pdf.

5. Suppose T_1 and T_2 are two random variables such that $T_1(s) \leq T_2(s)$ for all outcomes s in the sample space. Prove that $F_{T_1}(t) \geq F_{T_2}(t)$ for all real t. Hint: compare appropriate events about the random variables.

6. Chapter 2 Exercise 1(a,b). Be certain to clearly indicate the support of Y for each case.

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7. Let X be a continuous random variable, *not necessarily nonnegative*, with cdf F_X and pdf f_X.

   (a) Find the cdf for $Y = |X|$ and use it to find the pdf for Y. Be sure that both satisfy the necessary properties.

   (b) Apply the above to the standard normal density $f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$. (This gives the so-called half-normal pdf.)

8. Let $h(x) = xe^{-2x}$ for $x > 0$, and $h(x) = 0$ otherwise.

   (a) Find $c > 0$ such that $f(x) = ch(x)$ is a valid pdf.

   (b) Find the corresponding cdf. (Check that it satisfies the required properties of a cdf.)

   (c) Plot both the pdf and the cdf.

   (d) Suppose X has pdf $f(x)$. Find the cdf and pdf for $Y = \log(X)$. (This is the *natural* logarithm.)