Assignment 6

Partial solutions for selected problems

1. (a) By the result about linear transformations,

$$f_Y(y) = \frac{1}{|a|} f_X((y-b)/a) = \frac{\lambda}{2|a|} \mathrm{e}^{-\lambda|y-b|/|a|}, \quad \text{all } y.$$

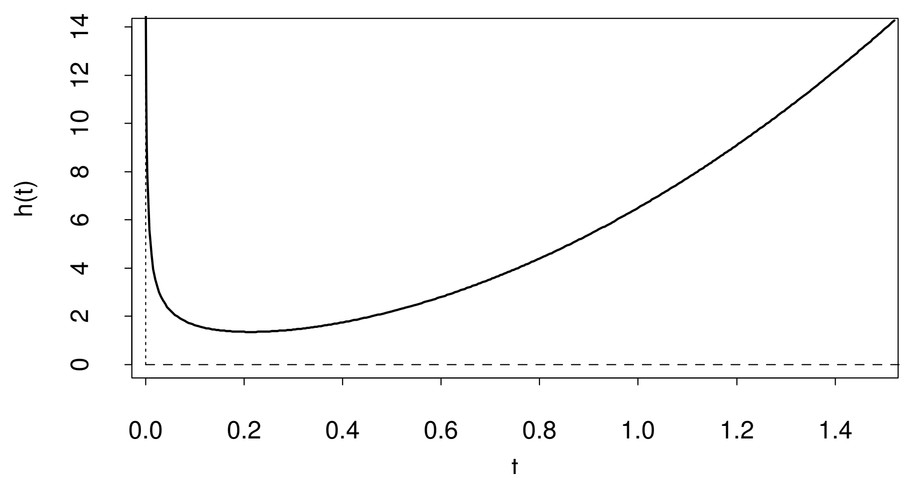
   (c) $M_Y(t) = \mathrm{e}^{bt} M_X(at) = \frac{\mathrm{e}^{bt}}{1-a^2t^2/\lambda^2}$.

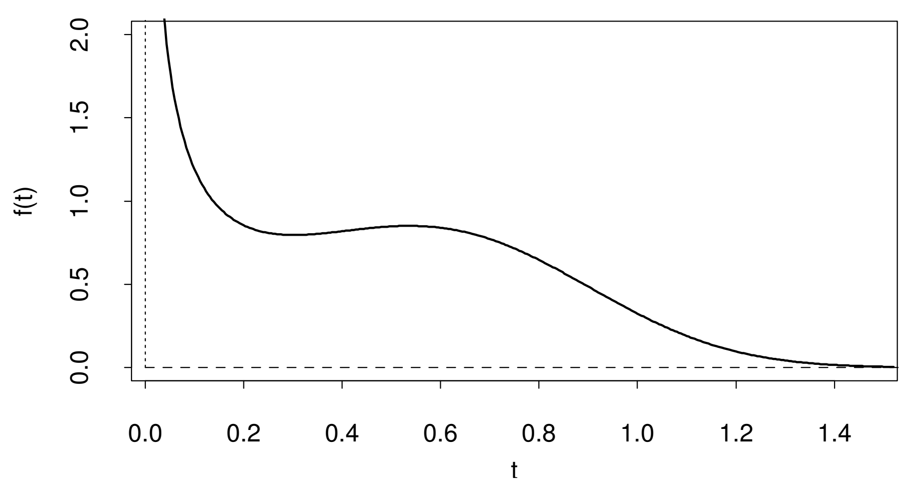
2. (a) $h(t) = \frac{x}{\beta(\beta+x)}$.

   (b) $h(t) = \frac{f(x)}{1-F(x)} = \frac{1+4\mathrm{e}^{-x}}{1+2\mathrm{e}^{-x}}$.

3. (a) $Q_T(p)$ solves $\exp(-H(t)) = 1-p$. That is, $Q_T(p) = H^{-1}(-\log(1-p))$.
   The Weibull$(2,\beta)$ quantile function is $(-\beta\log(1-p))^{1/2}$.

   (b) The minimum hazard rate is at $t = 48^{-.4} = 0.21257$ and the pdf is $f_T(t) = \frac{.5t^{-.5}+6t^2}{1-\mathrm{e}^{t^{.5}+2t^3}}$.
   See the plots below.





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4. There are more than one option for location and/or scale parameter, although some are more sensible than others.

   (a) These are indeed a location-scale family; pay attention to the indicator function. One choice is location $\frac{a+b}{2}$ (the midpoint) and scale $b-a$ (the interval width). Uniform$(-\sqrt{3}, \sqrt{3})$ has mean 0 and variance 1.

   (b) b is a location parameter and $|a|/\lambda$ is a scale parameter. [These are the preferred choices, being simplest, although the scale parameter could be anything proportional to $|a|/\lambda$.] Laplace$(0, \frac{1}{\sqrt{2}})$, that is, $f(x) = \frac{1}{\sqrt{2}}\mathrm{e}^{-\sqrt{2}|x|}$, has mean 0 and variance 1.

   (c) Scale family, but (according to the book's formulation) the scale parameter is $\sqrt{\beta}$, not β. Weibull$(2, \frac{1}{1-\pi/4})$ has variance 1.

5. Let $Y = \frac{X-c}{s}$. Then Y has the pdf with location 0 and scale 1 (and which does not depend on either c or s).

   (a) $\mu_X = \mathsf{E}(X) = c + s\mu_Y$.

   (b) $\mathsf{E}((X - \mu_X)^k) = s^k \mathsf{E}((Y - \mu_Y)^k)$, including the case $k = 2$ (variance).

6. Exer. 3.28. First observe that the supports do not depend on the parameters. You can identify the functions t_i and w_i by noting the following.

   (c) $f(x) \propto \exp((\alpha - 1)\log(x) + (\beta - 1)\log(1 - x))$.

   (d) $f(x) \propto (x!)^{-1} \exp(-\lambda x)$.

   (e) $f(x) \propto \binom{r+x-1}{x} \exp(\log(p)x)$.

7. Exer. 3.33(b). We have $t_1(x) = x$, $t_2(x) = x^2$, $w_1(\theta) = -\frac{2}{a\theta}$, $w_2(\theta) = \frac{1}{a\theta^2}$.

   [Even though they depend on only one parameter rather than the usual two, the examples in parts (a) and (c) are exponential families but actually <u>not curved</u>, despite what the problem suggests – check them out explicitly.]

8. (a) The support $[a, b]$ depends on the parameters.

   (b) We cannot express $\log(1 + \mathrm{e}^{-(x-\mu)/\beta})$ in terms of products of functions of (μ, β) and functions of x.