Assignment 2

Partial solutions for selected problems

1. Exercise 1.34.

   (a) $\frac{19}{30}$.

   (b) $\frac{10}{19}$.

3. (a) $\mathsf{E}(I|R) = 0.7266$, $\mathsf{E}(I|R^c) = 0.6374$. The drug appears to help.

   (b) The drug is not much help for patients in the early stages and actually is worse for patients in the later stages. The discrepancy with part (a) is because, for this experiment, too many in the early stages got the drug and too many in the later stages did not. It would be better if the experiment was designed so that the four groups had roughly equal numbers of patients.

4. Using counting rules, for any $k \le n$ and $i_1, \dots, i_k$,

$$\mathsf{P}\Big(\bigcap_{j=1}^{k} A_{i_j}\Big) = \frac{M^k \times N^{n-k}}{N^n} = \cdots = \prod_{j=1}^{k} \mathsf{P}(A_{i_j}),$$

   where the last equality comes from applying the first equality when $k = 1$. [It is not sufficient to show only that the probability of n Yes's is $(\frac{M}{N})^n$. Nor is it sufficient to say that since the first two are independent then all the rest must be.]

   Remark: since the result applies no matter what is meant by "Yes" (or what M is), it shows that whatever gets observed on the n individuals sampled by replacement will be independent.

5. (a) Pairwise independent only.

   (b) $\mathsf{P}(A \cap B \cap C) = \frac{1}{36} = \mathsf{P}(A)\mathsf{P}(B)\mathsf{P}(C)$, but A and C are not independent.

   (c) No to both. Use the example in part (a): A is disjoint from, not independent of, $B \cap C$. Consequently, A^c is not independent of $B^c \cup C^c$.

6. (b) binomial$(6, \frac{17}{100})$.

$$f_Y(y) = \binom{6}{y}(.17)^y(.83)^{6-y}1_{\{0,1,2,3,4,5,6\}}(y).$$

   The pmf and cdf must be defined for *every* real y, not just the integers. The pmf is positive only for integers $0, \dots, 6$, but the cdf is a step function with constant nonzero values between those integers.

7. hypergeometric(4,30,5).

$$f_X(x) = \frac{\binom{5}{x}\binom{25}{4-x}}{\binom{30}{4}} 1_{\{0,1,2,3,4\}}(x).$$

8. Use the geometric sum rule: $\sum_{x=m}^{\infty} q^x = \frac{q^m}{1-q}$ for $-1 < q < 1$.

   (a) $c = \frac{1}{4}$.

   (b) It easiest to first get $1 - F(x) = \sum_{j=x+1}^{\infty} f_X(j) = \frac{(1/3)^{x+1}}{2/3} + \frac{(2/3)^{x+1}}{4/3}$ for $x = 0, 1, 2, \dots$

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(c) Use the law of total probability:

$$\mathsf{P}(X = x) = \mathsf{P}(x \text{ rolls until 2, 3, 4 or } 5|T)\mathsf{P}(T) + \mathsf{P}(x \text{ rolls until 1 or } 6|H)\mathsf{P}(H).$$

(Or use the same method to get $\mathsf{P}(X > x)$ and deduce from that.)

[This is an example of a mixture of two different discrete distributions: X is like either a geometric(1/3) r.v. or a geometric(2/3) r.v., with probability 1/2 each. But the pmf of X is *not* one or the other function with probability 1/2 each – that would make the pmf itself random! Nor can there be two different pmfs for X.]