Printed Name: ______________________________

STAT 610 Exam I
Thursday, 26 September 2024

Instructions.

- You have 75 minutes.
- You may use a stand-alone calculator, but no other resources.
- Please write your solutions on separate sheets of paper, and *return them in order* with this sheet on top.
- You may use any results from class or homework as long as you explain clearly the result you are using (by name or description).
- To ensure credit, explain or show all of your steps.
- There are 60 points total.
- Attempt all parts of a question; some may not rely on successful completion of earlier parts.

1. (10 points) The mean and variance of $X \sim \text{binomial}(n, p)$ are np and $np(1-p)$, respectively. What is $\mathsf{E}(X(X+1))$? Try to simplify.

2. Let $Y \sim \text{uniform}(1, 3)$.

   (a) (10 points) Find the pdf for $V = \sqrt{Y}$.

   (b) (10 points) Find $\mathsf{E}(V)$ and $v_{.50}$ (the .50-quantile for V). Are they the same value?

3. 80% of connections to a certain server are of Type A and the other 20% are of Type B. Let T be the lifetime of a connection.

   For Type A, $T \sim \text{exponential}(1)$: $\mathsf{P}(T \leq t|\text{Type A}) = 1 - \mathrm{e}^{-t}$.
   For Type B, $T \sim \text{gamma}(2, .5)$: $\mathsf{P}(T \leq t|\text{Type B}) = 1 - (1 + 2t)\mathrm{e}^{-2t}$.

   (a) (10 points) Provide the cdf and pdf for a randomly chosen connection. That is, give the unconditional $\mathsf{P}(T \leq t)$ and then determine the pdf.

   (b) (10 points) The moment generating function for T is $M_T(s) = \frac{.8}{1-s} + \frac{.2}{(1-.5s)^2}$ for $s < 1$. Use this to find $\mathsf{E}(T)$.

4. (10 points) For a giveaway promotional event, a store randomly selects 40 pairs of shoes from its stock of 1000 pairs. The stock consists of 400 Hoka pairs, 350 New Balance pairs and 250 Saucony pairs. Give expressions for (a) the chance that x Hoka pairs, y New Balance pairs and z Saucony pairs are selected and for (b) the chance at least w Hoka or New Balance pairs are selected. Be complete.

(distribution formulas on next page)

Some common probability density functions and probability mass functions are provided below.

beta(α, β) $f(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}$, for $0 < x < 1$, $\alpha > 0$, $\beta > 0$.

binomial(n, p) $f(x) = \binom{n}{x} p^x (1-p)^{n-x}$, for $x = 0, 1, \ldots, n$; $0 < p < 1$.

exponential(β) $f(x) = \frac{1}{\beta} \mathrm{e}^{-x/\beta}$, for $x > 0$; $\beta > 0$.

gamma(α, β) $f(x) = \frac{1}{\beta^\alpha \Gamma(\alpha)} x^{\alpha-1} \mathrm{e}^{-x/\beta}$, for $x > 0$; $\alpha > 0$, $\beta > 0$.

geometric(p) $f(x) = p(1-p)^{x-1}$, for $x = 1, 2, \ldots$; $0 < p < 1$.

hypergeometric(n, N, M) $f(x) = \binom{M}{x}\binom{N-M}{n-x} / \binom{N}{n}$, for $x = 0, 1, \ldots, n$; $n, M, N > 0$.

negative binomial(k, p) $f(x) = \binom{k+x-1}{k-1} p^k (1-p)^x$, for $x = 0, 1, 2, \ldots$; $0 < p < 1$, $k > 0$.

normal(μ, σ^2) $f(x) = \frac{1}{\sqrt{2\pi}\sigma} \mathrm{e}^{-(x-\mu)^2/2\sigma^2}$, for $-\infty < x < \infty$; $-\infty < \mu < \infty$, $\sigma^2 > 0$.

Poisson(λ) $f(x) = \frac{\lambda^x}{x!} \mathrm{e}^{-\lambda}$, for $x = 0, 1, 2, \ldots$; $\lambda > 0$.

uniform(a, b) $f(x) = \frac{1}{b-a}$, for $a < x < b$; $a < b$.

Weibull(γ, β) $f(x) = \frac{\gamma}{\beta} x^{\gamma-1} \mathrm{e}^{-x^\gamma/\beta}$, for $x > 0$; $\gamma > 0$, $\beta > 0$.