STAT610 Assignment 6
due Wednesday, 15 October 2025

1. Let X have the *Laplace* distribution (recalling Problem 3 of Assignment 4), with pdf
$$f(x) = \frac{\lambda}{2}\mathrm{e}^{-\lambda|x|}, \quad \text{all } x.$$
Now suppose $a > 0$ and $b \in (-\infty, \infty)$. Let $Y = aX + b$.
    (a) Find the pdf for Y.
    (b) Show that X has mgf $M_X(t) = \frac{1}{1-t^2/\lambda^2}$. Hint: integrate two halves separately and then combine.
    (c) Use the mgf and a property of mgfs (Theorem 2.28 in the notes) to obtain the mgf for Y.

2. Determine the hazard functions for each of the following (see Section 3.4 in the notes).
    (a) The gamma$(2, \beta)$ distribution. (The cdf can be expressed explicitly for this case.) Use $\beta = 1, 2, 5$ for the plot (all together in one).
    (b) The distribution with pdf $f(x) = \frac{1}{3}\mathrm{e}^{-x} + \frac{4}{3}\mathrm{e}^{-2x}$ for $x > 0$. [This is a so-called *mixture* of two exponential pdfs.]

3. Let T be a positive random variable with hazard rate $h(t)$.
    (a) Find the quantile function for T and identify $\mathsf{med}(T)$ in terms of $H(t) = \int_0^t h(x)dx$. Apply to Weibull(γ, β) which has cdf $F_T(t) = 1 - \mathrm{e}^{-t^\gamma/\beta}$ for $t \geq 0$.
    (b) Consider the "U-shaped" $h(t) = .5t^{-.5} + 6t^2$. When is the failure rate at its lowest? Find the pdf.

4. Identify each of the following as defining a location family, a scale family or a location-scale family (if any). (Note: the given parameters are not necessarily location or scale.) Determine the member of the family with mean = 0 (if location family), variance = 1 (if scale family) or both (if location-scale family).
    (a) The uniform(a, b) distributions.
    (b) The Laplace distributions of Problem 1(a).
    (c) The Weibull(γ, β) distributions with $\gamma = 2$ fixed.

5. Suppose X has pdf $f_X(x) = \frac{1}{s}g((x-c)/s)$ from a location-scale family with location parameter c and scale parameter s (and "standard" pdf $g(y)$). Assume $\mathsf{E}(X^2) < \infty$.
    (a) Show that $\mathsf{E}(X)$ is linear in c and s, and that $\mathsf{Var}(X)$ is proportional to s^2 but independent of c.
    (b) How does the m-th *central moment* depend on c and s?

6. Chapter 3 Exercise 28(c–e).

7. Chapter 3 Exercise 33(b). Note: $\theta \in (-\infty, \infty)$. Also, plot $w_2(\theta)$ versus $w_1(\theta)$.

8. Show that the following are *not* exponential families.
    (a) The uniform(a, b) distributions.
    (b) The (location-scale) logistic(μ, β) distributions. (See Example 3.10 in the notes.)