610 Assignment 1

Partial solutions for selected problems

Note: I highly recommend that you do not just read the solutions but rather that you *fix your mistakes* by carefully redoing the problems you missed. This will help you avoid similar mistakes in the future. If you still have questions, please come see me or the grader.

These solutions are not meant to be complete. In some cases only the final answer is shown, with no derivation and not calculated entirely. In others, the main ideas are provided but they may not include all the necessary details or the full explanation and justification that would be expected of you. Some problems or parts are not shown.

These solutions might also suggest shortcuts or notational devices that could improve your presentation for later work.

1. Exer. 1.4. (a) and (c) are the same thing: union.

   Exer. 1.5(b) Note: $B \subset C$, which simplifies things slightly. By the multiplication rule, $\mathsf{P}(A \cap B \cap C) = \mathsf{P}(A|B \cap C)\mathsf{P}(B|C)\mathsf{P}(C) = \frac{1}{540}$.

2. (b) There are just 4 events in this σ-algebra: $\{\emptyset, \{a\}, \{b, c\}, \mathcal{S}\}$. Merely containing a does not make it one of these events: observing an event is not the same as observing precisely which outcome in the event is the actual experimental result.

3. (a) The sample space can be expressed in two different ways, but it is best to take the candies' perspective: there are $3^4 = 81$ equally likely outcomes (x_1, x_2, x_3, x_4) with $x_i \in \{1, 2, 3\}$, identifying which child gets each of the four candies. There are $\binom{4}{2} = 6$ choices for which two candies are given to the same child and 3 choices for that child. Then there are 2 ways to hand out the other two candies. So the first probability is $\frac{6\times3\times2}{81}$. Three outcomes are that all four candies are given to the same child. Hence the second probability is $\frac{3}{81}$.

   (b) $\dfrac{\binom{2}{x}\binom{3}{2-x}}{\binom{5}{2}}$, $x = 0, 1, 2$.

4. To do this by induction (recursion), you must be explicit in showing that the recursion works. The case $n = 2$ is given in Thm. 1.1.4.d in the book. Now assume the result holds for all cases $2, \ldots, n$. Then, for example, by using the n and 2 cases,

$$\left(\bigcap_{i=1}^{n+1} A_i\right)^c = \left(\bigcap_{i=1}^{n} A_i \cap A_{n+1}\right)^c = \left(\bigcap_{i=1}^{n} A_i\right)^c \cup A_{n+1}^c = \left(\bigcup_{i=1}^{n-1} A_i^c\right) \cup A_{n+1}^c = \bigcup_{i=1}^{n+1} A_i^c.$$

   So the result holds for the $n+1$ case. (Use a similar argument for the other DeMorgan rule.)

5. (b) Use an indicator function to integrate the value 0 as needed, in this case when $t < 0$. Specifically, for any real $a < b$,

$$\mathsf{P}(a < X \le b) = \frac{1}{10}\int_a^b \mathrm{e}^{-t/10} 1_{[0,\infty)}(t)\, \mathrm{d}t.$$

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6. (a) $1 - \mathsf{P}(\text{exactly 1 selection}) = 1 - \frac{K-j}{K}$.
   (b) $\frac{j}{K} \times \frac{K-j}{K-1}$.
   (c) Do not confuse "at least i" with "exactly i". There are at most $j+1$ selections. By considering what happens at each selection, the probability it takes exactly i selections is
$$\left(\prod_{m=1}^{i-1} \frac{j-m+1}{K-m+1}\right) \frac{K-j}{K-i+1},$$
   which may be simplified using factorials.

7. (a) $\binom{2309}{4}$.
   (b) For the event to occur, each game reduces the available words by 4, and then 4 more are to be selected from what remains for the next game. So the number of ways no word is repeated is $\prod_{i=0}^{k-1} \binom{2309-4i}{4}$, and the probability at least one word is repeated is $1 - \prod_{i=0}^{k-1} \frac{\binom{2309-4i}{4}}{\binom{2309}{4}}$.
   (d) $k = 15$ gives probability 0.5201.