$P(x|y) = \frac{P(y|x) \cdot P(x)}{P(y)}$ $\quad$ $P(x_t | z_{1:t}, u_{1:t}) = \frac{P(z_t | x_t, u_{1:t}, z_{1:t-1}) \cdot P(x_t | u_{1:t}, z_{1:t-1})}{P(z_t | u_{1:t}, z_{1:t-1})}$ ---

$= \left\langle \frac{\text{Likelihood of Observation} \cdot \text{Prior Belief}}{\text{Normalization Factor } \eta} \right\rangle$

Consider $\eta = P(z_t | u_{1:t}, z_{1:t-1})$

$P(x_t | z_{1:t}, u_{1:t}) = \eta \underbrace{P(z_t | x_t, u_{1:t}, z_{1:t-1})}_{\text{Sensor Model}} \cdot P(x_t | u_{1:t}, z_{1:t-1})$

The Likelihood of Observation given State & Control is Sensor Model

Considering the Sensor Model is a Markov Process

$P(z_t | x_t, u_{1:t}, z_{1:t-1}) = P(z_t | x_t) \rightarrow$ Current Measurement only depends on current state.

$P(x_t | z_{1:t}, u_{1:t}) = \eta P(z_t | x_t) \underbrace{P(x_t | u_{1:t}, z_{1:t-1})}_{\text{Applying Marginalization here}}$

$P(x_t | u_{1:t}, z_{1:t-1}) = \int P(x_t | x_{t-1}, z_{1:t-1}, u_t) \cdot P(x_{t-1} | z_{1:t-1}, u_{1:t}) \, dx_{t-1}$

$= \int P(x_t | x_{t-1}, u_t) \cdot \cancel{Bel} \, P(x_{t-1} | z_{1:t-1}, u_{1:t}) \, dx_{t-1}$

$= \int P(x_t | x_{t-1}, u_t) \, \underbrace{P(x_{t-1} | z_{1:t-1}, u_{1:t-1})}_{\text{Prior Belief}} \, dx_{t-1}$

$Bel(x_t) = \eta P(z_t | x_t) \cdot \int P(\cancel{Bel} \, P(x_t | x_{t-1}, u_t) \cdot Bel(x_{t-1}) \, dx_{t-1}$

---

2 Steps

Prediction $\overline{Bel}(x_t) = \int \overbrace{P(x_t | x_{t-1}, u_t)}^{\text{Action Model.}} \cancel{dx_{t-1}} \, Bel(x_{t-1}) \, dx_{t-1}$

Correction $Bel(x_t) = \eta \underbrace{P(z_t | x_t)}_{\text{Observation model}} \cdot \overline{Bel}(x_t)$