Surplus relativity & distribution

An alternative insight into the theory of surplus

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This paper examines the flaws in the current treatment of surplus under standard theory and proposes a new framework of thought to address such issues. Central to this new framework is the notion of surplus relativity which proposes that we cannot consider surplus in markets independently. In order to have a correct theoretical stance on surplus we must address the opportunity cost of choice.

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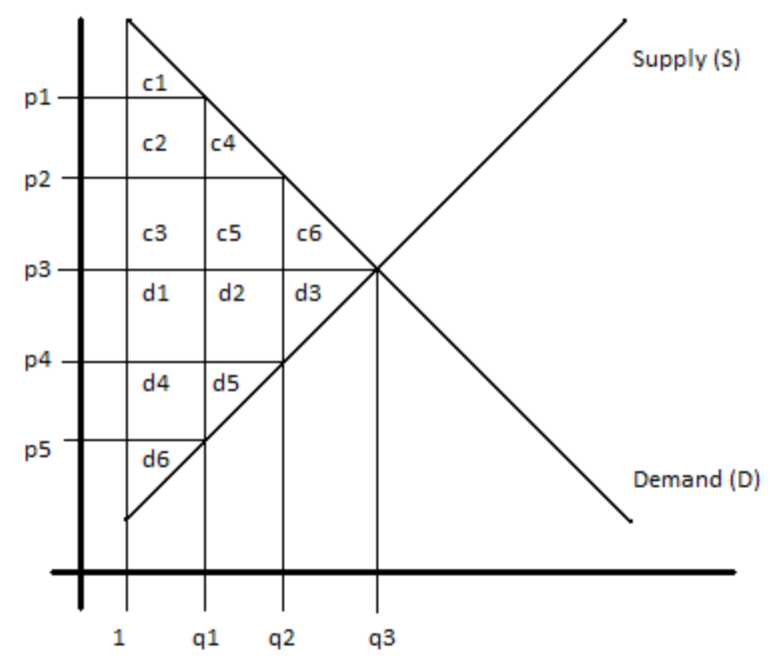
Introduction

Consider the diagram illustrated in fig.1. According to standard theory given a finitely divisible good the following supply and demand map will result in equilibrium (q3, p3). Consumer surplus is the difference between what consumers are willing and able to pay and what they actually pay to consume which is equal to $\sum_{i=1}^{6} c_i$. Producer surplus is the revenue producers receive over and above the minimum amount required to induce them to supply the good which is equal to $\sum_{i=1}^{6} d_i$.



Generic Supply & Demand Map Fig.1

Over the course of this paper we shall highlight situations in which such analysis of surplus is not theoretically sound. First we analyse cases in which the distribution of surplus deviates from standard theory; we find scenarios where consumer surplus could be $\sum_{i=1}^{6}(c_i + d_i)$, zero or ambiguous under equilibrium (q3, p3). We then assess to what extent surplus is distorted under standard theory. Finally we turn our attention to the misinterpretation of consumer surplus under standard theory. We show that in the vast majority of markets consumer surplus is underestimated.

1. Map segmentations

We define a segmentation of a (supply and demand) map as the process of turning a single parent map (we shall use the letter *A* to reference the parent map) into multiple child maps (referenced by $a_1,\ a_2,\ \ldots$)whose horizontal summation is equivalent to the parent map.

Specifically, a finite segmentation on map *A* is a set of finite maps $\{a_1, \ldots, a_n\}$ satisfying the following pair of equations, whereby the "+" operator indicates horizontal additional.

$$D_A = D_{a_1} + \ldots + D_{a_n} = \sum_{i=1}^{n} D_{a_i} \tag{1}$$

$$S_A = S_{a_1} + \ldots + S_{a_n} = \sum_{i=1}^{n} S_{a_i} \tag{2}$$

D_k and S_k represent the level of demand and supply in map k respectively.

Let map *A* be the map in fig 1 with the restriction on quantity such that Q = [1, q3] (3)

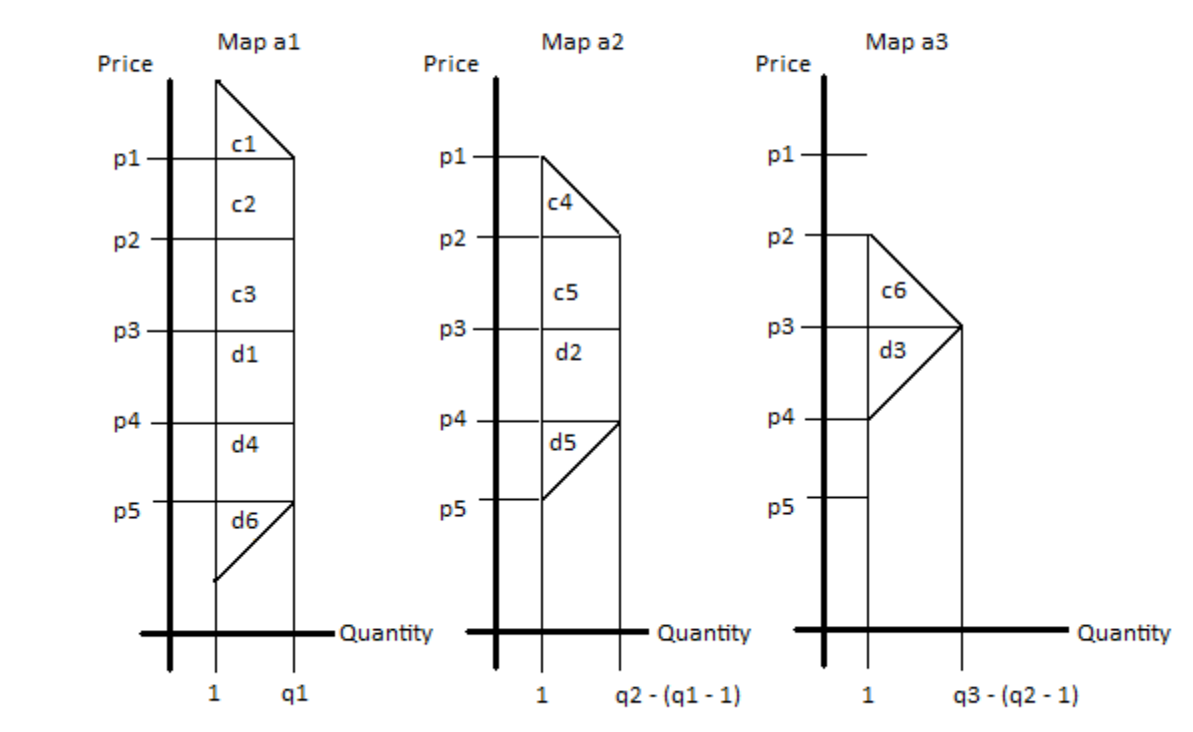
The motivation of this restriction will be discussed later. Then we can define a *three segmentation on A* to be the set of maps $\{a_1, a_2, a_3\}$ satisfying the following set of equations:

$$a_1 = \begin{cases} D_1 = \begin{cases} p = D,\ q = [1, q_1) \\ p = [p_5, p_1],\ q = q_1 \end{cases} \\ S_1 = \begin{cases} p = S(q),\ q = [1, q_1) \\ p = [p_5, p_1],\ q = q_1 \end{cases} \end{cases} \tag{4}$$

$$a_2 = \begin{cases} D_2 = \begin{cases} p = D,\ q = [1, q_2 - q_1 + 1) \\ p = [p_4, p_2],\ q = q_2 - q_1 + 1 \end{cases} \\ S_2 = \begin{cases} p = S,\ q = [1, q_2 - q_1 + 1) \\ p = [p_4, p_2],\ q = q_2 - q_1 + 1 \end{cases} \end{cases} \tag{5}$$

$$a_3 = \begin{cases} D_3 = \{\ p = D,\ q = [1, q_3 - q_2 + 1] \\ S_3 = \{\ p = D,\ q = [1, q_3 - q_2 + 1] \end{cases} \tag{6}$$

Note how (4-6) satisfy (1-2). To visually imagine this, draw the maps defined by (4-6) then horizontally add these maps, the result will be the map in fig 1 on the quantity restriction $q = [1, q_3]$ i.e. Map *A* as required. The segmentation of *A* into child maps is shown in fig. 2.



Segmentation of map A in maps {a1, a2, a3} Fig.2

1.1. Sequential Output Strategy (SOS)

Assumption[1] 1: demand does not vary between strategies.

Define a sequential output strategy as the rules governing the sequential release of output until price and output converge to the long run equilibrium. There will necessarily be a host of short term equilibria on route to reaching the long run equilibrium.

Let our firm release output sequentially. We will consider a *three sequence*; the firm will release three quantities of supply which in total will lead to the conventional equilibrium in fig.1. Let $m \subseteq \mathbb{Z}$ denote the range of the sequence, since we are considering a three sequence $m = [1,2,3]$. If $n \in m$ then *n* indicates the n^{th} part of sequence m, each element in m gives the firm a particular output strategy. Consider the following sequential output strategy:

[1] This assumption will be scrutinized and loosened later

$$n = 1 : Supply\ q = (q_1 - 1)$$

$$n = 2 : Supply\ q = (q_2 - q_1)$$

$$n = 3 : Supply\ q = (q_3 - q_2)$$

Whereby strategy $n + 1$ is undertaken once strategy n is complete i.e. supply more output only when the current market clears. Let us put this Sequential Output Strategy in action and assess its implications:

1. Executing strategy $n = 1 : Supply\ q = (q_1 - 1)$ the equilibrium formed in map $a_1$appears arbitrary since there exist a continuum of equilibria in the price interval $p = [p_5, p_1]$, however, since a_1 is a segment of map *A* we can consult *A* to find a unique equilibrium. To do this fix supply at $q_1 - 1$, the equilibrium price is p_1. The associated consumer welfare is given by c1 from fig.1 whilst producer welfare is (c2 + c3 + d1 + d4 + d6). Note how producer welfare is calculated here, we fixed supply at $q = (q_1 - 1)$ then compared the equilibrium price with what the producer was willing and able to supply *all units up to and including* $q = (q_1 - 1)$ at.

2. Once this market clears both supply and demand will decrease. Specifically, the resulting market formed will be the map $A - a_1 = a_2 + a_3$ where "-" is horizontal subtraction. Since strategy n is complete, strategy $n = 2 : Supply\ q = (q_2 - q_1)$ is undertaken. Using an analogous argument to the one above we find equilibrium price to be p_2. Since $(q_2 - q_1)$ is supplied, the only relevant segment of the resulting map is a_2. Consumer welfare in a_2 at equilibrium $(q_2 - q_1, p_2)$ is the area c4 whilst producer surplus is (c5 + d2 + d5).

3. Once a_2 is cleared adopt strategy $n = 3 : Supply\ q = (q_3 - q_2)$. The resulting map is $A - a_1 - a_2 = a_3$. With this map we do not need to consult *A* for a

distinct equilibrium since the equilibrium in map a_3 is already unique. Consumer welfare in a_3 at equilibrium $(q_3,\ p_3)$ is given by c6 whilst producer surplus is d6.

1.2. The implications of sequential output strategy[2]

In our example the sequential output strategy lead to falling prices until equilibrium was reached. The most significant implication of such strategy lies in the distribution of surplus for consumers and producers. For $n = 2, 3$ there was a disproportionate distribution of consumer and producer surplus. Specifically, producer surplus was disproportionately high in maps $a_1\ and\ a_2$ compared with the analysis of equilibrium $(q_3,\ p_3)$ in isolation. Table.1 highlights these facts.

Table.1	Surplus analysis on $(q_3,\ p_3)$ alone	Surplus under sequential output strategy
Consumer surplus	c1 + c2 + c3 + c4+ c5 + c6	c1 + c4 + c6
Producer surplus	d1 + d2 + d3 + d4 + d5 + d6	d1 + d2 + d3 + d4 + d5 + d6 + c2 + c3 + c5

It was only in map a_3 that producer and consumer surplus coincide with surplus analysis on equilibrium $(q_3,\ p_3)$ alone. This is no coincidence; we leave the general form of this result as a conjecture.

Conjecture 1: Given the finite segmentation of map *A* generating child maps $\{a_1,\ \dots,\ a_n\}$ with the application of sequential output strategy, the only map in which consumer and producer surplus coincide with surplus analysis on equilibrium $(q_3,\ p_3)$ in isolation is map a_n.

Analysing surplus based on the long run equilibrium $(q_3,\ p_3)$ in isolation overlooks the *process* by which the equilibrium is reached. As we have seen, the process by which

[2] Assuming the number of units supplied at each strategy is less than long run equilibrium output

equilibrium is reached is instrumental in defining the *distribution* of surplus. If the firm initially sets price or quantity to equilibrium values than traditional analysis holds but if the firm does not initially conform to equilibrium conditions, and as we have seen under sequential output strategy they have incentive not to, then the traditional treatment of surplus is incomplete. A complete treatment of the distribution of surplus would take into consideration:

1. The process of converging to the long run equilibrium (A.1)
2. Surplus post long run equilibrium (A.2)

Standard theory concerns itself with only the latter. To consider the extent to which concentration on post equilibrium analysis alone distorts the distribution of surplus we turn our attention to the lifespan of economic maps.

2. Map Longevity

Economic maps are implicitly referenced with time, for example, a map of the *annual* fruit market or a map of *weekly* grocery shopping in a particular area. We make the reference between a map and time explicit using the symbol *t*. If we have a map of an annual market then symbolically this is equivalent to the expression $A(1\ year)$. More generally, given an arbitrary time referenced map we denote it by $A(t)$. Let $A_i(t)$ fully describe a map where $i \in \mathbb{Z}_+$represents the number of maps that will exist through the lifespan of market[3] $A(t)$. Finally let t^* denote the time required for the long run equilibrium to be reached.

E.g. Consider $A_{10}(1)$: Market *A* is described yearly and the market is set to exist for 10 periods, its lifespan is thus 10*1 = 10 (years). Also let $t^* = 1$(year). It takes 1 year for the long run equilibrium to be reached therefore only one of the ten maps contains disproportionate surplus, the remaining nine maps fall in line with the usual surplus analysis. Thus method (A.2) alone would be just over[4] 90% accurate.

Let *k* denote the difference between the lifespan of a map and its equilibrium convergence time i.e. $k = i \bullet t - t^*$ and let the symbol $\bar{x}$ fix the arbitrary variable x then we can perform the following partial analysis:

[3] In reality knowing how long a certain market will exist is unclear, however, for our subsequent analysis we only require estimates.

[4] Conjecture 1 tells us that in the first map method [B] accurately gives surplus from at least the final segmented map of *A*

$$\lim_{i \to \infty} (i \bullet \bar{t} - \bar{t^*}) \to \infty \qquad (7)$$

$$\lim_{t \to \infty} (\bar{i} \bullet t - \bar{t^*}) \to \infty \qquad (8)$$

$$\lim_{t^* \to \infty} (\bar{i} \bullet \bar{t} - t^*) \to -\infty \qquad (9)$$

Equation (7) states that *k* tends to infinity as a result of an ever increasing array of maps. In this case[5]: $(7) \Rightarrow (A.2) \to (A.1) \bigcup (A.2)$ i.e. Method (A.2) of surplus analysis tends to the complete distributional method of surplus analysis.

Equation (9) states that *k* tends to negative infinity as a result of an ever increasing convergence time. In this case method (A.2) is arbitrarily inaccurate.

Equation (8) states that *k* tends to infinity as the time reference for the map tends to infinity, for example, suppose there is only one map $(i = 1)$ and its time reference is arbitrarily large then since convergence time is fixed, convergence time is relatively arbitrarily small which implies a market in which once the product is priced some value other than equilibrium price, it converges to the equilibrium price almost instantaneously. Thus (8) $\Rightarrow (A.2) \to (A.1) \bigcup (A.2)$. We can establish the following propositions based on the implications of this partial analysis:

Proposition 1: The longer the lifespan of map $A_i(t)$, where lifespan = $i \bullet t$, relative to equilibrium convergence time, given by t^*, the greater the accuracy of using method (A. 2) alone in determining the distribution of surplus.

Proposition 1.1: The accuracy of judging surplus from method (A.2) alone diminishes as the lifespan of map $A_i(t)$ relative to equilibrium convergence time reduces.

Proof: Proposition 1.1 is the contrapositive of theorem 1 therefore the two theorems are logically equivalent.

∎

3. Sequential price strategy (SPS)

The counterpart to the sequential output strategy is the sequential price strategy. Both strategies lead to the same long run equilibrium but sequential price strategy makes for

[5] Note: the OR symbol indicates the inclusion of both method (A) and (B)

intuitive analysis. First let us make some standard assumptions before delving into this new strategy.

Assumption 1: Consumers purchase if and only if their surplus from consumption is nonnegative.

Assumption 2: Given the supply and demand functions in figure 1 the firm realises the long run equilibrium in output is $(\mathrm{q}_3 - 1)$

Assumption 3: The objective of the firm is to maximise surplus

Under this strategy, the firm will initially produce and auction off units analogous to a Dutch auction. Note that if the firm were to produce more than $(q_3 - 1)$ units, its surplus will decrease thus highlighting the need for assumption 2. In our example the firm will initially set price to $p_1$for which units sold equals $(q_1 - 1)$. If we consider fig 1 in isolation then it appears that the market does not clear at this high price but the market does clear when considering map a_1. Once cleared the firm sets a lower price at p_2 this clears the map a_2 at quantity $q_2 - q_1 + 1$. Finally the firm sets the long run equilibrium price p_3 which clears the remainder of the total quantity produced.

The distribution of surplus under this strategy is equivalent to the distribution of surplus under its counterpart, the sequential output strategy. The firm once again obtains the lions share of surplus at the expense of consumers.

Now consider a variant of the sequential price strategy that starts off with a price higher than all consumers reservation price[6]. The price is then lowered in sufficiently small quantities such that price passes through every consumers reservation price until long run equilibrium is established. The implication of such strategy under assumption 1 is (intertemporal) first degree price discrimination, which in turn implies the firm captures all surplus. Let us describe this concept rigorously.

Let the reservation price of consumers be represented by the well-ordered set $R = \{p_1^r, \ldots, p_\Omega^r\}$ where p_1^r is the smallest reservation price and $p_\Omega^r$is the greatest reservation price such that $p_1^r \geq p^*$where $p^*$is the long run equilibrium price. A sequential price strategy describes a set of pricing actions for the firm to undertake. Let the pricing actions be represented by the well-ordered set $F = \{p_1^f, \ldots, p_\Omega^f\}$ where $p_1^f$is the minimum and $p_\Omega^f$is the maximum price to set. Then the sequence of pricing occurs by

[6] Equivalent to the maximum price a buyer is willing to pay for a good or service

initially setting $p = p^f_\Omega$ then lowering the price to the second largest element of F and so on until price converges to p^f_1. Let C^s and P^s refer to consumer and producer surplus as a proportion of total surplus.

Proposition 2: $R \subset F \;\Rightarrow$ Intertemporal first degree price discrimination.

Corollary 1: $R \subset F \;\Rightarrow \begin{cases} P^s = 1 \\ C^s = 0 \end{cases}$

The analysis done so far should have raised the following practical question: in reality how would the firm know when a particular map has been cleared and when to move on to another price level? A proxy for map clearance would be halting sales i.e.

$$\frac{d(Sales)}{dt} \to 0 \qquad (10)$$

But if condition (10) determines which strategy the firm adopts, in particular when to lower price, then consumers could alter the way act to benefit from such proxy. Specifically, consumers could form a counter strategy whereby condition (10) is *artificially* induced so as to reclaim surplus from the firm. We shall call such a strategy a counter-sequential price strategy.

Some time ago many people felt that leaving the light on, the tap running or the television on standby would not have significant consequences other than on utility bills. It was not until these consequences manifested themselves in the form of global warming that people began to realise the aggregate impact of their actions and only then understand the significance of their own individual contribution to it. Analogously, although consumer decisions appear to be largely independent of each other at this moment in time, if consumers knew the extent to which utilising interdependence would affect their wellbeing they may revise their actions accordingly. For this reason we shall consider counter strategies based on consumers utilizing this interdependence.

3.1. Counter-sequential price strategy (CSPS)

If consumers are to react to a sequential price strategy they must be motivated by more than nonnegative surplus alone. We will first consider the polar case where consumers seek to maximise surplus.

Assumption 1: The objective of the consumers is to maximise surplus

If condition (10) is induced then consumers would eventually observe a fall in price. Based on the knowledge that there is some minimum price for a given quantity of supply they could wait until price reaches this minimum and then consume so as to claim *all* surplus. Mathematically we have:

$$\frac{d(Sales)}{dt} \to 0 \Rightarrow \frac{dp}{dt} \leq 0 \qquad (11)$$

$$\lim_{t\to\infty}\left(\frac{dp}{dt}\right) = 0 \Rightarrow p = p_{a_i}{}^{min} \qquad (12)$$

Where $p_{a_i}{}^{min}$ is the minimum price required to supply a given quantity in map a_i. In every map a_i consumers wait until price is minimised until consuming. The impact on surplus is the exact opposite of intertemporal first degree price discrimination.

For example, consider the segmentations shown in fig 2. The counter strategy for (n = 1) would be waiting until price converges from p_1 to $p_6 = p_{a_1}{}^{min}$. For (n = 2) wait until price converges from p_2 to p_5 and finally for (n = 3) the sequential price strategy and its counterpart coincide thus no action is required. If this were to occur in the real world we would observe price rising until the equilibrium is reached. If every consumer's objective is to maximise surplus then they would *all* attempt to counter strategy (n = 1) since countering strategy (n > 1) would yield less surplus. But since demand outstrips supply at price $p_6 = p_{a_1}{}^{min}$ not all consumers will be able to counter strategy (n = 1). The explicit rivalry involved in executing the CSPS (n = 1) leads us to ask an important question: how do consumers interact with each other? We shall first consider the polar cases of consumer interaction: consumer unions and perfectly competitive consumers.

3.2. CSPS with consumer unions

If consumers could form a union and act in unison then Distributional issue only: show that total consumer is unaffected by who gets what surplus. How to distribute the gains of objective surplus? Compensation method: equality; everyone pays the average price. Under such agreement consumers are indifferent as to which SPS to counter; it resolves the issue of rivalrous counter strategies.

Practicalities: setting up the union, trust (contracts). Too much hassle.

3.3. CSPS with competitive consumers

Nash equilibrium tends to equilibrium price as consumers overcut each other's valuations. As the degree of competitiveness amongst consumers increases the Nash equilibrium in reservation price converges to equilibrium price. Implication: the distribution of surplus tends to standard theory.

The real world: consumers have limited information. Some guess equilibrium values better than others but on the whole the long run price is not known with certainty. Consumers may be aware of the interdependency of their economic actions but still disregard it when making economic decisions because they perceive their actions as having an insignificant impact on the market.

Theoretically this is fine but in reality how would consumers know p^{min} ? G

Issues to address: assumption issues

Good must be <u>durable</u>: although we are explicitly dealing with state-contingent issues the transition from one state to another in the real world evolves over time: arbitrarily long shelf-life.

Market must be <u>monopolised</u>: such that there is no competition

Interesting idea: Firm has a dynamic <u>monopoly</u>: There will be no imitation and distinct substitutes of the product for a given period of time. E.g. let t = [0, T] then there is no product depreciation in the time interval [0, T]. Implication: demand is unaffected in the closed interval t.

Intertemporal demand: why demand changes with time.

4. Higher level counter strategies and Game Theory

Based on (11) and (12) the firm could counter the counter strategy used by the consumers (CSPS): if it keeps the price fixed at a certain level for long enough it may fool consumers into believing p^{min} is reached thereby inducing consumption. We call such strategy by the firm a counter counter-sequential price strategy (CCSPS). However the consumers could counter the CCSPS analogously and so on. Let denote h the level of a counter strategy then the first counter strategy (CSPS) to the sequential price strategy refers to $h = 1$. The firms levelling response to the CSPS is the CCSPS, which refers to $h = 2$. In general h is a subset of the positive integers such that $h = \{1, 2, 3, \ldots\}$. As $h \to \infty$ there is stalemate where neither the firm nor the consumers are willing to compromise on surplus. Note that if h is odd than the consumers 'out-level' the firms' strategy but if h is even then the firm out levels the consumers' strategy. Situations of yielding to the others strategy will be called *fly* whereas never compromising will be called *fight*. Modeling this situation using a two player game where player 1 is the firm and player 2 is the group of consumers there emerges a game similar to that of the *battle of sexes*[7]. We first analyse a game in which the firm can alter price in sufficiently small quantities so as to cover all reservation prices[8].

Game 1		Group of consumers	
		Fight	*Fly*
Firm	*Fight*	A, 0*	B*, 0*
	Fly	0*, B*	C, D

Given[9] $A < 0, B > D,\ B > C$ there exist two pure strategy Nash equilibria (Fight, Fly) and (Fly, Fight). Game 1 captures the polar cases of consumer interaction: (C, D) may represent the respective distribution of surplus under a group of competitive consumers whereas (Fly, Fight) represents the payoffs for consumers working as a union. Theoretically the two PSNE should be consistent with the idea of a steady state. These equilibria suggest that the market gains are attributed to one party or another, not both. However, in reality we observe markets where people feel they obtain "value-for-money" (i.e. consumer surplus) whilst the firm simultaneously sell units above the minimum they are willing and able to supply at. In fact, the two PSNE are the exception in typical markets, not the rule. The problem of this poor explanation of reality may be because we are considering a group of consumers rather than assessing the situation pragmatically for each individual. If we consider Mixed Strategies this allows us to break up the group to distinguish the individuals that form it. In this situation the application of mixed strategies would give us an idea about the proportion of people adopting a certain strategy[10]. Game 1.1 is game 1 with the application of mixed strategies q for consumers and p for the firm.

[7] Reference some games

[8] See proposition 2.

[9] A<0 since if the firm does not sell any output it makes some loss. B is the total market surplus and since its partition is C+D we have B>C, B>D.

[10] Individuals are not limited to deterministic actions; they may select their strategy stochastically.

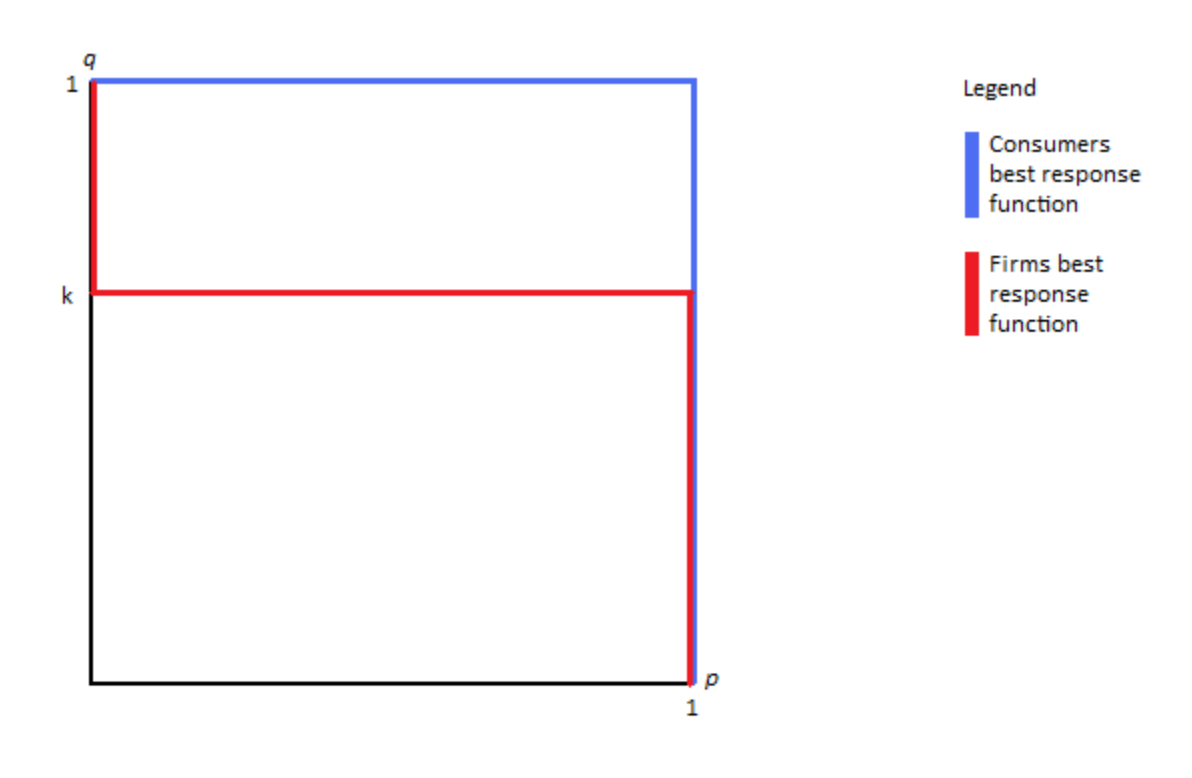
Game 1.1			Group of consumers	
			q	*(1-q)*
			Fight	*Fly*
Firm	*p*	*Fight*	A, 0*	B*, 0*
	(1-p)	*Fly*	0*, B*	C, D

Using the notation $B_i(j)$ to denote the best response function for player i given strategy j we find the best response functions for game 1.1 as follows:

$$B_1(q) = \begin{cases} p \in [0,1] \text{ } if \text{ } q = \frac{B-C}{B-A-C} \\ p = 1 \text{ } if \text{ } q < \frac{B-C}{B-A-C} \\ p = 0 \text{ } if \text{ } q > \frac{B-C}{B-A-C} \end{cases}$$

$$B_2(p) = \begin{cases} q \in [0,1] \text{ } if \text{ } p = 1 \\ q = 1 \text{ } if \text{ } p < 1 \end{cases}$$

The intersection of these best response functions are the set of mixed strategy Nash equilibria (MSNE). The set of pure strategy Nash equilibria are a subset of mixed strategy Nash equilibria i.e. PSNE ⊂ MSNE. Graphing the best response functions allows us to see where the MSNE lie.



Game 1.1 with mixed strategies Fig.3

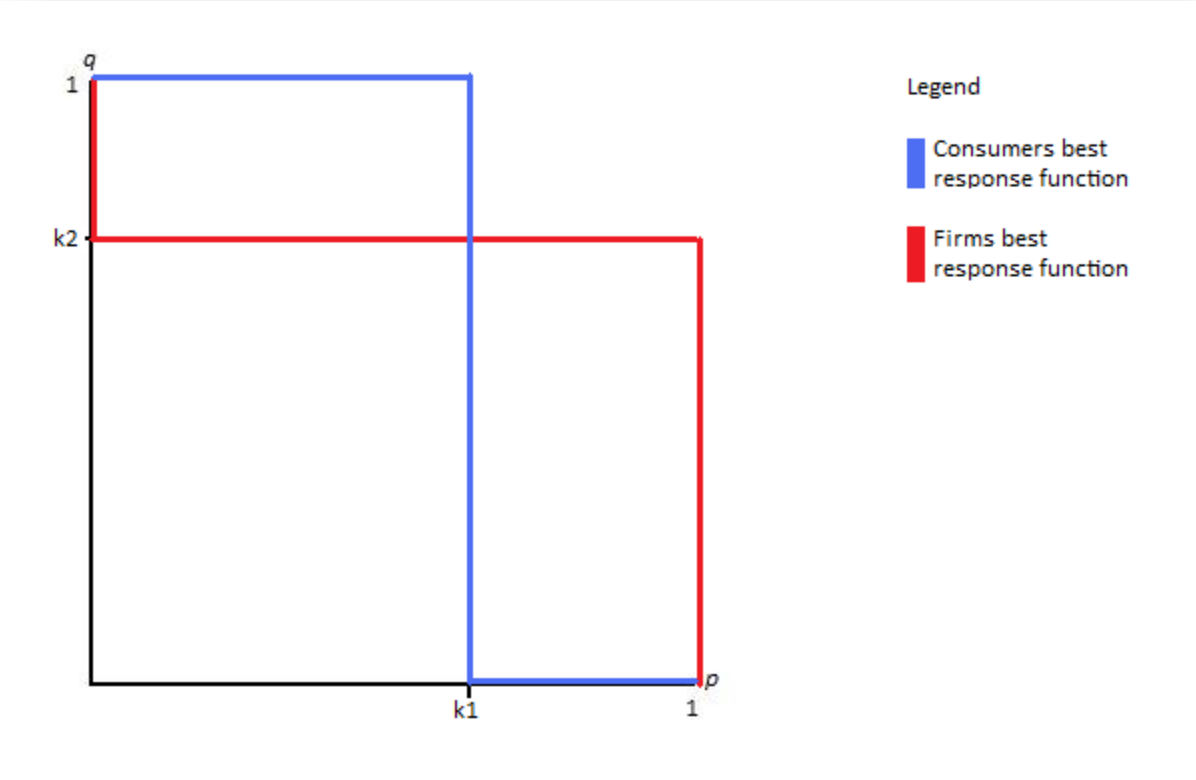
Fig 3 shows that there exists a continuum of MSNE in the interval (1, [0, k]) where $k = \dfrac{B-C}{B-A-C}$. In this interval the firm always fights in which case the market gains (loss) will be obtained by only one player (the firm). Once again the problem persists; the equilibria do not coincide with reality. The problem lies in the specification of game 1. We assumed that the firm can vary price in sufficiently small quantities so as to cover all reservation prices, but such logic is not consistent with reality since there exist costs in altering price among other issues. For this reason we shall assume the set of prices the firm charges (F) is a proper subset of the set of reservation prices (R) consumers hold i.e. $F \subsetneq R$.

Game 1.3			Group of consumers	
			q	*(1-q)*
			Fight	*Fly*
Firm	*p*	*Fight*	A, 0*	**Β*, γ***
	(1-p)	*Fly*	0*, B*	C, D

The payoffs of outcome (Fight, Fly) now change from (B, 0) to (ß, γ) where B > ß and γ > 0. This is because there will be some consumer(s) whose reservation price is above that of the initial price the firm sets and thus they accumulate some surplus. Since at least one consumer accumulates surplus the remainder for the firm falls. The best response functions are calculated as before and graphed in fig 4.

$$B_1(q) = \begin{cases} p \in [0,1] \; if \; q = \dfrac{\beta - C}{\beta - A - C} \\ p = 1 \; if \; q < \dfrac{\beta - C}{\beta - A - C} \\ p = 0 \; if \; q > \dfrac{\beta - C}{\beta - A - C} \end{cases}$$

$$B_2(p) = \begin{cases} q \in [0,1] \; if \; p = \dfrac{B - D}{B - D + \gamma} \\ q = 1 \; if \; p > \dfrac{B - D}{B - D + \gamma} \\ q = 0 \; if \; p < \dfrac{B - D}{B - D + \gamma} \end{cases}$$



Game 1.3 with mixed strategies Fig.4

Fig 4 shows the unique mixed strategy equilibrium lies at E* = (p, q) = (k1, k2) where $k1 = \dfrac{B - D}{B - D - \gamma}$ and $k2 = \dfrac{\beta - C}{\beta - A - C}$

We can interpret E* as proportions of the group of consumers playing fight or fly stochastically and for the firm, the extent to which they yield to consumers. An alternative and intuitive way of interpreting E* is through the idea of compromise. This opens up the realistic possibilities of interaction such as bargaining or satisficing. But is the idea of satisficing consistent with the idea of surplus maximisation? To answer this we shall take a deeper look into the theory of our consumers.

5. Theory of the Consumer

Consumers are persistently faced with an array of choices and they try to select the *best* option consistently. Let a particular individual be faced with the finite choice set $C = \{c_1, \ldots, c_n\}$ consisting of n choices. A consumer can order the elements of C to match his preference via (ordinal) utility. Suppose that element c_k is preferred to c_{k-1}then we can order preference as follows:

$$u(c_1) < \ldots < u(c_n)$$

If the utility of a particular choice is derived on the basis of its surplus then $u(c_i) = f(x)$ where x is the amount of surplus derived from choice c_i. The elements that generate the first and second greatest levels of utility are c_n and c_{n-1} respectively. Suppose that the consumer can only select one element of C then c_n will be selected and choice c_{n-1} represents its opportunity cost. Thus we have:

$$u(c_{n-1}) = f(b) < u(c_n) = f(a)$$

Where a > b. if the price of choice c_{n-1} were to fall then by definition its surplus increases thus $u(c_{n-1}) = f(b + d)$ where d is the increase in surplus attributed to the fall in price. If $b + d < a$ then the consumers choice will be unaffected, however, if $b + d > a$ then his choice will change to c_{n-1}.This suggests that one's choices are formed on the basis of crossing some threshold or optimisation rather than maximisation even though the consumers utility is derived from surplus. In light of the mixed strategy equilibrium E* = (k1, k2) is the notion of compromise with respect to surplus plausible?

5.1. Dynamic utility optimization

Utility is time dependent. Generally speaking the quicker one can consume a good the greater the utility, ceteris paribus, we shall call this concept time-value. Consider the choice c_n. Let this choice be the most preferred in the time interval $T = [0, S]$ and the individual can only select one choice. To define how utility alters with time let us introduce the function g such that $g_i\left(f(x),\ t\right)$ represents person $i's$ utility when consumption occurs at a specific point in time, where $t \in T$. Consider the dynamic utility function in the following three periods.

$$g_i\left(f(a),\ 0\right) = A$$

$$g_i\left(f\left(b\right),\ 1\right) = B$$

$$g_i\left(f(c),\ 2\right) = C$$

Such that $a < b < c$ i.e. surplus is increasing over time due to say falling prices. If S = 2 then surplus maximisation occurs when person i waits two periods before consuming. Than if $A < B < C$ consumer i favours the increase in surplus rather than time-value. Such mentality is consistent with the idea of surplus maximisation. However, if the values $A,\ B,\ C$ are ordered any other way then the consumer will not select a time that maximises surplus but a time that maximises dynamic utility, we call such preference time-preference. This is the main theoretical justification of a MSNE where compromise looks to play a part; consumers have their dynamic utility at heart which may induce outcomes that appear to be compromising. Dynamic utility is a more complete description of consumers' motivation since it can capture the ideas of surplus maximisation but is not limited to it as the above example shows.

5.2. Reality constraints on surplus maximisation

The notion of dogmatic surplus maximisation may be dismissed theoretically if consumers' exhibit time-preference. However, even if a consumer strictly prefers surplus value to time value the constraints of the real world may lead him to make choices that do not maximise surplus; we will take a look at two such cases. (1)Deadlines: Suppose that static surplus is maximised at $t = k$ i.e. $g_i\left(f(x^*),\ k\right) = X^*$ than if there is some deadline, d, by which consumption must occur and this is binding i.e. $d < k$ then the consumer does not have the opportunity to maximise surplus by definition. (2) Incomplete information: Consumers may overestimate the minimum price of a product due to a lack of information. Let the function $h_i(p^{MIN})$ denote person $i's$ hypothesised minimum price a good may fall to then if $h_i\left(p^{MIN}\right) > p^*$ where p^* is an equilibrium price in a particular map, consumer i will not maximise surplus.

6. Surplus relativity

The central idea of surplus relativity lies in the opportunity cost of choice. Pursing one choice may restrict an individual from another choice thus the opportunity cost of a choice presents a benchmark by which candidate choices are judged.

Let consumer $i's$ budget be represented by a budget function $B_i = f(R,\ T)$ such that the budget is a function of resources (including money) and time. Let us place the restriction on B_i by asserting $R = x$ and $t = (0,\delta]$ thus $B_i = f(x,\ (0,\delta])$ i.e. the budget for consumer i is currently[11] x. Furthermore we assume he is surplus oriented.

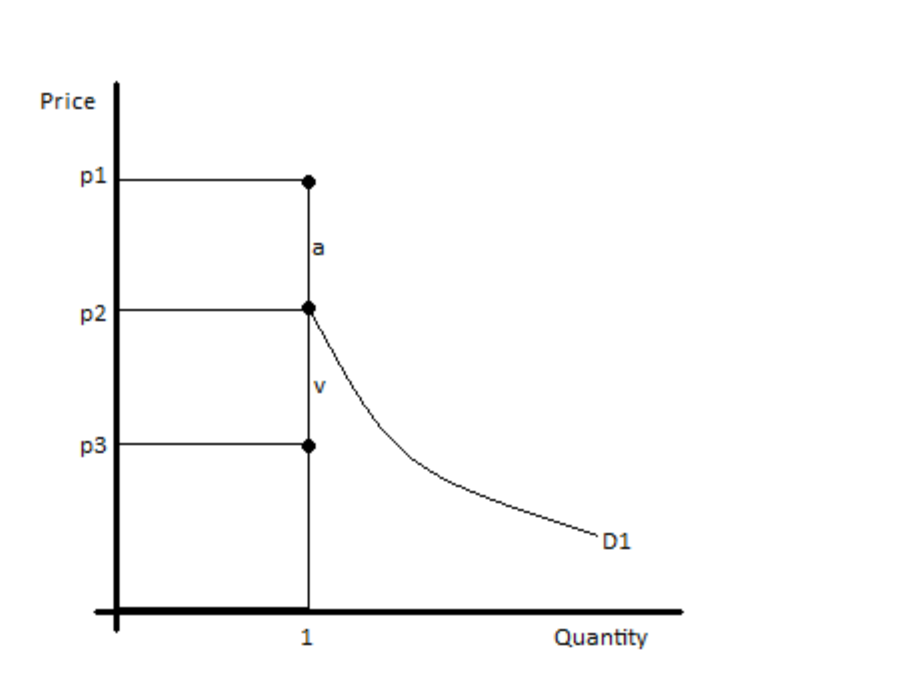
Suppose this consumer is faced with the following set of mutually exclusive[12] choices $C = \{c_1, \ldots,\ c_n\}$ that all cost y.

Given that the choice c_j where $j \in \{1,\ldots,n\}$ yields surplus equal to a. In order for some other choice, say c_{-j}, to be considered it must yield at least a. Let S denote a surplus deriving function then we require $S(c_j) = a \leq S\left(c_{-j}\right) = b$ for demand for c_{-j} to exist.

Therefore any demand that exists for c_{-j} encapsulates its opportunity cost of consumption. Consider the following map of c_{-j} shown in figure 5.

[11] The time interval is assumed to be arbitrarily small thus we can think of it representing the present.

[12] E.g. Choice 1 may be a weekend holiday and choice 2 may be tickets for a Saturday football match; since he cannot be at two places at once he can only select one choice.



Demand for consumer i Fig.5

Consumer $i's$ demand exists at point (1,p2) in the demand curve D1 for the market for c_{-j}.

Suppose that the firm sets some price p1> p > p2 then consumer i will not consume despite exhibiting positive surplus. Now suppose the firm sets price p = p3, then according to standard theory the consumer would only exhibit v as surplus, but due to surplus relativity he actually exhibits (a + v) surplus. If p = p2 then according to standard theory the consumer exhibits zero surplus but in reality he exhibits (p1 - p2 = a) surplus. Therefore consumer surplus is underestimated when the budget is limited in resource and time.

Let us loosen the restriction on the budget. Suppose $B_i = f(R,\ T)$ is sufficiently large in R and T such that $R \rightarrow \infty$ and $T = [0,\infty)$. This allows the consumer to select an arbitrary number of choices. Now if he is presented with the same choice set as before, choices will not be mutually exclusive because there is enough time and resource to undertake all choices. As before let element c_k be preferred to c_{k-1}then we can order preference as follows:

$$u(c_1) < \ldots < u(c_n)$$

Adjusting the ordinal utility function to account for changes in preference over time gives:

$$g_i(\ u(c_1), t_1) < \ldots < g_i(u(c_n), t_n) \qquad t_i \in T \qquad (13)$$

According to our theory of the consumer given the set of choices in (13) his actions will be based on dynamic utility optimisation. Denote the ordered set in (13) set G. The opportunity cost in set $G$is its second greatest element. Let c_j^odenote the opportunity cost in set G at iteration $j$and c_j^* denote an optimal choice at iteration j where iteration $j \in \mathbb{Z}_+$ refers to how many choices have been made. $j = 1$ refers to the first choice. At this juncture the consumer will select choice c_1^* over its opportunity cost c_1^o and c_1^* will be removed from set G. At $j = 2$choice c_2^* will be made where $c_1^o = c_2^*$ and now the new opportunity cost of c_2^* is the third greatest element of the original set G. We can see that the opportunity cost is falling as the number of iterations increases. Suppose that set Gcontains some element for which the individual is neutral i.e. some element that yields zero dynamic utility then as $j$gets sufficiently large the opportunity cost converges to zero[13] as shown below.

$$c_1^o = c_2^* > c_2^o = c_3^* > \ldots > \; c_\Omega^o = 0 \qquad (14)$$

It is only after set $G$is exhausted or emptied that consumer i considers choices on the basis of nonnegative surplus. Surplus relativity ceases to be relevant given a consumer has a sufficiently large budget in time and resource and given his choices are exhausted. In this case any new choice will be taken on the condition of positive surplus, however arbitrarily small and this is what standard theory proposes about the nature of demand. Thus in order to arrive at the conclusion that consumer surplus is the integral bound by demand and price we would have to simultaneously assume that (a) consumers have arbitrarily large budgets in resource and time and (b) all of their previous choices have been exhausted.

The extent to which these assumptions are applicable is market dependent. Markets in which the set of consumers are young retired billionaires may be represented somewhat well by standard theory. However, markets in which consumers have typical constraints on income and time may not be represented well and indeed these markets are the vast majority.

We should also pose the following question: Is set G ever exhausted? Such a question may belong in the realms of philosophy under the guise “can we be content at a period in time?” Let us play devil’s advocate and presume that set G can be exhausted. Even if this

[13] Consumers avoid actions that yield negative surplus thus such choices are not considered in set G

is true there may emerge some other set analogous to G, call it G', that contains more than one element i.e. cases in which demand grows faster than it can be satisfied. For example, a set of choices may be built up over time and thus assumption (b) breaks down. In such cases treatment of consumer surplus would need to address surplus relativity. It must be noted that surplus relativity is not only applicable in situations in which (a) and (b) do not hold but also when they are satisfied. This is because the standard theory of consumer surplus is a proper subset of surplus relativity, that is, the standard treatment of consumer surplus is a special case of surplus relativity.

7. Conclusion

The dynamics by which a price / output equilibrium is reached within a market influences the distribution of surplus. Consumer surplus is likely overestimated when looking at the equilibrium alone.