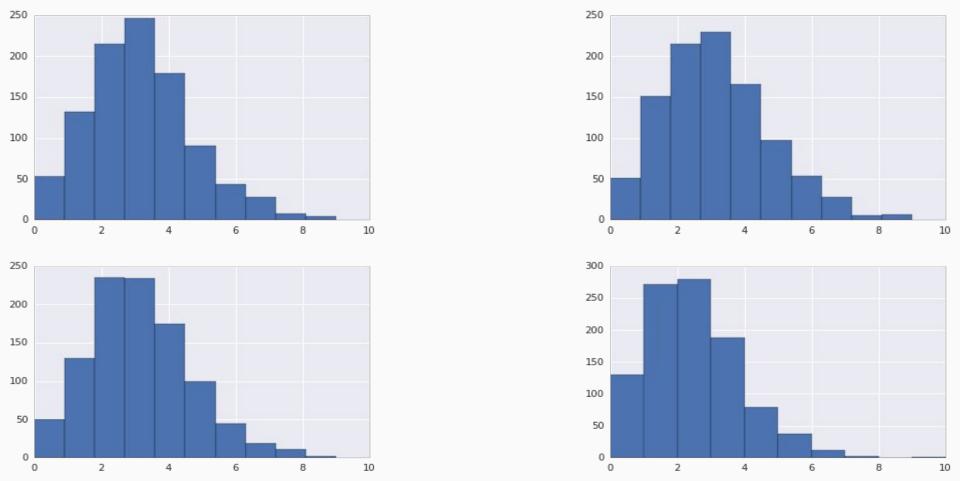
# Rental Car Problem

# **Problem Description**

- 2 Car Renting Locations (Maximum car capacity 6)
- Poisson Distribution for arrivals (mean 2 and 3)
- Poisson Distribution for returns (mean 2 and 1)
- 2\$ for moving car from one location to another
- Maximum 3 cars can be moved in a day
- 10\$ Reward for every rented car

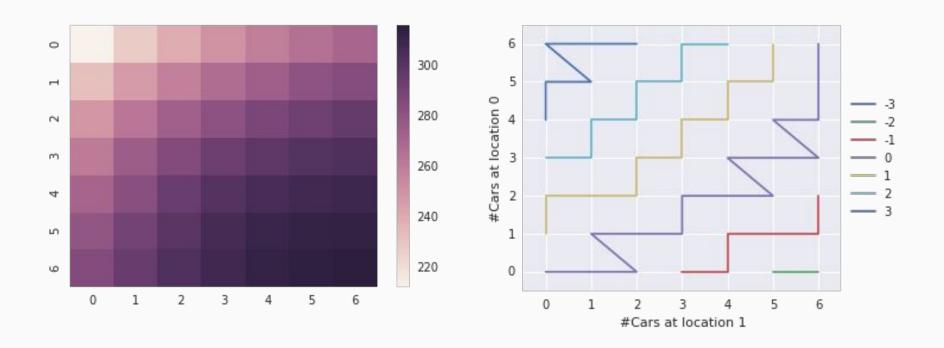


Poisson Distributions for Arrivals and Returns

# Approach

- State: (#Cars at Location0, #Cars at Location1)
- Action: #Cars moved at night
- Algorithm: Policy Iteration

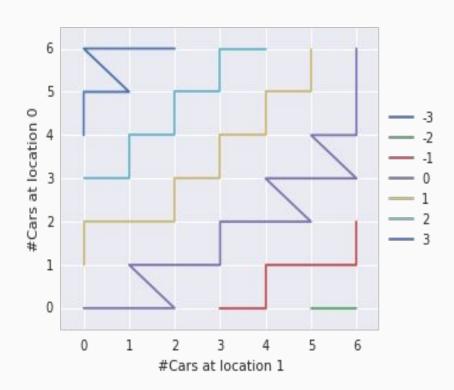
# Optimal Policy & State Values



## **Modified Version**

- Additional Cost of \$3 for parking if #Cars exceeds 3
- 1 car can be moved for free from Location 0 to 1

## What do you expect?



6 #Cars at location 0 6 0 #Cars at location 1

**Original Problem** 

**Modified Version** 

## Observations

- Free transportation of 1 car causes action of moving car from location 0 - 1 to be selected more.
- But additional parking cost didn't have any impact.

# Problem with current approach?

Infeasible for large State Space

#### Can you identify the bottleneck?

#### Policy iteration (using iterative policy evaluation)

- 1. Initialization  $V(s) \in \mathbb{R}$  and  $\pi(s) \in \mathcal{A}(s)$  arbitrarily for all  $s \in \mathcal{S}$
- 2. Policy Evaluation Repeat

$$\Delta \leftarrow 0$$

For each  $s \in S$ :

$$V(s) \leftarrow V(s)$$

$$V(s) \leftarrow \sum_{s',r} p(s', r|s, \pi(s)) [r + \gamma V(s')]$$

$$\Delta \leftarrow \max(\Delta, |v - V(s)|)$$

until  $\Delta < \theta$  (a small positive number)

3. Policy Improvement policy-stable  $\leftarrow true$ 

For each  $s \in S$ :

$$\frac{old\text{-}action \leftarrow \pi(s)}{\pi(s) \leftarrow \operatorname{argmax}_a \sum_{s',r} p(s',r|s,a) \left[r + \gamma V(s')\right]}$$

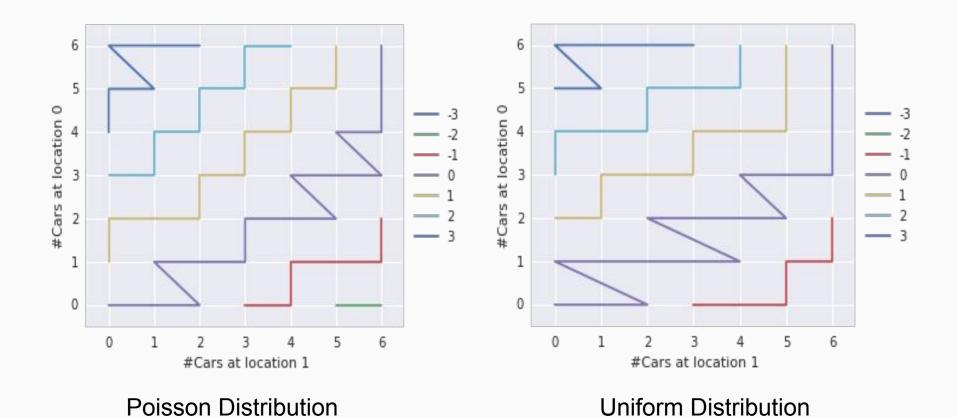
If  $old\text{-}action \neq \pi(s)$ , then  $policy\text{-}stable \leftarrow false$ 

If policy-stable, then stop and return  $V \approx v_*$  and  $\pi \approx \pi_*$ ; else go to 2

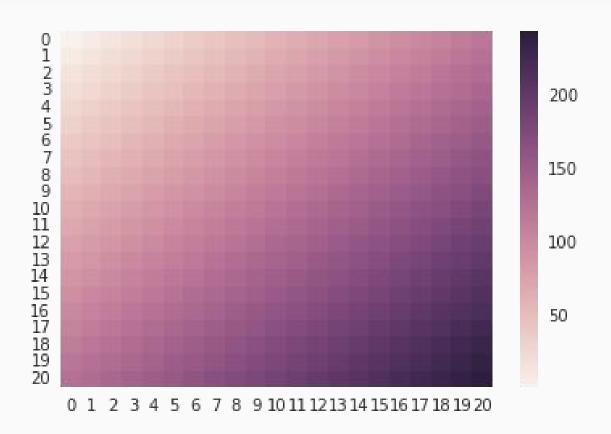
## Ideas ??

- State Values seem to be learnable by simple model
- Approximate expected poisson reward by stochastic sampling
- Can we use something else?

#### Policies for Poisson and Uniform distributions

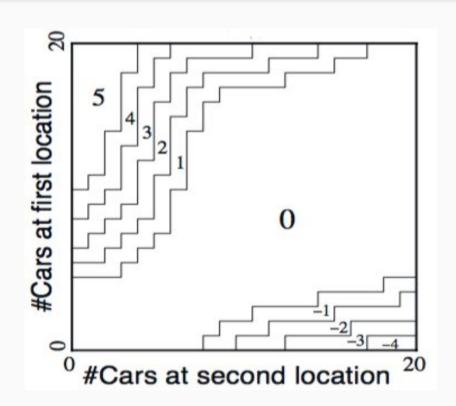


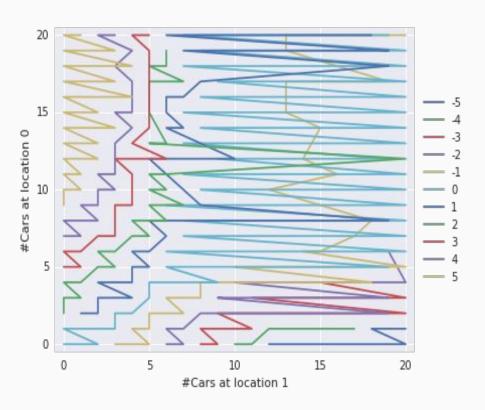
#### **Learnt State Values**



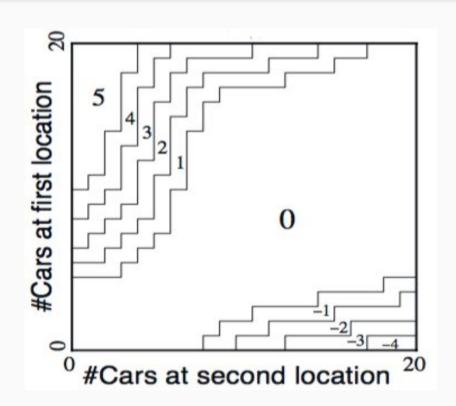
# Which expectation approximation technique do you expect to be better?

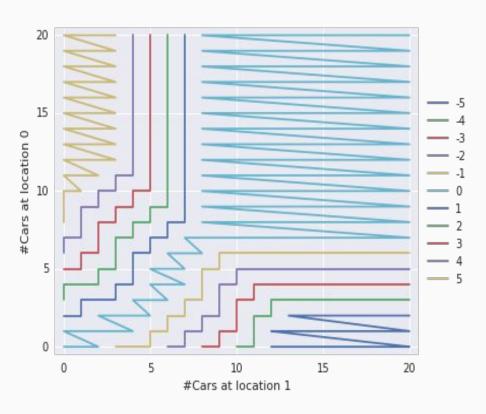
### Actual vs Estimate (Poisson sampling)



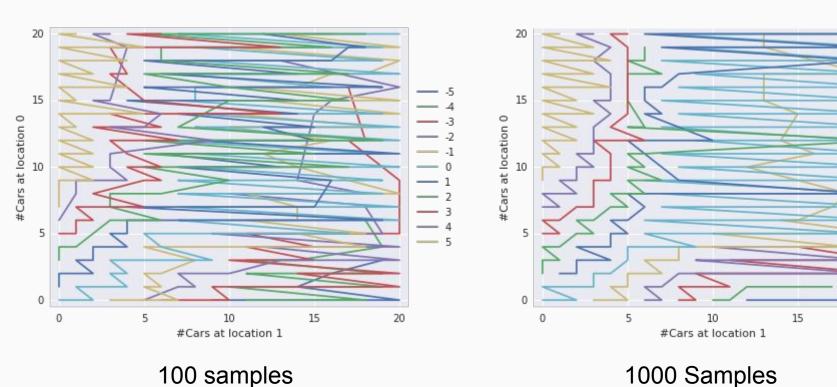


## Optimal vs Estimate (Uniform Distribution)





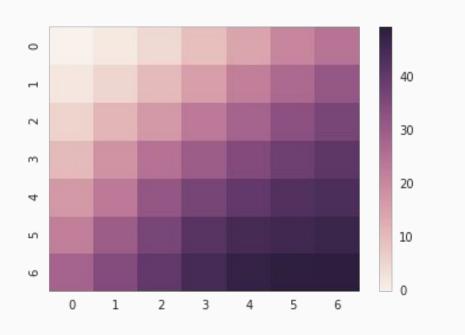
## Why was poisson approximation worse?

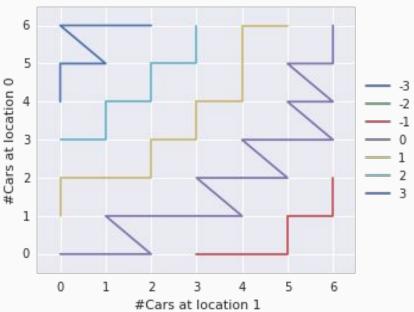


1000 Samples

20

# Optimal Policy & State Values (Discount 0)





# Why did discount have negligible effect on policy?

- State values reduce almost in same proportion.
- For larger state space discount might have significant effect

## What else could we have done?

Predicting policy values directly !!

# Key Takeaways

If environment model is know we can exploit structure in state - action space and apply supervised learning techniques to learn state values/policies to:

- Get good initial guesses for State Values and Policy
- Approximate solution for larger state action space