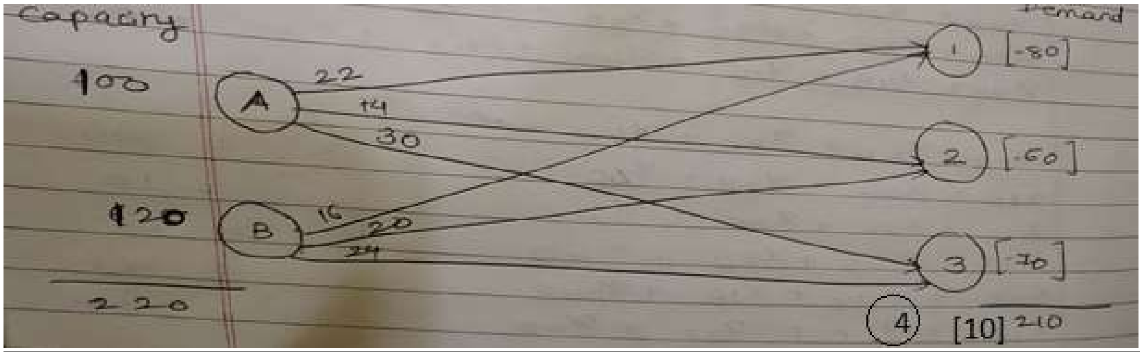
Assignment 4

Question 1:

Solution:



Decision Variable:

Let,

X_{ij} = AED distribution from plant i to warehouse j

i = Plants; i = A, B

j = Warehouse; j = 1, 2, 3

Objective Function:

To minimize the transportation cost of AEDs.

Min = 22 X_{A1} + 14 X_{A2} + 30 X_{A3} + 16 X_{B1} + 20 X_{B2} + 24 X_{B3} + 600 X_{A1} + 600 X_{A2} +600 X_{A3} + 625 X_{B1} + 625 X_{B2} + 625 X_{B3} + 0 X_{AD} + 0 X_{BD};

Constraints:

i. Supply/Capacity Constraints:

$X_{A1} + X_{A2} + X_{A3} + X_{AD} = 100$;
$X_{B1} + X_{B2} + X_{B3} + X_{BD} = 120$;

ii. Demand Constraints:

$X_{A1} + X_{B1} = 80$;
$X_{A2} + X_{B2} = 60$;
$X_{A3} + X_{B3} = 70$;

iii. Dummy Variable:

Production Capacity = 100 + 120 = 220
Demand = 80 + 60 + 70 = 210

Thus, create a dummy variable in demand section to equate the supply and demand.

Dummy Variable D = 10;
$X_{AD} + X_{BD} = 10$;

Mathematical Formulation of Linear Programming Problem:

Let,

X_{ij} = AED distribution from plant i to warehouse j
i = Plants; i = A, B
j = Warehouse; j = 1, 2, 3

Min = $22 X_{A1} + 14 X_{A2} + 30 X_{A3} + 16 X_{B1} + 20 X_{B2} + 24 X_{B3} + 600 X_{A1} + 600 X_{A2} + 600 X_{A3} + 625 X_{B1} + 625 X_{B2} + 625 X_{B3} + 0 X_{AD} + 0 X_{BD}$;

Subject To

$X_{A1} + X_{A2} + X_{A3} + X_{AD} = 100$;
$X_{B1} + X_{B2} + X_{B3} + X_{BD} = 120$;

$X_{A1} + X_{B1} = 80$;
$X_{A2} + X_{B2} = 60$;
$X_{A3} + X_{B3} = 70$;

$X_{AD} + X_{BD} = 10$;

And

$X_{ij} >= 0$;
i = A, B; j = 1, 2, 3;

<u>Question 2:</u>

<u>Solution:</u>

Decision Variable:

Let,

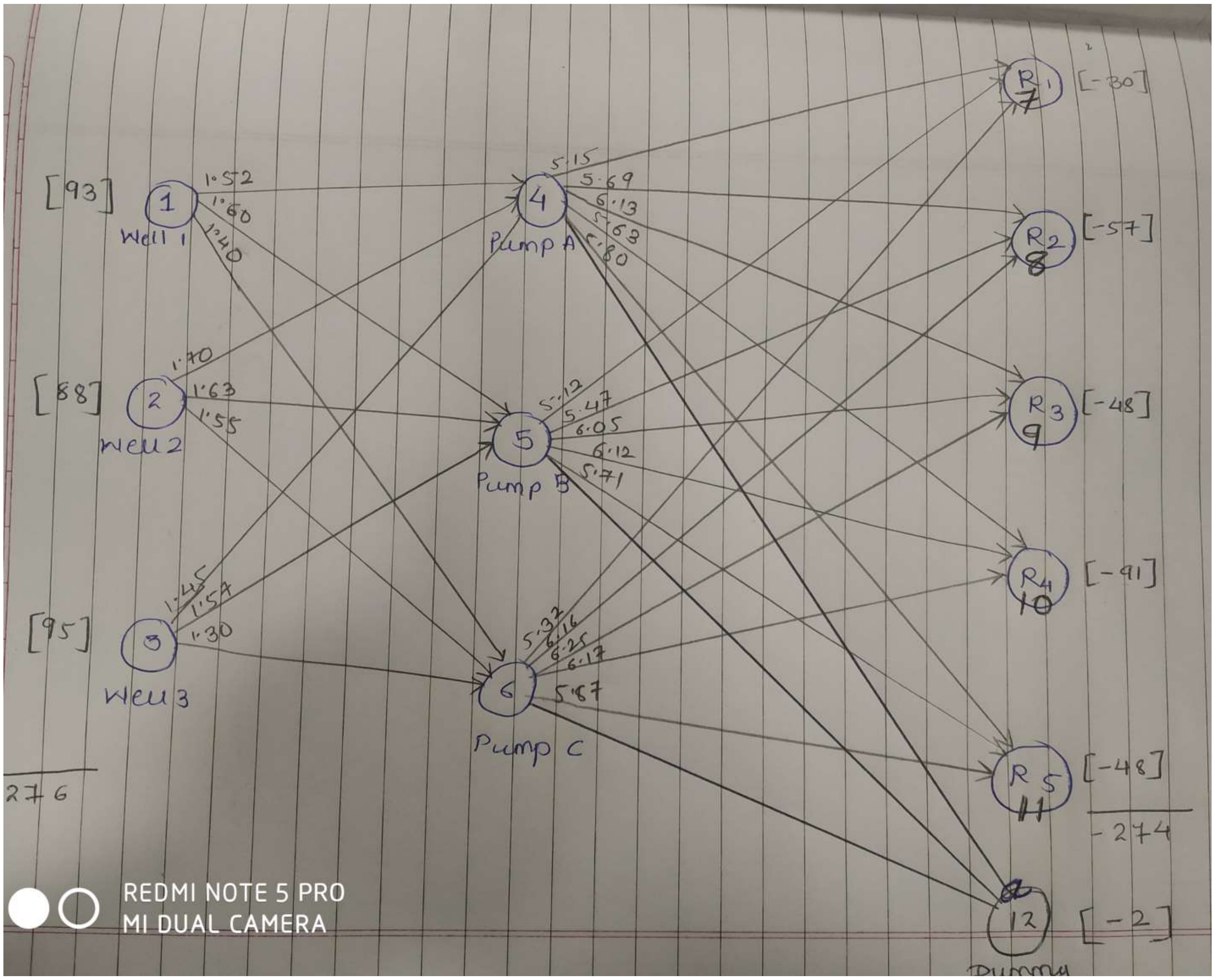
X_{ij} = Oil flow from well i to pump j in TBH.
X_{jk}= Oil flow from pump j to refineries k TBH.
i = Wells; i = 1, 2, 3
j = Pumps; j = 4, 5, 6
k = Refineries; k = 7, 8, 9

Objective Function:

To find the minimum providing cost of oil to the refineries.

$Min_z = 1.52 X_{14} + 1.60 X_{15} + 1.40 X_{16} + 1.70 X_{24} + 1.63 X_{25} + 1.55 X_{26} + 1.45 X_{34} + 1.57 X_{35} + 1.30 X_{36} + 5.15 X_{47} + 5.69 X_{48} + 6.13 X_{49} + 5.63 X_{410} + 5.80 X_{411} + 5.12 X_{57} + 5.47 X_{58} + 6.05 X_{59} + 6.12 X_{510} + 5.71 X_{511} + 5.32 X_{67} + 6.16 X_{68} + 6.25 X_{69} + 6.17 X_{610} + 5.87 X_{611}$;

Network Diagram for the problem:



Constraints:

i. Oil flow from wells to the pump

$X_{14} + X_{15} + X_{16} = 93;$
$X_{24} + X_{25} + X_{26} = 88;$
$X_{34} + X_{35} + X_{36} = 95;$

ii. Oil flow from pumps to the refineries

$X_{47} + X_{57} + X_{67} = 30;$
$X_{48} + X_{58} + X_{68} = 57;$
$X_{49} + X_{59} + X_{69} = 48;$
$X_{410} + X_{510} + X_{610} = 91;$
$X_{411} + X_{511} + X_{611} = 48;$
$X_{412} + X_{512} + X_{612} = 2;$

iii. Dummy variable

Production Capacity = 93 + 88 + 95 = 276 TBH
Demand = 30 + 57 + 48 + 91 + 48 = 274 TBH

Thus, create a dummy variable in demand section to equate the supply and demand.

Dummy Variable D = 2;

$X_{412} + X_{512} + X_{612} = 2;$

iv. Equality constraint

$X_{14} + X_{24} + X_{34} = X_{47} + X_{48} + X_{49} + X_{410} + X_{411} + X_{412};$
$X_{15} + X_{25} + X_{35} = X_{57} + X_{58} + X_{59} + X_{510} + X_{511} + X_{512};$
$X_{16} + X_{26} + X_{36} = X_{67} + X_{68} + X_{69} + X_{610} + X_{611} + X_{612};$

Mathematical Formulation of Linear Programming Problem:

Let,
X_{ij} = Oil flow from well i to pump j in TBH.
X_{jk}= Oil flow from pump j to refineries k TBH.
i = Wells; i = 1, 2, 3
j = Pumps; j = 4, 5, 6
k = Refineries; k = 7, 8, 9

$Min_z = 1.52\ X_{14} + 1.60\ X_{15} + 1.40\ X_{16} + 1.70\ X_{24} + 1.63\ X_{25} + 1.55\ X_{26} + 1.45\ X_{34} + 1.57\ X_{35} + 1.30\ X_{36} + 5.15\ X_{47} + 5.69\ X_{48} + 6.13\ X_{49} + 5.63\ X_{410}\ \ + 5.80\ X_{411} + 5.12\ X_{57} + 5.47\ X_{58} + 6.05\ X_{59} + 6.12\ X_{510} + 5.71\ X_{511} + 5.32\ X_{67} + 6.16\ X_{68} + 6.25\ X_{69} + 6.17\ X_{610} + 5.87\ X_{611};$

Subject To

$X_{14} + X_{15} + X_{16} = 93;$
$X_{24} + X_{25} + X_{26} = 88;$
$X_{34} + X_{35} + X_{36} = 95;$

$X_{47} + X_{57} + X_{67} = 30;$
$X_{48} + X_{58} + X_{68} = 57;$
$X_{49} + X_{59} + X_{69} = 48;$
$X_{410} + X_{510} + X_{610} = 91;$
$X_{411} + X_{511} + X_{611} = 48;$
$X_{412} + X_{512} + X_{612} = 2;$

$X_{14} + X_{24} + X_{34} = X_{47} + X_{48} + X_{49} + X_{410} + X_{411} + X_{412};$
$X_{15} + X_{25} + X_{35} = X_{57} + X_{58} + X_{59} + X_{510} + X_{511} + X_{512};$
$X_{16} + X_{26} + X_{36} = X_{67} + X_{68} + X_{69} + X_{610} + X_{611} + X_{612};$

And

$X_{ij} >= 0;$

$X_{jk} >= 0;$

i = 1, 2, 3; j = 4, 5, 6; k = 7, 8, 9;

Solution Network Diagram: