**Question 2:**

Back Savers is a company that produces backpacks primarily for students. They are considering offering some combination of two different models—the Collegiate and the Mini. Both are made out of the same rip-resistant nylon fabric. Back Savers has a long-term contract with a supplier of the nylon and receives a 5000 square-foot shipment of the material each week. Each Collegiate requires 3 square feet while each Mini requires 2 square feet. The sales forecasts indicate that at most 1000 Collegiate and 1200 Minis can be sold per week. Each Collegiate requires 45 minutes of labor to produce and generates a unit profit of $32. Each Mini requires 40 minutes of labor and generates a unit profit of $24. Back Savers has 35 laborers that each provides 40 hours of labor per week. Management wishes to know what quantity of each type of backpack to produce per week. Solve this problem graphically.

**Solution:**

**Given:**

Total shipment of material per week = 5000 sq. ft.

Material required for each Collegiate bag = 3 sq. ft.

Material required for each Mini bag = 2 sq. ft.

Sales forecast for Collegiate bag per week <= 1000 units

Sales forecast for Mini bag per week <= 1200 units

Cost Co-efficient:

Unit profit for Collegiate = $32

Unit profit for Mini = $24

Total labor hours = 35 labors \* 40 hours = 1400 hours

1. **To define decision variable:**

Let, X1 = Quantity of Collegiate Bags to be produced per week.

X2 = Quantity of Mini Bags to be produced per week.

1. **Objective Function:**

Maximum Total Profit gained by Collegiate and Mini bags per week.

Maxz = 32X1 + 24X2

1. **Constraints:**
2. Total available Material

3X1 + 2X2 <= 5000 sq. ft.

1. Sales Forecast

X1 <= 1000 units

X2 <= 1200 units

1. Labor Hours

(45/60) X1 + (40/60) X2 <= 1400 hours - (45 min and 40 min are converted into hours)

1. **Mathematical Formulation of Linear Programming Problem:**

Let,

X1 = Quantity of Collegiate Bags to be produced per week.

X2 = Quantity of Mini Bags to be produced per week.

Maxz = 32X1 + 24X2

Subject to

3X1 + 2X2 <= 5000 sq. ft.

X1 <= 1000 units

X2 <= 1200 units

(45/60) X1 + (40/60) X2 <= 1400 hours

And

X1 >= 0, X2 >= 0.

**Graphical Solution:**

Consider,

3X1 + 2X2 = 5000

Let X1 = 0 to get the Y intercept and later X2 = 0 to get the X intercept.

2 X2 = 5000

X2 = 2500

Now

3 X1 = 5000

X1 = 1666.67

Therefore, **X1 = 1666.67, X2 = 2500**

Chart, letter

Description automatically generated

Consider,

(45/60) X1 + (40/60) X2 = 1400 hours

Let X1 = 0 to get the Y intercept and later X2 = 0 to get the X intercept.

(45/60) X1 = 1400

X1 = 1866.67

Now,

(40/60) X2 = 1400

X2 = 2100

Therefore, **X1 = 1866.67, X2 = 2100**

Letter

Description automatically generated

Also, we have

**X1 = 1000**

**X2 = 1200**

Chart, letter

Description automatically generated

Feasible region for the plotted graph is as below shaded region.

Letter

Description automatically generated with medium confidence

Consider the point (2500, 2000) or (1500, 1500) which does not form an optimal solution.

Diagram

Description automatically generated with medium confidence

Since, the graph intersect the feasible region at a point (**1000, 974**) this point can be the feasible or optimal solution for the problem.

Diagram

Description automatically generated with medium confidence

Hence, according to objective function:

Maxz = 32 X1 +24 X2

**X1 = 1000, X2 = 974** as optimal solution,

Maxz = 32 \* 1000 + 24 \* 97,

Maximum Profit = **$55376**

Therefore, we have optimal solution for the problem where Bank Savers Company should produce **1000** units of collegiate bags and **975** units of Mini Bags to earn maximum profit of **$55,376.**