**QUESTION1:**

**(Computer Center Staffing)** You are the Director of the Computer Center for Gaillard College and responsible for scheduling the staffing of the center. It is open from 8 am until midnight. You have monitored the usage of the center at various times of the day and determined that the following numbers of computer consultants are required.

|  |  |
| --- | --- |
| Time of day | Minimum number of consultants required to be on duty |
| 8 am–noon | 4 |
| Noon–4 pm | 8 |
| 4 am–8 pm | 10 |
| 8 am–midnight | 6 |

Two types of computer consultants can be hired: full-time and part-time. The full-time consultants work for eight consecutive hours in any of the following shifts: morning (8 am – 4 pm), afternoon (noon – 8 pm), and evening (4 pm – midnight). Full-time consult- ants are paid $14 per hour.

Part-time consultants can be hired to work any of the four shifts listed in the table. Part-time consultants are paid $12 per hour. An additional requirement is that during every time period, at least one full-time consultants must be on duty for every part-time consultants on duty.

1. Determine a minimum-cost staffing plan for the center. In your solution, how many consultants will be paid to work full time and how many will be paid to work part-time? What is the minimum cost?
2. After thinking about this problem for a while, you have decided to recognize meal breaks explicitly in the scheduling of full-time consultants. In particular, full-time consultants are entitled to a one-hour lunch break during their eight-hour shift. In addition, employment rules specify that the lunch break can start after three hours of work or after four hours of work, but those are the only alternatives. Part-time consultants do not receive a meal break. Under these conditions, find a minimum-cost staffing plan. What is the minimum cost?

Hint: for this problem, you only need to formulate the LP problem without solving it.

**Solution:**

**Given:**

Types of consultants – Full time, Part-time

Number of hours for full time worker = 8 hours/shift

Number of hours for part-time worker = 4 hours/shift

Cost Co-efficient

Full time wages = $14/hour = $14 \* 8 per shift

Part-time wages = $12/hour = $12 \*4 per shift

**Decision Variable:**

Let,

X1 = Number of full-time consultants working on (8 am – 4 pm) Morning shift

X2 = Number of full-time consultants working on (noon – 8 pm) Afternoon shift

X3 = Number of full-time consultants working on (4 pm – midnight) Evening shift

Y1 = Number of Part-time consultants working on (8 am – noon) Morning shift

Y2 = Number of Part-time consultants working on (noon – 4 pm) Afternoon shift

Y3 = Number of Part-time consultants working on (4 pm – 8 pm) Evening shift

Y4 = Number of Part-time consultants working on (8 pm – midnight) Night shift.

**Objective Function:**

1. Minimum cost paid to work for each Full time and Part-time consultants.

Minz = (14 \* 8) (X1 + X2 + X3) + (12 \* 4) (Y1 + Y2 + Y3 + Y4)

**Constraints:**

**A picture containing timeline

Description automatically generated**

1. Minimum number of consultants required

X1 + Y1 >= 4, X1 + X2 + Y2 >= 8,

X2 + X3 + Y3 >= 10, X3 + Y4 >= 6

1. For every Part-time Consultants at least 1 Full time consultants

X1 >= Y1, X1 + X2 >= Y2,

X2 + X3 >= Y3, X3 >= Y4

**Mathematical Formulation of Linear Programming Problem:**

Let,

X1 = Number of full-time consultants working on (8 am – 4 pm) Morning shift

X2 = Number of full-time consultants working on (noon – 8 pm) Afternoon shift

X3 = Number of full-time consultants working on (4 pm – midnight) Evening shift

Y1 = Number of Part-time consultants working on (8 am – noon) Morning shift

Y2 = Number of Part-time consultants working on (noon – 4 pm) Afternoon shift

Y3 = Number of Part-time consultants working on (4 pm – 8 pm) Evening shift

Y4 = Number of Part-time consultants working on (8 pm – midnight) Night shift.

Minz = (14 \* 8) (X1 + X2 + X3) + (12 \* 4) (Y1 + Y2 + Y3 + Y4)

Subject To

X1 + Y1 >= 4, X1 + X2 + Y2 >= 8

X2 + X3 + Y3 >= 10, X3 + Y4 >= 6

X1 >= Y1, X1 + X2 >= Y2

X2 + X3 >= Y3, X3 >= Y4

And

X1, X2, X3 >= 0

Y1, Y2, Y3, Y4 >= 0

1. There is a 1-hour break for every Full-time worker after 3 or 4 hours of the shift. And the Part-time worker does not update.

Thus, we need to cut the cost of the break hour from the total cost.

**Objective function**

Thus, the objective function is updated as:

**Minz = (14 \* 8) (X1 + X2 + X3) – (14\*1) (X1 + X2 + X3) + (12 \* 4) (Y1 + Y2 + Y3 + Y4).**

**Constraints**

1. Minimum number of consultants required

X1 + Y1 >= 4, X1 + X2 + Y2 >= 8

X2 + X3 + Y3 >= 10, X3 + Y4 >= 6

1. For every Part-time Consultants at least 1 Full time consultants

X1 >= Y1, X1 + X2 >= Y2

X2 + X3 >= Y3, X3 >= Y4

**Mathematical Formulation of Linear Programming Problem:**

Let,

X1 = Number of full-time consultants working on (8 am – 4 pm) Morning shift

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Y3 = Number of Part-time consultants working on (4 pm – 8 pm) Evening shift

Y4 = Number of Part-time consultants working on (8 pm – midnight) Night shift.

Minz = (14 \* 8) (X1 + X2 + X3) – (14\*1) (X1 + X2 + X3) + (12 \* 4) (Y1 + Y2 + Y3 + Y4).

Subject To

X1 + Y1 >= 4, X1 + X2 + Y2 >= 8

X2 + X3 + Y3 >= 10, X3 + Y4 >= 6

X1 >= Y1, X1 + X2 >= Y2

X2 + X3 >= Y3, X3 >= Y4

And X1, X2, X3 >= 0 Y1, Y2, Y3, Y4 >= 0