REVISION

* Harmonic Oscillator:

$\hat{a}^\dagger |n\rangle = \sqrt{n+1}\, |n+1\rangle$

$\hat{a}\, |n\rangle = \sqrt{n}\, |n-1\rangle$

$\hat{x} = \sqrt{\frac{\hbar}{2m\omega}}\, (a + a^\dagger)$

$\hat{p} = i\sqrt{\frac{\hbar m\omega}{2}}\, (a^\dagger - a)$

* Angular Momentum:

$$\hat{L}_i = \sum_{j,k} \varepsilon_{ijk}\, \hat{x}_j\, \hat{p}_k \qquad [\hat{L}_i, \hat{L}_j] = i\hbar \sum_k \varepsilon_{ijk} \hat{L}_k$$

$$\hat{L}^2 |l,m\rangle = \hbar^2 l(l+1)\, |l,m\rangle \qquad \hat{L}_\pm = \hat{L}_x \pm i\hat{L}_y$$

$$L_z |l,m\rangle = \hbar m\, |l,m\rangle$$

$$L_+ |l,m\rangle = \hbar\sqrt{l(l+1) - m(m+1)}\, |l, m+1\rangle$$

$$L_- |l,m\rangle = \hbar\sqrt{l(l+1) - m(m-1)}\, |l, m-1\rangle$$

Addition of Angular Momentum:

$$\hat{J}_\mu = \hat{L}_\mu^{(1)} + \hat{L}_\mu^{(2)} \Rightarrow \hat{\vec{J}} = \hat{\vec{L}}^{(1)} + \hat{\vec{L}}_2^{(2)}$$

Operators in joint $\mathcal{H}$ $\quad \hat{L}_\mu^{(1)} \equiv \hat{L}_\mu^{(1)} \otimes \mathbb{1} \qquad \hat{L}_\nu^{(2)} \equiv \mathbb{1} \otimes L_\nu^{(2)}$

$$\vec{J}_{total} \equiv \vec{J} = \vec{L}^{(1)} + \vec{L}^{(2)} \Rightarrow [\hat{J}_i, \hat{J}_j] = i\hbar \sum_k \varepsilon_{ijk} \hat{J}_k$$

Hence $\hat{J}_\pm \equiv \hat{L}_\pm^{(1)} \otimes \mathbb{1} + \mathbb{1} \otimes \hat{L}_\pm^{(2)}$

Selection rules for CG coeffs. If two particles are in states l_1 & l_2 and the net state is $|j, m_j\rangle$, then

$$\left.\begin{array}{l} j \in \{|l_1 - l_2|, \ldots, l_1 + l_2\} \\ m_j = m_1 + m_2 \end{array}\right\} \text{selection rules}$$

$$|j, m_j\rangle = \sum C^{l_1\, l_2}_{m_1\, m_2}\, |l_1, m_1\rangle |l_2, m_2\rangle \quad \text{s.t. } m_j = m_1 + m_2$$

for fixed l_1 & l_2

find coeffs by repeatedly applying $J_\pm$

For a given $|j, m_j\rangle$, $m_j \in \{-j, -j+1, \ldots, j-1, j\}$ but $m_j = m_1 + m_2$ for CG coeffs

* Perturbation Theory:

Power Series Expans. → $(H^{(0)} - E_n^{(0)})\psi_n^{(1)} = (E_n^{(1)} - H')\psi_n^{(0)} \rightarrow \mathcal{O}(\lambda^1)$

↳ $(H^{(0)} - E_n^{(0)})\psi_n^{(2)} + (H' - E_n^{(1)})\psi_n^{(1)} = E_n^{(2)}\psi_n^{(0)} \rightarrow \mathcal{O}(\lambda^2)$

Non-degenerate: (nth Energy level is non-deg.)

$$E_n^{(1)} = \langle \psi_n^{(0)} | H' | \psi_n^{(0)} \rangle$$

$$\psi_n^{(1)} = \sum_{m \neq n} \frac{\langle \psi_m^{(0)} | H' | \psi_n^{(0)} \rangle}{(E_n^{(0)} - E_m^{(0)})} \psi_m^{(0)}$$

$\sqrt{1+x^2} = 1 + \frac{1}{2}x^2 - \frac{1}{8}x^4 + \dots$

$$E_n^{(2)} = \sum_{m \neq n} \frac{|\langle \psi_m^{(0)} | H' | \psi_n^{(0)} \rangle|^2}{(E_n^{(0)} - E_m^{(0)})}$$

Degenerate: (nth energy level is degenerate)

- Higher order corrections can only be written for "good" states.

For 2-fold degeneracy $\varphi_\pm = \alpha_\pm \psi_a^{(0)} + \beta_\pm \psi_b^{(0)}$ are good states.

$$\begin{pmatrix} H'_{aa} & H'_{ab} \\ H'_{ba} & H'_{bb} \end{pmatrix} \rightarrow \begin{matrix} \text{eigen}\vec{v} \rightarrow \text{"good" states} \\ \text{eigenvalues} \rightarrow 1^{st} \text{ order energies.} \end{matrix}$$

- Write H' matrix in the basis of degenerate eigenspace $(\psi_a^{(0)}, \psi_b^{(0)})$

(1) Find eigenvalues & eigen$\vec{v}$ of H' in this basis.

OR

(2) Find H'_{ij} elements, then $E_\pm^{(1)} = \frac{1}{2}\left(H'_{aa} + H'_{bb} \pm \sqrt{(H'_{aa} - H'_{bb})^2 + 4|H'_{ab}|^2} \right)$

<u>NOTE</u>: If $E_n^{(1)}$'s are also degenerate, then we can't use it to find "good" states, but the 1st order $E_n^{(1)}$ is correct regardless.

Theorem for "good" states Find a hermitian op. $\hat{A}$ s.t $[\hat{A}, \hat{H}'] = [\hat{A}, \hat{H}^{(0)}] = 0$. Then if ψ_a & ψ_b $\in$ degenerate eigenspace of $H^{(0)}$ are also eigenf of $\hat{A}$ with <u>distinct</u> eigenvalues, then $H'_{ab} = 0$ (and hence ψ_a, ψ_b are good states).

*** Relativistic Correction** $E_r^{(1)} = -\frac{1}{8m^3c^2}\langle p^4 \rangle = -\frac{1}{2mc^2}\left(E^2 + \langle V^2 \rangle - 2E\langle V \rangle\right)$