PHY101 Hello Session.

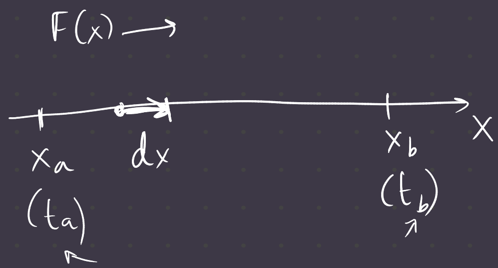
27-02-2022

Work & Energy

One-dimensional motion.

$$m\frac{d^2x}{dt^2} = F(x) \qquad \frac{dx}{dt} \equiv v(t)$$

$$\int_{x_a}^{x_b} m\frac{dv}{dt}\underbrace{dx}_{} = \int_{x_a}^{x_b} F(x)\,dx$$



$$dx = \left(\frac{dx}{dt}\right)dt = v\,dt$$

$$x \longrightarrow t$$

$$\int_{\cancel{v_a}\,t_a}^{\cancel{v_b}\,t_b} dt\, mv\underbrace{\frac{dv}{dt}}_{} = \int_{x_a}^{x_b} dx\, F(x)$$

$$\frac{1}{2}\left(v\frac{dv}{dt} + \frac{dv}{dt}v\right) = \frac{dv}{dt}v$$

$$\frac{d}{dt}\left(\frac{1}{2}v^2\right)$$

$$\int_a^b dx\,\frac{d}{dx}F(x) = F(b) - F(a)$$

$$\int_{t_a}^{t_b} dt\,\frac{d}{dt}\left(\frac{1}{2}mv^2\right) = \int_{x_a}^{x_b} dx\,F(x)$$

$$\frac{1}{2}mv(t_b)^2 - \frac{1}{2}mv(t_a)^2 = \int_{x_a}^{x_b} dx\,F(x) \Rightarrow \boxed{\frac{1}{2}mv_b^2 - \frac{1}{2}mv_a^2 = \int_{x_a}^{x_b} dx\,F(x) = W_{ba}}$$

$$v(t_b) = v_b, \qquad v(t_a) = v_a$$

$$K \equiv \frac{1}{2}mv^2 \qquad K_b - K_a = W_{ba}$$

↑ Kinetic energy

$$m\frac{d^2x}{dt^2} = F(x)$$

☆ Start with $\underline{\underline{v_0}} \longrightarrow v$ $\qquad \underline{\underline{x_0}} \longrightarrow x$

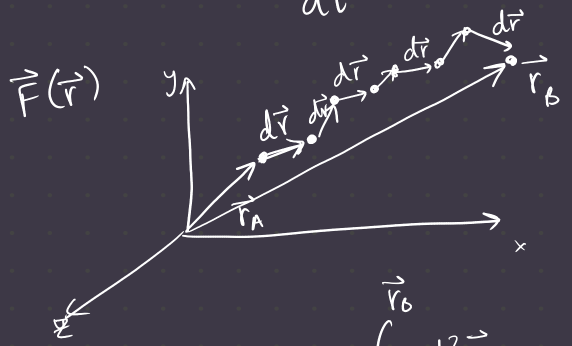
$$\frac{1}{2}mv^2 = \underbrace{\int_{x_0}^{x} dx\ F(x)}_{f(x)} + \frac{1}{2}mv_0^2$$

$$v \equiv v(x) = \frac{dx}{dt} \qquad \int_{x_0}^{x} \frac{dx}{v(x)} = \int_{t_0}^{t} dt$$

In higher dimensional

$$\vec{x} \equiv (x, y, z) \equiv \vec{r}$$

$$m\frac{d^2\vec{r}}{dt^2} = \vec{F}(\vec{r})$$



$$\underbrace{\int_{\vec{r}_A}^{\vec{r}_B} \vec{F}(\vec{r}) \cdot d\vec{r}} = m \int_{\vec{r}_A}^{\vec{r}_B} d\vec{r} \cdot \frac{d^2\vec{r}}{dt^2}$$

$\vec{r}(t_a) = \vec{r}_A$

$\vec{r}(t_B) = \vec{r}_B$

$$m\int_{\vec{r}_A}^{\vec{r}_B} \frac{d^2\vec{r}}{dt^2} \cdot d\vec{r} = m\int_{\vec{r}_A}^{\vec{r}_B} \frac{d\vec{v}}{dt} \cdot d\vec{r}$$

$$d\vec{r} = \left(\frac{d\vec{r}}{dt}\right) dt = \vec{v}\, dt$$

$$= m\int_{t_a}^{t_b} dt\; \underbrace{\vec{v} \cdot \frac{d\vec{v}}{dt}}_{= \frac{d}{dt}\left(\frac{1}{2}\vec{v}\cdot\vec{v}\right)} = \int_{t_a}^{t_b} dt\, \frac{d}{dt}\left(\frac{1}{2}mv^2\right)$$

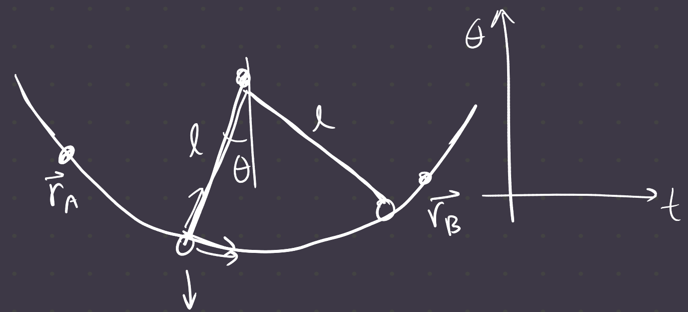
$$v^2 = v_x^2 + v_y^2 + v_z^2$$

$$\boxed{\underbrace{\frac{1}{2}mv(t_b)^2 - \frac{1}{2}mv(t_a)^2}_{\text{Energy}} = \underbrace{\oint_{\vec{r}_A}^{\vec{r}_B} d\vec{r} \cdot \vec{F}(\vec{r})}_{\text{Work}}}$$

Line integral

$\vec{v} \cdot \vec{v}$

$\vec{v}(t) \cdot \vec{v}(t)$



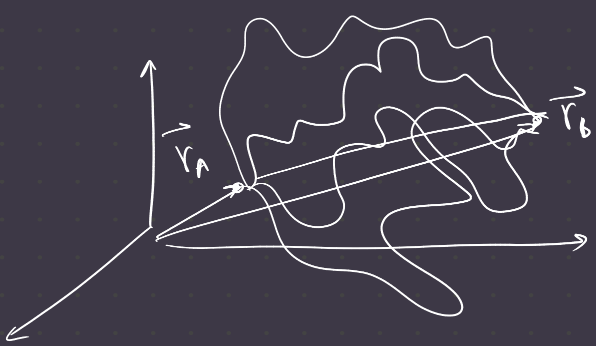
Useful in two cases —

- if motion is constrained.
- $\vec{F}$ is a "conservative force".

$\vec{r}(t)$

$$\oint_{\vec{r}_A}^{\vec{r}_B} d\vec{r} \cdot \vec{F}(\vec{r}) = f(\vec{r}_B) - f(\vec{r}_A)$$

A conservative force is defined as the one where $\oint \vec{F} \cdot d\vec{r}$ is INDEPENDENT of the path & only dependent upon the endpoints.



For conservative forces

$$\boxed{\oint_{\vec{r}_A}^{\vec{r}_B} d\vec{r} \cdot \vec{F}(\vec{r}) = -\left(U(\vec{r}_B) - U(\vec{r}_A)\right)}$$

$U \equiv$ pot^n energy

For a <u>conservative system</u>
$$K_b - K_a = \oint_{\vec{r}_A}^{\vec{r}_B} d\vec{r} \cdot \vec{F}(\vec{r}) = -\left[(U_b + C) - (U_a + C)\right]$$

$$K_b - K_a = -U_b + U_a \Rightarrow \underbrace{K_a + U_a}_{(\vec{x}_a, t_a)} = \underbrace{K_b + U_b}_{(\vec{x}_b, t_b)} = E$$

E ↑ total mechanical energy

$$\oint_{\vec{r}_A}^{\vec{r}_B} d\vec{r} \cdot F(\vec{r}) = -\left(U(\vec{r}_B) - U(\vec{r}_A)\right)$$
$$= -\left[\left(\underline{U(\vec{r}_B) + C}\right) - \left(U(\vec{r}_A) + C\right)\right]$$

$$E = K + (U + C)$$

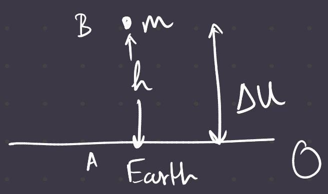
↑ arbitrary const.

$\underline{\underline{\Delta E}}$

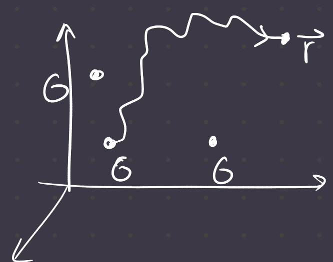
$$U(\vec{r}) \equiv -\oint_{G}^{\vec{r}} d\vec{r} \cdot \vec{F}(\vec{r})$$

G ↖ reference pt.

$$K = \frac{1}{2} m v^2$$



$U = mgh$



In 1D

$$\int_a^b dx \underbrace{\frac{df}{dx}} = f(b) - f(a) \qquad K_b - K_a = \int_{x_a}^{x_b} dx\, F(x)$$

$$\int_{x_a}^{x_b} dx\, F(x) = -U(x_b) + U(x_a)$$

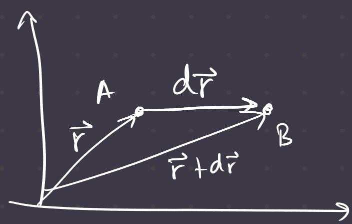
$$\int dx\, F(x) = -\int dx \frac{dU}{dx}$$

$$\int_{x_a}^{x_b} dx \frac{dU}{dx} = U(x_b) - U(x_a)$$

$$\boxed{F(x) = -\frac{dU(x)}{dx}}$$

- 3D

$$\int_{\vec{r}_A}^{\vec{r}_B} d\vec{r} \cdot \vec{F}(\vec{r}) = -\left(U(\vec{r}_B) - U(\vec{r}_A)\right)$$



$$\int_{\vec{r}}^{\vec{r}+d\vec{r}} d\vec{r}' \cdot \underbrace{\vec{F}(\vec{r}')} = -\left(U(\vec{r}+d\vec{r}) - U(\vec{r})\right)$$

$$\|$$

$$\vec{F}(\vec{r}) \cdot d\vec{r} = -\left(\underbrace{U(\vec{r}+d\vec{r}) - U(\vec{r})}\right)$$

$$U(\vec{r}+\Delta\vec{r}) - U(\vec{r}) = \underbrace{\frac{\partial U}{\partial x}} dx + \underbrace{\frac{\partial U}{\partial y}} dy + \underbrace{\frac{\partial U}{\partial z}} dz$$

$$dU$$

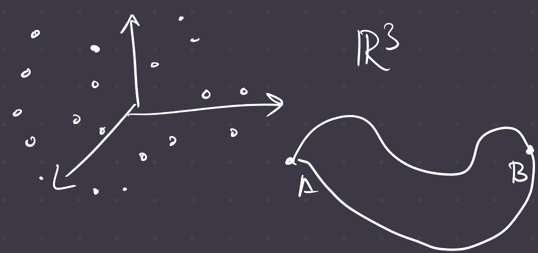
$$\vec{F}(\vec{r}) \cdot d\vec{r} = -\left(\frac{\partial U}{\partial x} dx + \frac{\partial U}{\partial y} + \frac{\partial U}{\partial z} dz\right)$$

$$U : \mathbb{R}^3 \longrightarrow \mathbb{R}$$

$$= -\left(\frac{\partial U}{\partial x}, \frac{\partial U}{\partial y}, \frac{\partial U}{\partial z}\right) \cdot (dx, dy, dx)$$

scalair

$$\vec{F}(\vec{r}) \cdot d\vec{r} = -\vec{\nabla} U \cdot d\vec{r} \quad \Rightarrow \quad \boxed{\vec{F}(\vec{r}) = -\vec{\nabla} U(\vec{r})}$$



How to find if F is conservative?

$$\oint_{\vec{r}_A}^{\vec{r}_B} d\vec{r} \cdot \vec{F}(\vec{r}) = -\left(U(\vec{r}_B) - U(\vec{r}_A)\right)$$

$$\boxed{\oint d\vec{r} \cdot \vec{F}(\vec{r}) = 0}$$

C → closed loop

$$\oint d\vec r \cdot \vec F(\vec r) \quad = \quad \int d\vec a \cdot (\vec\nabla \times \vec F)$$

summing of the $\vec F$ along the boundary = Summing of all the rotn curls in all the infinitesimal loop

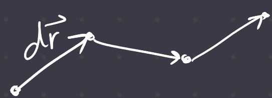
Griffiths Electrodynamics Ch-1

$$\boxed{\oint d\vec r \cdot \vec F(\vec r) \quad = \quad \int d\vec a \cdot (\vec\nabla \times \vec F)}$$

Stokes' Th^m

for conservative fields

$$\oint d\vec r \cdot \vec F(\vec r) = 0 = \int d\vec a \cdot (\vec\nabla \times \vec F)$$



$\boxed{\vec\nabla \times \vec F = 0}$ → if a force field is conservative