COLLISION THEORY.

$\nexists$ any external force $\vec{F}_{ext} = 0, \Rightarrow$ $\vec{F}_{ext} = \frac{d\vec{P}}{dt}$ $\quad \vec{P} = \sum_{i \in system} \vec{p}_i$

$\downarrow$

$\Rightarrow \vec{P}$ = constant of motion $\qquad = 0$

(3) want to find (3) 6 unknowns

known

$$\text{3 eq}^n\text{s} \rightarrow m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{v}_1' + m_2\vec{v}_2' \longrightarrow \text{Conservation of mom}^n$$

If on top of this, the interactions occur w.r.t. conservative forces

$\Rightarrow$ total energy is conserved.

$$\frac{1}{2}m_1|\vec{v}_1|^2 + \frac{1}{2}m_2|\vec{v}_2|^2 = \frac{1}{2}m_1|\vec{v}_1'|^2 + \frac{1}{2}m_2|\vec{v}_2'|^2$$

$\downarrow$ (1) $\qquad$ conservation of energy

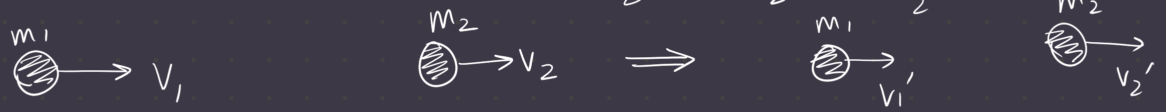
4 eq^n's & 6 unknowns.

Collisions in One dimension.

known

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2' \leftarrow$$

$$\frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \frac{1}{2}m_1 v_1'^2 + \frac{1}{2}m_2 v_2'^2 \leftarrow$$



Cons of mom^n + Cons of energy $\longrightarrow$ 2 eq^n's

2 unknowns

exactly solvable.

Elastic collision

Example

$p_i \qquad\qquad p_f$

$$\cancel{m_1}v - 3\cancel{m_1}v = \cancel{m_1}v_1' + 3\cancel{m_1}v_2'$$

$$-2v = v_1' + 3v_2' \leftarrow$$

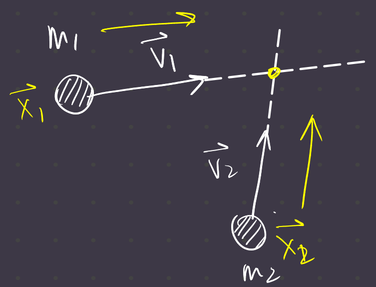
$$\hookrightarrow v^2 + 3(-v)^2 = v_1'^2 + 3v_2'^2 \rightarrow \text{Cons. of energy}$$

$$4v^2 = v_1'^2 + 3v_2'^2 \leftarrow$$

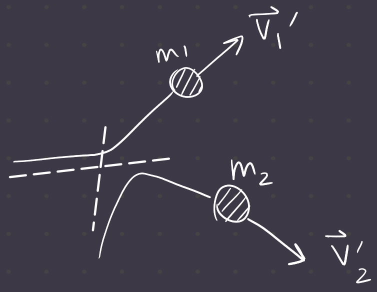
v_1' & $v_2' \rightarrow$ 2 eq^n s

Collisions in 3D (or 2D)

In lab frame $\longrightarrow$ COM frame.

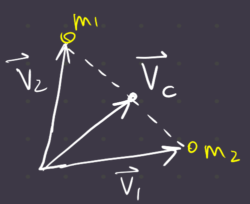


Before

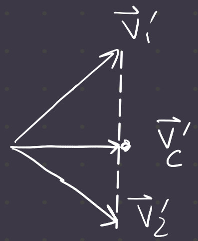


After

$$\dot{\vec{x}}_c = \frac{m_1\dot{\vec{x}}_1 + m_2\dot{\vec{x}}_2}{m_1+m_2}$$



$$\vec{V}_c = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1+m_2}$$



$$\vec{v}_c' = \frac{m_1\vec{v}_1' + m_2\vec{v}_2'}{m_1+m_2}$$

Why COM frame?

$$\vec{F}_{ext} = M\ddot{\vec{R}} \rightarrow \text{COM}$$

total mas

$$\vec{F}_{ext} = (m_1+m_2)\frac{d\vec{V}_c}{dt}$$

$$\frac{d\vec{V}_c}{dt} = \frac{d^2\vec{R}_{COM}}{dt^2} = \ddot{\vec{R}}$$

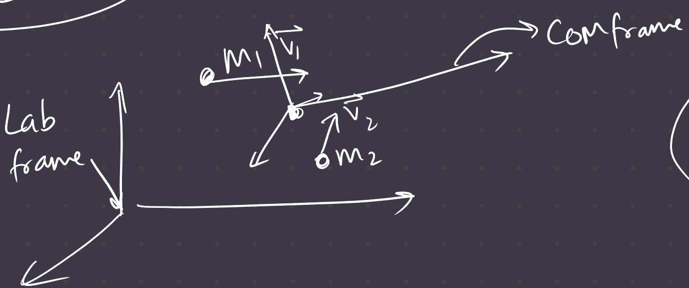
$\vec{F}_{ext} = 0$

$\Rightarrow$ $\vec{V}_c$ = constant of motion

if you shift to the COM frame

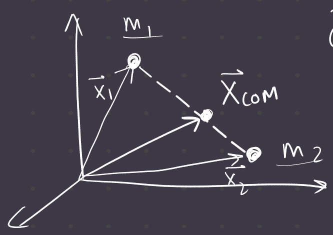
$\Rightarrow$ COM frame is an inertial frame.

$\vec{V}_c = 0$ in the COM.

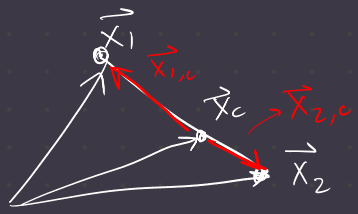


$$\vec{P}_{COM} = 0$$

$\vec V \equiv \vec V_1 - \vec V_2$



$$\frac{d}{dt}\vec X_c = \frac{d}{dt}\frac{m_1\vec x_1 + m_2\vec x_2}{m_1+m_2} \Rightarrow \vec V_c = \frac{m_1\vec V_1 + m_2\vec V_2}{m_1+m_2}$$



$\vec X_{1,c} = \vec X_1 - \vec X_c$ $\qquad$ $\vec X_{2,c} = \vec X_2 - \vec X_c$

$\vec V_{1,c} = \vec V_1 - \vec V_c$ $\qquad$ $\vec V_{2,c} = \vec V_2 - \vec V_c$

velocity of $m_{1,2}$ w.r.t. the COM frame.

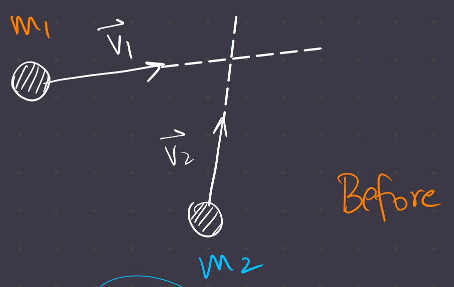
$\mu = \frac{m_1 m_2}{m_1+m_2}$

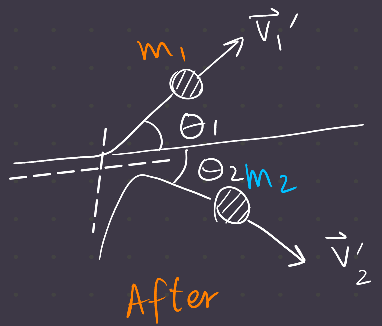
$\vec V \equiv \vec V_1 - \vec V_2$

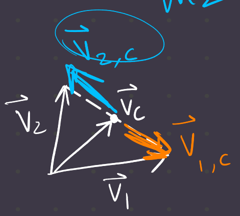
$\vec V_{1,c} = \frac{\mu}{m_1}\vec V$ $\qquad$ $\vec V_{2,c} = -\frac{\mu}{m_2}\vec V$

$$\vec P_c = m_1\vec V_{1c} + m_2\vec V_{2,c} = \mu\vec V - \mu\vec V = \underline{\underline{0}}$$

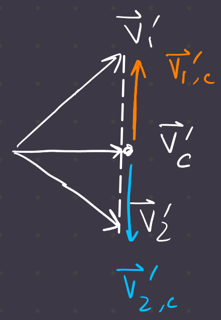
In lab frame





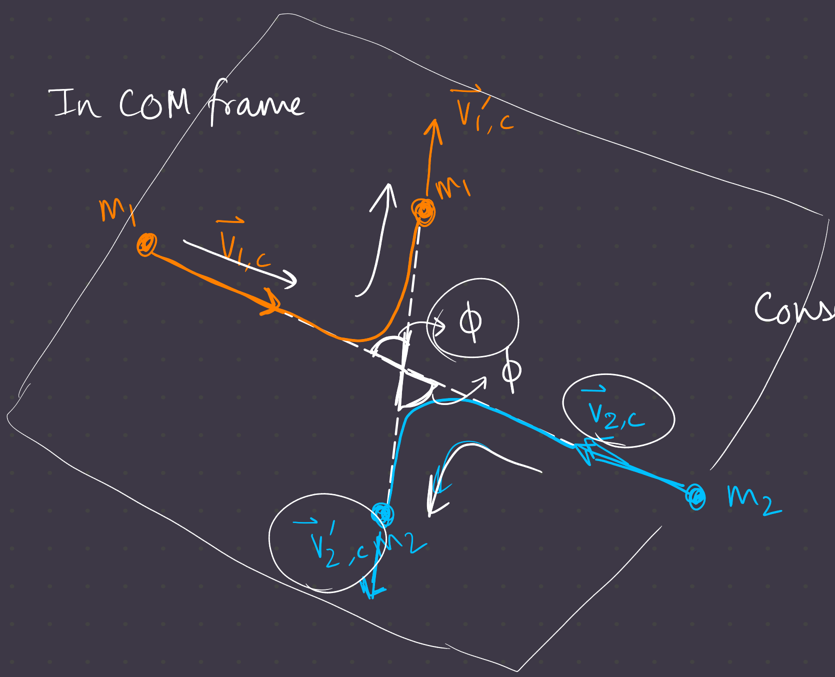


$$\vec{V} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1+m_2}$$



$$\vec{V}' = \frac{m_1\vec{v}_1' + m_2\vec{v}_2'}{m_1+m_2}$$

In COM frame



Conserve momentum in the two directions

$$m_1\vec{v}_{1,c} - m_2\vec{v}_{2,c} = 0$$
$$m_1\vec{v}_{1,c}' - m_2\vec{v}_{2,c}' = 0$$

If your ollision was elastic

$$\frac{1}{2}m_1 v_{1,c}^2 + \frac{1}{2}m_2 v_{2,c}^2 = \frac{1}{2}m_1 v_{1,c}'^2 + \frac{1}{2}m_2 v_{2,c}'^2$$

$\Downarrow$

$$v_{1,c} = v_{1,c}'$$
$$v_{2,c} = v_{2,c}'$$

6 unknowns + 4 eqⁿs

1 free parameter