

## 1. Asymptotic Analysis of Algorithm and Recurrence Relation

1) Let  $f(n) = n^2 \log n$  and  $g(n) = n(\log n)^{10}$  be two positive functions of  $n$ . Which of the following statement is correct ?

(a)  $f(n) = O(g(n))$  and  $g(n) \neq O(f(n))$

(b)  $g(n) = O(f(n))$  and  $f(n) \neq O(g(n))$

(c)  $f(n) \neq O(g(n))$  and  $g(n) \neq O(f(n))$

(d)  $f(n) = O(g(n))$  and  $g(n) = O(f(n))$

GATE-2001

2) If  $g = O(f)$  then find true statement from the following.

(a)  $f = O(g)$

(b)  $g = \theta(f)$

(c)  $f + g = \theta(g)$

(d)  $f + g = \theta(f)$

3) What does the following algorithm approximately ?

(assume  $m > 1, \epsilon > 0$ )

$x = m$  ;

$y = 1$  ;

while  $(x - y > \epsilon)$

{

$X = (x + y) / 2$  ;

$Y = m / x$  ;

}

print  $(x)$  ;

(a)  $\log m$

(b)  $m^2$

(c)  $m^{1/2}$

(d)  $m^{1/3}$

4) Consider the following C-program fragment in which I,j and n are Integer variables.

For(i=n, j=0 ; i>0 ; i/=2 , j+=i) ;

Let Val(j) denotes the value stored in the variable j after termination of the for loop. Which one of the following is true ?

(a)  $\text{val}(j) = \theta(\log n)$

(b)  $\text{val}(j) = \theta(\sqrt{n})$

(c)  $\text{val}(j) = \theta(n)$

(d)  $\text{val}(j) = \theta(n \log n)$

5) What is time complexity of the following recursive function :

Int Dosomething (int n)

{

  If (n<=2)

  Return 1 ;

Else

  Return (Dosomething(floor(sqrt(n)))+n) ;

}

(a)  $\theta(n^2)$

(b)  $\theta(n \log_2 n)$

(c)  $\theta(\log_2 n)$

(d)  $\theta(\log_2 \log_2 n)$

6) The recurrence equation

$$T(1) = 1$$

$$T(n) = 2T(n-1) + n, n \geq 2$$

Evaluates to

(a)  $2^{n+1}-n-2$

(b)  $2^n-n$

(c)  $2^{n+1}-2n-2$

(d)  $2^n+n$

Common data for Q.12 and Q.13

Consider the following C function :

```
Double foo (int n)
```

```
{
```

```
    int i ;
```

```
    double sum ;
```

```
    if (n==0) return 1.0 ;
```

```
    else
```

```
    {
```

```
        sum = 0.0 ;
```

```
        for (i=0; i<n; i++)
```

```
            sum += foo(i) ;
```

```
        return sum ;
```

```
    }
```

```
}
```

7) The space complexity of the above function is

a)  $O(1)$

(b)  $O(n)$

(c)  $O(n!)$

(d)  $O(n^n)$

**GATE 2005**

8) Suppose we modify the above function foo() and store the

values of foo(i),  $0 \leq i < n$ , as and when they are computed. With

With this modification, the time complexity for function foo() is

significantly reduced. The space complexity of the modified function would be :

(a)  $O(1)$

(b)  $O(n)$

(c)  $O(n^2)$

(d)  $O(n!)$

**GATE 2005**

9) Arrange the following function in increasing asymptotic order:

A.  $n^{1/3}$

B.  $e^n$

C.  $n^{7/4}$

D.  $n \log^9 n$

E.  $1.0000001^n$

a) A, D, C, E, B

(b) D, A, C, E, B

c) A, C, D, E, B

(d) A, C, D, B, E

**GATE 2008**

10) Consider the following program :

```
int Bar (int n)
{
    if (n<2) return ;
    else
    {
        int sum = 0 ;
        int I,j ;
        for (i=1; i<=4; i++)
            Bar (n/4) ;
        for (i=1; i<=n; i++)
        {
```

```

    for(j=1; j<=i; j++)
    {
        sum=sum+1 ;
    }
}
}
}

```

Now consider the following statements :

S1 : The time complexity of Bar(n) is  $\theta(n^2 \log n)$

S2 : The time complexity of Bar(n) is  $\Omega(n^2 \log n^2)$

S3 : The time complexity of Bar(n) is  $O(n^3 \log n^2)$

The number of correct assertions are \_\_\_\_.

- |       |       |
|-------|-------|
| (a) 0 | (b) 1 |
| (c) 2 | (d) 3 |

11) When  $n=2^{2k}$  for some  $k \geq 0$  , the recurrence relation  $T(n)=\sqrt{2}$

$T(n/2)+\sqrt{n}$ ,  $T(1)=1$  evaluates to :

- |                              |                       |
|------------------------------|-----------------------|
| (a) $\sqrt{n} (\log n + 1)$  | (b) $\sqrt{n} \log n$ |
| (c) $\sqrt{n} \log \sqrt{n}$ | (d) $n \log \sqrt{n}$ |

**GATE 2008**

12) Consider the following C function :

```

int fun1 (int n)
{
    int i,j,k,p,q=0 ;

```

```

for (i=1; i<n ; ++1)
{
    p=0 ;
    for(j=n; j>1; j=j/2)
    ++ p ;
    for(k=1; k<p; k=k*2)
    ++q ;
} return q ;
}

```

Which one of the following most closely approximates the return values of fun1 ?

- |                |                                       |
|----------------|---------------------------------------|
| (a) $n^3$      | (b) $n(\log n)^2$                     |
| (c) $n \log n$ | (d) $n \log(\log n)$ <b>GATE 2015</b> |

13) Consider the above C function. Find the time complexity and what is the value of 'q' ?

- |                |                      |
|----------------|----------------------|
| (a) $n^3$      | (b) $n(\log n)^2$    |
| (c) $n \log n$ | (d) $n \log(\log n)$ |

14) Which of the following statements True or False

- (a) If  $f(n)=O(g(n))$  then  $g(n)=O(f(n))$
- (b)  $f(n) + O(f(n)) = \theta(\min (f(n),g(n)))$
- (c)  $f(n) + o(f(n)) = \theta(f(n))$

(d)  $(\log n)!$  and  $(\log \log n)!$  are polynomially bounded

15) Consider the following true ?

$$T(n) = 2 T(\sqrt{n}) + 1, T(1) = 1$$

which one of the following is true ?

(a)  $T(n) = \theta(\log \log n)$

(b)  $T(n) = \theta \log n$

(c)  $T(n) = \theta(\sqrt{n})$

(d)  $T(n) = \theta(n)$  **GATE 2006**

16) Let  $f(n) = (n)$ ,  $g(n) = O(n)$  and  $h(n) = (n)$ . Then  $[f(n).g(n)] + h(n)$  is

\_\_\_\_\_ .

(a)  $\Omega(n)$

(b)  $O(n)$

(c)  $\theta(n)$

(d) None of these

17) Consider the following function :

```
find (int n)
{
    if (n < 2) then return ;
    else
    {
        sum = 0;
        for (i = 1; i ≤ 4 ; i++) find (n/2) ;
        for (i = 1; i ≤ n × n ; i++)
            sum = sum + 1 ;
    }
}
```

Assume that the division operation takes constant time and “sum” is global variable. What is the time complexity of “find(n)” ?

(a)  $\theta(n^2)$

(b)  $\theta(n^2 \log n)$

(c)  $\theta(n^3)$

(d) None of these

## 2. Divide and Conquer

1) Consider the following C-code :

```
int f(int x)
{
    if (x<1) return 1;
    else return f(x-1) + g(x) ;
}

int g(int x)
{
    if (x<2) return 1 ;
    else return
```

Of the following, which best describes the growth of f(x) as a function of x ?

(a) logarithm

(b) quadratic

(c) linear

(d) exponential

2) Consider the polynomial  $p(x) = a_0 + a_1x + a_2x^2 + a_3x^3$ , where  $a_i \neq 0$ ,



Vi . The minimum number of multiplications needed to evaluate p on an input x is

- (a) 3 (b) 4  
(c) 6 (d) 9 **GATE 2006**

3) Assume an array  $[1 \dots n]$  has n-elements, and every elements of an array is positive and less than or equal to n. An element is said to be “majority element”, if it is occurred in more than  $n/2$  position of an array. What is the time complexity to check whether the majority element exist or not in the given array ?

- (a)  $O(\log n)$  (b)  $O(n)$   
(c)  $O(n \log n)$  (d)  $O(n^2)$

4) The minimum number of comparision required to sort 5 elements is

- (a) 4 (b) 5  
(c) 6 (d) 7 **DRDO 2008**

5) Randomized quicksort is an extension of quicksort where the pivot is chosen randomly. What is the worst case complexity of sorting n numbers using randomized quicksort ?

- (a)  $O(n)$  (b)  $O(n \log n)$   
(c)  $O(n^2)$  (d)  $O(n!)$  **GATE 2001**

6) The usual  $\theta(n^2)$  implementation of insertion sort to sort an array uses linear search to identify the position where an

element is to be inserted into the already sorted part of the array. If, instead, we use binary search to identify the position, the worst case running time will

- (a) remain  $\theta(n^2)$  (b) becomes  $\theta(n(\log n)^2)$   
(c) becomes  $\theta(n \log n)$  (d) becomes  $\theta(n)$  **GATE 2003**

7) The minimum element in a max-heap represent by an array can be computed in time

- (a)  $\theta(n \log n)$  (b)  $O(n)$   
(c)  $\theta(n^2)$  (d)  $O(1)$  **DRDO 2009**

8) A weight-balanced tree is a binary tree in which for each node, the number of nodes in the left sub tree is at least half and at most twice the number of nodes in the right sub tree. The maximum possible height (number of nodes on the path from the root to the furthest leaf) of such a tree on  $n$  nodes is best described by which of the following ?

- (a)  $\log_2 n$  (b)  $\log_{4/3} n$   
(c)  $\log_3 n$  (d)  $\log_{3/2} n$  **GATE 2002**

9) Let  $T(n)$  be the number of different binary search trees on  $n$  distinct elements. Then  $T(n) = \sum_{k=1}^n T(k-1) \cdot T(x)$ , where  $x$  is

- (a)  $n-k+1$  (b)  $n-k$   
(c)  $n-k-1$  (d)  $n-k-2$  **GATE 2003**

10) What is the upper bound on the number of edge disjoint

spanning tree in a complete graph of  $n$  vertices ?

- (a)  $n$  (b)  $n-1$   
(c)  $\lceil n/2 \rceil$  (d)  $\lceil n/3 \rceil$  **DRDO 2009**

11) Given 2-sorted arrays each of  $n$ -elements and distinct. How much time it will take to find middle element of the union sorted array ?

- (a)  $O(1)$  (b)  $O(\log n)$   
(c)  $O(n)$  (d) None of these

Common data for Q.12 and Q.13

If a permutation  $a_1 \dots a_n$  of  $n$  distinct integers, an inversion is pair  $(a_i, a_j)$  such that  $i < j$  and  $a_i > a_j$ .

12) If all permutation are equally likely, what is the expected number of inversions in a randomly chosen permutation of  $1 \dots n$  ?

- (a)  $n(n-1)/2$  (b)  $n(n-1)/4$   
(c)  $n(n+1)/4$  (d)  $2n \lceil \log_2 n \rceil$  **GATE 2003**

13) What would be the worst case time complexity of the insertion sort algorithm, if the inputs are restricted to permutations of  $1 \dots n$  with at most  $n$  inversions ?

- (a)  $(n^2)$  (b)  $(n \log n)$   
(c)  $(n^{1.5})$  (d)  $(n)$  **GATE 2003**

14) Let  $F_k$  denote the  $k^{\text{th}}$  Fibonacci number with  $F_k = F_{k-1} + F_{k-2}$  for  $k \geq 2$ ,  $F_0 = F_1 = 1$ .

Consider the following variation of an merge sort, which assume that the number of elements in it's list argument  $L$  is a Fibonacci number  $F_k$ .

Algorithm FibMergeSort ( $L$ )

$L$  is a list of items from a totally ordered set, whose length is a Fibonacci number  $F_k$ .

```
{  
  If  $L$  contains only 1 element, then return  $L$ ;  
  else  
  {  
    divide  $L$  into  $L_1$  (the first  $F_{k-1}$  items) and  
     $L_2$  (the remaining  $F_{k-2}$  items)  
    sorted  $L_1$  := FibMergeSort( $L_1$ )  
    sorted  $L_2$  := FibMergeSort( $L_2$ )  
    sorted  $L$  := Merge (sorted $L_1$ , sorted  $L_2$ )  
    return sorted  $L$  ;  
  }  
}
```

}

Assuming that the “divide” step in FibMergeSort takes constant time (no comparision) and Merge behaves similar to the merge in the normal merge sort. Identify which of the following expression most closely matches the total number of comparisions performed by FibMergeSort when initially given a list of  $F_k$  elements ?

(a)  $O(k \log k)$

(b)  $O(K^2)$

(c)  $O(k F_k)$

(d)  $O(F_k \log k)$

15) Let  $s$  be a sorted array of  $n$  integers. Let  $t(n)$  denote the time taken for the most efficient algorithm to determine if there are two elements with sum less than 1000 in  $s$ . Which of the following statements is true ?

(a)  $t(n)$  is  $O(1)$

(b)  $n \leq t(n) \leq n \log_2 n$

(c)  $n \log_2 n \leq t(n) < (n/2)$

(d)  $t(n) = (n/2)$

**GATE 2000**

16) The cube root of a natural number  $n$  is defined as the largest natural number  $m$  such that  $m^3 \leq n$ . The complexity of computing the cube root of  $n$  ( $n$  is represented in binary notation) is

(a)  $O(n)$  but not  $O(n^{0.5})$

(b)  $O(n^{0.5})$  but not  $O(\log n)^k$  for any constant  $k > 0$

(c)  $O(\log n)^k$  for some constant  $k > 0$ , but not  $O(\log \log n)^m$  for any constant  $m > 0$

(d)  $O(\log \log n)^n$  for some constant  $k > 0.5$ , but not  $O(\log \log n)^{0.5}$

**GATE 2003**

17) In the following C function, let  $n \geq m$ .

```
int gcd (n,m)
{
    if(n%m==0) return m;
```

```

n = n%m ;
return gcd (m,n) ;
}

```

How many recursive calls are made by this function ?

- (a)  $\theta(\log_2 n)$  (b)  $\Omega(n)$   
(c)  $\theta(\log_2 \log_2 n)$  (d)  $\theta(\sqrt{n})$  **GATE 2007**

18) If we use Radix Sort to sort  $n$  integers in the range  $(n^{k/12}, n^k)$ , for some  $k > 0$  which is independent of  $n$ , the time taken would be

- (a)  $\theta(n)$  (b)  $\theta(kn)$   
(c)  $\theta(n \log n)$  (d)  $\theta(n^2)$  **GATE 2008**

19) An element in an array  $X$  is called a leader if it is greater than all elements to the right of it in  $X$ . The best algorithm to find all leaders in an array,

- (a) Solves it in linear time using a left to right pass of the array  
(b) Solves in linear time using a right to left pass  
(c) Solves it using divide and conquer in time  $\theta(n \log n)$   
(d) Solves it in time  $\theta(n^2)$  **GATE 2006**

20) Which one of the following statements are correct regarding Bellman-Ford shortest path algorithm ?

P : Always finds a negative edge weight cycle if one exists.

Q : Find whether any negative edge weight cycle reachable from the source.

(a) P only

(b) Q only

(c) Both P and Q

(d) neither P nor Q

21) If one uses straight two-way merge sort algorithm to sort the following elements in ascending order :

20 , 47 , 15 , 8 , 9 , 4 , 40 , 30 , 12 , 17

Then the order of these elements after second pass of the algorithm is

(a) 8 , 9 , 15 , 20 , 47 , 4 , 12 , 17 , 30 , 40

(b) 8 , 15 , 20 , 47 , 4 , 9 , 30 , 40 , 12 , 17

(c) 15 , 20 , 47 , 4 , 8 , 9 , 12 , 30 , 40 , 17

(d) 4 , 8 , 9 , 15 , 20 , 47 , 12 , 17 , 30 , 40

22) Let  $A[1 \dots n]$  be an array storing a bit (1 or 0) at each location, and  $f(m)$  is a function whose time complexity is  $\theta(m)$ . Consider the following program fragment written in a C like language :

```
counter = 0 ;  
for (i=1 ; i<n ; i++)  
{  
    if (A[i]==1)  
        counter ++ ;  
    else  
    {  
        f(counter) ;
```

```

    counter = 0 ;
}
}

```

The complexity of this program fragment is

- |                   |                                     |
|-------------------|-------------------------------------|
| (a) $\Omega(n^2)$ | (b) $\Omega(n \log n)$ and $O(n^2)$ |
| (c) $\theta(n)$   | (d) $O(n)$ <b>GATE 2004</b>         |

23) The number of elements that can be stored in  $\theta(\log n)$  time using heap sort is

- |                                    |                                       |
|------------------------------------|---------------------------------------|
| (a) $\theta(1)$                    | (b) $\theta(\sqrt{\log n})$           |
| (c) $\theta(\log n / \log \log n)$ | (d) $\theta(\log n)$ <b>GATE 2013</b> |

24) Suppose there are  $\log n$  sorted list of  $n/\log n$  elements each. The time complexity of producing a sorted list of all these element is :  
(Hint : use a heap data structure)

- |                        |  |
|------------------------|--|
| (a) $O(n \log \log n)$ | (b) $\theta(n \log n)$                 |
| (c) $\Omega(n \log n)$ | (d) $\Omega(n^{3/2})$ <b>GATE 2005</b> |

25) Consider the following “Max-heapify” algorithm. Array has size atleast  $n$  and  $1 \leq i \leq n$ . After applying the Max-heapify rooted at  $A[i]$  is max heap. [Assume that except root  $A[i]$ , all it's children satisfies heap property]



Max-heapify (int A[], int n, int i)

```
{  
    int p, m ;  
    p = i ;  
    while (X)  
    {  
        if (Y && Z)  
            m = 2p+1 ;  
        else m = 2p ;  
        if (a[p] < A[m])  
        {  
            Swap (A[p],A[m]) ;  
            p = m ;  
        }  
    }  
    else  
    return ;  
}
```

Find missing statements at X, Y and Z respectively to apply the heapify for subtree rooted at A[i].

(a)  $p \leq n$  ,  $(2p+1) \geq n$  ,  $A[2p+1] > A[2p]$

(b)  $2p \leq n$  ,  $(2p+1) \leq n$  ,  $A[2p+1] > A[2p]$

(c)  $2p \leq n$ ,  $(2p+1) \geq n$ ,  $A[2p+1] < A[2p]$

(d)  $p \leq n$ ,  $(2p+1) \leq n$ ,  $A[2p+1] < A[2p]$

26) Consider the following sorting algorithm.

Sorting (A, low, high)

{

if (low == high) return ;

if (low+1 = high)

{

if ( $A[\text{low}] > a[\text{high}]$ )

Swap ( $A[\text{low}], A[\text{high}]$ );

return ;

}

$t_1 = \text{low} + \frac{(\text{high} - \text{low} + 1)}{3}$  ;

3

$t_2 = \text{low} + 2 \frac{(\text{high} - \text{low} + 1)}{3}$  ;

3

Sorting (A, low,  $t_2$ ) ;

Sorting (A,  $t_1$ , high) ;

Sorting (A, low,  $t_2$ ) ;

}

What is the running time of Sorting (A,1,n) function :

(a)  $\theta(n^{1.7})$

(b)  $\theta(n^{2.7})$

(c)  $\theta(n^{3.7})$

(d)  $\theta(n^{0.7})$

27) Consider a modification to merge sort in which  $m/k$  sublists each of length  $k$  are sorted using insertion sort and then merged using standard merge procedure. Then find total time complexity of modified merger sort.

### 3. Greedy Technique and Dynamic Programming

1) Adjacency list is preferred over adjacency matrix when the graph is

(a) Planer

(b) Dense

(c) Clique

(d) None of these **DRDO 08**

2) A complete n-ary tree in which each node has n children or no children. Let I be the number of internal nodes and L be the number of leaves in a complete n-ary tree. If  $L=41$ , and  $I=10$ , what is the value of n ?

(a) 3

(b) 4

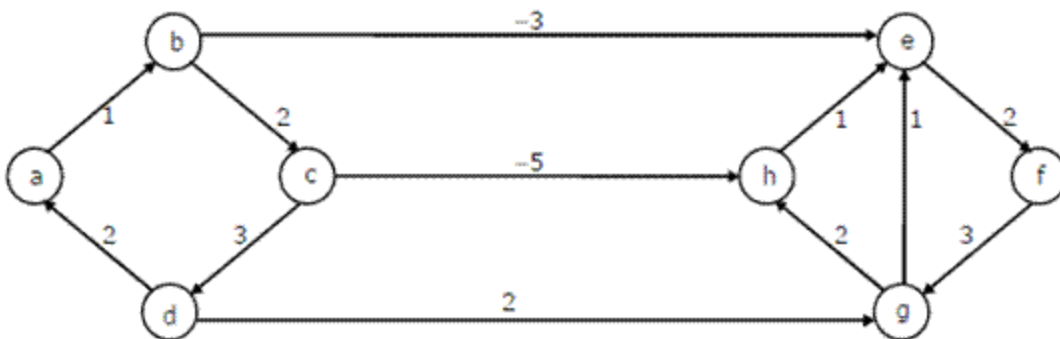
(c) 5

(d) 6

**GATE 2007**

3) Dijkstra's single source shortest path algorithm when run from vertex a in the above graph, computes the correct shortest path distance to

**GATE 2008**



(a) only vertex a

(b) only vertices a, e, f, g, h

(c) only vertices a, b, c, d

(d) All the vertices

4) Let  $w$  be the minimum weight among all edge weights in an undirected connected graph. Let  $e$  be a specific edge of weight  $w$ .

Which of the following FALSE ?

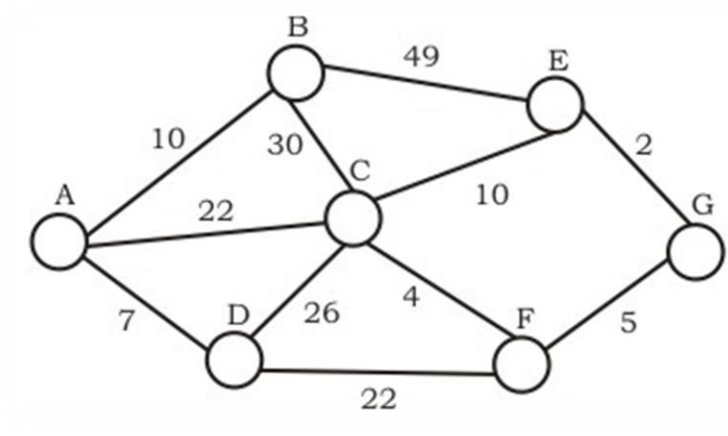
- (a) There is a minimum spanning tree containing  $e$ .
- (b) If  $e$  is not in a minimum spanning tree  $T$ , then in the cycle formed by adding  $e$  to  $T$ , all edges have the same weight.
- (c) Every minimum spanning tree has an edge of weight  $w$ .
- (d)  $e$  is present in every minimum spanning tree GATE 2007

5) The values of  $l(i, j)$  could be obtained by dynamic programming based on the correct recursive definition of  $l(i, j)$  of the form given above, using an array  $L[M, N]$ , where  $M = m+1$  and  $N = n+1$ , such that  $L[i, j] = l(i, j)$ . Which one of the following statements would be TRUE regarding the dynamic programming solution for the recursive definition of  $l(i, j)$  ?

- (A) All elements  $L$  should be initialized to 0 for the values of  $l(i, j)$  to be properly computed
- (B) The values of  $l(i, j)$  may be computed in a row major order or column major order of  $L(M, N)$
- (C) The values of  $l(i, j)$  cannot be computed in either row major order or column major order of  $L(M, N)$
- (D)  $L[p, q]$  needs to be computed before  $L[r, s]$  if either  $p < r$  or  $q < s$

6) Consider the following statements :

- I. For every weighted graph and any two vertices  $s$  and  $t$ , Bellman-ford algorithm starting at  $s$  will always return a shortest path to



t.

II. At the termination of the Bellman-Ford algorithm, even if graph

has negative weight cycle, a correct shortest path is found for a

vertex for which shortest path is well-defined.

Which of the above statements are true ?

7) Consider the following statements :

I. Let  $T$  be a minimum spanning tree of a graph  $G$ . Then for any vertices  $u$  and  $v$  the path from  $u$  to  $v$  in  $T$  is the shortest path from  $u$  to  $v$  in the graph  $G$ .

II. Suppose that average edge weight for a graph  $G$  is  $A_{avg}$ . Where  $n$  is number of vertices in graph  $G$ .

Which of the above statements are true ?

(a) Only I

(b) Only II

(c) both I and II

(d) None of these

8) Consider the undirected graph below :

Using Prim's algorithm to construct a minimum spanning tree starting with node  $A$ , which one of the following sequences of edges represents a possible order in which the edges would be added to construct the minimum spanning tree?

(A)  $(E, G), (C, F), (F, G), (A, D), (A, B), (A, C)$

(B) (A, D), (A, B), (A, C), (C, F), (G, E), (F, G)

(C) (A, B), (A, D), (D, F), (F, G), (G, E), (F, C)

(D) (A, D), (A, B), (D, F), (F, C), (F, G), (G, E) **GATE 2004**

9) Let  $G$  be a weighted undirected graph and  $e$  be an edge with maximum weight in  $G$ . Suppose there is a minimum weight spanning tree in  $G$  containing the edge  $e$ . Which of the following statements is always TRUE?

(A) There exists a cutset in  $G$  having all edges of maximum weight.

(B) There exists a cycle in  $G$  having all edges of maximum weight

(C) Edge  $e$  cannot be contained in a cycle.

(D) All edges in  $G$  have the same weight

10) Consider the tree arcs of a BFS traversal from a source node  $W$  in an unweighted, connected, undirected graph. The tree  $T$  formed by the tree arcs is a data structure for computing.

(A) the shortest path between every pair of vertices.

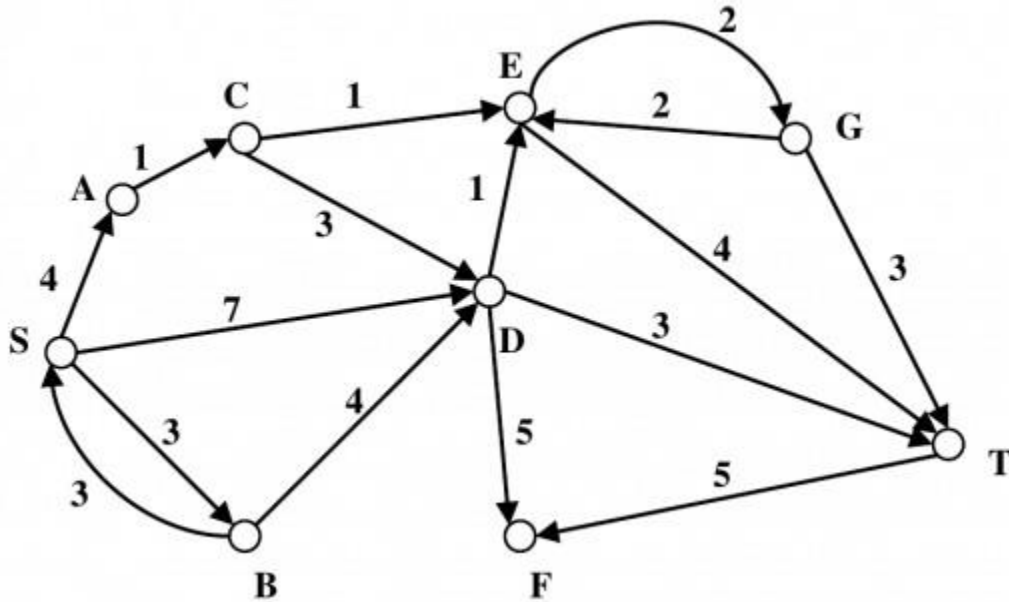
(B) the shortest path from  $W$  to every vertex in the graph.

(C) the shortest paths from  $W$  to only those nodes that are leaves of  $T$ .

(D) the longest path in the graph. **GATE 2014**

11) Consider the directed graph shown in the figure below. There are multiple shortest paths between vertices  $S$  and  $T$ . Which one will be reported by Dijkstra's shortest path algorithm? Assume that, in any iteration, the shortest path to a vertex  $v$  is updated

only when a strictly shorter path to  $v$  is discovered.



(a) SDT  
(c) SACDT

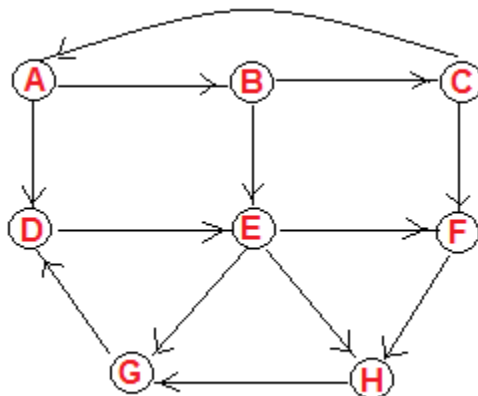
(b) SVDT  
(d) SACET **GATE 2012**

12)  $G=(V,E)$  is an undirected simple graph in which each edge has a distinct weight, and  $e$  is a particular edge of  $G$ . Which of the following statements about the minimum spanning trees (MSTs) of  $G$  is/are TRUE ?

- I. If  $e$  is the lightest edge of some cycle in  $G$ , then every MST of  $G$  include  $e$ .
- II. If  $e$  is the heavistedge of some cycle in  $G$ , then every MST of  $G$  excludes  $e$ .

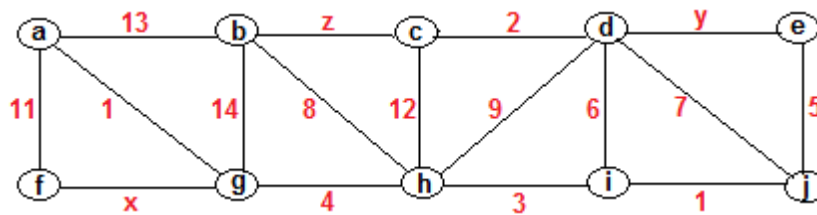
a) I only  
c) Both I and II  
(b) II only  
(d) Neither I nor II **GATE 2016**

13) Find the number of strong components in the following graph :





- 14) Suppose the maximum spanning tree of the following edge weighted graph contains the edges with weights  $x, y$  and  $z$ .



What is the maximum value of  $x+y+z$  ?

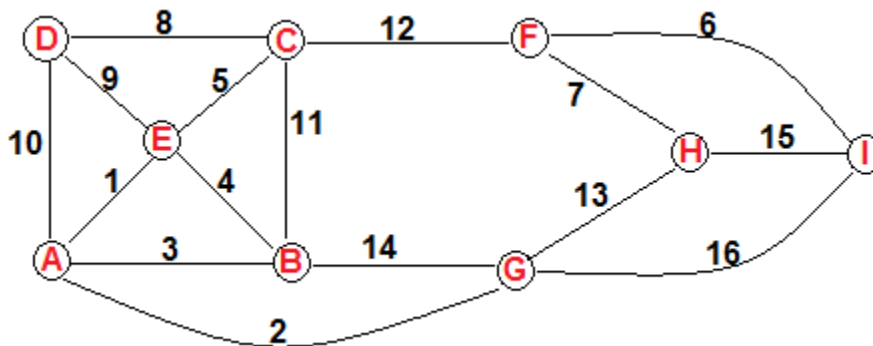
- 15) Suppose you want to move from 0 to 100 on the number line. In each step, you either move right by a unit distance or you take a shortcut. A shortcut is simply a pre-specified pair of integers  $i, j$  with  $i < j$ . Given a shortcut  $i, j$  if you are at position  $i$  on the number line, you may directly move to  $j$ . Suppose  $T(k)$  denotes the smallest number of steps needed to move from  $k$  to 100. Suppose further that there is at most 1 shortcut involving any number, and in particular from 9 there is a shortcut to 15. Let  $y$  and  $z$  be such that  $T(9) = 1 + \min(T(y), T(z))$ . Then the value of the product  $yz$  is \_\_\_\_.

(A) 50  
(C) 150

(B) 100  
(D) 200

GATE 2014

- 16) Consider the weighted undirected graph below :



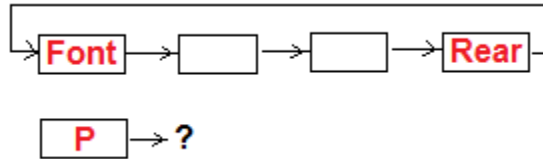
Assume Prim's algorithm and Kruskal's algorithm are executed on the above graph to find the minimum spanning tree. For a particular edge

$(e_j)$  which is included in minimum spanning tree and the position of an edge in minimum spanning tree is denoted by  $e_{pi}$ . Where  $1 \leq e_{pi} \leq 8$  (where position defines the order in which edges are included in the MST). Then what is the maximum value of  $|(e_{pi})_{\text{prim's}} - (e_{pi})_{\text{krushkal's}}|$  ?

- 17) Complexity of krushkal algorithm for finding the minimum algorithm cost spanning tree of an undirected graph contain  $n$ -vertices and  $m$ -edges, if the edges are already sorted.
- 18) Given a sequence of  $n$ -real numbers  $a_1, a_2, a_3 \dots a_n$  then to find contiguous subsequence  $a_i, a_{i+1}, a_{i+1}, \dots, a_j$ . Such that its sum is  $\max^m$ .
- 19) Let  $T$  be a MST of  $G$ . Suppose that we decreased the weight of one of the edge present in  $G$  but not in  $T$ . Then how much time will take to construct for the modified graph  $G$  ?
- 20) Let  $G$  be a undirected graph of  $n$ -node. Any two of the following statements implies the 3<sup>rd</sup>. Is it True/False ?
  - i)  $G$  is connected
  - ii)  $G$  don't have cycle
  - iii)  $G$  contain  $n-1$  edges

#### 4. Linked List, Hashing, Stack, Queue and Array

- 1) A circularly linked list is used to represent a Queue. A single variable p is used to access the Queue. To which node should p point such that both the operations Enqueue and Dequeue can be performed in constant time ?



- (a) rear node  
(b) front node  
(c) not possible with a single pointer  
(d) node next to front

GATE 2004

- 2) The following C function takes a singly-linked list of integers as a parameter and rearranges the elements of the list. The function is called with the list containing integers 1,2,3,4,5,6,7 in the given order. What will be the contents of the list after the function completes execution ?

```
struct nodes
{
    int value ;
    struct node *next ;
} ;
void rearrange (struct node *list)
{
    struct node *p, *q ;
    int temp ;
    if (!list || !list → next) return ;
    p=list;
    q= list→next ;
    while (q)
    {
        temp = p→value ;
        p→value = q→value ;
        q→value = temp ;
        p= q→next ;
    }
```

- ```

    q=p ? p→next : 0 ;
  }
}

```
- (a) 1,2,3,4,5,6,7 (b) 2,1,4,3,6,5,6  
 (c) 1,3,2,5,4,7,6 (d) 2,3,4,5,6,7,1 **GATE 2008**
- 3) Suppose each set is represented as a linked with elements in arbitrary order. which of the operation among union, intersection, membership, cardinality will be the slowest ?  
 (a) union only (b) intersection, membership  
 (c) membership, cardinality (d) union, intersection **GATE 2004**
- 4) A double linked list facilitates the determination of the predecessor of a given item. Which of the following operation utilizes this property of a doubly-linked list to it's greatest advantage ?  
 (a) accessing an item (b) recovering a lost pointer  
 (c) copying a list (d) merging two list
- 5) Let P be a singly linked list. Let Q be the pointer to an intermediate node x in the list. What is the worst-case time complexity of the best known algorithm to delete the node x from the list?  
 (A)  $O(n)$  (B)  $O(\log^2 n)$   
 (C)  $O(\log n)$  (D)  $O(1)$  **GATE 2004**
- 6) Which of the following is the best choices as m is the hash function,  $h(k)=k.\text{mod } m$  ?  
 (a) 61 (b) 701  
 (c) 81 (d) 1031 **GATE 2000**
- 7) Consider the following function.
- ```

void madeeasy(int n)
{
  enqueue (Q,0);
  enqueue (Q,1);
  for (i=0; i<n; i++)
  {

```

```

x = dequeue (Q) ;
y = dequeue (Q) ;
enqueue (Q,y) ;
enqueue (Q,x+y) ;
print (x) ;
}
}

```

What is the functionality of above function madeeasy ?

- (a) prints number from 0 to n-1
- (b) prints numbers from n-1 to 0
- (c) prints first n Fibonacci numbers
- (d) prints first n Fibonacci numbers in reverse order

8) Choosing the hash function randomly from a class of hash functions such that it is independent of the keys to be stored, is termed as :

- (a) perfect hashing
  - (b) simple uniform hashing
  - (c) universal hashing
  - (d) none
- GATE 2000

9) Given the hash function  $h(k,i) = (h'(k) + i + i^2) \bmod 11$  and  $h'(k) = k \bmod 11$ . What is the number of collision to store the following keys ?

- (a) 3
  - (b) 2
  - (c) 11
  - (d) None
- GATE 2000

10) Consider an open address hash table with a total of 10,000 slot, containing 9800 entries. What is expected number of probes in successful search ?

- (a) 2
  - (b) 3
  - (c) 4
  - (d) 4.5
- GATE 2000

11) To evaluate an expression without any embedded function call

- (a) one stack is enough
  - (b) two stack is enough
  - (c) as many stack as the height of the expression tree are needed
  - (d) a turning machine is needed in the general case
- GATE 2002

- 12) Let  $S$  be a stack of size  $n \geq 1$ . Starting with the empty stack, suppose we push the first  $n$  natural number in sequence, and then perform  $n$  pop operations. Assume that push and pop operations take  $X$  seconds each, and  $Y$  seconds elapse between the end of one such stack operation and the start of the next operation. For  $m \geq 1$ , define the stack-life of  $m$  as the time elapsed from the end of push( $m$ ) to the start of the pop operation that removes  $m$  from  $S$ . The average stack-life of an element of this stack is
- (a)  $n.(X+Y)$  (b)  $3Y + 2X$   
(c)  $n.(X+Y) - Y$  (d)  $Y - 2X$  GATE 2003
- 13) A single array  $A[1..MAXSIZE]$  is used to implement two stacks. The two stacks grow from opposite ends of the array. Variables  $top1$  and  $top2$  ( $top1 < top2$ ) point to the location of the topmost element in each of the stacks. If the space is to be used efficiently, the condition for "stack full" is
- (A) ( $top1 = MAXSIZE/2$ ) and ( $top2 = MAXSIZE/2+1$ )  
(B)  $top1 + top2 = MAXSIZE$   
(C) ( $top1 = MAXSIZE/2$ ) or ( $top2 = MAXSIZE$ )  
(D)  $top1 = top2 - 1$  GATE 2004
- 14) What is the maximum size of the operator stack during the conversion of the infix expression  $A+B*C-D/F$  to postfix ?
- (a) 1 (b) 2  
(c) 3 (d) 4 DRDO 2008
- 15) What is maximum size of the operand stack while evaluating the postfix expression  $6\ 2\ 3\ +\ -\ 3\ 8\ 2\ /\ +\ *?$
- (a) 1 (b) 2  
(c) 3 (d) 4 DRDO 2008
- 16) A Circular queue has been implemented using singly linked list where each node consists of a value and a pointer to next node. We maintain exactly two pointers **FRONT** and **REAR** pointing to the front node and rear node of queue. Which of the following statements is/are correct for circular queue so that insertion and

deletion operations can be performed in  $O(1)$  i.e. constant time.

I. Next pointer of front node points to the rear node.

II. Next pointer of rear node points to the front node.

(A) I only

(B) II only

(C) Both I and II

(D) Neither I nor II

GATE 2017

17) Suppose you are given an implementation of a queue of integers. The operations that can be performed on the queue are:

i. `isEmpty(Q)` — returns true if the queue is empty, false otherwise.

ii. `delete(Q)` — deletes the element at the front of the queue and returns its value.

iii. `insert(Q,i)` — inserts the integer `i` at the rear of the queue.

Consider the following function :

```
void f (queue Q)
{
    int i ;
    if (!isEmpty(Q))
    {
        i = delete(Q);
        f(Q);
        insert(Q, i);
    }
}
```

What operation is performed by the above function f ?

- A. Leaves the queue Q unchanged
  - B. Reverses the order of the elements in the queue Q
  - C. Deletes the element at the front of the queue Q and inserts it at the rear keeping the other elements in the same order
  - D. Empties the queue Q
- GATE 2007

18) Suppose a circular queue of capacity  $(n-1)$  elements is implemented with an array of  $n$  elements. Assume that the insertion and deletion operations are carried out using REAR and FRONT as array index variables, respectively.

Initially,  $\text{REAR} = \text{FRONT} = 0$ . The conditions to detect queue full and queue empty are

- (a) full:  $(\text{REAR} + 1) \bmod n == \text{FRONT}$   
empty:  $\text{REAR} == \text{FRONT}$
  - (b) full:  $(\text{REAR} + 1) \bmod n == \text{FRONT}$   
empty:  $(\text{FRONT} + 1) \bmod n == \text{REAR}$
  - (c) full:  $\text{REAR} == \text{FRONT}$   
empty:  $(\text{REAR} + 1) \bmod n == \text{FRONT}$
  - (d) full:  $(\text{FRONT} + 1) \bmod n == \text{REAR}$   
empty:  $\text{REAR} == \text{FRONT}$
- GATE 2012

19) Consider the following keys that are hashed into the hash table in the order given using the hash function  $h(i) = (2i + 5) \bmod 11$

12, 44, 13, 88, 23, 94, 11, 39, 20, 16, 5

Assume hash table has location from 0 to 10.

If hash table uses chaining to handle the collisions, how many locations are left without hashing any element into it ?

20) Consider the implementation of multiple stacks in single array S of size P from 0 to P-1. Number of stacks are Q. The following function PUSH(), used to push data x on to a particular stack i where  $T_i$  is used as top variable for stack i (i indicates stack number).

```
PUSH (S, P, Q,  $T_i$ , x)
{
    if (___A___)
```



```

{
    print("stack overflow");
    exit (1) ;
}
else
    Ti ++ ;
    S(Ti) = x ;
}

```

Stack 0 stores elements from 0 to Q-1, stack 1 stores from q to 2Q-1, and similarly other stack will store elements. Which of the following is the correct expression to replace A in the above function ?

- (a)  $T_i = P/Q \times i - 1$                       (b)  $T_i = (P/Q \times i + 1)$   
 (c)  $T_i = (P/Q \times (i-1) - 1)$               (d)  $T_i = (P/Q \times (i+1) - 1)$

## 5. Tree and Graph

- 1) Among the following statement, identify the false statement.
  - (a) We must balance a left-of-left unbalanced AVL tree by rotating the out-of-balance node to the right.
  - (b) The inorder traversal of a binary search tree produces an ordered list.
  - (c) when a module calls a subroutine recursively, in each call, all of the information is popped in the same order when subroutine are terminated one after another and finally the control is returned to the calling module.
  - (d) A recursion algorithm has two elements : Each call either solves only part of the problem or it reduces the size of the problem.
- 2) Suppose a binary tree has only 3 node A,B and C. The post-order traversal of the tree is B-A-C. The exact preorder traversal for the tree is :
  - (a) C-A-B                                      (b) A-B-C
  - (c) C-B-A                                      (d) a definite pre-order traversal cannot be determined.
- 3) The most efficient algorithm for finding the number of connected components in an undirected graph on n vertices and m edges has time complexity ?
  - (a)  $\theta(n)$                                       (b)  $\theta(m)$

- 4) Which of the following statement is false ?
- (a) The depth of any DFS (Depth First Search) tree rooted at a vertex is at least as much as the depth of any BFS tree rooted at the same vertex.
  - (b) if all edges in a graph have distinct weight then the shortest path between two vertices is unique
  - (c) For a directed graph, the absence of back edge in a DFS tree means graph has no cycle.
  - (d) BFS take  $O(V^3)$  time in a graph  $G(V,E)$  if graph is represented with an adjacency matrix.
- 5) Consider the tree arcs of a BFS traversal from a source node  $w$  in an weighted, connected, undirected graph with equal edge weights. The tree  $T$  formed by the tree arcs is a data structure for computing.
- (a) The shortest path between every pair of vertices
  - (b) The shortest path from  $w$  to every vertex in the graph
  - (c) the shortest path from  $w$  to only those nodes that are leaves of  $T$ .
  - (d) the longest path in the graph.
- 6) In delete operation of binary search tree, we need inorder successor (or predecessor) of a node when a node to be deleted where it has both left and right child. Which of the following is true about inorder successor needed in delete operation ?
- (a) Inorder successor is always either leaf node or a node with empty right child.
  - (b) Inorder successor maybe a an ancestor of the node.
  - (c) Inorder successor is always a leaf node.
  - (d) Inorder successor is always either a leaf node or a node with empty left child.
- 7) The worst case running time complexity to search for an element in a balanced binary search tree with  $(2n)!$  elements ?
- (a)  $O(n^n)$
  - (b)  $O(n \log n)$

(c)  $O(n)$

(d)  $O(n^2)$

- 8) Consider the pseudocode given below. The function DoSomething() takes as argument a pointer to the root of an arbitrary tree represented by the leftMostChild-rightSibling representation. Each node of the tree is of type treeNode.

```
typedef struct treeNode* treeptr;

struct treeNode
{
    treeptr leftMostChild, rightSibling;
};

int DoSomething (treeptr tree)
{
    int value=0;
    if (tree != NULL)
    {
        if (tree->leftMostChild == NULL)
            value = 1;
        else
            value = DoSomething(tree->leftMostChild);
        value = value + DoSomething(tree->rightSibling);
    }
    return(value);
}
```

When the pointer to the root of a tree is passed as the argument to DoSomething, the value returned by the function corresponds to the

- (A) number of internal nodes in the tree.
- (B) height of the tree.
- (C) number of nodes without a right sibling in the tree.
- (D) number of leaf nodes in the tree.

**GATE 2014**

- 9) If the graph  $G(V,E)$  is represented using adjacency matrix then

k to find universal sink (in-degree is  $V-1$  and out-degree is 0).

10) Binary search can be carried out on a set of ordered data items stored in a

- (a) array
- (b) stack
- (c) queue
- (d) list

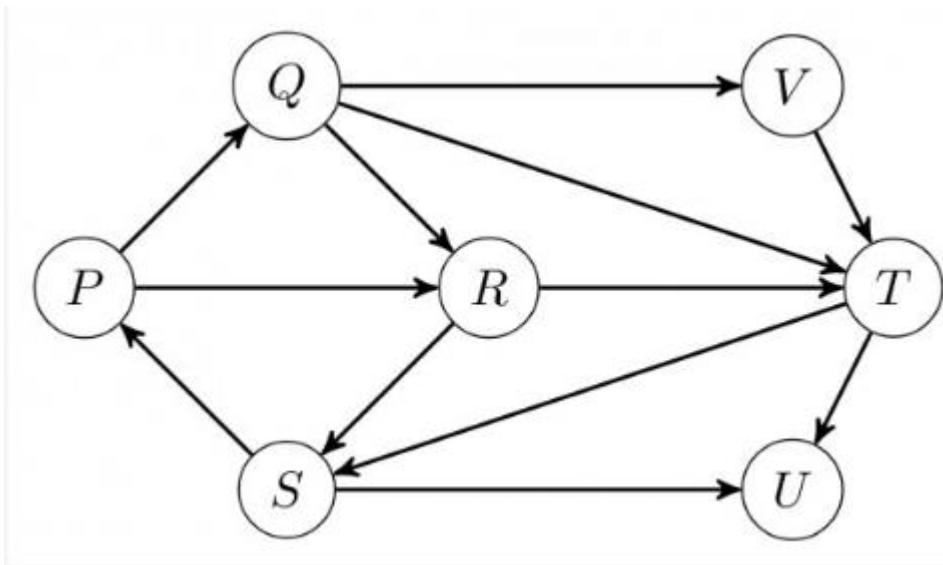
DRDO 2008

11) How many distinct BSTs can be constructed with 3 distinct key ?

- (a) 4
- (b) 5
- (c) 6
- (d) 9

GATE 2008

12) Which of the following is the correct decomposition of the directed graph given below into its strongly connected components ?



- (A) {P, Q, R, S}, {T}, {U}, {V}
- (B) {P, Q, R, S, T, V}, {U}
- (C) {P, Q, S, T, V}, {R}, {U}
- (D) {P, Q, R, S, T, U, V}

GATE 2006

13) You are given the postorder traversal, P, of a binary search tree on the n elements 1, 2, ..., n. You have to determine the unique binary search tree that has P as its postorder traversal. What is the time complexity of the most efficient algorithm for doing

this?

(A)  $O(\log n)$

(B)  $O(n)$

(C)  $O(n \log n)$

(D) none of the above, as the tree

cannot be uniquely determined.

**GATE 2008**

14) We have a binary heap on  $n$  elements and wish to insert  $n$  more elements (not necessarily one after another) into this heap. The total time required for this is

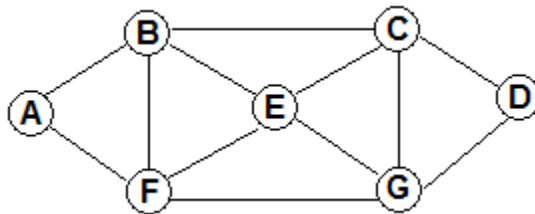
(A)  $(\log n)$

(B)  $(n)$

(C)  $(n \log n)$

(D)  $(n^2)$  **GATE 2008**

15) Consider the following graph :



The maximum and minimum size of queue and stack required when performing BFS and DFS respectively on the above graph \_\_\_\_.

16) What is the maximum possible height of an AVL tree with 20 nodes ?

**DRDO 2008**

17) Assume the preorder traversal of binary tree is "abc". How many total different binary trees are possible whose postorder

traversal is “cba” with the given preorder traversal ?

18) Consider a rooted  $n$  node binary tree represented using pointer.

The best upper bound on the time required to determine the number of subtrees having exactly 4 nodes is  $O(n^a \log^b n)$ . Then value of  $a+10b$  is \_\_\_\_\_. GATE 2014

19) Given an adjacency-list represented of directed graph  $G(V,E)$ .

Then how much time does it take to compute the out-degree of each vertex.