TUTORIAL-2

Sol 1. $j = 1$, $i = 0$

while(i<n){ i = i + j; j++;}

j = 1, 2, 3, 4, 5, 6

i = 0, 1, 3, 7, 12, .

$TC = O(n)$

Sol 2.

```
void iteration(int n)
{   int f = 1;
    for(i=2; i<=n ; i++)
        f *= i;
    cout<<f;
}
```

TC = O(n)
SC = O(1)

```
int recursing(int n)
{   if (n < 2)
        return 1;
    return n * recursing(n-1);
}
```

TC = O(n)
SC = O(n)

SC is O(n) as stack is created in the memory while recurrence function was called till reaching the base condition.

Sol 3. $n \log n$ ⇒

```
for(i=0; i<n; i++)
    for(j=0; j*j<n; j++)
        sum = sum + j;
```

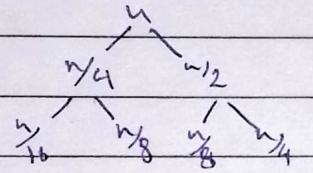
n^3 ⇒

```
for(i=0; i<n; i++)
    for(j=0; j<n; j++)
        for(k=0; k<n; k++)
            sum = sum + k;
```

$\log(\log n)$ ⇒

```
i = n
while(i > 0)
    i = √i ;
    i--;
```

Sol4. $T(n) = T(n/4) + T(n/2) + cn^2$



n^2

$\frac{n^2}{16} + \frac{n^2}{4} = \frac{5n^2}{16}$

$\frac{n^2}{256} + \frac{n^2}{64} + \frac{n^2}{64} + \frac{n^2}{16} = \frac{25n^2}{256}$

$$T(n) = c\left(n^2 + \frac{5n^2}{16} + \frac{25n^2}{256}\right)$$

$$r = \frac{5}{16} \Rightarrow 5n = \frac{1}{1-r}$$

$$T(n) = cn^2\left(1 + \frac{5}{16} + \frac{25}{256} + \cdots\right)$$

$$= c n^2 \left(\frac{1}{1-5/16}\right) = cn^2 \frac{16}{11}$$

$$TC = O(n^2)$$

Sol5.

```
int fun(int n)
    for(int i=1; i<n; i++)          — O(n)
        for(j=1; j<n; j+=i)         — log(n)
            // O(1)                 — O(1)
```

$TC = O(n \log n)$

Sol6. $i = 2, 2^k, 2^{k^2}, 2^{k^3}, 2^{k^4}, \ldots 2^{k^x}$

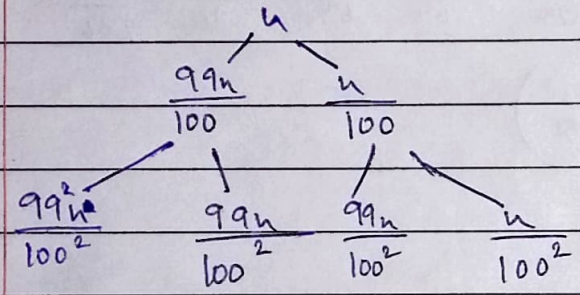
$n = 2^{k^x}$

$\log n = k^x \log 2$

$\frac{\log \log n}{\log 2} = x \log k \Rightarrow x = \frac{\log \log n}{\log 2 \times \log k}$

$$TC = O(\log(\log n))$$

Sol 7.



Taking longest branch = $\frac{99n}{100}$

$$TC = \log \frac{100n}{99} = \log n$$

$$n = \left(\frac{99}{100}\right)^k \Rightarrow k = \log\left(\frac{100n}{99}\right)$$

$$T(n) = n\left(\log_{\frac{100}{99}}\right)^n \quad /100$$

$$TC = O(n \log n)$$

Sol 8. (a) $100 < \log\log n < \log n < \sqrt{x}\,(\text{sqrt}(n)) < n < n\log n < n^2 < 2^n < 2^{2^n} < 4^n < n!$

(b) $1 < \log\log n < \sqrt{\log n} < \log n < \log 2n < \log n < n < 2n < 4n < n\log n < n^2 < \log(n!) < 2^{2^n} < n!$

(c) $96 < \log 8^n < \log_2 n < 5n < n.\log(n) < n\log_2 n < 8n^2 < 7n^3 < \log(n!) < 8^{2n} < n!$