Design and Analysis of Algorithms

Tutorial-4

Master Theorem

The Master Theorem applies to recurrences of the following form: T (n) = aT (n/b) + f (n)

where $a \geq 1$ and $b > 1$ are constants and f(n) is an asymptotically positive function. There are 3 cases:

1. If $f(n) = O(n^{\log_b a-\varepsilon})$ for some constant $\varepsilon > 0$, then $T(n) = \Theta(n^{\log_b a})$.
2. If $f(n) = \Theta(n^{\log_b a} \log^k n)$ with[1] $k \geq 0$, then $T(n) = \Theta(n^{\log_b a} \log^{k+1} n)$.
3. If $f(n) = \Omega(n^{\log_b a+\varepsilon})$ with $\varepsilon > 0$, and f(n) satisfies the regularity condition, then $T(n) = \Theta(f(n))$. Regularity condition: $af(n/b) \leq cf(n)$ for some constant $c < 1$ and all sufficiently large n.

Practice Problems

For each of the following recurrences, give an expression for the runtime T(n) if the recurrence can be solved with the Master Theorem. Otherwise, indicate that the Master Theorem does not apply.

1. $T(n)=3T(n/2)+n^2$	12. $T(n)=sqrt(n)T(n/2)+logn$
2. $T(n)=4T(n/2)+n^2$	13. $T(n)=3T(n/2)+n$
3. $T(n)=T(n/2)+2^n$	14. $T(n)=3T(n/3)+sqrt(n)$
4. $T(n)=2^nT(n/2)+n^n$	15. $T(n)=4T(n/2)+cn$
5. $T(n)=16T(n/4)+n$	16. $T(n)=3T(n/4)+nlogn$
6. $T(n)=2T(n/2)+ nlogn$	17. $T(n)=3T(n/3)+n/2$
7. $T(n)=2T(n/2)+n/logn$	18. $T(n)=6T(n/3)+n^2logn$
8. $T(n)=2T(n/4)+n^{0.51}$	19. $T(n)=4T(n/2)+n/logn$
9. $T(n)=0.5T(n/2)+1/n$	20. $T(n)=64T(n/8)-n^2logn$
10. $T(n)=16T(n/4)+n!$	21. $T(n)=7T(n/3)+n^2$
11. $T(n)=4T(n/2)+logn$	22. $T(n)=T(n/2)+n(2-cosn)$