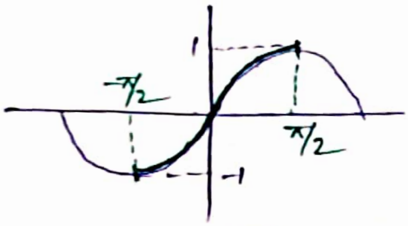
Inverse Trigonometry Function

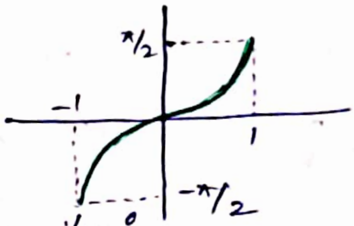
All trigonometric fn are periodic & hence many one therefore their inverse does not exist.
we curtail their domain without compromising on their Range

$f : [-\pi/2, \pi/2] \to [-1, 1]$ $\qquad$ $f^{-1} : [-1, 1] \to [-\pi/2, \pi/2]$

$f(x) = \sin(x)$ $\qquad$ $f(x) = \sin^{-1}(x)$



inverse $\longrightarrow$ Exists



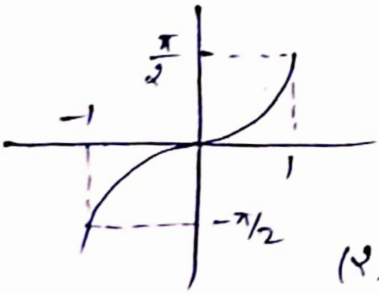
Trign fn (Angle daalo) = Value milagi

Inv. Trig. fn(Value daalo) = Angle milega

Inv fn Domain (values) $\qquad$ Range of Inverse (Angles)

$\boxed{\sin^{-1}x \text{ stands for angle in } [-\pi/2, \pi/2] \text{ whose sine is } x}$

* Domain, Range & Graph of inverse :



$y = \sin^{-1}x$

Domain $\equiv [-1, 1]$

Range $\equiv [-\pi/2, \pi/2]$

(1) Inc fn i.e $x_1 > x_2 \Leftrightarrow \sin^{-1}x_1 > \sin^{-1}x_2 \ \forall x_1, x_2$

(2) Odd fn $\sin^{-1}(-x) = -\sin^{-1}x$

(3) $\sin^{-1}x\big|_{max} = \pi/2$, $\sin^{-1}x\big|_{min} = -\pi/2$

(4) Vertical tangents at $x = -1, 1$

$$\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$$

(5) A periodic

(6) Point of inflection at $(0,0)$

~~(5) A periodic~~

~~(6) Point of inflection.~~

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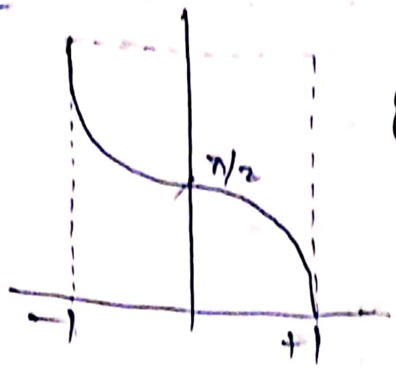

cos⁻¹x stands for angle in [0, π] whose cosine is x

Domain [-1, 1]

Range [0, π]

$f^{-1}(x) = \cos^{-1}(x)$

{ one-one, onto }

(1) Dec. $x_1 > x_2 \Leftrightarrow \cos^{-1}x_1 < \cos^{-1}x_2 \;\forall\; x_1, x_2 \in [-1, 1]$

(2) Neither odd nor even

(3) V.T at $x = -1, 1$

$$\frac{d(\cos^{-1}x)}{dx} = \frac{-1}{\sqrt{1-x^2}}$$

(4) A periodic (periodic nahi)

(5) Point of inflexion at $(0, \pi/2)$

(6) $\cos^{-1}x\big|_{MAX} = \pi$, $\cos^{-1}x\big|_{MIN} = 0$

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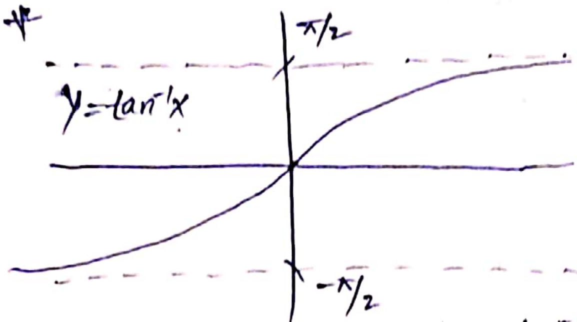


tan⁻¹x denotes angle in $(-\pi/2, \pi/2)$ whose tan is x

$f: (-\pi/2, \pi/2) \longrightarrow R$

(1) Inc $x_1 > x_2 \Leftrightarrow \tan^{-1}x_1 > \tan^{-1}x_2 \;\forall\; x_1, x_2 \in R$

(2) odd fn $\tan^{-1}(-x) = -\tan^{-1}x$

(3) $\lim_{x\to\infty} \tan^{-1}x = \pi/2$, $\lim_{x\to-\infty} \tan^{-1}(x) = -\pi/2$

(4) Point of inflection at (0,0)

(5) $\tan^{-1}x\big|_{max} \Rightarrow$ DNE $\to \pi/2$

$\tan^{-1}x\big|_{MIN} \Rightarrow$ DNE $\to -\pi/2$

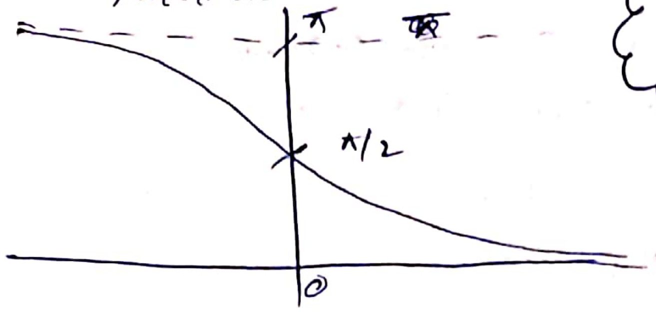
* $$\frac{d(\tan^{-1}x)}{dx} = \frac{1}{1+x^2}$$

(6) A periodic

(7) NO. V.T

$\boxed{\begin{array}{l}\text{Domain : } R \\ \text{Range : } (-\pi/2, \pi/2)\end{array}}$

* ~~$f:(0,\pi)\to R$~~



$y = \cot^{-1}x$

"$y=\cot^{-1}x$ denotes an angle in $(0,\pi)$ whose cot is x"

(1) dec: $x_1 > x_2 \Leftrightarrow \cot^{-1}x_1 < \cot^{-1}x_2$ $\forall\ x_1, x_2 \in R$

(2) Neither Odd Nor even

(3) A periodic

(4) Point of inflection at $(0, \pi/2)$

(5) NO V.T

(6) $\lim_{x\to+\infty} \cot^{-1}x = 0$, $\lim_{x\to-\infty} \cot^{-1}x = \pi$

(7) $\cot^{-1}x|_{MAX} = DNE \to \pi$

$\cot^{-1}x|_{MIN} = DNE \to 0$

Domain: R

Range: $(0,\pi)$

$$\left\{\frac{d(\cot^{-1}x)}{dx} = \frac{-1}{1+x^2}\right\}$$

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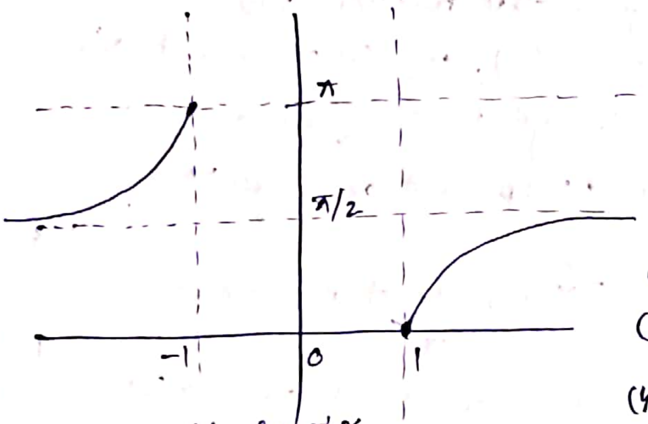

$y = \sec^{-1}x$

~~$f:([0,\pi]-\{\pi/2\})\to(-\infty,-1]\cup[1,\infty)$~~

Domain: $R-(-1,1)$

or

$(-\infty,-1]\cup[1,\infty)$

Range: $[0,\pi]-\left\{\frac{\pi}{2}\right\}$

(1) inc in $(-\infty,-1]\cup[1,\infty)$

$x_1 > x_2 \Leftrightarrow \sec^{-1}(x_1) > \sec^{-1}(x_2)$

$\forall\ x_1, x_2 \in (-\infty,-1]$ or $x_1, x_2 \in [1,\infty)$

(2) Neither odd nor even

(3) A periodic

(4) $\frac{d(\sec^{-1}x)}{dx} = \frac{1}{|x|\sqrt{x^2-1}}$ V.T at $x=-1, 1$

(5) No inflexion point

(6) $\lim_{x\to+\infty} \sec^{-1}(x) = \pi/2$

$\lim_{x\to-\infty} \sec^{-1}(x) = \pi/2$

"$\sec^{-1}x$ denotes an angle in $[0,\pi]-\{\pi/2\}$"

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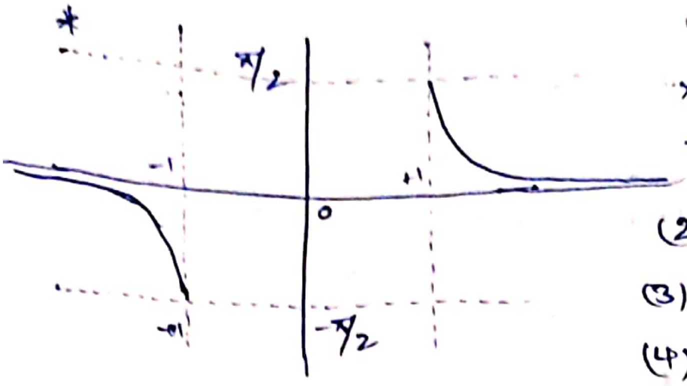


$y = \text{cosec}^{-1}(x)$

~~$f : [-\frac{\pi}{2}, \frac{\pi}{2}] - \{0\} \to R - (-1,1)$~~

Domain: $R - (-1,1)$

Range: $[-\frac{\pi}{2}, \frac{\pi}{2}] - \{0\}$

(1) Dec in $R-(-1,1)$

$x_1 > x_2 \implies \text{cosec}^{-1}(x_1) < \text{cosec}^{-1}(x_2)$

$\forall\, x \in (-\infty, -1]$ or $[1, \infty)$

(2) odd function

(3) A periodic

(4) $\dfrac{d(\text{cosec}^{-1}x)}{dx} = \dfrac{-1}{|x|\sqrt{x^2-1}}$

(5) No inflection point

(6) $\lim_{x\to\infty} \text{cosec}^{-1}(x) = 0$

$\lim_{x\to-\infty} \text{cosec}^{-1}(x) = 0$

$\text{cosec}^{-1}(x)$ denotes angle in $[-\pi/2, \pi/2] - \{0\}$

KUCH IMPORTANT POINT

(i) All the Inverse trigonometry functions represent an angle

(ii) If $x > 0$, then all six inverse trigonometric functions viz $\sin^{-1}(x)$, $\cos^{-1}(x)$, $\tan^{-1}(x)$, $\sec^{-1}(x)$, $\text{cosec}^{-1}(x)$, $\cot^{-1}(x)$ represent an acute angle, i.e all six have their range in 1st quadrant.

(iii) If $x < 0$, then $\sin^{-1}(x)$, $\tan^{-1}x$ & $\text{cosec}^{-1}x$ represent an angle from $-\pi/2$ to 0 (IVth quadrant)

(iv) If $x < 0$, then $\cos^{-1}(x)$, $\cot^{-1}(x)$ & $\sec^{-1}x$ represent an obtuse angle (IInd quadrant)

(v) IIIrd quadrant is never used in the range of any inverse trigonometry function.

NOTE: (i) $\sin^{-1}x\big|_{max} = \pi/2$ & $\sin^{-1}x\big|_{min} = -\pi/2$

(ii) $\cos^{-1}x\big|_{max} = \pi$ & $\cos^{-1}x\big|_{min} = 0$

ITF always denotes an angle

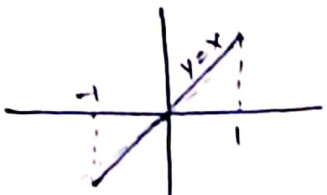
NICHOD:

f(x)	Domain	Range
(1) $\sin^{-1}x$	$\|x\| \leq 1$	$[-\frac{\pi}{2}, \frac{\pi}{2}]$
(2) $\cos^{-1}x$	$\|x\| \leq 1$	$[0, \pi]$
(3) $\tan^{-1}x$	$x \in R$	$(-\frac{\pi}{2}, \frac{\pi}{2})$
(4) $\sec^{-1}x$	$\|x\| \geq 1$	$[0, \pi] - \{\frac{\pi}{2}\}$ or $[0, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \pi]$
(5) $\text{cosec}^{-1}x$	$\|x\| \geq 1$	$[-\frac{\pi}{2}, \frac{\pi}{2}] - \{0\}$
(6) $\cot^{-1}(x)$	$x \in R$	$(0, \pi)$

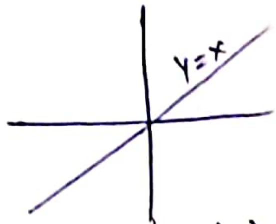
Properties of Inverse Trigonometric function

P-1

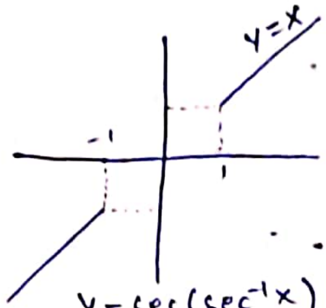
* $y = \sin(\sin^{-1}x) = x$, $x \in [-1,1]$, $y \in [-1,1]$
* $y = \cos(\cos^{-1}x) = x$, $x \in [-1,1]$, $y \in [-1,1]$, y is aperiodic
* $y = \tan(\tan^{-1}(x)) = x$, $x \in R$, $y \in R$
* $y = \cot(\cot^{-1}x) = x$, $x \in R$, $y \in R$
* $y = \operatorname{cosec}(\operatorname{cosec}^{-1}x) = x$, $|x| \geq 1$, $|y| \geq 1$
* $y = \sec(\sec^{-1}x) = x$, $|x| \geq 1$, $|y| \geq 1$



$f(x) = \sin(\sin^{-1}x)$
$g(x) = \cos(\cos^{-1}x)$
~~h(x)=x~~



$y = \tan(\tan^{-1}x)$
$y = \cot(\cot^{-1}x)$
$y = x$



$y = \sec(\sec^{-1}x)$
$y = \operatorname{cosec}(\operatorname{cosec}^{-1}x)$

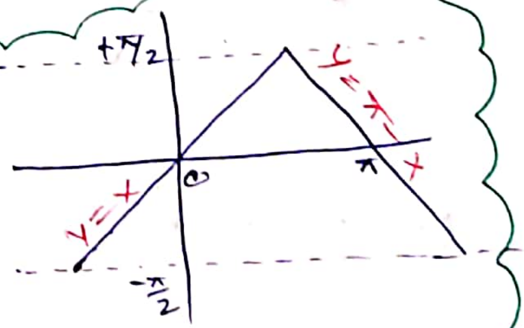
P-2

* $y = \sin^{-1}(\sin x)$, $x \in R$, $y \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

$\sin^{-1}(\sin x) \neq x$ (always)

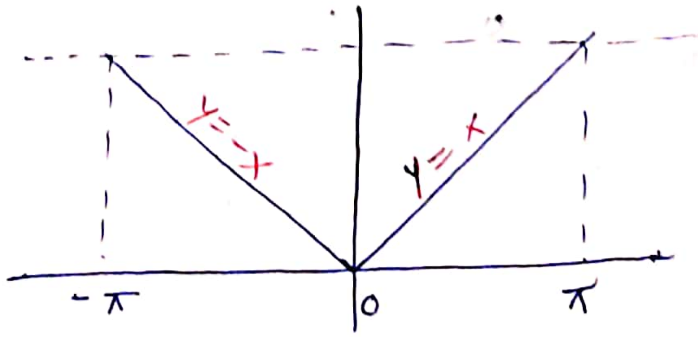
$$f(x) = \begin{cases} \sin^{-1}(\sin x) = x , & -\frac{\pi}{2} \leq x \leq \pi/2 \\ \sin^{-1}(\sin x) = \pi - x , & \pi/2 < x \leq 3\pi/2 \end{cases}$$

$$\sin^{-1}(\sin x) = \begin{cases} x , & -\frac{\pi}{2} \leq x \leq \pi/2 \\ \pi - x , & \frac{\pi}{2} < x \leq 3\pi/2 \end{cases}$$

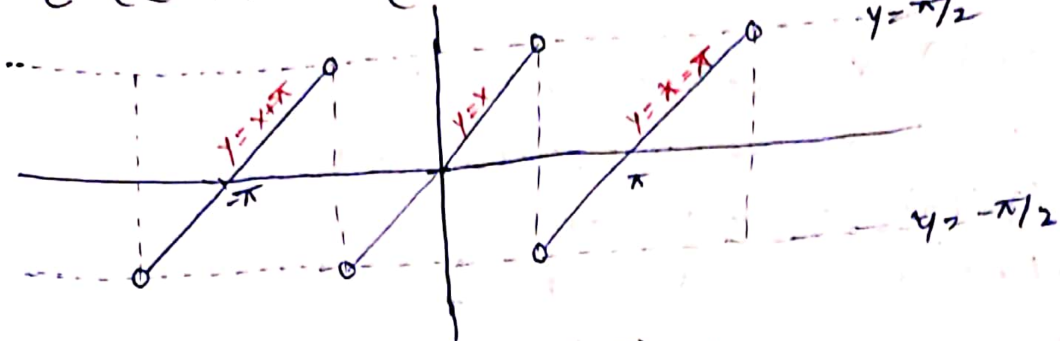


$$\cos^{-1}(\cos x) = \begin{cases} -x , & -\pi \leq x \leq 0 \\ x , & 0 \leq x \leq \pi \end{cases}$$

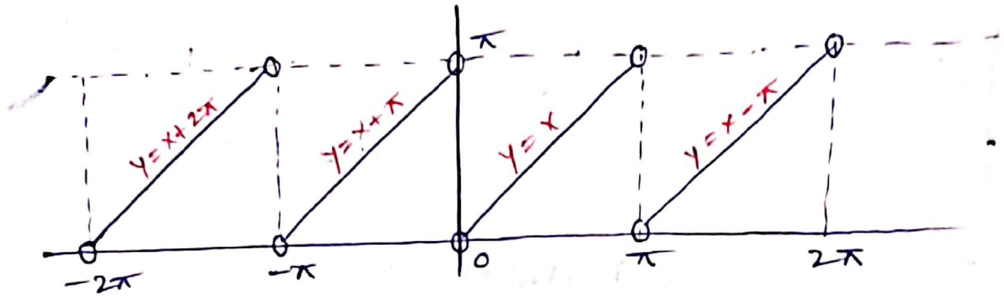
NICHOD!!



* $y = \tan^{-1}(\tan x)$, $x \in R - \left\{ (2n-1)\frac{\pi}{2}, n \in I \right\}$; $y \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$



* $y = \cot^{-1}(\cot x)$, $x \in R - \{n\pi\}$, $y \in (0, \pi)$



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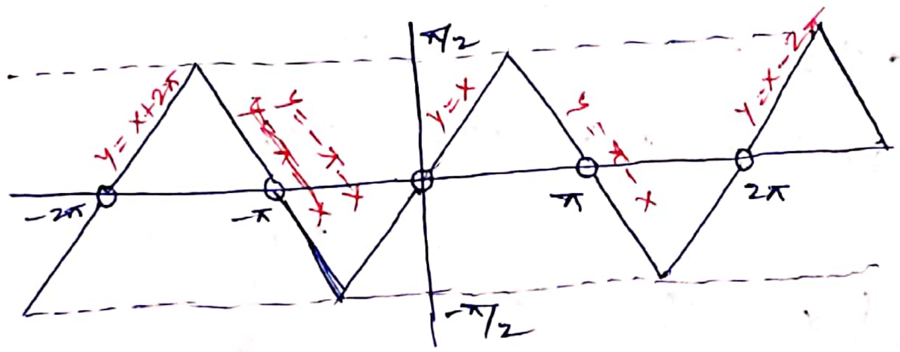
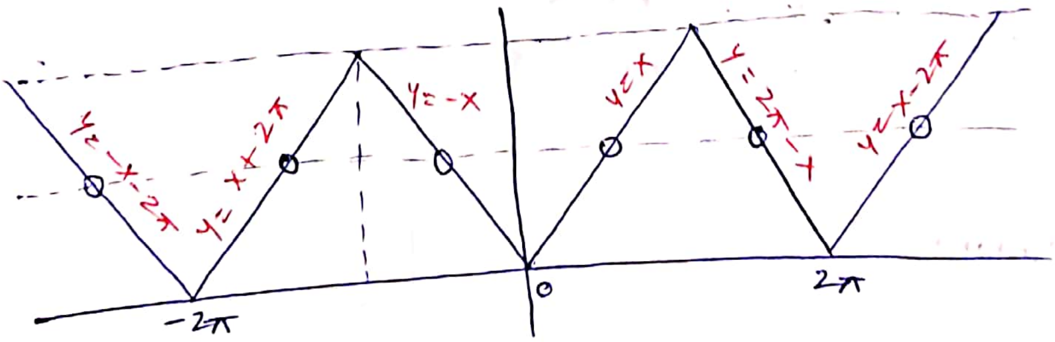
$$y = \operatorname{cosec}^{-1}(\operatorname{cosec} x) = \begin{cases} x, & -\frac{\pi}{2} \le x \le \pi/2, \ x \ne 0 \\ \pi - x, & \frac{\pi}{2} < x \le 3\pi/2, \ x \ne \pi \end{cases}$$



* $\sec^{-1}(\sec x)$ $x \in R - (2n+1)\pi/2$

$y \in [0, \pi] - \{\pi/2\}$



P-3

(1) $\operatorname{cosec}^{-1}x = \sin^{-1}\left(\frac{1}{x}\right)$; $|x| \geq 1$, $\sin^{-1}x = \operatorname{cosec}^{-1}\left(\frac{1}{x}\right), |x| \leq 1, x \neq 0$

(2) $\sec^{-1}x = \cos^{-1}\left(\frac{1}{x}\right)$; $|x| \geq 1$, $\cos^{-1}x = \sec^{-1}\left(\frac{1}{x}\right), |x| \leq 1, x \neq 0$

(3) $\cot^{-1}x = \begin{cases} \tan^{-1}\left(\frac{1}{x}\right), & x > 0 \\ \pi + \tan^{-1}\left(\frac{1}{x}\right), & x < 0 \end{cases}$, $\tan^{-1}x = \begin{cases} \cot^{-1}\left(\frac{1}{x}\right), & x > 0 \\ -\pi + \cot^{-1}\left(\frac{1}{x}\right), & x < 0 \end{cases}$

P-4

(1) $\sin^{-1}(-x) = -\sin^{-1}x$, $|x| \leq 1$

(2) $\tan^{-1}(-x) = -\tan^{-1}x$, $x \in R$

(3) $\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1}x$, $|x| \geq 1$

(4) $\cot^{-1}(-x) = \pi - \cot^{-1}x$, $x \in R$

(5) $\sec^{-1}(-x) = \pi - \sec^{-1}x$, $|x| \geq 1$

(6) $\cos^{-1}(-x) = \pi - \cos^{-1}x$, $|x| \leq 1$

P-5

(1) $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$, $|x| \leq 1$

(2) $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$, $x \in R$

(3) $\operatorname{cosec}^{-1}x + \sec^{-1}x = \frac{\pi}{2}$, $|x| \geq 1$

NOTE: $\tan^{-1}x + \tan^{-1}(1/x) = \begin{cases} \frac{\pi}{2}, & x > 0 \\ -\frac{\pi}{2}, & x < 0 \end{cases}$

P-6

* $\tan^{-1}x + \tan^{-1}y = \begin{cases} \tan^{-1}\left(\frac{x+y}{1-xy}\right), & x, y \geq 0, \ xy < 0 \\ \pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right), & x, y \geq 0 \ \ xy > 1 \end{cases}$

* $\tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$, $x, y \geq 0$

SIMPLIFYING ITF USING elementary Substitution:

* $\sqrt{a^2 - x^2} \Rightarrow x = a\sin\theta, a\cos\theta$
* $\sqrt{x^2 - a^2} \rightarrow x = a\sec\theta, a\operatorname{cosec}\theta$
* $\sqrt{a^2 + x^2} \rightarrow x = a\tan\theta, a\cot\theta$
* $\sqrt{\frac{a-x}{a+x}} \rightarrow x = a\cos(2\theta)$

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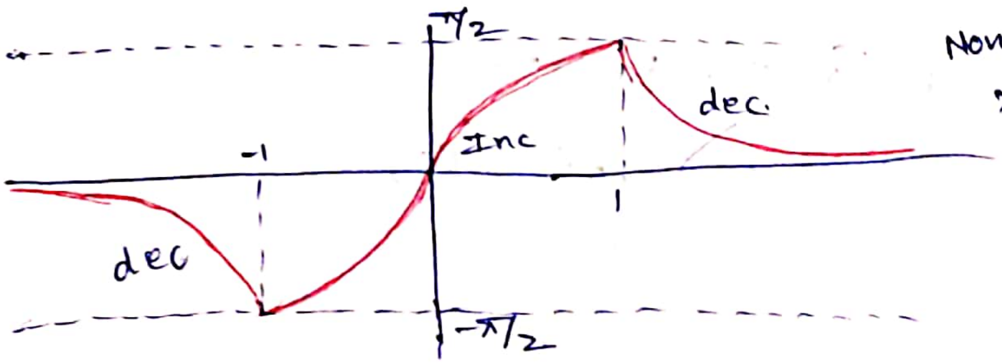
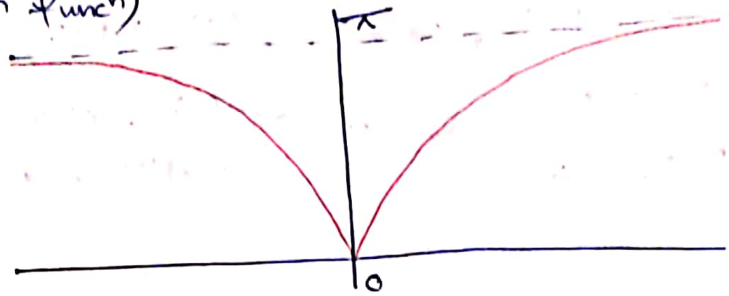
* $\sin^{-1}\left(\frac{2x}{1+x^2}\right) = \begin{cases} -\pi - 2\tan^{-1}x, & x < -1 \\ 2\tan^{-1}x, & -1 \leq x \leq 1 \\ \pi - 2\tan^{-1}x, & x > 1 \end{cases}$



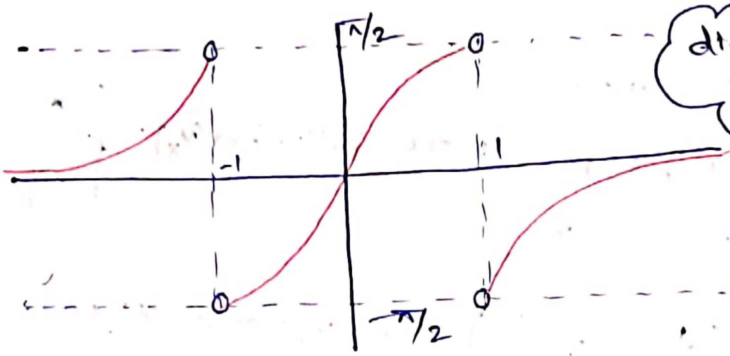
Non diff$^{\text{n}}$ at $x = -1, 1$

* $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = \begin{cases} -2\tan^{-1}x, & x < 0 \\ 2\tan^{-1}x, & x \geq 0 \end{cases}$

(Even funcn)



* $\tan^{-1}\left(\frac{2x}{1-x^2}\right) = \begin{cases} \pi + 2\tan^{-1}x, & x < -1 \\ 2\tan^{-1}x, & -1 < x < 1 \\ 2\tan^{-1}x - \pi, & x > 1 \end{cases}$



discontinuity at $x = -1, 1$

* **NOTE!**

(1) $\tan^{-1}(1) + \tan^{-1}(2) + \tan^{-1}(3) = \pi$ (remember)

(2) $\cot^{-1}(1) + \cot^{-1}(2) + \cot^{-1}(3) = \pi/2$ (remember)

Summation of Series:

$\tan^{-1}\left(\frac{x-y}{1+xy}\right) = \tan^{-1}x - \tan^{-1}y \Rightarrow$ with it's help we will make a Telescoping series

Q $S = \tan^{-1}\left(\frac{x}{1+1\cdot 2\cdot x^2}\right) + \tan^{-1}\left(\frac{x}{1+2\cdot 3x^2}\right) + \cdots + \tan^{-1}\left(\frac{x}{1+n(n+1)x^2}\right)$

$T_n = \tan^{-1}\left(\frac{nx + x - nx}{1 + nx\cdot(nx+x)}\right) = \tan^{-1}(nx+x) - \tan^{-1}(nx)$

$T_1 = \tan^{-1}(2x) - \tan^{-1}(x)$

$T_2 = \tan^{-1}(3x) - \tan^{-1}(2x)$

$T_3 = \tan^{-1}(4x) - \tan^{-1}(3x)$

$\vdots$

$T_n = \tan^{-1}((n+1)x) - \tan^{-1}(nx)$

$\sum T_n = S_n = \tan^{-1}((n+1)x) - \tan^{-1}(x)$