Lecture 14

1 Diffusion into semi-finite solids

Diffusion into a solid that has a finite face and an infinite body.

THe solution for this is:

$$\frac{c_x - c_0}{c_s - c_0} = 1 - erf(\frac{x}{2\sqrt{Dt}}) \tag{1}$$

Where:

1. t is the diffusion time
2. x is the position
3. c_x the concentration of the diffusing species
4. c_s surface concentration of diffusing species
5. c_0 the initial bulk concentration

Example 5.4:

$$\begin{aligned} c_x &= 0.6\% \\ c_0 &= 0.2\% \\ c_s &= 1.0\% \\ X &= 1\text{mm} \\ \frac{c_x - c_0}{c_s - c_0} &= 0.5 \\ D &= 2.98 \times 10^{-11}\frac{m^2}{s} \end{aligned} \tag{2}$$

$$\begin{aligned} \frac{c_x - c_0}{c_s - c_0} &= 1 - erf(\frac{x}{2\sqrt{Dt}}) \\ erf(\frac{x}{2\sqrt{Dt}}) &= 1 - \frac{c_x - c_0}{c_s - c_0} \\ &= 0.5 \\ t &= 3.68 \times 10^4 s \end{aligned} \tag{3}$$

Example 5.5:
Recalculate above with master plot:

$$\begin{aligned} \frac{c_x - c_0}{c_s - c_0} &= 0.5 \\ \Rightarrow \frac{x}{\sqrt{Dt}} &= 0.95 \\ \Rightarrow t &= 3.72 \times 10^4 s \end{aligned} \tag{4}$$

2 Diffusivity Equations

$$D = D_0 E^{\frac{-Q}{RT}} \tag{5}$$

Where:

1. D_0 preexponential constant
2. Q the activation energy per mole of diffusing species or for the motion of one diffusing species (J/mol, J, or eV)
3. R the universal gas constant

4. T the absolute temperature

In general diffusion occurs faster in BCC than in FCC structure.

We can also have diffusivity data for non-metals. Note that for non-metals the diffusivity rate is many orders of magnitude lower than that of metals.

Example 5.6:

$$\begin{aligned} D &= D_0 E^{\frac{-Q}{RT}} \\ D_0 &= 20 \times 10^{-6} \\ Q &= 1.42 \times 10^5 \\ D &= ? \end{aligned} \tag{6}$$

$$\begin{aligned} \frac{c_x - c_0}{c_s - c_0} &= 1 - erf(\frac{x}{2\sqrt{Dt}}) \\ \frac{0.35 - 0.20}{1 - 0.2} &= 1 - erf(\frac{4\text{mm}}{2\sqrt{D(49.5\text{hours})}}) \\ \Rightarrow D &= 2.6 \times 10^{-11} \frac{m^2}{s} \\ T &= \frac{Q}{k(\ln D_0 - \ln D)} \\ &= 1260.2\,\mathrm{K} \end{aligned} \tag{7}$$

3 Diffusion in Semiconducting Materials

We need to be able to dope our semiconductos to yeild special electronic properties.

1. Pre-Deposition
   - Impurity atoms (e.g. P or B) are diffusted into silicon often from a gas phase. 900 °C to 1000 °C
   - The concentration of the dopant in gas phase and at silicon surface are kept constant
   - Semi-infinite diffusion model can be applied.
2. Drive-in Diffusion
   - This treatment is used to transport atoms farther into stilicon to provide more suitable concentration distribution without increasing impurity content
   - Temperature is increased upto 1200 °C
   - Treatment applies a SiO_2 layer to prevent escape of impurity.

4 Drive-in Diffusion

1. The impurity atoms introduced are confined to a very thin layer of the silicon
2. Then the solution to Flick's second law is:

$$C(x,t) = \frac{Q_0}{\sqrt{\pi D_d t}} e^{\frac{-x^2}{4D_d t}} \tag{8}$$

   Where:
   - $C(x,t)$ is the impurity concentration at position and time.
   - D_d is the diffusion coefficient in the drive-in step
   - Q_0 is the total amount of impurities per unity area in the solid that were introduced during the pre-deposition treatment which is:

$$Q_0 - 2C_s\sqrt{\frac{D_p t_p}{\pi}} \tag{9}$$

     Where

- C_s is the surface concentration for the pre-deposition step
- D_p is the diffusion coefficient in the pre-deposition step
- t_p is the pre-deposition treatment time

Junction Depth, x_j: The Depth at which tht ediffusing impurity concentration is just equal to the background concentration of that impurity in the silicon.

$$x_j = \sqrt{(4D_d t_d)\ln(\frac{Q_0}{C_B\sqrt{\pi D_d t_d}})} \tag{10}$$

Example 5.7:
a)

$$\begin{aligned} Q_0 &= 2C_s\sqrt{\frac{D_p t_p}{\pi}} = 3.44\times 10^{18}\frac{\text{atoms}}{m^2} \\ t_p &= 30\text{mm}\times 60\frac{s}{m} \\ C_s &= 3\times 10^{26} \\ D_p &= D_0 e^{\frac{-q}{kt}} \\ &= 5.73\times 10^{-20}\frac{m^2}{s} \end{aligned} \tag{11}$$

b)

$$\begin{aligned} x_j &= \sqrt{(4D_d t_d)\ln(\frac{Q_0}{C_B\sqrt{\pi D_d t_d}})} \\ &= 2.19\times 10^{-6} m \\ &= 2.19\mu m \end{aligned} \tag{12}$$

$$D_d = D_0 e^{\frac{q}{kT_d}} = 1.51\times 10^{-17}\frac{m^2}{s} \tag{13}$$

c)

$$C(x,t) = \frac{Q_0}{\sqrt{\pi D_d t}} e^{\frac{-x^2}{4D_d t}} = 5.90\times 10^2 3\frac{\text{atoms}}{m^3} \tag{14}$$