Options Pricing

-By

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Overview

This document reports our reading exercise for options and different models for predicting option prices. Initially discussing Options and how they are priced, we then move to discuss different models like Binomial and Black Scholes for predicting option prices

Goals

1. Understanding of options, and how they are priced
2. Working of Binomial and Black Scholes model to price models

Options

Options are financial instruments based on the value of underlying securities, such as stocks, indexes, and exchange-traded funds (**ETFs**).

Purpose: Options offer the buyer the right, but not the obligation, to buy (call option) or sell (put option) the underlying asset at a specified price (strike price) before or on a certain date (expiration date).

There are two types of options based on the right to sell or buy:

- **Call Options:** Gives the holder the right to buy the underlying asset at the strike price. Profitable if the underlying asset's price rises above the strike price.
- **Put Options:** Gives the holder the right to sell the underlying asset at the strike price. Profitable if the underlying asset's price falls below the strike price.

Options are further divided into two parts based on the time of execution details:

- **American Options:** Can be exercised anytime before expiration.
- **European Options:** Can only be exercised on the expiration date.

Components of Option Pricing

Intrinsic Value

Intrinsic value is the portion of an option's premium that represents the difference between the current price of the underlying stock and the option's strike price.

- **Call Options**: Intrinsic value is calculated as the current stock price minus the strike price. For example, if a stock is trading at $49 and the strike price is $45, the intrinsic value is $4.
- **Put Options**: Intrinsic value is calculated as the strike price minus the current stock price. If the stock price is below the strike price, the option has intrinsic value; otherwise, it does not.

Extrinsic Value

Time value is the portion of the premium that exceeds intrinsic value, representing the potential for the option to gain value before expiration.

The longer the time until expiration, the higher the time value, as there is more opportunity for the stock price to move in a favorable direction.

Time value decreases as the expiration date approaches, a process known as **time decay**.

Factors influencing Option Pricing

Stock price movement

Volatility plays a significant role in option pricing as it measures the extent of price fluctuations of the underlying security. Higher volatility typically increases the potential for an option to expire in the money, which in turn affects the option's premium.

- **Call Options**: Profit when the underlying stock price rises above the strike price.
- **Put Options**: Profit when the underlying stock price falls below the strike price.

Volatility

Volatility refers to the rate at which the stock price fluctuates. Volatility is often calculated using daily, weekly, or monthly price data. The standard deviation of these returns provides a percentage measure of the stock's past volatility.

Implied Volatility (IV): Measures the market's expectation of future volatility. Higher implied volatility increases the option's premium because it raises the likelihood of the stock price moving beyond the strike price.

Certain events can significantly impact implied volatility and, consequently, option prices:

- **Earnings Announcements**: Prior to earnings reports, implied volatility often increases due to uncertainty about the company's performance. After the announcement, implied volatility typically drops, a phenomenon known as "volatility crush."
- **Economic Data Releases**: Major economic indicators, such as employment reports or interest rate decisions, can cause volatility spikes due to their impact on market expectations.
- **Corporate Actions**: Events like mergers, acquisitions, or product launches can lead to increased volatility due to the anticipated impact on the company's future performance.

Time to Expire

The time to expiry, or time until the option contract expires, is a critical factor in option pricing. The longer the time to expiry, the higher the premium of the option, as it provides more time for the underlying asset's price to move in a favorable direction

- Options with more time until expiry have higher premiums due to the greater chance of the stock price moving favorably.

Delta

Delta measures the sensitivity of an option's price to changes in the underlying stock's price.

- A delta of 1.0 indicates the option price will move dollar-for-dollar with the stock price.
- For puts, delta is negative, reflecting the inverse relationship with the stock price.

Theta

Theta represents the rate of time decay of an option's premium.

- As expiration approaches, theta accelerates, eroding the time value of the option. This necessitates timely decision-making for option traders to avoid significant losses due to time decay.

Options profitability

Options traders can profit by being option buyers or option writers. Options allow for potential profit during volatile times, regardless of market direction. This is possible because options can be traded in anticipation of market appreciation or depreciation. As long as the prices of assets like stocks, currencies, and commodities are moving, there is an options strategy that can take advantage of it.

- **Call Option Buyer**: Profits if the underlying asset rises above the strike price before expiry. The profit is the difference between the stock price and the option strike price at expiration or when the position is closed.
- **Put Option Buyer**: Profits if the underlying asset's price falls below the strike price before expiry.

Options Buying vs Writing

- **Buying Options:** The buyer starts with a buy (buy-to-open) order and closes with a sell (sell-to-close) order. Buyers can achieve substantial returns if the trade works out due to potentially significant price movements. However, the probability of success is typically lower, as successful buyers win fewer trades but with larger relative payouts.
- **Writing Options:** The writer starts with a sell (sell-to-open) order and closes with a buy (buy-to-close) order. Writers have limited profit potential tied to the premiums received. However, they often win trades more frequently but with smaller profits.

Option Pricing models

Binomial Option Pricing Model

The binomial option pricing model, developed in 1979, is an options valuation method that uses an iterative procedure, allowing for the specification of nodes, or points in time, between the valuation date and the option's expiration date.

The binomial option pricing model assumes **two possible outcomes**—hence, the binomial part of the model. The two outcomes are a move up or a move down. The major advantage of a binomial option pricing model is its mathematical simplicity, though it can become complex in a multi-period model.

The binomial model allows for the calculation of the asset and the option for multiple periods along with the range of possible results for each period. This multi-period view allows users to visualize the change in asset price from period to period and evaluate the option based on decisions made at different points in time. For U.S-based options, which can be exercised at any time before expiration, the binomial model provides insight into when exercising the option may be advisable and when it should be held for longer periods.

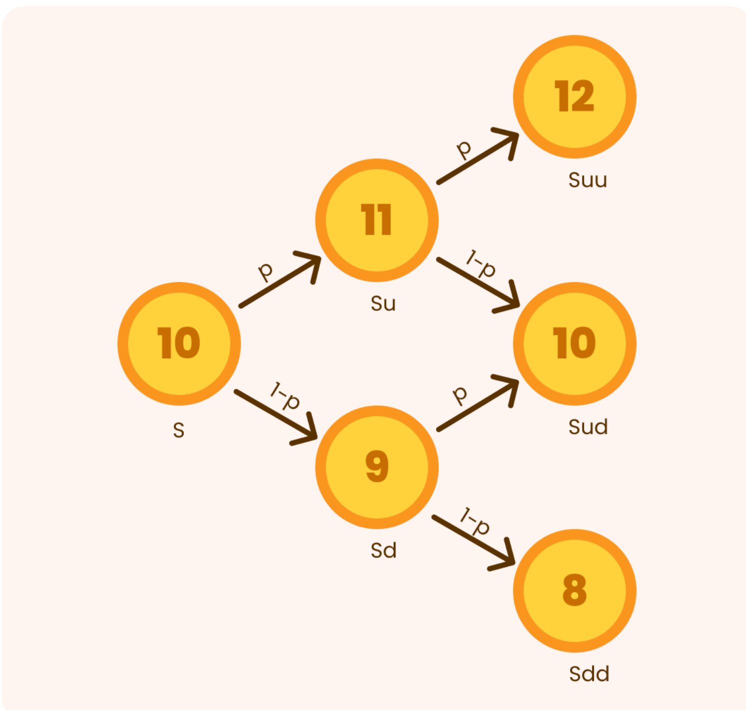
By looking at the binomial tree of values, a trader can determine in advance when a decision on an exercise may occur. If the option has a positive value, there is the possibility of exercise, whereas if the option has a value less than zero, it should be held for longer periods.

Price calculation with Binomial Model

The basic method of calculating the binomial option model is to use the same probability each period for success and failure until the option expires. However, a trader can incorporate different probabilities for each period based on new information obtained as time passes.

A binomial tree is a useful tool when pricing American options and embedded options. Its simplicity is both an advantage and a disadvantage. The tree is easy to model mechanically, but the problem lies in the possible values the underlying asset can take in one period of time. In a binomial tree model, the underlying asset can only be worth exactly one of two possible values, which is not realistic, as assets can be worth any number of values within a given range.

Example



S: The initial stock price, here represented as $10.

u: The up factor, which represents the multiplier for an upward movement in price.

d: The down factor, which represents the multiplier for a downward movement in price.

P: The probability of the stock price moving up.

1-P: The probability of the stock price moving down.

Black Scholes Option Pricing Model

The Black-Scholes model, also known as the Black-Scholes-Merton (BSM) model, is a cornerstone of modern financial theory. This mathematical equation estimates the theoretical value of derivatives based on other investment instruments, accounting for time and various risk factors. Developed in 1973 by Fischer Black, Robert Merton, and Myron Scholes, the Black-Scholes model remains one of the best methods to price an options contract.

The Black-Scholes model posits that instruments such as stock shares or futures contracts have a lognormal distribution of prices following a random walk with constant drift and volatility. The equation uses this assumption, factoring in other variables to derive the price of a European-style call option. The model requires six variables:

- Volatility
- The price of the underlying asset
- The strike price of the option
- The time until the expiration of the option
- The risk-free interest rate
- The type of option (call or put)

Black-Scholes Assumptions

The model makes several assumptions:

- No dividends are paid out during the option's life.
- Market movements are random and unpredictable.
- There are no transaction costs in buying the option.
- The risk-free rate and volatility of the underlying asset are constant.
- The returns of the underlying asset are normally distributed.
- The option is European and can only be exercised at expiration.

$$C = SN(d_1) - Ke^{-rt}N(d_2)$$

where:

$$d_1 = \frac{ln\frac{S}{K} + (r + \frac{\sigma_v^2}{2})t}{\sigma_s\sqrt{t}}$$

and

$$d_2 = d_1 - \sigma_s\sqrt{t}$$

Benefits:

- Provides a stable, theoretical framework for pricing options.
- Helps investors manage risk by understanding exposure.
- Assists in portfolio optimization by measuring expected returns and risks.
- Enhances market efficiency and transparency.
- Streamlines pricing processes across markets and jurisdictions.

Limitations:

- Limited to pricing European options; does not account for early exercise of American options.
- Assumes constant dividends, risk-free rates, and volatility, which may not be realistic.
- Ignore transaction costs, taxes, and the possibility of arbitrage opportunities.
- Relies on assumptions that may not materialize in actual market conditions.

Monte Carlo Option Pricing Model

The Monte Carlo option pricing model is a powerful and flexible numerical method used to estimate the value of options and other financial derivatives. Unlike analytical models like Black-Scholes, which rely on a closed-form solution, the Monte Carlo method uses statistical sampling to simulate a large number of possible outcomes for the underlying asset's price. This method is particularly useful for pricing complex derivatives or options with path-dependent features where traditional models might struggle.

How Monte Carlo Simulation Works

Monte Carlo simulations rely on the concept of randomness to simulate the future paths of an asset's price. The basic idea is to generate numerous potential scenarios for how the underlying asset might evolve over time, calculate the option's payoff for each scenario, and then average these payoffs to estimate the option's fair value. The process involves several key steps:

1. **Modeling the Asset's Price Dynamics**:
   - The underlying asset's price is typically modeled using stochastic processes, such as Geometric Brownian Motion (GBM). The asset price $S(t)$ evolves according to the stochastic differential equation:

$$dS(t) = \mu S(t)dt + \sigma S(t)dW(t)$$

   where:

     - μ is the drift rate (expected return).
     - σ is the volatility of the asset.
     - dW(t) represents the Wiener process or Brownian motion, introducing randomness into the price path.

2. **Simulating Price Paths**:

   - The time to expiration is divided into small intervals, and the price at each interval is simulated using a discretized version of the SDE:

$$S_{t+\Delta t} = S_t \exp\left(\left(\mu - \frac{\sigma^2}{2}\right)\Delta t + \sigma\sqrt{\Delta t}Z\right)$$

Here, Z is a random variable drawn from a standard normal distribution. This simulation is repeated many times (often tens or hundreds of thousands) to generate a wide range of possible future price paths.

2. **Calculating Option Payoffs**:
    - For each simulated price path, the option's payoff at maturity is calculated. For example, for a European call option, the payoff is

$$\max(S(T) - K, 0)$$

3. **Discounting Payoffs to Present Value**:
    - Since the option's value is based on the present value of expected future payoffs, each payoff is discounted back to the present using the risk-free rate r:

$$\text{Present Value} = \text{Payoff} \times e^{-rT}$$

4. **Averaging the Payoffs**:
    - The final step involves averaging all the discounted payoffs from the different simulations to obtain the estimated option price:

$$\text{Option Price} = \frac{1}{N}\sum_{i=1}^{N} \text{Present Value}_i$$

Advantages of Monte Carlo Simulation

- **Flexibility**: The Monte Carlo method can handle a wide range of options and derivatives, including those with complex features like path-dependency, barriers, or American-style options.
- **Versatility**: It can be applied to various financial instruments and is not limited by the assumptions required in analytical models.
- **Accuracy**: Increasing the number of simulations improves the accuracy of the result, making it a robust tool for pricing.

Limitations of Monte Carlo Simulation

- **Computational Intensity**: Monte Carlo simulations can be computationally expensive, especially for options requiring a large number of simulations to achieve accurate results.

- **Complexity**: Implementing the method for more sophisticated options can be complex and time-consuming.
- **Assumptions**: While flexible, the method still relies on certain assumptions (e.g., constant volatility), which might not hold in real markets.