Solving a system of linear equations with weights as the hilbert matrix.
Calculations performed upto 3 decimal places.

$$A = \text{hilbert}(3) \begin{bmatrix} 1 & 0.5 & 0.333 \\ 0.5 & 0.333 & 0.25 \\ 0.333 & 0.25 & 0.2 \end{bmatrix} \qquad b = \begin{bmatrix} 1.833 \\ 1.083 \\ 0.783 \end{bmatrix}$$

$$I = [1 \;\; 2 \;\; 3] \qquad \text{Scale Vector} \quad S = \begin{bmatrix} 1.0 \\ 0.5 \\ 0.333 \end{bmatrix}$$

Iteration 1: $\dfrac{|A_{i1}|}{S_i}$ $i = 1,2,3$ $= \begin{bmatrix} 1/1 \\ 0.5/0.5 \\ 0.333/0.333 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $\therefore$ first highest = 1 = pivot

- swap(I[1], I(pivot]) = swap(I[1], I[1]) ⇒ I = [1, 2, 3]
- Find Multiplier for Row 2. = A[row = 2, column = 1] / A[pivot, column 1]

  = 0.5/1 = 0.5.

  Multiply pivot row by 0.5 & subtract from 2.
- Similarly for Row 3 = A[row = 3, column = 1] / A[pivot, column 1]

  = 0.333/1 = 0.333.

  Multiply pivot row by 0.333 & subtract from 3.

$$= \left[\begin{array}{ccc|c} 1 & 0.5 & 0.333 & 1.833 \\ 0 & 0.333 - 0.25 & 0.25 - 0.333 \times 0.5 & 1.083 - 1.833 \times 0.5 \\ 0 & 0.25 - 0.25 \times 0.333 & 0.2 - 0.333 \times 0.333 & 0.783 - 1.833 \times 0.333 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & 0.5 & 0.333 & 1.833 \\ 0 & 0.083 & 0.083 & 0.166 \\ 0 & 0.083 & 0.089 & 0.173 \end{array}\right]$$

Iteration 2: $\dfrac{|A_{i2}|}{S_i}$ $i = 2,3$ = $\begin{bmatrix} 0.083/0.5 \\ 0.083/0.333 \end{bmatrix} = \begin{bmatrix} 0.166 \\ 0.249 \end{bmatrix}$ pivot = row 3.

swap.(I[2], I[3]) ⇒ I = [1, 3, 2]

· Find Multiplier for Row 2 = A[Row = 2, column = 2] / A[Pivot, col = 2]

= 0.083 / 0.083 = 1.

· Multiply pivot row by multiplier & subtract from 2.

$$\left[\begin{array}{ccc|c} 1 & 0.5 & 0.333 & 1.833 \\ 0 & \cancel{0.083}\ 0 & -0.006 & -0.007 \\ 0 & 0.083 & 0.089 & 0.173 \end{array}\right]$$

Back Substitution : reverse order of I = [1, 3, 2].

row 2 ; $x_3 = -0.007 / -0.006 = 1.167$

row 3 ; $x_2 = [0.173 - (1.167 \times 0.089)] / 0.083 = 0.833$

row 1 ; $x_1 = [1.833 - (1.167 \times 0.333 + 0.833 \times 0.5)] / 1 = 1.028$

$\therefore X = \begin{bmatrix} 1.028 \\ 0.833 \\ 1.167 \end{bmatrix}$

Condition number of hilbert (3) = 524.056.

From the thumb rule: if Condition number is 10^k then we may loose upto k digits of accuracy. Over and above the Numerical loss.

And we can see here that our results are off by 2 digits of precision (A little more than 2 but that can be attributed to the numerical error.).

∴ Because of the high ~~hilbert~~ Condition number. The precision we are loosing by keeping upto 3 decimal places is causing a ~~har~~ Large change in the output. over and above the numerical error.