Becs-114.1100 Computational Science / Laskennallinen tiede. Fall 2015.

Assignment 9. Monte Carlo -simple sampling.
Due Tue 24.11.2015 11 pm.

Web page: `https://mycourses.aalto.fi/course/view.php?id=4367`

computer = programming task
pencil and paper = solve on paper

Problem 1. (3 points)
(a) (*pencil and paper*) Suppose that you use the Monte Carlo method to estimate the value of a given quantity A. One simulation run gives you a single numerical value denoted by A_i and by running the simulation n times you get the set $\{A_i\}_{i=1}^n$. How can you obtain a reliable estimate of the true value of A and how can you calculate an estimate of the error? How is the central limit theorem related to the estimation?

(b) (*computer*) Write a program that computes an estimate of π using the "hit-or-miss" Monte Carlo method. The program should also calculate an error estimate σ (as you described in part (a)) and the absolute deviation from the correct value ($\pi = 3.141592654$).

Do this by generating N uniformly distributed random points inside the square defined by $-1 \leq x \leq 1$ and $-1 \leq y \leq 1$. Calculate the number of points which are inside the circle $x^2 + y^2 = 1^2$. For each value of N, perform $n = 1000$ independent measurements. The resulting average value

$$\pi_{est} = \bar{\pi} = \frac{1}{n}\sum_{i=1}^{n}\pi_i$$

is your MC estimate. The absolute error is $\Delta = |\pi_{est} - \pi|$.

Plot your estimate of π, the error estimate $\log(\sigma)$ and the average absolute error $\log(\Delta)$ as a function of $\log(N)$ for $N = 1000 - 30000$ (do a series of runs increasing N by 1000 at each round). If the calculation takes too long for testing purposes, use smaller values of N for testing. Plot the error and the error estimate in a single figure. What relation does the error estimate follow? What about the absolute error?

Problem 2. (*computer*) (0 points - do not hand in)
Write a program which uses the "sample-mean" method to compute the integral

$$I = \int_0^2 \left\{ \int_3^6 \left[\int_{-1}^1 (yx^2 + z\ln y + e^x)dx \right] dy \right\} dz$$

Include the calculation of an error estimate in your program. Show the results as a function of N (number of random points). The correct answer is $I \approx 49.9213$.

Problem 3 (*computer*) (2 points)
(a) Write a program which uses importance sampling to compute the integral

$$I = \int_1^2 (2x^8 - 1)dx$$

by using the weight function $w(x) = Cx^8$. Plot the result as a function of N.

(b) Compute the same integral using the sample mean method. Plot the absolute error $|I_{est} - I|$ as a function of N for the results obtained using the sample mean method and for the results obtained in (a) using importance sampling. Which method is better?

Problem 4. (*computer*) (0 points - do not hand in)
Write a program which simulates the process of radioactive decay. You are given a sample of $N = 20000$ radioactive nuclei each of which decays at a rate p per second. What is the half-life of the sample if $p = 0.4$? Calculate the estimate of the half-life $\langle t_{1/2} \rangle$ from m independent measurements and include an estimation of the error in your program. Increase m until you reach an accuracy of at least 0.005.