**Becs-114.1100 Computational Science / Laskennallinen tiede. Fall 2015.**

Assignment 6. Splines.
Due Tue 03.11.2015 11 pm.

Web page: `https://mycourses.aalto.fi/course/view.php?id=4367`

*computer* = programming task
*pencil and paper* = solve on paper

---

**Problem 1.** (*pencil and paper*) (0 points - do not hand in)
Determine the natural cubic spline that interpolates the function $f(x) = x^6$ over the interval $[0,2]$ using knots 0, 1 and 2.

**Problem 2.** (*computer*) (0 points - do not hand in)
Write a program which determines a *natural cubic spline* $S(x)$ based on a table of values and evaluates the spline function at a given value of $x$. Test the code by determining the natural cubic spline interpolant for $f(x) = \sin(x) + 0.04x^2$ at the following 15 *unevenly* spaced knots in the interval $[-9.7, 12.3]$:

| | | | | | |
|---|---|---|---|---|---|
| $x_0 \cdots x_4$ | -9.7000 | -7.3000 | -5.4000 | -5.0000 | -3.0100 |
| $x_5 \cdots x_9$ | -2.1300 | -1.2000 | -0.5600 | 0.0000 | 1.2000 |
| $x_{10} \cdots x_{14}$ | 4.5000 | 6.7000 | 9.9000 | 10.0000 | 12.3000 |

Evaluate the spline at 100 equally spaced points in the interval $[-9.7, 12.3]$. Print out the value of $S(x)$ and also the absolute error $|S(x) - f(x)|$. Present your results graphically ($S(x)$ and $f(x)$ vs. $x$ and $|S(x) - f(x)|$ vs. $x$).

**Problem 3.** (*computer*) (0 points - do not hand in)
Write a program to estimate $f'(x)$ for any $x$ in $[a,b]$ assuming that we know only the values of $f(x)$ at knots $a = t_0 < t_1 < \ldots < t_n = b$. Test your program using the set of data points $\{x, f(x)\}$ given in Problem 2 (use the given $x$ values and the corresponding values of $f(x) = \sin(x) + 0.04x^2$).

Evaluate the derivative of the spline function $S'(x)$ at 100 equally spaced points in the interval $[-9.7, 12.3]$. Present your results graphically in a plot showing $S'(x)$ and $f'(x) = \cos(x) + 0.08x$ as a function of $x$.

**Problem 4.** (2 points)
**(a)** (*computer*) Use your programs developed in Problems 2 and 3 to determine the natural cubic interpolant $S(x)$ and the derivative $S'(x)$ for a set of experimental data that is available in MyCourses (titanium.dat).

Present your results graphically (plot of $S(x)$ vs. $x$ and $S'(x)$ vs. $x$).

**(b)** (MATLAB) Examine what functions can be found in the Matlab Spline toolbox. Repeat part (a) using Matlab functions. Compare the results. Are there differences? What is causing them?