[redacted]

e). [redacted] :

$$K_1(x_i, x_j) = \phi_1(x_i)^T \phi_1(x_j) = \sum_{l=1}^{d} \phi_{1l}(x_i) \cdot \phi_{1l}(x_j).$$

$$K_2(x_i, x_j) = \phi_2(x_i)^T \phi_2(x_j) = \sum_{m=1}^{d} \phi_{2m}(x_i) \cdot \phi_{2m}(x_j).$$

so $$K(x_i, x_j) = K_1(x_i, x_j) \cdot K_2(x_i, x_j) = \left[\sum_{l=1}^{d} \phi_{1l}(x_i) \cdot \phi_{1l}(x_j)\right]\left[\sum_{m=1}^{} \phi_{2m}(x_i) \cdot \phi_{2m}(x_j)\right]$$

Expanding out a few terms.

$$= \left[\phi_{11}(x_i) \cdot \phi_{11}(x_j) + \phi_{12}(x_i) \cdot \phi_{12}(x_j) + \dots + \phi_{1d}(x_i) \phi_{1d}(x_j)\right]$$ has d terms

*

$$\left[\phi_{21}(x_i) \cdot \phi_{21}(x_j) + \phi_{22}(x_i) \cdot \phi_{22}(x_j) + \dots + \phi_{2d}(x_i) \cdot \phi_{2d}(x_j)\right]$$ has d terms.

= when we find the product of the above two terms it is going to be. a. term with dxd. terms.

d terms

$$= \Big[\phi_{11}(x_i)\phi_{11}(x_j)\phi_{21}(x_i)\phi_{21}(x_j) + \phi_{11}(x_i)\phi_{11}(x_j)\phi_{22}(x_i)\phi_{22}(x_j) + \dots + \phi_{11}(x_i)\phi_{11}(x_j)\phi_{2d}(x_i)\phi_{2d}(x_j)$$

d terms

$$+ \phi_{12}(x_i)\phi_{12}(x_j)\phi_{21}(x_i)\phi_{21}(x_j) + \dots\dots$$

$$\vdots$$

$$+ \phi_{1d}(x_i)\phi_{1d}(x_j) \cdot \phi_{21}(x_i)\phi_{21}(x_j) + \dots \quad + \phi_{1d}(x_i)\phi_{1d}(x_j)\phi_{2d}(x_i)\phi_{2d}(x_j)\Big]$$

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Individual terms among these
~~These~~ $d \times d$ terms can be ~~written~~ re-written as.

$$\Big[\left(\overline{\phi_{11}(x_i)\,\phi_{21}(x_i)}\right)\left(\overline{\phi_{11}(x_j)\,\phi_{21}(x_j)}\right) + \left(\overline{\phi_{11}(x_i)\,\phi_{22}(x_i)}\right)\left(\overline{\phi_{11}(x_j)\,\phi_{22}(x_j)}\right) + \cdots + \left(\overline{\phi_{11}(x_i)\,\phi_{2d}(x_i)}\right)\left(\overline{\phi_{11}(x_j)\,\phi_{2d}(x_j)}\right)$$

$$+\left(\overline{\phi_{12}(x_i)\,\phi_{21}(x_i)}\right)\left(\overline{\phi_{12}(x_j)\,\phi_{21}(x_j)}\right) + \left(\overline{\phi_{12}(x_i)\,\phi_{22}(x_i)}\right)\cdot\left(\overline{\phi_{12}(x_j)\cdot\phi_{22}(x_j)}\right) + \cdots + \left(\overline{\phi_{12}(x_i)\,\phi_{2d}(x_i)}\right)\left(\overline{\phi_{12}(x_j)\,\phi_{2d}(x_j)}\right)$$

$$+ \cdots$$

$$\vdots$$

$$+\left(\overline{\phi_{1d}(x_i)\,\phi_{21}(x_i)}\right)\left(\overline{\phi_{1d}(x_j)\,\phi_{21}(x_j)}\right) + \left(\overline{\phi_{1d}(x_i)\cdot\phi_{22}(x_i)}\right)\left(\overline{\phi_{1d}(x_j)\cdot\phi_{22}(x_j)}\right) + \cdots + \left(\overline{\phi_{1d}(x_i)\,\phi_{2d}(x_i)}\right)\left(\overline{\phi_{1d}(x_j)\,\phi_{2d}(x_j)}\right)\Big]$$

This sum can be re-written as a dot-product of a vector $\phi(x_i)$ with itself. i.e. $\phi(x_i)^T\phi(x_j)$
~~such that~~ where $\phi(x_j)$ is defined as follows.

$$\Big[\phi_{11}(x)\cdot\phi_{21}(x),\quad \phi_{11}(x)\,\phi_{22}(x),\quad \ldots,\quad \phi_{11}(x)\,\phi_{2d}(x),$$

$$\phi_{12}(x)\,\phi_{21}(x),\quad \phi_{12}(x)\cdot\phi_{22}(x),\quad \ldots,\quad \phi_{12}(x)\cdot\phi_{2d}(x),$$

$$\vdots$$

$$\phi_{1d}(x)\cdot\phi_{21}(x),\quad \ldots\ldots,\quad \ldots,\quad \phi_{1d}(x)\cdot\phi_{2d}(x)\Big]^T$$

This is exactly what $\phi(x)$ would be, if it were defined as
$\phi_1(x) \otimes \phi_2(x)$. Hence if $K(x_i, x_j) = K_1(x_i, x_j) \times K_2(x_i, x_j)$ then $K(x_i, x_j)$ can be written as $\phi(x_i)^T\phi(x_j)$ where $\phi(x) = \phi_1(x) \otimes \phi_2(x)$.