Question 2: To show, $h(x) = \text{sign}\left(\| \phi(x) - c_- \|^2 - \| \phi(x) - c_+ \|^2 \right)$. can be re-written as.

$$\text{sign}\left(\sum_{i=1}^{m} \alpha_i K(x, x_i) + b \right).$$

we know that $\| \phi(x) - c_- \|^2 = \langle \phi(x) - c_- , \phi(x) - c_- \rangle$.

so $h(x) = \text{sign}\left(\langle \phi(x) - c_- , \phi(x) - c_- \rangle - \langle \phi(x) - c_+ , \phi(x) - c_+ \rangle \right)$.

Using the property of linearity of inner product according to which. $\langle x+y, z \rangle = \langle x, z \rangle + \langle y, z \rangle$ and $\langle x, y+z \rangle = \langle x, y \rangle + \langle x, z \rangle$.

$$h(x) = \text{sign}\left(\langle \phi(x), \phi(x) - c_- \rangle - \langle c_-, \phi(x) - c_- \rangle - \{ \langle \phi(x), \phi(x) - c_+ \rangle - \langle c_+, \phi(x) - c_+ \rangle \} \right).$$

$$= \text{sign}\left(\langle \phi(x), \phi(x) \rangle - \langle \phi(x), c_- \rangle - \{ \langle c_-, \phi(x) \rangle - \langle c_-, c_- \rangle \} - \{ \langle \phi(x), \phi(x) \rangle - \langle \phi(x), c_+ \rangle \} + \{ \langle c_+, \phi(x) \rangle - \langle c_+, c_+ \rangle \} \} \right)$$

$$= \text{sign}\left(\cancel{\langle \phi(x), \phi(x) \rangle} - \langle \phi(x), c_- \rangle - \langle c_-, \phi(x) \rangle + \langle c_-, c_- \rangle - \cancel{\langle \phi(x), \phi(x) \rangle} + \langle \phi(x), c_+ \rangle + \langle c_+, \phi(x) \rangle - \langle c_+, c_+ \rangle \right)$$

also $\langle a,b\rangle \equiv \langle b,a\rangle$.

$\therefore h(x) = \text{sign}\left(2\langle\phi(x), c_+\rangle - 2\langle\phi(x), c_-\rangle + \langle c_-, c_-\rangle - \langle c_+, c_+\rangle\right)$

$= \text{sign}\left(2\langle\phi(x), c_+ - c_-\rangle + \langle c_-, c_-\rangle - \langle c_+, c_+\rangle\right)$ — ①

$\cancel{= \text{sign}\left(2\langle\phi(x), c_+ - c_-\rangle\right.} +$

c_- and c_+ are vectors. $\therefore$ $\langle c_-, c_-\rangle = \left[\frac{1}{m_-}\sum_{i\in I^-}\phi(x_i)\right]^T\left[\frac{1}{m_-}\sum_{i\in I^-}\phi(x_i)\right]$

$$= \frac{1}{m_-^2}\sum_{i,j\in I^-}\phi(x_i)^T\cdot\phi(x_j) = \frac{1}{m_-^2}\sum_{i,j\in I^-}\phi(x_i)^T\phi(x_j)$$ ~~where $x_I = x_i \forall i\in I^-$~~

$$= \frac{1}{m_-^2}\sum_{i,j\in I^-}\langle\phi(x_i), \phi(x_j)\rangle$$

$$= \frac{1}{m_-^2}\sum_{i,j\in I^-}K(x_i, x_j)$$ using kernel trick. — ②

similarly. $\langle c_+, c_+\rangle = \frac{1}{m_+^2}\sum_{i,j\in I^+}K(x_i, x_j)$. — ③.

and.

$$\langle\phi(x), c_+ - c_-\rangle = \phi(x)^T\left[\frac{1}{m_+}\sum_{i\in I^+}\phi(x_i) - \frac{1}{m_-}\sum_{i\in I^-}\phi(x_i)\right]$$

$$= \frac{1}{m_+}\sum_{i\in I^+}\phi(x)^T\phi(x_i) - \frac{1}{m_-}\sum_{i\in I^-}\phi(x)^T\phi(x_i).$$ — ①

$$= \sum_{i\in m}\alpha_i\phi(x)^T\phi(x_i).$$

$$\alpha_i = \begin{cases}\frac{1}{m_+} & \text{if } y_i = +1\\ -\frac{1}{m_-} & \text{if } y_i = -1\end{cases}$$

$$\therefore h(x) = \text{sign}\left(2\sum_{i=1}^{m}\alpha_i\,\phi(x)^T\phi(x_i) + \frac{1}{m_+^2}\sum_{i,j\in I^+}K(x_i, x_j) + \frac{1}{m_-^2}\sum_{i,j\in I^-}K(x_i,x_j)\right)$$

b

$$h(x) = \text{sign}\left(\cancel{2}\cdot\sum_{i=1}^{m} \alpha_i\, \phi(x)^T \phi(x_i) + b\right).$$

taking the 2. out, common doesn't affect the sign.

$$b = \frac{1}{2m_+^2} \sum_{i,j \in I^+} K(x_i, x_j) + \frac{1}{2m_-^2} \sum_{i,j \in I^-} K(x_i, x_j)$$

$$\alpha_i = \begin{cases} \dfrac{1}{m_+} & \text{if } y_i = +1. \\ \dfrac{-1}{m_-} & \text{if } y_i = -1. \end{cases}$$