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Date	Unit No.	Lecture No.	Faculty	Subject Name	Subject Code	Main Topics:-

Random Variable

Experiment A various process which give us some observations known as experiment.

Two types of Experiment

↳ Deterministic ⟶ when outcome is certain.

↳ Random ⟶ when outcome is not certain.

Sample space → Collections of all possible outcomes of an experiment.

Probability →

↳ classical ⟶ $\dfrac{\text{Favourable outcomes}}{\text{Total no. of outcomes}} = \dfrac{m}{n}$

↳ ~~Statical~~ Statistical ⟶ $P(A) = \lim_{n \to \infty} \dfrac{m}{n} = \dfrac{\text{favourable}}{\text{Total}}$

Probability of Sample Space = 1.

Main Ideas, Questions & Summary:

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Random Variable

we mean a real number X connected with the outcomes a random experiment

Eg

Tossing of 2 coins

outcome → getting head

outcome	HH	HT	TH	TT
value of x	2	1	1	0

X (Random variable) = No. of head

$X(HH) = 2$

$X(HT) = X(TH) = 1$

$X(TT) = 0$

★ In other words random variable X is a function which is defined from sample space to real number

$$X : S \longrightarrow R$$

Eg: Rolling a Dice the random variable X represent the number that turns up.

$$X = 1, 2, 3, 4, 5, 6$$

Eg:→ Tossing a coin thrice A random variable X denotes the no. of tails that turns up.

X =	HHH	HTH	HHT	THH	TTT	TTH	THT	HTT
	0	1	1	1	3	2	2	2

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we have 2 types of random variable

- Discrete Random Variable (DRV)
- Continous Random Variable (CRV)

→ If a random variable takes atmost a countable number a countable no. of values it is called a discrete Random variable.

→ If X is a random Variable which can take all values in an interval then X is called a continous random variable.

Eg Height of a person, price of a house

Discrete Random Variable

Probabity distribution of Discrete Random Variable

It list all the possible values that the random variable can assume & their corresponding ~~values~~ probabilities.

Eg:→ (1) Rolling a single dice

$X=x$	1	2	3	4	5	6
$P(x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

Main Ideas, Questions & Summary:

(2) when tossing a coin X denotes the number of heads obtain

$X=x$	0	1
$P(x)$	1/2	1/2

Probability mass function (PMF)

Let X be a DRV such that $P(X=x_i) = p_i$ then p_i is said to be pmf if it satisfy following conditions.

(1) $p_i \geq 0 \ \forall i$

(2) $\sum_i p_i = 1$

Eg

Probability distribution of no. of vehicles owned by families as ~~no of vehicles owned~~

No. of vehicles owned (x)	0	1	2
$P(x)$	0.015	0.235	0.425

Is this a pmf

↳ No becaus $\sum_i p_i \neq 1$

Eg check whether the following f^n serves as pmf

(1) $P(X=x) = \frac{x-2}{2} \quad \forall x=1,2,3,4$

$X=x$	1	2	3	4
$P(x)$	-0.5	0	0.5	1

Not Pmf $p_i, i=1 < 0$

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(2)

x	1	2	3	4
$P(x)$	$\frac{1}{25}$	$\frac{4}{25}$	$\frac{9}{25}$	$\frac{16}{25}$

Not a Pmf

$\sum_i p_i \neq 1$

$P(X=x) > 0 \ \forall x = 1,2,3,4$

$\sum_{x=1}^{4} P(X=x) = \frac{30}{25} = 1.2 > 1$

(3)

Four bad oranges are mixed accidently mixed with 16 good oranges find the P(X) of the no. of bad oranges in a draw of two oranges.

X denotes the no. of bad oranges in a draw of 2 oranges

GG
GB
BG
BB

$P(X=0) = \frac{^{16}C_2}{^{20}C_2} \Rightarrow \frac{15\times 16\times \cancel{14!}\ \cancel{18!}\times \cancel{2!}}{\cancel{14!}\ \cancel{2!}\ 20\times 19\times \cancel{18!}}$

$\frac{20!}{(20-2)!\times 2!}$

$\frac{3\cancel{15}\times 4\cancel{16}}{\cancel{20}_5\times 19} = \frac{12}{19}$

$P(X=1) = 1 \text{ bad } 1 \text{ Good} = \frac{^{4}C_1\times {^{16}C_1}}{^{20}C_2}$

$\frac{16\times 4}{\cancel{20}\times 19} = \frac{32}{95} \leftarrow = \frac{4\times 16\times 2}{\cancel{20}_5\times 19}$

Main Ideas, Questions & Summary:-

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$P(X=2) = \dfrac{{}^{G}C_{2}\ \ ^{16}C_2}{^{20}C_2} = \dfrac{3}{95}$

$X=x$	0	1	2
$P(x)$	$\frac{12}{19}$	$\frac{32}{95}$	$\frac{3}{95}$

$= 1$

$\dfrac{60+32+3}{95} = 1$

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DISTRIBUTION FUNCTION

let X be a discrete random variable then its discrete distribution f^n or CDF (commulative Distribution f^n) is defined as

$$F(x) = \sum_{x_i \leq x} p_i$$

i.e. $F(x_j) = P(X \leq x_j) = \sum_{i \leq j} p_i$

$\times$ Commulative Distribution Function (CDF)

Properties

1. $0 \leq F(x) \leq 1$ or $F(x)$ also lies b/w 0 and 1
2. $F(x)$ is a monotonic non-decreasing f^n of x. i.e $x_1 \leq x_2$
   $P(-\infty < x \leq x_1) \leq P(-\infty < x \leq x_2)$
3. $\lim_{x \to -\infty} f(x) = 0$ i.e $f(x) = 0$ for x sufficiently small
4. $\lim_{x \to \infty} f(x) = 1$ i.e $f(x) = 1$ $\forall$ x sufficiently large

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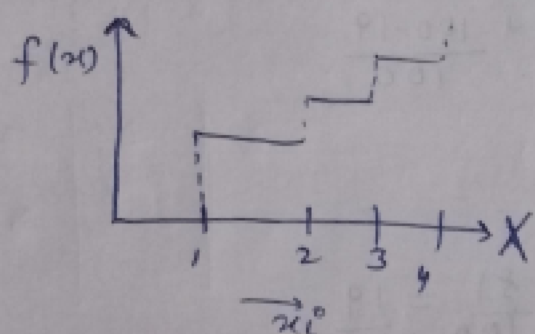
$f(x)$ has a +ve jump as $i = 1, 2, --$ to $P(x_i)$ & $f(x)$ is constant for all $x_i \le x \le x_{i+1}$

$$f(x) = f(x_i) \quad \forall \; x_i \le x \le x_{i+1}$$

$$f(x_{i+1}) = f(x_i) + P(x_{i+1})$$

for any distribution point

$$P(x_i) = f(x_i) - f(\bar{x_i}) = p(x_i)$$



A random variable X has the following probability distribution

x	0	1	2	3	4	5	6	7
$P(x)$	0	K	$2K$	$2K$	$3k$	k^2	$2k^2$	$7k^2 + k$

① Find K

② Evaluate $P(X<6)$, $P(X \ge 6)$ (or) $P(0<x<5)$

③ Determine distribution function

④ If $P(X \le C) > \frac{1}{2}$ Find min value of C

⑤ Find $P\left(\frac{1.5 < X < 4.5}{X > 2}\right)$

$8k+k^2$

$\frac{8}{10}+\frac{1}{100}$

(1) $\sum_{i=0} p_i = 1$

$10k^2+9k-1=0$

$10k^2+10k-k-1=0$

$10k(k+1)-1(k+1)=0$

$k=1/10 \quad k=-1$

$$\boxed{k=1/10}$$

(2)

a) $P(X<6)=1-P(X\geq 6)$

$=1-[P(X=6)+P(X=7)]$

$=1-2k^2-7k^2-k$

$=1-\frac{2}{100}-\frac{7}{100}-\frac{1}{10}$

$=\frac{100-9-10}{100} + \frac{100-19}{100}$

$$\boxed{P(X<6)=\frac{81}{100}}$$

b) $P(X\geq 6)=1-\frac{81}{100}=\frac{19}{100}$

c) $P(0<x<5)$

$\rightarrow P(X=1)+P(X=2)+P(X=3)+P(X=4)$

$\rightarrow$

$8k=\frac{8}{10}=\frac{4}{5}$

(3)

X	$f(x)=P(X\leq x)$
0	$0=0$
1	$k=1/10$
2	$2k+k+0=3k=3/10$
3	$2k+3k=5k=5/10$
4	$8k=8/10$
5	$8k+k^2=81/100$
6	$8k$ [illegible]
7	$10k^2+9k=1$

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(4) If $(P(X \leq c) > 1/2)$ $c = ?$

From distribution fn it is clear

$F(3) = P(X \leq 3) = \frac{5}{10} = 0.5$

$F(4) = P(X \leq 4) = \frac{8}{10} = 0.8 > 1/2$ (min.)

$F(5) = P(X \leq 5) = \frac{81}{100} = 0.81 > 1/2$ & so on

Here the min. value c for which $P(X \leq C) > \frac{1}{2}$ is 4. $\boxed{c = 4}$

(5)
$$P\left(\frac{1.5 < x < 4.5}{X > 2}\right) = \frac{P[(1.5 < x < 4.5) \cap (X > 2)]}{P(X > 2)}$$

$$= \frac{P(2 < X < 4.5)}{1 - P(X \leq 2)}$$

$$= \frac{P(3) + P(4)}{1 - [P(0) + P(1) + P(2)]}$$

$$= \frac{5k}{1 - [0 + 3k]} = \frac{5k}{1 - 3k}$$

$$= \frac{\frac{5}{10}}{1 - \frac{3}{10}} \Rightarrow \frac{5 \times 10}{10 \times 7} = 5/7$$

Ideas, Questions & Summary

(2) Find the probability distribution of boys & girls in families with 3 children assuming equal probabilities of boys & girls also give the distribution f^n. (CDF)

Binomial distribution

$$P(X=x) = {}^{n}C_{r}\, p^{r} q^{n-r}$$

$p \rightarrow$ success
$q \rightarrow$ failure

$X = 0, 1, 2, 3$

$P(\text{Boys}) = P(\text{Girls}) = 1/2 = 0.5$

$p = 0.5$
$q = 0.5$

$$P(X=x) = {}^{3}C_{r} \left(\tfrac{1}{2}\right)^{r} \left(\tfrac{1}{2}\right)^{3-r}$$

$P(X=0) = {}^{3}C_{0} \times 1 \times \frac{1}{8}$
$= \frac{1}{8}$

$P(X=1) = {}^{3}C_{1} \times \frac{1}{2} \times \frac{1}{4}$
$= 3 \times \frac{1}{8}$

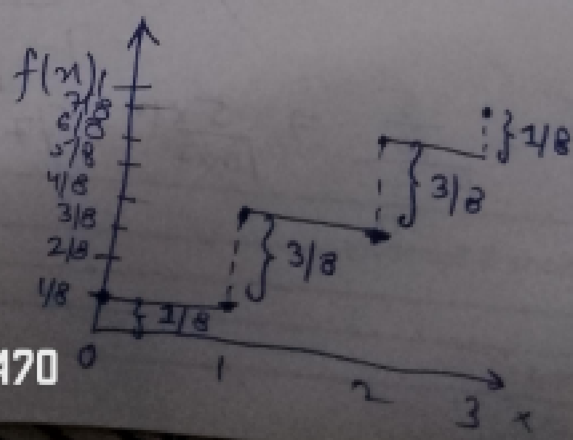
$P(X=2) = {}^{3}C_{2} \times \frac{1}{4} \times \frac{1}{2}$

$P(X=3) = {}^{3}C_{3} \times \frac{1}{8} =$

X	0	1	2	3
P(X)	$\frac{1}{8}$	3/8	3/8	1/8

X	$f(x) = P(X \le x)$
0	1/8
1	4/8 = 1/2
2	7/8
3	1

or
$$f(x) = \begin{cases} \frac{1}{8}, & X=0 \\ \frac{4}{8}, & X=1 \\ 7/8, & X=2 \\ 1, & X=3 \end{cases}$$



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Q.1.

Main Idea

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$\frac{3!}{2! \times 1!}$

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CRV

continous random variable can assume only value over an interval or intervals because the no. of values contained in any interval is infinite.

Probability density function (PDF)

The f^n ($f(x)$) for a continous RV. (x) is said to be (PDF) if it satisfied the following cond^n

(i) $f(x) \geq 0 \quad -\infty < x < \infty$

(ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

However probability in which random variable lies

$$P(a \leq x \leq b) = \int_a^b f(x)\,dx$$

Q. The diameter of an electric cable say (x) is assumed to be a CRV with PDF $= f(x) = 6x(1-x)$; $0 \leq x \leq 1$.

(i) Check above given is a PDF.

(ii) Determine a no. 'b' such that Probability $P(x<b) = P(x>b)$.

(i) $f(x) \geq 0 \;\forall\; 0 \leq x \leq 1$ PDF ✓

$\int_0^1 f(x)\,dx \qquad 6\int_0^1 x - x^2 \Rightarrow 6\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1$

$6\left[\frac{1}{2} - \frac{1}{3}\right] \qquad 6 = 6\left(\frac{1}{6}\right) = 1$

Hence it is PDF.

Ideas, Questions & Summary:

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$$P(X<b) = \int_0^b f(x)\,dx = 6\int_0^b (x-x^2)\,dx = 6\left[\frac{x^2}{2}-\frac{x^3}{3}\right]_0^b$$
$$= 6\left(\frac{b^2}{2}-\frac{b^3}{3}\right) = 3b^2-2b^3$$

$P(X>b) =$

$$P(X<b) + P(X>b) = 1$$
$$P(X<b) = 1-P(X>b)$$

if both are equal

$$2P(X<b) = 1$$
$$2\times(3b^2-2b^3) = 1$$
$$6b^2-4b^3-1=0$$

$$4b^3-6b^2+1=0$$

$b=\frac{1}{2}$

$$\begin{array}{c|cccc} \frac{1}{2} & 4 & -6 & 0 & 1 \\ & \downarrow & 2 & -2 & -1 \\ \hline & 4 & -4 & -2 & 0 \end{array}$$

$4b^2-4b-2$

$$(2b-1)(4b^2-4b-2)=0$$
$$(2b-1)\,\underbrace{2(2b^2-2b-1)}_{\text{don't come in range of } [0,1]}=0$$

$b = 1/2$

Distribution Function (CDF)

So let 'x' be a continous random variable with pdf $f(x)$ then $F(x)$ is called the distribution f^n or CDF of 'x' whose

$$F_X(x) = P(X\le x) = \int_{-\infty}^{x} f(x)\,dx \quad , -\infty<x<\infty$$

Properties

(i) $0 \le F(x) \le 1 \quad , -\infty<x<\infty$

(ii) $F(x) = \int_{-\infty}^{x} f(x)\,dx$

i.e. $\frac{d}{dx}(F(x)) = f(x) \ge 0$

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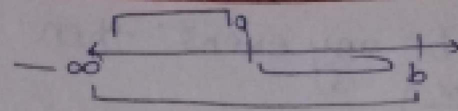

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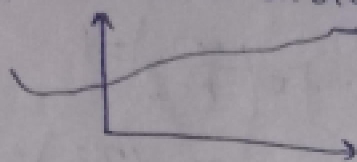
$f(x)$ is non-decreasing f^n



(iii) $F(-\infty) = \lim_{x \to -\infty} f(x) = \int_{-\infty}^{\infty} f(x)\,dx = 0$

(ii) $F(+\infty) = \lim_{x \to +\infty} f(x) = \int_{-\infty}^{\infty} f(x)\,dx$

(iv) $f(x)$ is a continous f^n of x on the right having at most countable discounties.

(v) $P(a \leq x \leq b) = \int_a^b f(x)\,dx = \int_{-\infty}^{b} f(x)\,dx - \int_{-\infty}^{a} f(x)\,dx$

$P(x \leq b) - P(X \leq a) = f(b) - f(b)$

i.e. $P(a \leq x \leq b) = f(b) - f(a)$

↳ capital distribution f^n

↳ small PDF f^n

Q. Let 'x' be a C.R.V with PDF

$$f(x) = \begin{cases} ax & , 0 \leq x \leq 1 \\ a & , 1 \leq x \leq 2 \\ -ax + 3a & , 2 \leq x \leq 3 \\ 0 & , \text{otherwise} \end{cases}$$

① Determine the constant 'a'

② Find $P(X \leq 1.5)$

③ Determine CDF and hence find $P(X \leq 2.5)$

Ideas, Questions & Summary:

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Let A be any event then conditional distribution $F_X\left(\frac{x}{A}\right)$

$$F_X\left(\frac{x}{A}\right) = P\left\{X \le \frac{x}{A}\right\} = \frac{P\{X \le x \cap X \le A\}}{P(A)}$$

<u>Properties</u>

(i) $0 \le F_X\left(\frac{x}{A}\right) \le 1$

(ii) $F_X\left(\frac{-\infty}{A}\right) = 0 \quad F_X\left(\frac{\infty}{A}\right) = 1$

(iii) $$P\left\{\frac{x_1}{A} \le X \le \frac{x_2}{A}\right\} = F_X\left(\frac{x_2}{A}\right) - F_X\left(\frac{x_1}{A}\right)$$

$$= \frac{P\{x_1 \le X \le x_2 \cap X \le A\}}{P(A)}$$

$$f_X\left(\frac{x}{A}\right) = \frac{d}{dx} F_X\left(\frac{x}{A}\right)$$

Q (i) $X \le x_1$

<u>conditional distribution</u> $$F_X\left(\frac{x}{X \le x_1}\right) = \frac{P\{X \le x \cap X \le x_1\}}{P\{X \le x_1\}}$$

$$= \begin{cases} \dfrac{P\{X \le x\}}{P\{X \le x_1\}} & , \text{if } x \le x_1 \\ \dfrac{P\{X \le x_1\}}{P\{X \le x_1\}} & , \text{if } x \ge x_1 \\ \dfrac{F_X(x)}{F(x_1)} & , \text{if } x < x_1 \\ 1 & , \text{if } x \ge x_1 \end{cases}$$

condition density

$$f_X\left(\frac{x}{X \le x_1}\right) = \begin{cases} \dfrac{f_X(x)}{f_X(x_1)} & ; x < x_1 \\ 0 & , x > x_1 \end{cases}$$

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As we know that

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_0^1 ax\,dx + \int_1^2 a\,dx + \int_2^3 (-ax+3a)\,dx = 1$$

$$\left[\frac{ax^2}{2}\right]_0^1 + [ax]_1^2 + \left[-\frac{ax^2}{2}+3ax\right]_2^3 = 1$$

$$\frac{a}{2} + 2a - a + \left[-\frac{9a}{2} + 9a + 2a - 6a\right] = 1$$

$$\frac{3a}{2} + \left[-4a + \frac{9a}{2} = \frac{a}{2}\right]$$

$2a = 1$ $\quad a = 1/2$

(ii) $P(X \le 1.5)$

$$\int_0^1 ax\,dx + \int_1^{1.5} a\,dx = \left[\frac{ax^2}{2}\right]_0^1 + [ax]_1^{1.5}$$

$$= \frac{a}{2} + \frac{3a}{2} - a$$

$$= \frac{3a}{2} - \frac{a}{2} = a = 1/2$$

(iii) CDF

$\int_{-\infty}^{x} f(x)\,dx \rightarrow$ for $x < 0$ $\quad F(x) = 0$

$\rightarrow$ For $0 \le x \le 1$ $\quad F(x) = \int_0^x f(x)\,dx = \int_0^x ax\,dx$

$$\frac{ax^2}{2} = \frac{x^2}{4}$$

$\rightarrow$ For $1 \le x \le 2$ $\quad F(x) = \int_0^1 ax\,dx + \int_1^2 a\,dx = \left[\frac{ax^2}{2}\right]_0^1 + [ax]_1^x$

$$= \frac{a}{2} + ax - a$$

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$$\Rightarrow \frac{x}{2} - \frac{a}{2}$$

$$\frac{x}{2} - \frac{1}{4}$$

→ For $2 \le x \le 3$

$$F(x) = \int_0^1 f(x)dx + \int_1^2 f(x)dx + \int_2^x f(x)dx$$

$$\left[\frac{ax^2}{2}\right]_0^1 + [ax]_1^2 + \left[-\frac{ax^2}{2} + 3ax\right]_2^x$$

$$\frac{a}{2} + 2a - a \quad -\frac{ax^2}{2} + 3ax + 2a - 6a$$

$$\cancel{\frac{3a}{2} + 2a - \frac{ax^2}{2} = \frac{7a}{2}}$$

$$\frac{a}{2} + a - \frac{ax^2}{2} + 3ax - 4a$$

$$\frac{a}{2} - 3a + \frac{3x}{2} - \frac{x^2}{4}$$

$$-\frac{5a}{2} + \frac{3x}{2} - \frac{x^2}{4}$$

→ for $x \ge 3$

$$F(x) = \int_0^\infty f(x)\,dx = 1$$

CDF →

$$F(x) = \begin{cases} 0, & x < 0 \\ \frac{x^2}{4}, & 0 \le x \le 1 \\ \frac{x}{2} - \frac{1}{4}, & 1 \le x \le 2 \\ -\frac{x^2}{4} + \frac{3x}{2} - \frac{5}{4}, & 2 \le x \le 3 \\ 1, & x \ge 3 \end{cases}$$

next

find $P(X \le 2.5) = F(2.5)$

$$-\frac{x^2}{4} + \frac{3x}{2} - \frac{5}{4}$$

$$-\frac{25}{16} + \frac{15}{4} - \frac{5}{4} = \frac{-25 + 60 - 20}{16} = \frac{15}{16} = 0.9375$$

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Two Dimension R.V.

S be the sample space associated with a random experiment E. Let $X = X(S)$ & $Y = Y(S)$ be 2 f^ns each assigning a real no. to each outcome then (X, Y) is called a 2 dim. R.V.

Joint PMF (Probability mass f^n)

for a 2 two Dim. D.R.V. $P_{ij} = P(X = x_i, Y = y_j)$ then P_{ij} is known as Joint PMF. If

i) $P_{ij} \geq 0 \;\forall\; i, j$

ii) $\sum_j \sum_i P_{ij} = 1$

The set $\{x_i, y_j, P_{ij}\}$, $i = 1, 2, \ldots n$, $j = 1, 2, 3, \ldots n$ is called joint Prob. distribution of (X, Y).

Q. 3 balls are drawn at random without replacement from a box containing 2 white, 3 Red & 4 black balls. If X denotes the no. of white balls drawn. Y denote no. of red balls drawn. find joint probability distribution of X, Y.

W	R	B
2	3	4

X↓ \ Y→	0	1	2	3
0	1/21	3/14	1/7	1/84
1	1/7	2/7	1/14	0
2	1/21	1/28	0	0

$$P(X=0, Y=0) = \frac{{}^4C_3}{{}^9C_3} = \frac{4}{\frac{3 \times 4 \times 7}{\cancel{3 \times 2 \times 1}}} = \frac{1}{21}$$

$$P(X=0, Y=1) = \frac{{}^3C_1 \cdot {}^4C_2}{{}^9C_3} = \frac{3 \times 4 \times 3}{\frac{2 \times 9 \times 8 \times 7}{3 \times 2 \times 1}} = \frac{3}{14}$$

Ideas, Questions & Summary:-

$$\frac{4 \times 3}{3 \times 11}$$

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$P(X=0, Y=2) = \frac{{}^3C_2 \cdot {}^4C_1}{{}^9C_3} = \frac{3! \times 4 \times 3! \times 6 \times 1!}{2! \times 1! \times 9 \times 8 \times 7 \times 6} = \frac{1}{7}$

$P(X=0, Y=3) = \frac{{}^3C_3}{{}^9C_3} = \frac{1 \times 3 \times 2}{9 \times 8 \times 7} = \frac{1}{84}$

$P(X=1, Y=0) = \frac{{}^2C_1 \times {}^4C_2}{{}^9C_3} = \frac{2 \times 4 \times 3 \times 3 \times 2}{2 \times 9 \times 8 \times 7} = \frac{1}{7}$

$P(X=1, Y=1) = \frac{{}^2C_1 \times {}^3C_1 \times {}^4C_1}{{}^9C_3} = \frac{2 \times 3 \times 4 \times 3 \times 2}{9 \times 8 \times 7} = 2/7$

$P(X=1, Y=2) = \frac{{}^2C_1 \times {}^3C_2}{{}^9C_3} = \frac{2 \times 3 \times 3 \times 2}{9 \times 8 \times 7} = 1/14$

$P(X=1, Y=3) = 0$

$P(X=2, Y=0) = \frac{{}^2C_2 \times {}^4C_1}{{}^9C_3} = \frac{1 \times 4 \times 3 \times 2}{9 \times 8 \times 7} = 1/21$

$P(X=2, Y=1) = \frac{{}^2C_2 \times {}^3C_1}{{}^9C_3} = \frac{1 \times 3 \times 3 \times 2}{9 \times 8 \times 7} = 1/28$

$P(X=2, Y=2) = 0$

$P(X=2, Y=3) = 0$

$^3C_2 = (^3C_1) = 3$ $\quad$ $^4C_3 = 4$ $\quad$ $^4C_4 = 1$

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Unit No.	Lecture No.	Faculty	Subject Name	Subject Code	Main Topics:-

Joint Probability Density f^n

If (X,Y) is a 2-dim. CRV such that $P\{x - \frac{dx}{2} \le X \le x + \frac{dx}{2}$ and $y - \frac{dy}{2} \le Y \le y + \frac{dy}{2}\} = f(x,y)\,dx\,dy$ then $f(x,y)$ is called joint PDF of (X,Y) if

(i) $f(x,y) \ge 0$; $\forall -\infty < x < \infty, -\infty < y < \infty$

(ii) $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dx\,dy = 1$

Joint PDF of 2-D R.V. be given by $f(x,y) = xy^2 + \frac{x^2}{8}$, $0 \le x \le 2$, $0 \le y \le 1$ check whether it is a joint PDF of (X,Y).

here $f(x,y) \ge 0$

$$\int_0^1\int_0^2 \left(xy^2 + \frac{x^2}{8}\,dx\right)dy \qquad \int_0^1 \left[\frac{x^2y^2}{2} + \frac{x^3}{24}\right]_0^2 dy$$

$$\int_0^1 \left[\frac{4 \times y^2}{2} + \frac{8}{24}\right] dy$$

$$\left.\frac{2y^3}{3} + \frac{y}{3}\right]_0^1 \rightarrow \frac{2}{3} + \frac{1}{3} = 1$$

H.P. it's a JPDF

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Q. The Joint PMF of (X,Y) is given by $P(x,y) = k(2x+3y)$
$x = 0,1,2 \quad y = 1,2,3$ find.

(1) k

(2) Joint PDF of (X,Y).

$X\downarrow \backslash Y\rightarrow$	1	2	3
0	$3k = 3/72$	$6k = 1/12$	$9k = 1/8$
1	$5k = 5/72$	$8k = 1/9$	$11k = 11/72$
2	$7k = 7/72$	$10k = 10/72$	$13k = 13/72$

$72k = 1$

$\boxed{k = 1/72}$

Marginal PMF & CDF (Discrete)

marginal probability mass f^n of X is defined as

$$P(X = x_i) = \sum_j P_{ij} = P_{i1} + P_{i2} + \text{-----} P_{in} = P_{i*}$$

and collection of pairs $\{x_i, P_{i*}\}$, $i = 1,2, \text{----} m$ is called marginal probability distribution of X. similarly for Y

$$P(Y = y_j) = \sum_i P_{ij} = P_{j1} + P_{j2} + \text{-----} P_{jn} = P_{*j}$$

and collection of pairs $\{y_j, P_{*j}\}$, $j = 1, 2, \text{---} n$ is called marginal PDF of Y.

Marginal Distribution F^n of X (CDF)

$$F_X(x) = \sum_{y=j} P(X \leq x, Y = y) = \sum_{\substack{j \\ i \leq k}} P_{ij}$$

similarly

marginal D F^n of Y

$$F_Y(y) = \sum_{x=i} P(X = x, Y \leq y) = \sum_i P_{ij}$$

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marginal PDF & CDF (continous)

$$f_X(x) = \int_{-\infty}^{\infty} f(x,y)\,dy$$

$$f_Y(y) = \int_{-\infty}^{\infty} f(x,y)\,dx$$

$$F_X(x) = \int_{-\infty}^{x} \left\{ \int_{-\infty}^{\infty} f(x,y)\,dy \right\} dx$$

$$F_Y(y) = \int_{-\infty}^{y} \left\{ \int_{-\infty}^{\infty} f(x,y)\,dx \right\} dy$$

conditional P.F

(1) DRV

The condition Pro. F^n of x given $Y = y_j$

$$P\left(\frac{X = x_i}{Y = y_j}\right) = \frac{P_{ij}}{P_{*j}}$$

similarily

The condional Prob. F^n of Y given $X = x_i$

$$P\left(\frac{Y = y_j}{X = x_i}\right) = \frac{P_{ij}}{P_{i*}}$$

(2) CRV

$$f\left(\frac{x}{y}\right)dx = \frac{f(x,y)\,dx}{f_y(y)} \longrightarrow \text{marginal PDF}$$

$$f\left(\frac{y}{x}\right)dy = \frac{f(x,y)\,dy}{f_x(x)} \longrightarrow \text{marginal PDF}$$

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Q. The Joint PMF of (x,y) is given by $P(x,y) = k(2x+3y)$, $x = 0,1,2$ $y = 1,2,3$ Find

(1) K

(2) marginal Prob. distribution of X

(3) " " " of Y

(4) Conditional distr. of X given $Y=1$

(5) Conditional dis. of y given $X=2$

(6) The Prob. distr. of $(X+Y)$

Sol (i) $K = 1/72$

$X \downarrow$ \ $Y \rightarrow$	1	2	3
0	$3K = 1/24$	$6K = 1/12$	$9K = 1/8$
1	$5K = 5/72$	$8K = 1/9$	$11K = 11/72$
2	$7K = 7/72$	$10K = 10/72$	$13K = 13/72$

$72K = 1$

$\boxed{K = 1/72}$

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$\frac{4+3+5}{12} = \frac{12}{12} = 1$

Unit No.	Lecture No.	Faculty	Subject Name	Subject Code	Main Topics:-

marginal Probability of X

$P(X=0) = P_{0*} = P_{01}+P_{02}+P_{03} = \frac{3}{72}+\frac{6}{72}+\frac{9}{72} = \frac{18}{72} = \frac{2}{8} = \frac{1}{4}$

$P(X=1) = P_{1*} = P_{11}+P_{12}+P_{13} = \frac{5}{72}+\frac{8}{72}+\frac{11}{72} = \frac{24}{72} = \frac{1}{3}$

$P(X=2) = P_{2*} = P_{21}+P_{22}+P_{23} = \frac{7}{72}+\frac{10}{72}+\frac{13}{72} = \frac{30}{72} = \frac{10}{24} = \frac{5}{12}$

$x=i$	p_i
0	1/4
1	1/3
2	5/12
Total	1

marginal probability of Y

$P(Y=1) = P_{*1} = P_{01}+P_{11}+P_{21} = \frac{15}{72}$

$P(Y=2) = P_{*2} = P_{02}+P_{12}+P_{22} = 24/72$

$P(Y=3) = P_{*3} = P_{03}+P_{13}+P_{23} = 33/72$

$y=i$	p_i
1	15/72
2	24/72
3	33/72
Total	1

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(iv) conditional Distribution of X given Y=1

$P_{01} + P_{11} + P_{21} = 15k = \frac{15}{72}$

$$P\left\{\frac{x=i}{y=1}\right\} = \frac{P_{i1}}{P_{*1}} = \frac{P_{i1}}{15/72}$$

$x=i$	$\frac{P_{i1}}{P_{*1}} = \frac{P_{i1}}{15/72}$
0	$\frac{P_{01}}{P_{*1}}$ $\frac{3k}{15k} = \frac{1}{5}$
1	$\frac{P_{11}}{P_{*1}}$ $\frac{5k}{15k} = \frac{1}{3}$
2	$\frac{P_{21}}{P_{*1}}$ $\frac{7k}{15k} = \frac{7}{15}$
Total =>	1

(v) Conditional distribution of Y given X=2

$$P\left\{\frac{y=i}{x=2}\right\} = \frac{P_{2i}}{P_{2*}}$$

$y=i$	$\frac{P_{2i}}{P_{2*}} = \frac{P_{2i}}{30k}$
1	$\frac{P_{21}}{P_{2*}} = \frac{7k}{30k} = \frac{7}{30}$
2	$\frac{P_{22}}{P_{2*}} = \frac{10k}{30k} = \frac{1}{3}$
3	$\frac{P_{23}}{P_{2*}} = \frac{13k}{30k} = \frac{13}{30}$

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Date	Unit No.	Lecture No.	Faculty	Subject Name	Subject Code	Main Topics:-

(vi) The Probability dist. of $(x+y)$

let $z = x+y$

$x+y$ can have $1, 2, 3, 4, 5$

x	0	0	1	0	1	2	1	2	2
y	1	2	1	3	2	1	3	2	3
z	1	2		3			4		5

$$P(z=1) = P(x=0, y=1) = P_{01} = 3k = 3/72$$

$$P(z=2) = P(x=0, y=2 + x=1, y=1) = P_{02} + P_{11} = 6k + 5k = 11k = 11/72$$

$$P(z=3) = P(x=0, y=3 + x=1, y=2 + x=2, y=1) = P_{03} + P_{12} + P_{21} = 9k + 8k + 7k = 24k = \frac{24}{72}$$

$$P(z=4) = P(x=1, y=3 + x=2, y=2) = P_{13} + P_{22} = 11k + 10k = 21k = \frac{21}{72}$$

$$P(z=5) = P(x=2, y=3) = P_{23} = 13k = \frac{13}{72}$$

$z = x+y$	1	2	3	4	5
$P(x+y)$	3/72	11/72	24/72	21/72	13/72

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Q. The Joint PDF of a 2-D RV (X,Y) is given by
$f(x,y) = xy^2 + \frac{x^2}{8}$, $0 \le x \le 2$, $0 \le y \le 1$.
Compute $P(X>1)$, $P(Y<1/2)$, $P(X>1, Y<1/2)$,
$P(Y<1/2 / X>1)$, $P(X<Y)$, $P(X+Y \le 1)$

Sol.

(i) $P(X>1)$

$$\int_1^2 \int_0^1 \left(xy^2 + \frac{x^2}{8} dx\right) dy = \int_1^2 \left[\frac{xy^3}{3} + \frac{x^2}{8}y\right]_0^1 dx$$

$$\left(\frac{x}{3} + \frac{x^2}{8}\right) dx$$

$$\left.\frac{x^2}{6} + \frac{x^3}{24}\right]_1^2 \Rightarrow \frac{4}{6} + \frac{1}{3} - \frac{1}{6} - \frac{1}{24}$$

$$\frac{3}{6} + \frac{1}{3} - \frac{1}{24} = \frac{12+8-1}{24} = \frac{19}{24}$$

(ii) $P(Y<1/2)$

$$\int_0^{1/2} \int_0^2 \left(xy^2 + \frac{x^2}{8} dy\right) dx = \int_{1/2}^1 \left[\frac{x^2y^2}{2} + \frac{x^3}{24}\right]_0^2 dy$$

$$\int_{1/2}^1 2y^2 + \frac{1}{3} dy \qquad \left[\frac{2y^3}{3} + \frac{y}{3}\right]_0^{1/2}$$

$$\frac{2}{3} + \frac{1}{3} - \frac{2}{24} - \frac{1}{6}$$

$$\frac{2}{24} + \frac{1}{6} \Rightarrow \frac{2+4}{24} = \frac{1}{4}$$

$$1 - \left[\frac{2+4}{24}\right]$$

$$1 - \frac{1}{4} = \frac{3}{4}$$

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(iii) $P(X>1,\ Y<\frac{1}{2})$ $\quad 1\le x\le 2 \quad 0\le y\le \frac{1}{2}$

$$\int_0^{1/2}\int_1^2 \left(xy^2+\frac{x^2}{8}\,dy\right)dx \Rightarrow \int_0^{1/2}\left[\frac{x^2y^2}{2}+\frac{x^3}{24}\right]_1^2 dy$$

$$\cancel{\int_0^{1/2} 2y^2+\frac{1}{3}\,dy} \Rightarrow \cancel{\left[\frac{2y^3}{3}+\frac{y}{3}\right]_0^{1/2}}$$

$$\int_0^{1/2} 2y^2+\frac{1}{3}-\frac{y^2}{2}-\frac{1}{24}\,dy \qquad \cancel{\frac{2}{24}+\frac{1}{6}}$$

$\frac{2}{48}=\frac{1}{24}$

$$\left[\frac{2y^3}{3}+\frac{y}{3}-\frac{y^3}{6}-\frac{y}{24}\right]_0^{1/2} \Rightarrow \frac{1}{12}\frac{2}{24}+\frac{1}{6}-\frac{1}{48}-\frac{1}{48}$$

$$\frac{1}{12}+\frac{1}{6}-\frac{1}{24}=\frac{2+4-1}{24}$$

$$\Rightarrow \frac{5}{24}$$

(iv) $\cancel{P\left[\frac{0<y<\frac{1}{2}}{1<x<2}\right]}$ $\quad \cancel{\frac{P[(0<y<\frac{1}{2})\cap(1<x<2)]}{P(1<x<2)}}$

$$P\left[\frac{0<y<\frac{1}{2}}{1<x<2}\right]=\frac{P\left(y<\frac{1}{2},\ x>1\right)}{P(x>1)}=\frac{\frac{5}{24}}{19/24}=\frac{5}{19}$$

(v) $P(X<Y)$

$\downarrow$ $0<y<1$

$0<x<y$

$$\int_0^1\int_0^y \left(xy^2+\frac{x^2}{8}\,dy\right)dx \Rightarrow \left[\frac{x^2y^2}{2}+\frac{x^3}{24}\right]_0^y$$

$$\int_0^1 \frac{y^4}{2}+\frac{y^3}{24}\,dy$$

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$$\left[\frac{y^5}{10}+\frac{y^4}{24\times4}\right]_0^1 \Rightarrow \frac{1}{10}+\frac{1}{96}=\frac{96+10}{960}$$

$$\Rightarrow \frac{106}{960}$$

$$\frac{49}{240} \Leftarrow \frac{98}{480} \quad \frac{53}{480}$$

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(vi) $P(x+y<1)$

$x = 1-y$

$$\int_0^1 \int_0^{1-y} \left(xy^2 + \frac{x^2}{8}\, dy\right) dx$$

$$\int_0^1 \left[\frac{x^2y^2}{2} + \frac{x^3}{24}\right]_0^{1-y} dy$$

$$\int_0^1 \frac{(1-y)^2 y^2}{2} + \frac{(1-y)^3}{24}\, dy$$

$$\frac{(y^2+1-2y)\, y^2}{2}$$

$$\frac{y^4+y^2-2y^3}{2} + \frac{(1-y)^3}{24}$$

$$\left[\frac{y^5}{10} + \frac{y^3}{6} - \frac{2y^4}{8} + \frac{(1-y)^4}{24\times 4} \times -1\right]_0^1$$

$$\frac{1}{10} + \frac{1}{6} - \frac{1}{4} + 0 + \frac{1}{24\times 4} = \frac{26}{960} = \frac{13}{480} \text{ Ans.}$$

$\frac{1}{10} + \cancel{\frac{1}{24}} + \frac{1}{96}$ $\qquad$ $\frac{1}{10} + \frac{2}{24}\frac{1}{12}$ $\qquad$ $\frac{6+5}{60} = \frac{11}{60}$

$\frac{1}{24} + \frac{1}{96} + \frac{1}{10}$

$\frac{5}{96} + \frac{1}{10}$ $\qquad$ $\frac{5+96}{960} = \frac{99}{960}$

$\frac{1}{10} - \frac{1}{12} + \frac{1}{96}$

$\frac{1}{10} + \frac{[-8+1]}{96}$ $\qquad$ $\frac{1}{10}$

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Two RV X & Y has Joint PDF $f_{xy}(x,y) = \begin{cases} A e^{-(2x+y)}, & x,y \geq 0 \\ 0, & \text{else} \end{cases}$

(i) evaluate A

(ii) Find Marginal PDF's

(iii) Find Joint CDF

(iv) Find distribution fn & conditional PDF's

$$\int_0^\infty \left(\int_0^\infty A\, e^{-(2x+y)}\, dy \right) dx = 1 \qquad e^{-2x} \cdot e^{-y}$$

$$\int_0^\infty \int_0^\infty A \left[e^{-(2x+y)} \right]_0^\infty dy\, dx$$

$$\int_0^\infty \left[-\frac{A\, e^{-2x}}{2} \cdot e^{-y} \right]_0^\infty dy$$

$$\int_0^\infty \left[-\frac{A}{2} \cancel{\frac{e^{-y}}{e^{\infty}}} + \frac{A}{2} e^{-y} \times e^0 \right] dy$$

$$\left[\frac{-A e^{-y}}{2} \right]_0^\infty = 1$$

$$\frac{-A}{2e^{y}} + \frac{A}{2} e^0 = 1$$

$$\boxed{A = 2}$$

(ii)(A) Marginal PDF of X

$$\int_0^\infty 2 e^{-(2x+y)}\, dy \Rightarrow \left[\frac{2 \times e^{-(2x+y)}}{-1} \right]_0^\infty = \frac{-2}{2 e^{\infty}} + \frac{2 \cdot 1\, e^{-2x}}{2}$$

$$= 2 e^{-2x}$$

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(ii) B Marginal PDF of y

$$\int_0^\infty 2\, e^{-(2x+y)}\, dx = \left[\frac{2\times e^{-(2x+y)}}{-2}\right]_0^\infty = -e^{\infty} + e^{-y}$$

$$= e^{-y} \text{ Ans}$$

$\boxed{X \text{ and } Y \text{ are statistical independent}} \Rightarrow f_{xy}\ \boxed{f(x,y) = f(x)\cdot f(y)}$

$= 2e^{-2x}\cdot e^{-y} = 2e^{-(2x+y)}$

(iii) Joint CDF of x

$$F_{(x,y)}x = \int_{-\infty}^{y}\left\{\int_{-\infty}^{x} f(x,y)\,dy\right\} dx$$

$$= \int_0^y \int_0^x 2e^{-(2x+y)}\, dx\, dy \quad , \; x, y \geq 0$$

$$\int_0^y \left[\frac{2e^{-(2x+y)}}{-2}\right]_0^x dy$$

$$\int_0^y -\frac{2e^{-(2x+y)}}{2} + e^{-(y)}\; dy$$

$$\left[\frac{+e^{-(2x+y)}}{+1} + \frac{e^{-y}}{-1}\right]_0^y$$

$$e^{-(2x+y)} - e^{-y} - e^{-2x} + 1$$

$$e^{-2x}\cdot e^{-y} - e^{-y} - e^{-2x} + 1$$

$$e^{-y}\left[e^{-2x} - 1\right] - 1\left[e^{-2x} - 1\right]$$

$$(e^{-y} - 1)(e^{-2x} - 1)$$

$$\begin{cases} (e^{-y}-1)(e^{-2x}-1), & x, y \\ 0 & , \text{ else} \end{cases}$$

(iv)(A) Distribution f^n

$$f_X(x) = \int^{x}\int_{-\infty}^{\infty} f_{xy}(x,y)\,dy\,dx$$

$$f_Y(y) = \int_0^{y}\int_{-\infty}^{\infty} f_{xy}(x,y)\,dx\,dy$$

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$x \to y$ की change
$y \to x$ की change

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$$f_{x,y}(x,y) = \int_0^x \int_0^\infty \left(2e^{-(2x+y)}\,dy\right) dx$$

$$\left[-2e^{-(2x+y)}\right]_0^\infty$$

$$-2e^{-(2x+\infty)} + 2e^{-(2x+0)}\,dx$$

$$\int_0^x \left[\cancel{-\frac{2}{e^\infty}} + \cancel{\frac{2}{2e^\infty}} \; 2e^{-(2x-\infty)}\right] dx$$

$$\cancel{\left[\frac{2e^{-(2x-\infty)}}{-2}\right]_0^x}$$

$$\cancel{-e^{-(2x+\infty)} + e^{(0-\infty)}}$$

$$-e^\infty + e^\infty = 0$$

$$\int_0^x 2e^{-2x}\,dx \quad \left[\frac{2e^{-2x}}{-2}\right]_0^x = -e^{-2x} + 1$$

$$f_y(y) = \int_0^\infty \int_0^y \left(2e^{-(2x\pm y)}\right) dx\, dy$$

$$\int_0^\infty \left[\frac{2e^{-(2x+y)}}{-1}\right]_0^y dy \qquad \left[-2e^{-(2x+y)} + 2e^{-(2x)}\right]_0^\infty dx$$

$$\left[\frac{-2e^{-(2x+y)}}{-2} + \frac{2e^{-2x}}{-2}\right]_0^\infty$$

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$$\int_0^y \left[\int_0^\infty 2e^{-(2x+y)}\,dx\right] dy$$

$$\left[\frac{2e^{-(2x+y)}}{-2}\right]_0^\infty \quad \int_0^y \frac{e^{-y}}{-1}\,dy \qquad x - e^{-y} + 1$$

$$f_Y(y) = \begin{cases} (1-e^{-y}) & y \geq 0 \\ 0, & \text{else} \end{cases}$$

(iv) B

Conditional CDF

$$f\left(\frac{x}{y}\right)dx = \frac{f(x,y)\,dx}{f_Y(y)} \qquad f(y/x)\,dy = \frac{f(x,y)\,dy}{f_X(x)}$$

$$f(x/y)\,dx = \frac{2e^{-(2x+\cancel{y})}}{\cancel{e^{-y}}} \qquad f(y/x)\,dy = \frac{\cancel{2e^{-2x}} \cdot e^{-y}}{\cancel{2e^{-2x}}}$$

$$\begin{cases} = 2e^{-2x}, & x \geq 0 \\ 0 & \text{, else} \end{cases} \qquad \begin{cases} e^{-y}, & y \geq 0 \\ 0; & \text{else} \end{cases}$$

Q. The Joint PDF of a bivariate R.V (x,y) is $f_{xy}(x,y) = \begin{cases} kx^2y^2 & , 0<x<2, \ 0<y<2 \\ 0 & \text{, else} \end{cases}$

(i) find K
(ii) Are X & Y independent

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$$\int_0^2 \int_0^2 k x^2 y^2 \, dx\, dy \quad \Rightarrow \left[\frac{k x^3 y^2}{3}\right]_0^2 dy \quad \left[\frac{8}{3} k \frac{y^3}{3}\right]_0^2$$

$$\frac{64}{9} k = 1$$

$$\boxed{k = 9/64}$$

$$f_X(x) = \int_{-\infty}^{\infty} f_{xy}(x,y)\, dy = \int_0^2 k x^2 y^2 \, dy = \left[\frac{k x^2 y^3}{3}\right]_0^2$$

$$\frac{9 \times x^2 \times 8}{64 \times 3} = \frac{3x^2}{8}$$

$$f_Y(y) = \int_0^2 k x^2 y^2 \, dx$$

$$\Rightarrow \left[k \times \frac{x^3}{3} y^2\right]_0^2 \Rightarrow k \times \frac{8}{3} \times y^2 \Rightarrow \frac{9 \times 8}{64 \times 3} y^2 = \frac{3y^2}{8}$$

EXPECTED VALUE OF RV

let X be a R.V. then expected value or mean value of 'x' is defined as

$$\bar{X} = E(X) = \begin{cases} \sum x_i p_i & \text{if } X \text{ is DRV with pmf } p_i \\ \int_{-\infty}^{\infty} x f(x)\, dx & \text{if } X \text{ is CRV with PDF } f(x) \end{cases}$$

EXPECTED VALUE OF F^n of RV

let X be a R.V. then expected value of f^n $g(x)$ if R.V. (x) defined as

$$E\{g(x)\} = \begin{cases} \sum g(x_i) p_i & \text{if } x \text{ is DRV} \\ \int_{-\infty}^{\infty} g(x) f(x)\, dx & \text{if } x \text{ is CRV} \end{cases}$$

VARIANCE OF R.V

$$Var(X) = \sigma_x^2 = E(x-\bar{x})^2 = E(x^2) - \{E(x)\}^2$$

$$= \begin{cases} \sum (x-\bar{x})^2 p_i , & DRV \\ \int_{-\infty}^{\infty} (x-\bar{x})^2 f(x) dx , & CRV \end{cases}$$

Expected value of f^n of 2-Dim. R.V.

Let (x,y) be 2-Dim R.V. & $g(x,y)$ be a f^n of (x,y) then

$$E\{g(x,y)\} = \begin{cases} \sum_j \sum_i g(x_i, y_j) p_{ij} , & (x,y) \text{ is DRV} \\ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x,y) f(x,y) dx dy ; & (x,y) \text{ is CRV} \end{cases}$$

Correlation of x, y

$$Cov(x,y) = E(xy) - E(x) E(y)$$

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NOTE :→

$E(X+Y) = E(X) + E(Y)$

$E(X.Y) = E(X).E(Y)$ if X, Y are independent R.V.

$$E(X.Y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xy\, f(x,y)\, dx\, dy$$

$$E(X+Y) = \int_{-\infty}^{\infty} x f_x(x) dx + \int_{-\infty}^{\infty} y f_y(y) dy = E(X) + E(Y)$$

Q. A R.V. 'X' has the following probability distribution.
① Calculate the value of K ② Variance of X

x_i	-2	-1	0	1	2	3
$f(x_i)$	0.1	K	0.2	2K	0.3	K

$\sum P_i = 1$

$0.6 + 4K = 1$

$4K = 0.4K$

$\boxed{K = 0.1}$

x_i	P_i	$P_i x_i$	$P_i x_i^2$
-2	0.1	-0.2	0.4
-1	0.1	-0.1	0.1
0	0.2	0	0
1	0.2	0.2	0.2
2	0.3	0.6	1.2
3	0.1	0.3	0.9

$\sum P_i x_i = 0.8$ $\quad \sum P_i x_i^2 = 2.8$

$var(X) = \cancel{E(X^2) - E(X^2)} = E(X^2) - \{E(X)\}^2 = 2.8 - 0.64 = 2.16$

Q. If the joint PDF of (x,y) is given by $f(x,y) = 24y(1-x)$ $0 \le y \le x \le 1$
Find $E(x,y)$.

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Unit No.	Lecture No.	Faculty	Subject Name	Subject Code	Main Topics:-

$E(x,y) = \iint$

Kunal Gupta

2 cards are drawn successively with replacement from a well-shuffled 52 cards, find the mean & variance of no. of ~~aces~~ aces.

$x = 0,1,2$

x	0	1	2
$P(x)$	188/221	24/169	

48 NA, 4 A $\rightarrow \dfrac{^{48}C_2}{^{52}C_2}$ $\quad \dfrac{^{48}C_1 \times {^4C_1}}{^{52}C_2}$ $\quad \dfrac{^4C_2}{^{52}C_2}$

$\dfrac{4 \times 3 \times 2}{2 \times 52 \times 51}$

$x = 0,1,2$

x	0	1	2
$P(x)$	$\frac{188}{221}$	$\frac{32}{221}$	$\frac{1}{221}$

$\dfrac{^{48}C_2}{^{52}C_2} = \dfrac{188}{221}$

$\dfrac{^{48}C_1 \times 4}{^{52}C_2} \Rightarrow \dfrac{48 \times 4 \times 2}{52 \times 51} = \dfrac{32}{221}$

$\dfrac{^4C_2}{^{52}C_2} \Rightarrow \dfrac{4 \times 3}{52 \times 51}$

$E(x) = 0 + \dfrac{32}{221} + \dfrac{2}{221} = \dfrac{34}{221} = \dfrac{2}{13}$

$\text{Var } x = E(x^2) - \{E(x)\}^2 = \dfrac{36}{221} - \left\{\dfrac{2}{13}\right\}^2$

Main Ideas, Questions & Summary:

Library / Website Ref.:-

Q. Two R.V X & Y have the following PDFs of f^n $f(x,y) = 2-x-y$

$0 \le x \le 1$ $\quad 0 \le y \le 1$ find ① Marginal PDF of x, y

③ Variance of x & y ② Conditional Density of x/y

④ Correlation of x & y

(i) ~~for x~~

$$\int_0^1 (2-x-y)\,dy = \left[2y - xy - \frac{y^2}{2}\right]_0^1 \Rightarrow 2 - x - \frac{1}{2} = \frac{3}{2} - x$$

for y

$$\int_0^1 (2-x-y)\,dx = \left[2x - \frac{x^2}{2} - xy\right]_0^1 = 2 - \frac{1}{2} - y = \frac{3}{2} - y$$

(ii) ~~for x~~

$$f_{xy}\left(\frac{x}{y}\right) = \frac{f_{xy}(x,y)}{f_y(y)} = \frac{2-x-y}{\frac{3}{2}-y}$$

for y

$$f_{x/y}\left(\frac{x}{y}\right) = \frac{f_{xy}(x,y)}{f_x(x)} = \frac{2-x-y}{\frac{3}{2}-x}$$

(iii)

$$E(x) = \int_0^1 x f(x)\,dx = \int_0^1 (x)\left(\frac{3}{2}-x\right)dx \Rightarrow \int_0^1 \left[\frac{3}{2}x - x^2\right]dx$$

$$\left[\frac{3}{2}\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 \Rightarrow \frac{3}{4} - \frac{1}{3} = \frac{9-4}{12} = 5/12$$

$$E(y) = \int_0^1 y f_y(y)\,dy = \int_0^1 \frac{3}{2}y - y^2\,dy = \left[\frac{3y^2}{4} - \frac{y^3}{3}\right]_0^1 = \frac{3}{4} - \frac{1}{3} = 5/12$$

~~Var(x)=~~

$$E(x^2) = \int_0^1 x^2 f(x)\,dy \Rightarrow \int_0^1 \left(\frac{3}{2}x^2 - x^3\right)dx = \left[\frac{3}{2}\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1 \Rightarrow \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$

~~Var(x)~~

$$E(y^2) = \frac{1}{4} \qquad Var(x) = E(x^2) - \{E(x)\}^2$$

$$= \frac{1}{4} - \frac{25}{144} \Rightarrow \frac{36-25}{144} = \frac{11}{144}$$

$Var(y) = 11/144$

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Unit No.	Lecture No.	Faculty	Subject Name	Subject Code	Main Topics:-

(iv) $\mathrm{Cov}(x,y) = E(xy) - E(x)E(y)$

$$E(xy) = \int_0^1\int_0^1 (xy)(2-x-y)\,dx\,dy$$

$$= \frac{1}{6}$$

$$\mathrm{Cov}(x,y) = \frac{1}{6} - \frac{5}{12}\times\frac{5}{12} = \frac{1}{6} - \frac{25}{144} = \frac{24-25}{144} = \frac{-1}{144}$$

$\boxed{\text{Moments}}$ (μ_n)

It is the tendency of a force to rotate a body about a point.

→ ~~Moment~~ About origin

$$\mu_n' = E(x^n) = \begin{cases} \sum x_i^n p_i & \rightarrow \text{D.R.V.} \\ \int_{-\infty}^{\infty} x^n f(x)\,dx & \rightarrow \text{C.R.V.} \end{cases}$$

→ About centre

$$\mu_n = E(X-\mu_x)^n = \begin{cases} \sum (x_i-\mu_n)^n p_i & \rightarrow \text{D.R.V} \\ \int_{-\infty}^{\infty} (x-\mu_n)^n\,dx & \rightarrow \text{C.R.V.} \end{cases}$$

→ Arbitary Point

$$\mu_n'' = E(x-a)^n = \begin{cases} \sum (x_i-a)^n p_i \\ \int_{-\infty}^{\infty} (x-a)^n f(x)\,dx \end{cases}$$

Main Ideas, Questions & Summary:- ~~$E\{(x)^n\}$ & $E\{(x-\mu_n)^n\}$~~

Library / Website Ref.:-

$E\{|x^n|\}$ & $E|x-\mu_n|^n$ are absolute movement of x.

3/09/24

Karl Pearson's β & γ coefficient

(i) β coefficient

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3}, \quad \beta_2 = \frac{\mu_4}{\mu_2^2}$$

- β_1 = measurement of skewness
- β_2 = measurement of kurtosis

(ii) γ coefficient

$\gamma_1 = +\sqrt{\beta_1}$

$\gamma_2 = \beta_2 - 3$

If $\beta_2 = 3$, $\gamma_2 = 0 \Rightarrow$ mesokurtic

$\beta_2 > 3$, $\gamma_2 > 0 \Rightarrow$ leptokurtic

$\beta_2 < 3$, $\gamma_2 < 0 \Rightarrow$ platykurtic

Q. Calculate first 4 moments the mean of the following distribution & calculate β_1 & β_2.

x	1	2	3	4	5	6	7	8	9
y	1	6	13	25	30	22	9	5	2

x_i	y_i	x_i-5	$y_i(x_i-5)$	$y_i(x_i-5)^2$	$y_i(x_i-5)^3$	$y_i(x_i-5)^4$
1	1	-4	-4	16	-64	256
2	6	-3	-18	72	-162	486
3	13	-2	-26	52	-104	208
4	25	-1	-25	25	-25	25
5	30	0	0	0	0	0
6	22	1	22	22	22	22
7	9	2	18	36	72	144
8	5	3	15	45	135	405
9	2	4	8	32	128	512
	113	0	-10	282	2	2058

$* \ \mu_1 = 0$

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$$\mu_1'' = \frac{\Sigma y_i(x_i - 5)}{\Sigma y_i} = \frac{-10}{113} = \cancel{-0.08} - 0.088$$

$$\mu_2'' = \frac{\Sigma y_i(x_i-5)^2}{\Sigma y_i} = \frac{282}{113} = 2.496$$

$$\mu_3'' = \frac{\Sigma y_i(x_i-5)^3}{\Sigma y_i} = \frac{2}{113} = 0.018$$

$$\mu_4'' = \frac{\Sigma y_i(x_i-5)^4}{\Sigma y_i} = \frac{2058}{113} = 18.212$$

$\mu_1 = 0$ $\quad * \ \boxed{\mu_2 = \mu_2'' - \mu_1''^2} \Rightarrow 2.496 - (-0.088)^2$

$\Rightarrow 2.488$

$$* \ \boxed{\mu_3 = \mu_3'' - 3\mu_2''\mu_1'' + 2\mu_1''^3} = 0.018 - 2.496 \times 3 \times -0.088 + 2 \times (-0.88)^3$$

$$= 0.67558180 56$$

$$\mu_4 = \mu_4'' - 4\mu_3''\mu_1'' + 6\mu_2''\mu_1''^2 - 3\mu_1''^4$$

$$= 18.212 - 4 \times 0.0018 \times -0.088 + 6 \times 2.496 \times (-0.088)^2 - 3 \times (-0.088)^4$$

$$= 18.3341302 35392$$

Main Ideas, Questions & Summary:

Library / Website Ref.:-

$\mu_1 = 0$
$\mu_2 = 2.5$
$\mu_3 = 0.68$
$\mu_4 = 18.3$

$$\beta_1 = \frac{(0.68)^2}{(2.5)^3} = 0.03$$

$$\beta_2 = \frac{18.3}{(2.5)^2} = 3$$

(5)

Q. Find the first 4 moments of a distribution about the value '4' of a variable are $-1.5, 17, -30, 108$. Find
1. moment about origin
2. moment about mean
3. moment about point $x=2$
4. β_1, β_2 hence give your conclusion

$\mu_1'' = -1.5 \qquad \mu_2'' = 17 \qquad \mu_3'' = -30 \qquad \mu_4'' = 108$

(ii) $\mu_1 = 0$

$\mu_2 = \mu_2'' - (\mu_1'')^2 = 17 - (-1.5)^2 = 14.75$

$\mu_3 = \mu_3'' - 3\mu_2''\mu_1'' + 2\mu_1''^3 = -30 - 3\times 17\times -1.5 + 2\times(-1.5)^3$
$= 39.75$

$\mu_4 = \mu_4'' - 4\mu_3''\mu_1'' + 6\mu_2''\mu_1''^2 - 3\mu_1''^4$

$= 108 - 4\times -30\times -1.5 + 6\times 17\times(-1.5)^2 - 3\times(-1.5)^4$
$= 142.3125$

(iii) $y_i(x_i - 4) = -1.5 \qquad y_i(x_i - 2)$

$y_i(x_i-4)^2 = 17$

$\cancel{17x_i - 4\times 17 = -1.5x_i + 6}$

$18.5x_i = 6 + 68$

$18.5x_i = 74$

$\boxed{x_i = 4}$

$17x_i - 4\times 17 = -1.5\times(x_i^2 + 16 - 8x_i)$

$17x_i - 68 = -1.5x_i^2 - 24 + 12x_i$

$1.5x_i^2 - 5x_i - 44 = 0$

$15x_i^2 - 50x_i - 440 = 0$

$y_i x + 8 = -1.5 \qquad 3x_i^2 - 10x_i - 88 = 0$

$y_i = \frac{1.5}{8}$

$\boxed{x_i = -4}$

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(ii) about origin

$\mu_1' = \bar{x} = 2.5$

$\mu_1'' = \bar{x} - a$

$-1.5 = \bar{x} - 4$

$\bar{x} = 2.5$

$\mu_2 = \mu_2' - \mu_1'^2$

$\boxed{\mu_2' = 21}$

$\mu_3 = \mu_3' - 3\mu_1'\mu_2' + 2\mu_1'^3$

$\boxed{\mu_3' = 166}$

$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$

$\boxed{\mu_4' = 1132}$

$$\boxed{\begin{aligned} \mu_1' &= 2.5 \\ \mu_2' &= 21 \\ \mu_3' &= 166 \\ \mu_4' &= 1132 \end{aligned}}$$

(iii) $\mu_1'' = \bar{x} - a$

$= 2.5 - 2 = 0.5$

$\mu_2'' = \mu_2 + (\mu_1'')^2 = 15$

$\mu_3'' = \mu_3 + 3\mu_2''\mu_1'' - 2\mu_1''^3 = 62$

$\mu_4'' = \mu_4 + 4\mu_3''\mu_1'' - 6\mu_2''\mu_1''^2 + 3\mu_1''^4 = 244$

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3} = \frac{(39.75)^2}{(14.75)^3} = 0.4923$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{1132}{(14.75)^2} = 0.6541$$

(i) $\beta \neq 0 \rightarrow$ non symmetric curve or skewed curve

(ii) $\beta_2 < 3 \rightarrow$ platyukurtic curve

POORNIMA

Date	Unit No.	Lecture No.	Faculty	Subject Name	Subject Code	Main Topics:-
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Probability Distribution

1. Binomial distribution
2. Poission distribution

→ DRV

3. Normal distribution
4. Uniform distribution
5. Exponential distribution

→ CRV

Binomial Distribution

→ 'n' is the no. of independent trials - Each trial has 2 outcomes called success & failure denoted by p & q.

- $P(X=r) = {}^nC_r \, p^r q^{n-r} \quad r = 0,1,2,\ldots$

X is said to be binomial distribution with parameters n and p.

→ Mean ⇒ np

$$E(X) = \mu_1' = \sum x p(x) = \sum_{r=0}^{n} r \, {}^nC_r \, p^r q^{n-r}$$

$$\Rightarrow 0 + 1\,{}^nC_1 \, p q^{n-1} + 2\,{}^nC_2 \, p^2 q^{n-2} + \ldots + np^n$$

$$\Rightarrow npq^{n-1} + (n)(n-1)p^2 q^{n-2} + \frac{(n-1)(n-2)}{2!} p^3 q^{n-3} \ldots + np^n$$

$$\Rightarrow np\left[q^{n-1} + \frac{(n-1)}{1!} p q^{n-2} + \frac{(n-1)(n-2)}{2!\,(n)} p^2 q^{n-3} \ldots\ldots + p^{n-1}\right]$$

$$\Rightarrow (np)\,[\underbrace{q+p}_{1}]^{n-1} \Rightarrow np \text{ ANS}$$

Main Ideas, Questions & Summary:

Library / Website Ref.:-

$$\Rightarrow E(x^2) = \mu_2' = \sum x^2 p(x) = \sum_{r=0} r^2 \, {}^nc_r \, p^r q^{n-r}$$

$$= \sum_{r=0}^{n} (r(r-1)+r) \, {}^nc_r \, p^r q^{n-r}$$

$$= \sum_{r=0}^{n} r(r-1) \, {}^nc_r \, p^r q^{n-r} + \underbrace{\sum_{r=0}^{n} r \, {}^nc_r \, p^r q^{n-r}}_{\mu_1' = \boxed{np}}$$

$$\Rightarrow (n)(n-1)p^2 q^{n-2} + (n)(n-1)(n-2)p^3 q^{n-3} ---- + (n)(n-1)p^n + np$$

$$\Rightarrow (n)(n-1)p^2 \left[q^{n-2} + \frac{(n-2)}{1!} p q^{n-3} + \frac{(n-2)(n-3)}{2!} q^{n-4} + ------ + p^{n-2} \right] + np$$

$$\Rightarrow (n)(n-1)p^2 \left[q+p \right]^{n-2} + np$$

$$\boxed{\begin{matrix} p+q=1 \\ q=1-p \end{matrix}}$$

$$\Rightarrow (n)(n-1)p^2 + np$$

$$\Rightarrow np[(n-1)p + 1] \Rightarrow np[np - p + 1] \Rightarrow (np)(np+q)$$

$$\boxed{E(x^2) = (np)(np+q)}.$$

$$\Rightarrow var(x) = E(x^2) - \{E(x)\}^2$$

$$= (np)[np+q] - (np)^2$$

$$\Rightarrow (np)(\cancel{n}p + q - \cancel{n}p)$$

$$\Rightarrow npq$$

$$\boxed{Var(x) = npq}.$$

→ Standard deviation $= \sqrt{var} = \sqrt{npq}$.

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Date	Unit No.	Lecture No.	Faculty	Subject Name	Subject Code	Main Topics:-

Moment & Moment Generating Function $[M_x(t)]$

- $M_x(t) = (q+pe^t)^n$

$$\mu_1' = \left[\frac{d}{dt} M_x(t)\right]_{t=0} = np$$

$$\mu_2' = \left[\frac{d^2}{dt^2} M_x(t)\right]_{t=0} = n^2p^2 + npq$$

$$\mu_3' = \left[\frac{d^3}{dt^3} M_x(t)\right]_{t=0} = np + (n)(n-1)(n-2)p^3 + 3n(n-1)p^2$$

$$\mu_4' = \left[\frac{d^4}{dt^3} M_x(t)\right]_{t=0} = n(n-1)(n-2)(n-3)p^4 + 6(n)(n-1)(n-2)p^3 + 7n(n-1)p^2 + np$$

x ——— x ——— x

Q. 8 coins are tossed simultaneously. Find the probability of getting atleast 6 heads.

$n=8$ $\quad P(x) = {}^8C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{8-r}$

$p = \frac{1}{2}$

$\boxed{r=6,7,8}$ $\quad q=\frac{1}{2}$ $\quad$ ~~P(x)=~~ $\quad \boxed{r=6}$

$${}^8C_6\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^2 \Rightarrow \frac{8\times 7}{2} \times \left(\frac{1}{2}\right)^6 \times \left(\frac{1}{2}\right)^2$$

$$= 28 \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4}$$

$$= \frac{7}{64}$$

Main Ideas, Questions & Summary:

Library / Website Ref.:-

$P(X \geq 6) = P(6) + P(7) + P(8)$

$= \frac{7}{64} + {}^8C_7 \left(\frac{1}{2}\right)^7 \left(\frac{1}{2}\right) + {}^8C_8 \left(\frac{1}{2}\right)^8$

$= \frac{7}{64} + 8 \times \frac{1}{16} \times \frac{1}{2^4} + \frac{1}{2^8}$

$= \frac{7}{64} + \frac{1}{32} + \frac{1}{256} =$ ~~$\frac{28}{256}$~~

$= 0.1 + 0.03 + 0.003$

$= 0.1445$

Q. 2 Players A & B play tennis game. Their chances of winning a game are in the ratio 3:2. Find A chance of winning atleast 2 games out of 4 games played.

$p = \frac{3}{5}$ $\quad q = \frac{2}{5}$ $\quad n = 4$ $\quad {}^4C_r \left(\frac{3}{5}\right)^r \left(\frac{2}{5}\right)^{4-r}$

$\boxed{2,3,4}$

~~$P(X) = P(2)$~~ $P(X \geq 2) = 1 - P(X < 2)$

$= 1 - [P(1) + P(0)]$

$= 1 - \left[{}^4C_1 \cdot \frac{3}{5} \times \left(\frac{2}{5}\right)^3 + \left(\frac{2}{5}\right)^4\right]$

$\Rightarrow 1 - \left[4 \times \frac{3}{5} \times \frac{8}{125} + \frac{16}{125 \times 5}\right]$

$\boxed{625}$

$0.15 \qquad 0.025$

$= 0.82$

$\frac{y!}{(y-0)!\,(0)!}$

POORNIMA

Date	Unit No.	Lecture No.	Faculty	Subject Name	Subject Code	Main Topics:-

Fitting of B.D.

Fitting is that to find frequencies for a given frequency distribution

If $N = \Sigma f_i$ where f_i = freq. of data

$\bar{x} = \frac{\Sigma x_i f_i}{\Sigma f_i} = \frac{\Sigma f x}{N}$ in B.D. we take $n = N$

$\bar{x} = np = Np$

$p = \frac{\bar{x}}{N} = \frac{\Sigma f_i x_i}{N^2}$

By expanding $N(p+q)^n$ we find corresponding frequencies for $x = 0, 1, 2, \_\_\_ n$ for the fitted B.D.

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Curve Fitting

- To express the given data, a no. of eqn of various types can be obtained by this method. The method of finding such eqn of best fit is called curve fitting.

Least square method

The principle of least square method is that the sum of the diff. b/w observed value of result & the expected value of y_i ($i = 1, 2, ---- n$) should be minimum.

Ideas, Questions & Summary:

Library / Website Ref.:-

① st. line

$y = a+bx$ form

formula → ① $\Sigma y = na + b\Sigma x$

② $\Sigma xy = a\Sigma x + b\Sigma x^2$

② fit 2nd degree parabola $y = a+bx+cx^2$

$$\sum_{i=1}^{n} y_i = na + b\sum_{i=1}^{n} x_i + c\sum_{i=1}^{n} x^2$$

$$\sum_{i=1}^{n} x_i y_i = a\sum_{i=1}^{n} x_i + b\sum_{i=1}^{n} x_i^2 + c\sum_{i=1}^{n} x_i^3$$

$$\sum_{i=1}^{n} x_i^2 y_i = a\sum_{i=1}^{n} x_i^2 + b\sum_{i=1}^{n} x_i^3 + c\sum_{i=1}^{n} x_i^4$$

①. Fit a st. line $y = a+bx$

x	5	15	25	35	45	50
y	10	18	20	25	32	45
$\Sigma x \cdot y$	50	270	500	875	1440	2250

Total = 5385

$n = 6$
$\Sigma x = 175$
$\Sigma y = 150$
$\Sigma xy = 5385$
$\Sigma x^2 = 6625$

$\Sigma y = na + b\Sigma x$

$150 = 6a + 175b$ — ①

$\Sigma xy = a\Sigma x + b\Sigma x^2$

$5385 = 175a + 6625b$ — ②

on solving ① & ② we get

$a = 5.75$ $b = 0.66$

so st. line to be fitted in curve

$y = 5.75 + 0.66x$

Shot on AWESOME A70

POORNIMA

Date	Unit No.	Lecture No.	Faculty	Subject Name	Subject Code	Main Topics:-

Q. Fit a Parabola $y = a + bx + cx^2$

x	y	xy
0	12	0
1	10.5	10.5
2	10	20
3	8	24
4	7	28
5	8	40
6	7.5	
7	8.5	180.5
8	9.5	
$\Sigma x_i = 36$	$\Sigma y_i = 81$	$303 = \Sigma x_i y_i$

$\Sigma x^2 = 1+4+9+16+25+36+49+64+ = 204$

$\Sigma x_i = 36$

$\Sigma y_i = 81$

$\Sigma y_i = a + b\Sigma x_i + c\,\Sigma x_i^2$

$\boxed{81 = a + 36b + 204c}$ — (1)

$\Sigma x_i y_i = a\Sigma x_i + b\Sigma x_i^2 + c\,\Sigma x_i^3$

$\boxed{303 = 36a + 204b + 1296c}$ — (2)

$\Sigma x_i^2 y_i = a\Sigma x_i^2 + b\Sigma x_i^3 + c\,\Sigma x_i^4$

$\boxed{1729 = 204a + 1296b + 8772c}$ — (3)

⇓

By solving

$a = 12.09 \qquad b = -1.85 \qquad c = 0.183$

Main Ideas, Questions & Summary:

Library / Website Ref.:-

Correlation statistical investigation are based on prediction preferable on the basis of mathematical eqns.
prediction may be treated in 2 ways:-

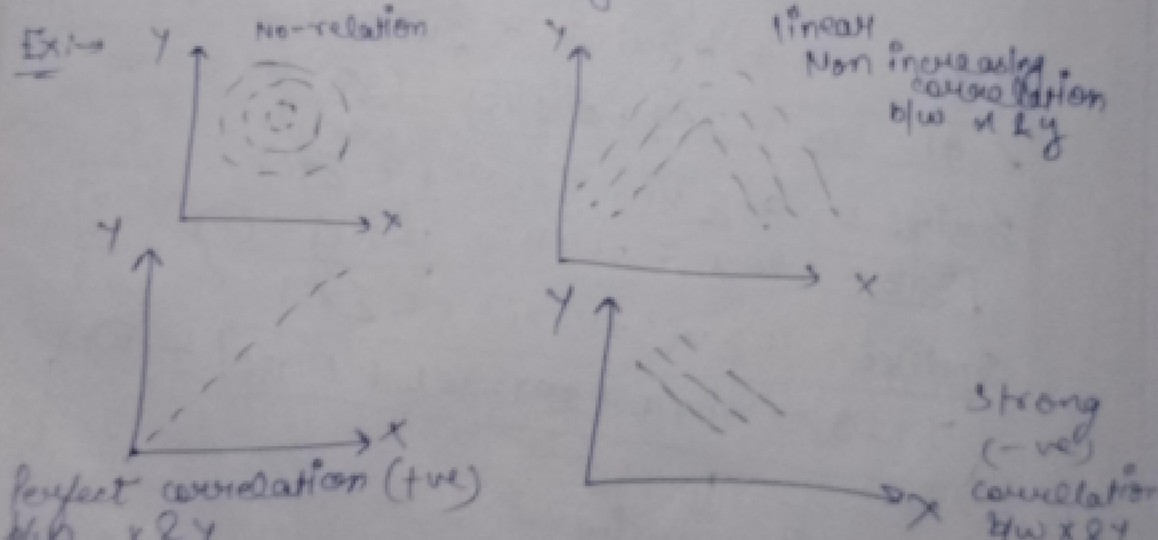
1. By computing correlation coefficients
2. By using Regression analysis

→

→ Measure of correlation → By —
(1) Scattered diagram
Karl Pearson's
(2) coefficient of correlation

(1) **Scattered Diagram:** → It is a simple way to represent bivariate data in form of diagrams, the diagrams are called as scattered diagram.

Ex:→



(2) **Karl Pearson's coefficient of correlation**

$$r(X,Y) = r_{xy} = \frac{cov(x,y)}{\sigma_x \sigma_y}$$

or

$$= \frac{\frac{1}{n}\sum xy - \bar{x}\bar{y}}{\sqrt{\frac{1}{n}\sum x^2 - \bar{x}^2}\sqrt{\frac{1}{n}\sum y^2 - (\bar{y})^2}}$$

POORNIMA

Date	Unit No.	Lecture No.	Faculty	Subject Name	Subject Code	Main Topics:-

Q. Calculate the correlation for the following height in (inches) of ~~fauter~~ father X & their son Y

X	65	66	67	67	68	69	70	72
Y	67	68	65	68	72	72	69	71

X	Y	X^2	Y^2	XY
65	67			
66	68			
67	65			
67	68			
68	72			
69	72			
70	69			
72	71			
544	552	37028	38132	37560

$\bar{x} = \frac{\Sigma x}{n}$

$n = 8$

$\Sigma xy = 37560$

$\Sigma x = 544$

$\Sigma y = 552$

$\Sigma x^2 = 37028$

$\Sigma y^2 = 38132$

$\bar{x} = \frac{\Sigma x}{n} = 68$

$\bar{y} = \frac{\Sigma y}{n} = 69$

Acc. to formula

$$r = \frac{\left[\frac{1}{n}\left[\Sigma xy\right] - \bar{x}\bar{y}\right]}{\sqrt{\frac{1}{n}\cdot\Sigma x^2 - \bar{x}^2}\ \sqrt{\frac{1}{n}\cdot\Sigma y^2 - \bar{y}^2}}$$

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$$r = \frac{\frac{1}{8} \times 37560 - (68)(69)}{\sqrt{\frac{1}{8} \times 37028 - 68)^2} \cdot \sqrt{\frac{1}{8} \times 38132 - (69)^2}}$$

$$\boxed{r = 0.6032}$$

Linear Regression :→ If there exist a linear relationship b/w two variates X & Y, the dots of the scattered diagram are concentrated around the st. line called the line of regression.

The line of regression is a st. line which gives the best fit in the least square sense to the given distribution.

If the st. line is so choosen that the sum of square of deviation is parallel to the axis of Y is minimized. It is called the line of regression of Y on X. If the sum of square of deviation is parallel to the x-axis is minimized, the resulting st. line is the line of regression of X on Y.

Two eqn

Y on X

$$y - \bar{y} = \frac{\mu_{11}}{\sigma_x^2}(x - \bar{x})$$

X on Y

$$x - \bar{x} = \frac{\mu_{11}}{\sigma_y^2}(y - \bar{y})$$

Both are passing through $(\bar{x}, \bar{y})$

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The term $\frac{\mu_{11}}{\sigma_x^2}$ or $\frac{\mu_{11}}{\sigma_y^2}$ can also be defined as

$$r\frac{\sigma_y}{\sigma_x} = b_{yx} \qquad r\frac{\sigma_x}{\sigma_y} = b_{xy}$$

Q. Solve the line of Regression for X & Y

X	Y	X^2	Y^2	XY
1	10			
2	12			
3	16			
4	28			
5	25			
6	36			
7	41			
Σ 28	128	140	4886	822

$$\bar{x} = \frac{\Sigma x}{n} = 4$$

$$\bar{y} = \frac{\Sigma y}{n} = 24$$

$$\mu_{11} = cov(x,y) = \frac{1}{n}\Sigma xy - \bar{x}\bar{y}$$
$$= \frac{1}{7}\times 822 - 4\times 24$$
$$= 21.428$$

$$\sigma_x = \sqrt{\frac{1}{n}\Sigma x^2 - \bar{x}^2}$$
$$= \sqrt{\frac{1}{7}\times 140 - (4)^2} = \boxed{2}$$

$$\sigma_y = \sqrt{\frac{1}{n}\Sigma y^2 - \bar{y}^2} = \sqrt{\frac{1}{7}\times 4886 - (24)^2} = \boxed{11.04}$$

$$r_{xy} = \frac{cov(x,y)}{\sigma_x \sigma_y} = \frac{\mu_{11}}{\sigma_x \sigma_y} = \frac{21.428}{(2)(11.04)} = 0.97$$

Main Ideas, Questions & Summary

→ y on x

$$y-\bar{y} = \left(\frac{\mu_{11}}{\sigma_x^2}\right) \text{ or } r\,\frac{\sigma_y}{\sigma_x}\,(x-\bar{x})$$

$$y-24 = \frac{21.428}{(2)^2}\times(x-4) \text{ or } y-24 = 0.97\times\frac{11.04}{2}\times(x-4)$$

$$y = 5.36x + 2.57$$

x on y

$$x-\bar{x} = \frac{\mu_{11}}{\sigma_y^2}\,(y-\bar{y})$$

$$x-4 = \frac{21.428}{(11.04)^2}\times(y-24)$$

or

$$x-4 = \frac{0.97\times 2}{11.04}\times(y-24)$$

$$\boxed{x = 0.176y - 0.224}$$

Rank Correlation

Although the data is measured in several cases is numeric in nature but there may be some cases when data turns out to be quantitative or non-numeric in nature.

For eg:→ Appearance, Beauty, Honesty, Intelligentia, etc.

In such cases data is ranked acc to a particular characteristic instead of taking numeric measurement on them.

The spearman's rank correlation coefficient for non-repeated rank is given as

$$\boxed{\rho = 1 - \frac{6\sum d_i^2}{n(n^2-1)}}, \quad d_i = x_i - y_i$$

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Rank correlation repeated ranks

$\boxed{\frac{m(m^2-1)}{12}}$ is to be added to d^2

m = no. of times a rank is repeated

Q. The ranks of some ten students in 2 subjects A and B are given below:→ — Calculate correlation coefficient.

Rank in (A)	5	2	9	8	1	10	3	4	6	7
Rank in (B)	10	5	1	3	8	6	2	7	9	4

x_i	y_i	$d_i = x_i - y_i$	d_i^2
5	10	-5	25
2	5	-3	9
9	1	8	64
8	3	5	25
1	8	-7	49
10	6	4	16
3	2	1	1
4	7	-3	9
6	9	-3	9
7	4	3	9
		0	216

36
100
114
250

$$\rho = 1 - \frac{6\sum d_i^2}{n(n^2-1)}$$

$$\rho = 1 - \frac{\cancel{6} \times \cancel{216}}{\cancel{5}(\cancel{10})(\cancel{99})}$$

$$\rho = 1 - 1.30891$$

$$\boxed{\rho = -0.309}$$

Q. obtain the rank correlation for the following data

x	85	74	85	50	65	78	74	60	74	90
y	78	91	78	58	60	72	80	55	68	70
d_i	7	-17	7	-8	5	+6	-6	5	6	+20
x_i =	2.5	6	2.5	10	8	4	6	9	6	1
y_i =	3.5	1	3.5	9	8	5	2	10	7	6

← Direct Nhi Nikalte

→ 90 bada tha to Sabse to 1

→ 85 se tie 2nd and 3rd ke liye $\frac{2+3}{2} = 2.5$

for x_i

$\frac{5+6+7}{3}$ for 74

neric t

eate → $\frac{3+4}{2}$ = for 78

d_i =	-1	5	-1	1	0	-1	4	-1	-1	-5
d_i^2	1	25	1	1	0	1	16	1	1	25

$= 0$

$= 72$

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for observation of X

Rank 2.5 is repeated twice $m=2$ $\frac{m(m^2-1)}{12} = \frac{(2)(3)}{12} = \frac{1}{2}$

Rank 6 is repeated thrice $m=3$ $\frac{m(m^2-1)}{12} = \frac{(3)(8)}{12} = 2$

for observation of Y

Rank 3.5 is repeated twice, $m=2$ $\frac{m(m^2-1)}{12} = \frac{2\times3}{12} = \frac{1}{2}$

Total correlation factor $= \frac{1}{2}+2+\frac{1}{2} = 3$

sum to the $d_i^2 = \Sigma d_i^2 + cf = 72+3 = 75$

spearman's rank correlation coefficient is $\rho = 1 - \frac{6[\Sigma d_i^2 + c.f.]}{(n)(n^2-1)}$

$$\rho = 1 - \frac{6\times75}{10\times99} = 0.545$$

Q. Two R.V. have the least square regression lines with eq^ns $3x+2y-26=0$ & $6x+y-31=0$. Find the mean values & the coefficient of correlation b/w X & Y.

as we know that regression eq^ns passes through mean values i.e $(\bar{x},\bar{y})$.

$3\bar{x}+2\bar{y}=26$
$6\bar{x}+\bar{y}=31$

$3\bar{x}+2\bar{y}=26$
$12\bar{x}+2\bar{y}=62$
$-9\bar{x}=-36$
$\boxed{\bar{x}=4}$

$12+2\bar{y}=26$
$2\bar{y}=14$
$\boxed{\bar{y}=7}$

(i) $\bar{x}=4$
$\bar{y}=7$

(ii) $3x+2y=26$

$y = \frac{26-3x}{2} = 13 - \frac{3}{2}x$

$x = \frac{31}{6} - \frac{y}{6}$

$b_{yx} = \frac{\mu_{11}}{\sigma_x^2} = -\frac{3}{2}$ — ①

$b_{xy} = \frac{\mu_{11}}{\sigma_y^2} = -\frac{1}{6}$ — ②

$\frac{\mu_{11}}{\sigma_x^2} = r\frac{\sigma_y}{\sigma_x} = b_{yx}$

$\frac{\mu_{11}}{\sigma_y^2} = r\frac{\sigma_x}{\sigma_y} = b_{xy}$

$b_{yx}\cdot b_{xy} =$

① × ②

$b_{yx}\, b_{xy} = \frac{\mu_{11}}{\sigma_x^2} \times \frac{\mu_{11}}{\sigma_y^2} = -\frac{3}{2}\times\frac{-1}{6} = \frac{1}{4}$

$\boxed{r^2 = \frac{1}{4}}$ $r = \pm\frac{1}{2}$

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Both coefficent are -ve $\boxed{r = -ve}$

both +ve $\boxed{r = +ve}$

1 +ve , 1 -ve r = greater wala sign

$\boxed{r = -\frac{1}{2}}$ → Because both b_{xy} & b_{yx} are -ve

H.W.

Q. If θ is the acute angle b/w the 2 regression lines of case of 2 variable x & y show that

$$\tan\theta = \frac{1 - r^2}{r} \times \frac{\sigma x \, \sigma y}{(\sigma x^2 + \sigma y^2)}$$

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Poisson Distribution

It is a distribution which is a limiting case of Binomial distribution under the following condition

(i) when no. of trials 'n' is very large i.e $n \to \infty$

(ii) Probability of success 'p' is very small i.e $p \to 0$

If both the above conditions are satisfied then we use the poisson's distribution in place of binomial distribution

$$P(X=x) = \frac{(\lambda)^x}{\lfloor r} e^{-\lambda} \qquad \lambda = \text{mean value}$$

variance $= \sigma^2 = \lambda$

Standard deviation $= +\sqrt{\lambda}$

Fitting $\quad \frac{P(x+1)}{P(x)} = \frac{\lambda}{x+1}$

It is basically we are finding frequencies for a given frequency distribution

Q. Find the probability that atmost five defective fuses will be found in a box of 200 fuses if experience shows that 2% of such fuses are defective

No. of independent trials = 200 (v. large)

$\boxed{n=200}$

$p = 2\% = 0.02$ $\quad \boxed{\lambda = np} \to$ mean same as binomial

$q = 0.98$ $\quad \lambda = 200 \times \frac{2}{100} = 4$

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$r = 0,1,2,3,4,5$

$P(X \leq 5) = P(0) + P(1) + P(2) + P(3) + P(4) + P(5)$

$= \frac{(\lambda)^0 e^{-4}}{\lfloor 0} + \frac{\lambda^1 e^{-4}}{\lfloor 1} + \frac{\lambda^2 e^{-4}}{\lfloor 2} + \frac{\lambda^3 e^{-4}}{\lfloor 3} + \frac{\lambda^4 e^{-4}}{\lfloor 4} + \frac{\lambda^5 e^{-4}}{\lfloor 5}$

$= \frac{(4)^0 e^{-4}}{\lfloor 0} + \frac{4 \times e^{-4}}{\lfloor 1} + \frac{(4)^2 e^{-4}}{\lfloor 2} + \frac{(4)^3 e^{-4}}{\lfloor 3} + \frac{(4)^4 e^{-4}}{\lfloor 4} + \frac{(4)^5 e^{-4}}{\lfloor 5}$

$= e^{-4}\left[1 + \frac{4}{1} + \frac{16}{2} + \frac{64}{6} + \frac{256}{24} + \frac{1024}{120}\right]$

$= e^{-4}[42.86]$

$= 0.018 \times 42.86$

$= 0.7851$

Q. A car firm has two cars which is hire out day by day the no. of demands for a car on each day is distributed as a poissons variate with mean 1.5. calculate the proportion of days on which (i) neither car is used (ii) some demands is refused

$\lambda = 1.5$

$n = 2$

$p(x = r) = \frac{(\lambda)^r e^{-\lambda}}{\lfloor r}$

(i) Neither car is used $r = 0$

$(\lambda) = 1.5 \quad r = 0 \qquad P(X=0) = \frac{e^{-1.5} (1.5)^0}{\lfloor 0} = 0.22313016$

(ii) Some demand is refused

$P(X > 2) = 1 - P(X \leq 2) = 1 - [P(X=0) + P(X=1) + P(X=2)]$

$= 1 - \left[0.2231 + \frac{e^{-1.5} \times (1.5)^1}{\lfloor 1} + \frac{e^{-1.5} (1.5)^2}{\lfloor 2}\right]$

$= 1 - [0.2231 + 0.3346 + 0.2510]$

Main Ideas, Questions & Summary: $= 1 - [0.8087] = 0.19127$

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Q. A skilled typist kept a record of mistakes for 300 days →

Mistakes/day	0	1	2	3	4	5	6
No. of days	143	90	42	12	9	3	1

Compute the frequency of poission distribution

Soln

x	f	fx	$P(X=x)=\frac{1^x e^{-1}}{\lfloor x}$	freq. = $300\times P(X=x)$
0	143	0	$P(X=0)=0.411$	123
1	90	90	$P(X=1)=0.365$	110
2	42	84	$P(X=2)=0.163$	49
3	12	36	$P(X=3)=0.048$	14
4	09	36	$P(X=4)=0.011$	3
5	03	15	$P(X=5)=0.002$	0.6 around 1
6	01	6	$P(X=6)=0.0003$	0
	300	267		

$\lambda=\frac{\Sigma fx}{\Sigma f}$

Q. Fit a Binomial Distribution

x	0	1	2	3	4	5
f	2	14	20	34	22	8

$P(X=x)={}^nC_r\, p^r q^{n-r}$ $\qquad f = P(x)\times 100$

x	f	fx
0	2	0
1	14	14
2	20	40
3	34	102
4	22	88
5	8	40
	100	284

$\lambda=\frac{284}{100}=np$

$=\frac{284}{100\times 6}=0.473$ $\qquad q=0.527$

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Q. The no. of admission each day is found to have a poission dist. with mean 4. Find probability that on a particular day there will be no admission

$$P(x=x) = \frac{\lambda^x e^{-\lambda}}{\lfloor x} = \frac{e^{-4} \times (4)^0}{\lfloor 0} = 0.0183$$

Q. In a book of 600 pages there are 60 errors. Assuming poission law for the no. of errors page. Find the probability that a randomly choosen 4 page will contain no error,

$$\lambda = \frac{60}{600} = \frac{1}{10}$$

$$P(x=0) = \frac{e^{-1/10} \times (\frac{1}{10})^0}{0!} = e^{-0.1} = 0.90$$

For 4 pages

$$(\text{Data})^4 = (0.90)^4 = 0.67$$

Q. letter are receive in a office for each 100 days assuming the following. Find expected frequencies

$e^{-4} = 0.183$

x	0	1	2	3	4	5	6	7	8	9	10
f	1	4	15	22	21	20	8	6	1	0	1

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x	f	fx	$P(X=x)=\frac{e^{-\lambda}\lambda^{x}}{x!}$	$f = 100\times P(X=x)$
0	1		0.183	$100\times 0.183 = 2$
1	4		0.073	7.3
2	15		0.14765	14.65
3	22		0.1953	19.53
4	21		0.1853	18.53
5	20		0.1562	15.62
6	8		0.1041	10.41
7	6		0.0595	5.95
8	2		0.0297	2.97
9	0		0.0132	1.3
10	1		0.00529	0.529

mean : $\frac{\Sigma fx}{\Sigma f} = \frac{400}{100} = 4$

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24/09						

Chebyshev's Inequality

If x is a RV with mean μ and variance σ^2 then

(1) $P\left[|x-\mu| \geq k\sigma\right] \leq \frac{1}{k^2}$

(2) $P\left[|x-\mu| < k\sigma\right] \geq 1-\frac{1}{k^2}$

x ——— x

Tut-4

Sol 1. $p = 1/5$ $\quad q = \frac{4}{5}$ $\quad n=6$ $\quad r = 2,3,4,5,6$

$r = 0,1$