

DIWALI ASSIGNMENT ON CALCULUS

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THE MAGNETIC BUNGEE PROBLEM

The magnetic bungee is a newly invented technology that can be used for recreational purposes. We are only going to plan a simple magnetic bungee.

INTRODUCTION

Bungee jumping is a sport that involves jumping down from a tall structure while connected to a large elastic cord. Bungee jumping is a risky sport as it involves great height. If the cord is not strong enough to support the weight of a person, it can claim a person's life. If people get killed in a sport, the popularity of the sport will be reduced. So while creating a structure, we need to be very careful and take proper estimates so that there are no casualties.

We are going to focus on an entirely new type of bungee called magnetic bungee. Magnetic force is used to support the weight of the person and ensure that he will not fall down. We need to create a magnetic bungee that is safe to use and does not cause any casualties so that it becomes popular for recreation.

A FORMAL STATEMENT OF THE PROBLEM

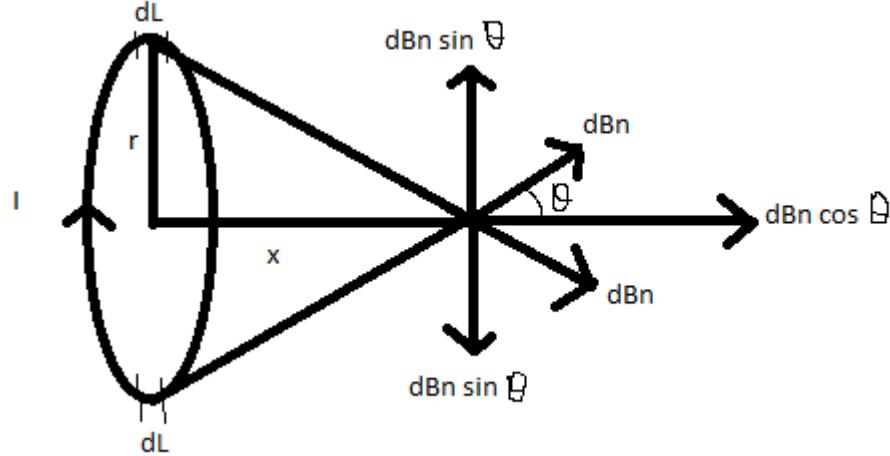
We have to assume some given physical quantities to solve the given problem. First, we need to understand how we can create one of the simplest bungees.

We consider a tall building of height 81 metres above the ground and a circular ring of radius a with centre 40 metres above the ground.

We need to create the safest possible bungee.

AN ANALYTICAL SOLUTION

We need to find out the magnetic field due to the ring.



Let the differential equation magnetic field dB due to the small element of the ring be given by the Biot Savart Law:

$$dB = \frac{\mu_0 i dl \sin \alpha}{4\pi r^2}$$

i = current through the ring

dl is the small element that we have considered in the ring.

r = is the radius of the ring

We know that $\sin \alpha = 1$

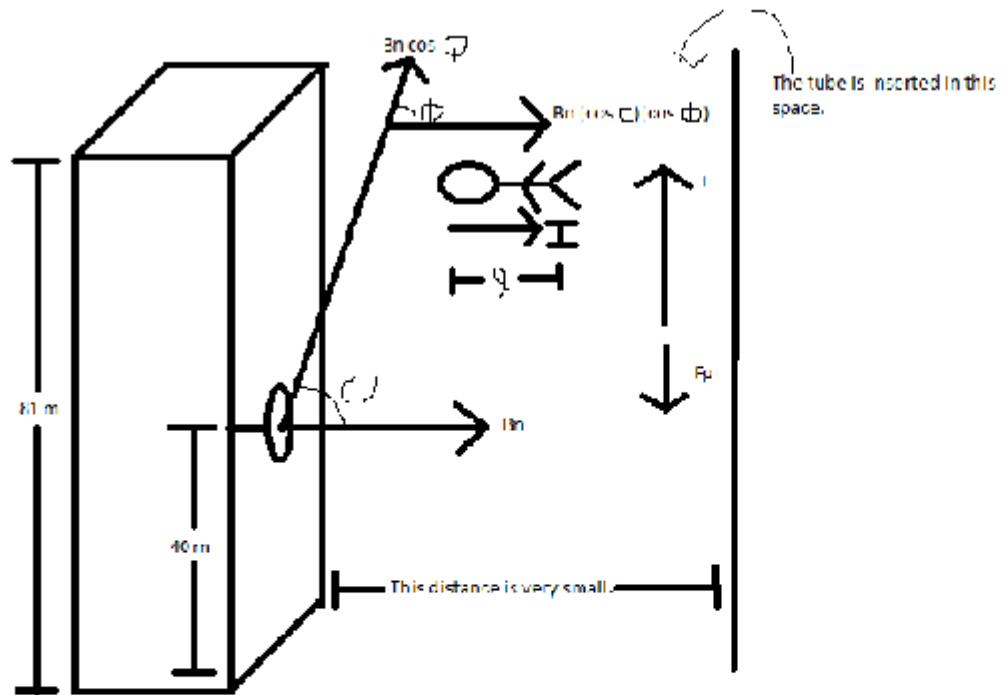
Integrating the expression we get:

$$\int dB = \int \frac{\mu_0 i dl}{4\pi r^2}$$

$$B = \frac{\mu_0 i l}{4\pi r^2}$$

As $l = 2\pi r$,

$$B = \frac{\mu_0 i * 2\pi r}{4\pi r^2}$$



We can divide the magnetic field into 2 components $B\cos\theta$ and $B\sin\theta$.

The $\sin\theta$ components cancel out each other and the $\cos\theta$ components add up.

Final expression for magnetic field:

$$B_n = \frac{\mu_0 i r^2}{2(r^2 + x^2)^{3/2}}$$

x = the distance of the man from the centre of the ring.

We find now keep the current carrying ring at a height of 40 metres above the ground.

Now we have to find out the maximum safe distance of the person from the ring for which the person will not reach the ground.

We will wrap a wire of length l around the person who will fall down.

The force on the person due to the magnetic field is given by

$$F = Ilb$$

where

$$b = B_n \cos^2 \Phi$$

I = current through the straight insulated wire belt around the man.

l = length/height of the man

Thus the force on the man due to the magnetic field is given by:

$$F = \frac{Il\mu_0 ir^2 \cos^2 \Phi}{2(r^2 + x^2)^{3/2}}$$

The force on the man due to gravitational force is given by:

$$F_g = mg$$

The net force on the person is given by:

$$f = F - F_g$$

$$\Rightarrow f(x) = \frac{Il\mu_0 ir^2 \cos^2 \Phi}{2(r^2 + x^2)^{3/2}} - mg$$

We will find out the value of x for which the force is minimum, the minimum force should be greater than zero in all cases.

We will differentiate the function with respect to x .

$$f'(x) = -\frac{3}{2} \frac{IL\mu_0 ir^2 \cos^2 \Phi 2x}{2(r^2 + x^2)^{5/2}}$$

For the function to have the maximum or minimum value,

$$f'(x) = 0$$

$$\Rightarrow -\frac{3}{2} \frac{IL\mu_0 ir^2 \cos^2 \Phi 2x}{2(r^2 + x^2)^{5/2}} = 0$$

On solving the given equation, we will get

$$x = 0$$

Now we have to use the second derivative test to check out whether the point $x = 0$ is a point of maximum or minimum value

$$f''(x) = \frac{3}{2} \frac{2(r^2 + x^2)^{5/2} IL\mu_0 ir^2 \cos^2 \Phi 2 - 5/2(r^2 + x^2)^{3/2} 4x}{(2(r^2 + x^2)^{5/2})^2}$$

By placing $x = 0$, we can clearly see that $f''(x) > 0$, thus the point $x = 0$ is a point of minimum value.

The minimum force f is given by

$$f(0) = \frac{Il\mu_0 ir^2 \cos^2 \Phi}{2(r^3)} - mg$$

Now we must ensure that whenever the man is very close to the ring below the height of 80 metres, the man should always be very close to the ring. When we drop the man from 81 metres, the man will gain some downward velocity.

To ensure that the man is safe at all times, we must ensure that there is an upward force acting on the man whenever the man is at a height of less than 80 metres from the ground.

$$f > 0$$

$$\begin{aligned} &\Rightarrow \frac{Il\mu_0 ir^2 \cos^2 \Phi}{2(r^3)} - mg > 0 \\ &\Rightarrow \frac{Il\mu_0 ir^2 \cos^2 \Phi}{2(r^3)} > mg \end{aligned}$$

Considering the realistic values of the given variables,

$$m = 100kg$$

$$g = 10m/s^2$$

$$mg = 1000$$

Thus we need to fulfil the condition:

$$\begin{aligned} &\frac{Il\mu_0 ir^2 \cos^2 \Phi}{2(r^3)} > 1000 \\ &\Rightarrow \frac{Il\mu_0 i \cos^2 \Phi}{2r} > 1000 \end{aligned}$$

μ_0 is a constant whose value is $4\pi 10^{-7}$.

$\cos \Phi$ is close to zero.

Let us assume that $\cos \Phi = 10^{-3}$ as the angle is close to $\frac{\pi}{2}$.

Let $r = 1m$

On substituting all the values in the given equation, we get

$$iI > 10^7$$

Thus we are able to create a safe magnetic bungee when the given condition is satisfied.

Thus the problem is solved.

APPLICATIONS OF THIS PROBLEM

This problem can be used in medical fields to remove substances with magnetic properties from the human body that have entered by mistake

This problem can also be used to repel meteors that may have magnetic properties which would otherwise cause massive damage to well developed cities if it hits the earth.

CONCLUSION

We can make a human fly beside a building with the use of magnetic field. Magnetic field has a lot of scope in the future.

References:

<https://www.youtube.com/watch?v=8M6M1ZDo8N8>

<https://www.scienceabc.com/sports/first-ever-fake-wireless-cordless-magnetic-bungee-jump-real.html>

<https://en.wikipedia.org/wiki/Biot>